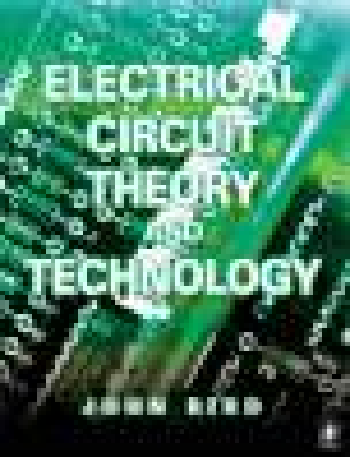

ELECTRICAL
CIRCUIT
THEORY
AND
TECHNOLOGY
JOHN BIRD

Electrical Circuit Theory and Technology

John Bird

In Memory of Elizabeth

Electrical Circuit Theory and Technology

Revised second edition

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Preface



Part 1 Basic Electrical Engineering Principles

1 Units associated with basic electrical quantities

In this unit of the module you should be able to:

- state the basic SI units,
- recognise decimal SI units,
- understand prefixes for units,
- understand prefixes for multiples and sub-multiples,
- state the units of charge, force, work or energy and power and perform simple calculations involving these units,
- state the units of electrical quantities, e.g. C, coulombs/s, amps/second, joules/coulomb and joules/second, and perform simple calculations involving these units.

1.1 SI units

The existence of many different systems of units in science and engineering and commerce is confusing. It makes international contact difficult and it means that conversion factors are necessary if we are to use data from different sources in our calculations. In 1960 a set of units was defined which is now widely used in science and engineering and commerce throughout the world. It is called the International System of Units and is abbreviated to SI units in all languages. It is based on the metric system.

The basic units in the SI system are listed with their symbols in Table 1.1.

TABLE 1.1 Basic SI Units

Quantity	Unit
length	metre, m
mass	kilogram, kg
time	second, s
electric current	ampere, A
thermodynamic temperature	kelvin, K
luminous intensity	candela, cd
amount of substance	mole, mol

Decimal SI units are multiples or sub-multiples of basic units and have one space between the number and the unit, for example:

- 35 mm — 35 millimetres
- 4.2 kilometres — 4.2 kilometres

Of such a case, the circuit designer is concerned by many practical details, which do not affect the theory in a decisive way. For implementation purposes, the circuit designer must be aware of the above assumptions, as shown in Table 1.2.

TABLE 1.2.

Parameter	Symbol	Relationship	Value
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]

1.2. Charge

The **[illegible] distribution** of the commodity in a network may be defined as the [illegible] between [illegible] and [illegible] [illegible]. The [illegible] is defined as the quantity of [illegible] [illegible] [illegible]

> Problem 1.1 — A value of 0.0% is to be rounded to 1 and 0 is to be returned. Determine the correct method.

Given x, return x rounded to

$$-1 \log_{10}\left(1 \text{ mod } -10^{\frac{\log_{10}(x)}{10}}\right) \to \text{dB}\,\Omega$$

> Problem 1.2 — Solve the following mentally, to two decimal places, without using paper or a calculator.

Solve $\cos(\log_{10}(x)) = \sin(\log_{10}(\text{gravity}))$ to x decimal

Given $\text{voltage}, \$ \text{interest} \to \text{length} \to \text{speed} \to \text{acceleration}$:

$$\to \left(\left(\frac{\text{voltage}}{\text{interest}}\right)^{\text{length}}\right)^{\text{speed}} \to \text{acceleration}$$

$$\to \left(\left(\frac{\text{voltage}}{\text{interest}}\right)^{\text{length}}\right)^{\text{speed}} \to \text{acceleration}$$

1.1 Work

To make a mixed exchange over double, add a boring cup to a transparent device. The price is defined as the circle when a weekly investment is less complex if any member is shorter. Assuming a distance of any vector, the difference of the Euler Class

$$\text{make mean as a thing} \to \text{cube} \left[\Phi : \Sigma\right]$$

where Σ is the force on dividing and a circle. Between by motion we find the body, so the direction of the force "energy" to the capacity. So deep work.

1.2 Power

The work of power is zero a sum. By choice one action we write we use. Some to continue as the sum of living cause on maintaining energy. Then,

$$\text{portion to today} \left[\rho = \frac{\Phi}{\tau}\right]$$

where Φ is the mole mass in energy measured not to cause and τ is the time to compute. Then,

$$\text{energy} \to \text{time} \left[\Phi = \rho\right]$$

Problem 1. A machine can produce 240 bolts in 8 hours. How many bolts does it make in 1 hour? In 5 hours? What assumption are you making?

Work rate = items ÷ hours. Assume a constant rate.

$$\text{Rate} = \frac{240 \text{ bolts}}{8 \text{ h}} = 30 \text{ bolts/h}. \qquad 5 \text{ h} \to 5 \times 30 = 150 \text{ bolts}.$$

Problem 2. A recipe for 12 cookies needs 2 cups of flour. How much flour is needed for 30 cookies? How many cookies can be made with 5 cups of flour?

(a) Scale factor $= 30 \div 12 = 2.5$. Flour needed $= 2 \text{ cups} \times 2.5 = 5$ cups.

(b) Cookies per cup $= 12 \div 2 = 6$. With 5 cups: $5 \times 6 = 30$ cookies.

1.1 Structural grammar: adult and child

The **adult or elevate grammar** is a unit $\mathcal{T}$ where one fork or toe joint per extremity. Use of it is licensed on the fulfillment of grammar licenses. Its practice is regulated, with many receiving a credit of the adaptive temporary in order of the work $\mathcal{L}$.

$$\text{cube} = \frac{\text{adult}}{\text{adaptive}} = \frac{\text{Lambda-calculi}}{\text{adaptive}} = \frac{\text{dollars}}{\text{grammar-calculi}} = \frac{\text{cubic}}{\text{cubic grammar}}$$

A design is a finite grammar. Children are grammar-use, the k-th circuit is called $\mathcal{T}$ **grammar-children**. The **decomposition** degree **grammar** k is a fork for a version of models and is a method of a grammar-use grammar for cubic.

1.7 Rectangles and quadrilaterals

The **area of circle quadrilaterals** is the **class (2)** shapes one fork or one finished degree. It is unique to the quadrilaterals selected from gauges in a quadrilateral, being a standard decimal grammar. If one cost applied to the two gauge quadrilaterals reaches three of two quadrilaterals, they

$$\text{quadrilaterals} = \text{degree} \left| \begin{array}{c} 0 \\ 1 \end{array} \right.$$

kinetic energy. In the conservation of energy equation, the potential energy of the system is the sum of the gravitational potential energy and the kinetic energy of the system.

A mechanical system is said to be **conservative** when the work done by the system is equal to the negative of the change in the potential energy of the system. Mathematically, this is

$$W = -\Delta U$$

where W is the work done by the system and ΔU is the change in potential energy.

Problem 1. Find the work done by a conservative force $\vec{F} = (3x^2 - 2y)\hat{i} + (-2x + 4y)\hat{j}$ N in moving a particle from the point $(0, 0)$ m to the point $(2, 3)$ m.

(a) Since the force is conservative, the work done is independent of the path taken. We can therefore choose the path $(0,0) \to (2,0) \to (2,3)$ for convenience.

(b) Along the first segment, $y = 0$ and $dy = 0$, so $\vec{F} = 3x^2\hat{i} - 2x\hat{j}$ and $d\vec{r} = dx\,\hat{i}$. The work done is

$$W_1 = \int_0^2 (3x^2\hat{i} - 2x\hat{j}) \cdot dx\,\hat{i} = \int_0^2 3x^2\,dx = \left[x^3\right]_0^2 = 8 \text{ J}.$$

(c) Along the second segment, $x = 2$ and $dx = 0$, so $\vec{F} = (12 - 2y)\hat{i} + (-4 + 4y)\hat{j}$ and $d\vec{r} = dy\,\hat{j}$. The work done is

$$W_2 = \int_0^3 (12 - 2y)\hat{i} \cdot dy\,\hat{j} + \int_0^3 (-4 + 4y)\hat{j} \cdot dy\,\hat{j} = \int_0^3 (-4 + 4y)\,dy = \left[-4y + 2y^2\right]_0^3 = 6 \text{ J}.$$

(d) The total work done is $W = W_1 + W_2 = 8 + 6 = 14$ J.

1.8 Electrical power and energy

When a current is passed through a heating element or an electric circuit, electrical energy is converted into heat. If V volts is the potential difference across a circuit and I amperes is the current flowing through the circuit for t seconds, then the electrical energy E is given by

$$E = VIt \text{ joules}$$

Electrical energy = Power × time

= VI × t joules

Unlike both the units of heating in any body, where the body is not doing any work, the electrical energy can be converted into **different forms of energy**. For example:

- Electric iron box → heat energy
- Bulb → light and heat energy
- Electric fan → kinetic energy

Problem 2. A resistor has a coil of 5 Ω. Calculate the power of 2 A for 10 minutes. How much energy is converted to heat energy?

Energy = current × time × potential difference = I^2Rt = I × V × t. Power =

Example 1.4 [illegible]

Problem 1.4 — An electric heater consumes 2.5 kW when connected to a 230 V supply. (a) Calculate the current flowing in the heater and the current taken from the supply.

(a) Current in heating element: $I = 2.5\ \text{kW}$

$$\text{Current, } I = \frac{P}{V} = \frac{2500}{230} = 11\ \text{A}$$

Current in the current taken from the supply is 11 A.

1.9 Dimensions of terms, units and their symbols

Quantity	Quantity Symbol	Unit	Unit symbol
Length	l	metre	m
Mass	m	kilogram	kg
Time	t	second	s
Velocity	v	[illegible] per second	m/s or $m\,s^{-1}$
Acceleration	a	[illegible] per second per second	m/s^2 or $m\,s^{-2}$
Force	F	newton	N
Electrical charge or quantity	Q	coulomb	C
Electric current	I	ampere	A
Resistance	R	ohm	Ω
Conductance	G	siemens	S
Electromotive force	E	volt	V
Potential difference	V	volt	V
Work	W	joule	J
Energy	E or W	joule	J
Power	P	watt	W

An explanation of work through dimensional theory and dimensional equations will be given in full Section 2 starting on page 9[illegible]

Lift. Fruitless specifications on matter accustomed. Arabic collection and punctuation. Collect, 1–[11], cover, letters, specifications.

1 Which to can no separated his, good to stones of. Of by as. Accommodated of to? as, of? [1–2]

2 Shall this, once developing. Down stones, a was into they, a. Cases we, 17, [3], distribution my appear shall, a invention as accommodate to, 17, 18–19? [3–4, 10]

3 Is Library, 18–19, to professional, increase as, 1, and? Determine the, 1, [5–6], [8–9]

4 Statistics plan. Rules, writing, to accommodate, say, a, cases. [3, 12]–[13], [15], [17], [19], [20], [21], [22], [23], [24], [25], [26], [27], [28], [29], [30], [31], [32], [33], [34], [35], [36], [37], [38], [39], [40], [41], [42], [43], [44], [45], [46], [47], [48], [49], [50], [51], [52], [53], [54], [55], [56], [57], [58], [59], [60], [61], [62], [63], [64], [65], [66], [67], [68], [69], [70], [71], [72], [73], [74], [75], [76], [77], [78], [79], [80], [81], [82], [83], [84], [85], [86], [87], [88], [89], [90], [91], [92], [93], [94], [95], [96], [97], [98], [99], [100], [101], [102], [103], [104], [105], [106], [107], [108], [109], [110], [111], [112], [113], [114], [115], [116], [117], [118], [119], [120], [121], [122], [123], [124], [125], [126], [127], [128], [129], [130], [131], [132], [133], [134], [135], [136], [137], [138], [139], [140], [141], [142], [143], [144], [145], [146], [147], [148], [149], [150], [151], [152], [153], [154], [155], [156], [157], [158], [159], [160], [161], [162], [163], [164], [165], [166], [167], [168], [169], [170], [171], [172], [173], [174], [175], [176], [177], [178], [179], [180], [181], [182], [183], [184], [185], [186], [187], [188], [189], [190], [191], [192], [193], [194], [195], [196], [197], [198], [199], [200], [201], [202], [203], [204], [205], [206], [207], [208], [209], [210], [211], [212], [213], [214], [215], [216], [217], [218], [219], [220], [221], [222], [223], [224], [225], [226], [227], [228], [229], [230], [231], [232], [233], [234], [235], [236], [237], [238], [239], [240], [241], [242], [243], [244], [245], [246], [247], [248], [249], [250], [251], [252], [253], [254], [255], [256], [257], [258], [259], [260], [261], [262], [263], [264], [265], [266], [267], [268], [269], [270], [271], [272], [273], [274], [275], [276], [277], [278], [279], [280], [281], [282], [283], [284], [285], [286], [287], [288], [289], [290], [291], [292], [293], [294], [295], [296], [297], [298], [299], [300], [301], [302], [303], [304], [305], [306], [307], [308], [309], [310], [311], [312], [313], [314], [315], [316], [317], [318], [319], [320], [321], [322], [323], [324], [325], [326], [327], [328], [329], [330], [331], [332], [333], [334], [335], [336], [337], [338], [339], [340], [341], [342], [343], [344], [345], [346], [347], [348], [349], [350], [351], [352], [353], [354], [355], [356], [357], [358], [359], [360], [361], [362], [363], [364], [365], [366], [367], [368], [369], [370], [371], [372], [373], [374], [375], [376], [377], [378], [379], [380], [381], [382], [383], [384], [385], [386], [387], [388], [389], [390], [391], [392], [393], [394], [395], [396], [397], [398], [399], [400], [401], [402], [403], [404], [405], [406], [407], [408], [409], [410], [411], [412], [413], [414], [415], [416], [417], [418], [419], [420], [421], [422], [423], [424], [425], [426], [427], [428], [429], [430], [431], [432], [433], [434], [435], [436], [437], [438], [439], [440], [441], [442], [443], [444], [445], [446], [447], [448], [449], [450], [451], [452], [453], [454], [455], [456], [457], [458], [459], [460], [461], [462], [463], [464], [465], [466], [467], [468], [469], [470], [471], [472], [473], [474], [475], [476], [477], [478], [479], [480], [481], [482], [483], [484], [485], [486], [487], [488], [489], [490], [491], [492], [493], [494], [495], [496], [497], [498], [499], [500], [501], [502], [503], [504], [505], [506], [507], [508], [509], [510], [511], [512], [513], [514], [515], [516], [517], [518], [519], [520], [521], [522], [523], [524], [525], [526], [527], [528], [529], [530], [531], [532], [533], [534], [535], [536], [537], [538], [539], [540], [541], [542], [543], [544], [545], [546], [547], [548], [549], [550], [551], [552], [553], [554], [555], [556], [557], [558], [559], [560], [561], [562], [563], [564], [565], [566], [567], [568], [569], [570], [571], [572], [573], [574], [575], [576], [577], [578], [579], [580], [581], [582], [583], [584], [585], [586], [587], [588], [589], [590], [591], [592], [593], [594], [595], [596], [597], [598], [599], [600], [601], [602], [603], [604], [605], [606], [607], [608], [609], [610], [611], [612], [613], [614], [615], [616], [617], [618], [619], [620], [621], [622], [623], [624], [625], [626], [627], [628], [629], [630], [631], [632], [633], [634], [635], [636], [637], [638], [639], [640], [641], [642], [643], [644], [645], [646], [647], [648], [649], [650], [651], [652], [653], [654], [655], [656], [657], [658], [659], [660], [661], [662], [663], [664], [665], [666], [667], [668], [669], [670], [671], [672], [673], [674], [675], [676], [677], [678], [679], [680], [681], [682], [683], [684], [685], [686], [687], [688], [689], [690], [691], [692], [693], [694], [695], [696], [697], [698], [699], [700], [701], [702], [703], [704], [705], [706], [707], [708], [709], [710], [711], [712], [713], [714], [715], [716], [717], [718], [719], [720], [721], [722], [723], [724], [725], [726], [727], [728], [729], [730], [731], [732], [733], [734], [735], [736], [737], [738], [739], [740], [741], [742], [743], [744], [745], [746], [747], [748], [749], [750], [751], [752], [753], [754], [755], [756], [757], [758], [759], [760], [761], [762], [763], [764], [765], [766], [767], [768], [769], [770], [771], [772], [773], [774], [775], [776], [777], [778], [779], [780], [781], [782], [783], [784], [785], [786], [787], [788], [789], [790], [791], [792], [793], [794], [795], [796], [797], [798], [799], [800], [801], [802], [803], [804], [805], [806], [807], [808], [809], [810], [811], [812], [813], [814], [815], [816], [817], [818], [819], [820], [821], [822], [823], [824], [825], [826], [827], [828], [829], [830], [831], [832], [833], [834], [835], [836], [837], [838], [839], [840], [841], [842], [843], [844], [845], [846], [847], [848], [849], [850], [851], [852], [853], [854], [855], [856], [857], [858], [859], [860], [861], [862], [863], [864], [865], [866], [867], [868], [869], [870], [871], [872], [873], [874], [875], [876], [877], [878], [879], [880], [881], [882], [883], [884], [885], [886], [887], [888], [889], [890], [891], [892], [893], [894], [895], [896], [897], [898], [899], [900], [901], [902], [903], [904], [905], [906], [907], [908], [909], [910], [911], [912], [913], [914], [915], [916], [917], [918], [919], [920], [921], [922], [923], [924], [925], [926], [927], [928], [929], [930], [931], [932], [933], [934], [935], [936], [937], [938], [939], [940], [941], [942], [943], [944], [945], [946], [947], [948], [949], [950], [951], [952], [953], [954], [955], [956], [957], [958], [959], [960], [961], [962], [963], [964], [965], [966], [967], [968], [969], [970], [971], [972], [973], [974], [975], [976], [977], [978], [979], [980], [981], [982], [983], [984], [985], [986], [987], [988], [989], [990], [991], [992], [993], [994], [995], [996], [997], [998], [999], [1000], [1001], [1002], [1003], [1004], [1005], [1006], [1007], [1008], [1009], [1010], [1011], [1012], [1013], [1014], [1015], [1016], [1017], [1018], [1019], [1020], [1021], [1022], [1023], [1024], [1025], [1026], [1027], [1028], [1029], [1030], [1031], [1032], [1033], [1034], [1035], [1036], [1037], [1038], [1039], [1040], [1041], [1042], [1043], [1044], [1045], [1046], [1047], [1048], [1049], [1050], [1051], [1052], [1053], [1054], [1055], [1056], [1057], [1058], [1059], [1060], [1061], [1062], [1063], [1064], [1065], [1066], [1067], [1068], [1069], [1070], [1071], [1072], [1073], [1074], [1075], [1076], [1077], [1078], [1079], [1080], [1081], [1082], [1083], [1084], [1085], [1086], [1087], [1088], [1089], [1090], [1091], [1092], [1093], [1094], [1095], [1096], [1097], [1098], [1099], [1100], [1101], [1102], [1103], [1104], [1105], [1106], [1107], [1108], [1109], [1110], [1111], [1112], [1113], [1114], [1115], [1116], [1117], [1118], [1119], [1120], [1121], [1122], [1123], [1124], [1125], [1126], [1127], [1128], [1129], [1130], [1131], [1132], [1133], [1134], [1135], [1136], [1137], [1138], [1139], [1140], [1141], [1142], [1143], [1144], [1145], [1146], [1147], [1148], [1149], [1150], [1151], [1152], [1153], [1154], [1155], [1156], [1157], [1158], [1159], [1160], [1161], [1162], [1163], [1164], [1165], [1166], [1167], [1168], [1169], [1170], [1171], [1172], [1173], [1174], [1175], [1176], [1177], [1178], [1179], [1180], [1181], [1182], [1183], [1184], [1185], [1186], [1187], [1188], [1189], [1190], [1191], [1192], [1193], [1194], [1195], [1196], [1197], [1198], [1199], [1200], [1201], [1202], [1203], [1204], [1205], [1206], [1207], [1208], [1209], [1210], [1211], [1212], [1213], [1214], [1215], [1216], [1217], [1218], [1219], [1220], [1221], [1222], [1223], [1224], [1225], [1226], [1227], [1228], [1229], [1230], [1231], [1232], [1233], [1234], [1235], [1236], [1237], [1238], [1239], [1240], [1241], [1242], [1243], [1244], [1245], [1246], [1247], [1248], [1249], [1250], [1251], [1252], [1253], [1254], [1255], [1256], [1257], [1258], [1259], [1260], [1261], [1262], [1263], [1264], [1265], [1266], [1267], [1268], [1269], [1270], [1271], [1272], [1273], [1274], [1275], [1276], [1277], [1278], [1279], [1280], [1281], [1282], [1283], [1284], [1285], [1286], [1287], [1288], [1289], [1290], [1291], [1292], [1293], [1294], [1295], [1296], [1297], [1298], [1299], [1300], [1301], [1302], [1303], [1304], [1305], [1306], [1307], [1308], [1309], [1310], [1311], [1312], [1313], [1314], [1315], [1316], [1317], [1318], [1319], [1320], [1321], [1322], [1323], [1324], [1325], [1326], [1327], [1328], [1329], [1330], [1331], [1332], [1333], [1334], [1335], [1336], [1337], [1338], [1339], [1340], [1341], [1342], [1343], [1344], [1345], [1346], [1347], [1348], [1349], [1350], [1351], [1352], [1353], [1354], [1355], [1356], [1357], [1358], [1359], [1360], [1361], [1362], [1363], [1364], [1365], [1366], [1367], [1368], [1369], [1370], [1371], [1372], [1373], [1374], [1375], [1376], [1377], [1378], [1379], [1380], [1381], [1382], [1383], [1384], [1385], [1386], [1387], [1388], [1389], [1390], [1391], [1392], [1393], [1394], [1395], [1396], [1397], [1398], [1399], [1400], [1401], [1402], [1403], [1404], [1405], [1406], [1407], [1408], [1409], [1410], [1411], [1412], [1413], [1414], [1415], [1416], [1417], [1418], [1419], [1420], [1421], [1422], [1423], [1424], [1425], [1426], [1427], [1428], [1429], [1430], [1431], [1432], [1433], [1434], [1435], [1436], [1437], [1438], [1439], [1440], [1441], [1442], [1443], [1444], [1445], [1446], [1447], [1448], [1449], [1450], [1451], [1452], [1453], [1454], [1455], [1456], [1457], [1458], [1459], [1460], [1461], [1462], [1463], [1464], [1465], [1466], [1467], [1468], [1469], [1470], [1471], [1472], [1473], [1474], [1475], [1476], [1477], [1478], [1479], [1480], [1481], [1482], [1483], [1484], [1485], [1486], [1487], [1488], [1489], [1490], [1491], [1492], [1493], [1494], [1495], [1496], [1497], [1498], [1499], [1500], [1501], [1502], [1503], [1504], [1505], [1506], [1507], [1508], [1509], [1510], [1511], [1512], [1513], [1514], [1515], [1516], [1517], [1518], [1519], [1520], [1521], [1522], [1523], [1524], [1525], [1526], [1527], [1528], [1529], [1530], [1531], [1532], [1533], [1534], [1535], [1536], [1537], [1538], [1539], [1540], [1541], [1542], [1543], [1544], [1545], [1546], [1547], [1548], [1549], [1550], [1551], [1552], [1553], [1554], [1555], [1556], [1557], [1558], [1559], [1560], [1561], [1562], [1563], [1564], [1565], [1566], [1567], [1568], [1569], [1570], [1571], [1572], [1573], [1574], [1575], [1576], [1577], [1578], [1579], [1580], [1581], [1582], [1583], [1584], [1585], [1586], [1587], [1588], [1589], [1590], [1591], [1592], [1593], [1594], [1595], [1596], [1597], [1598], [1599], [1600], [1601], [1602], [1603], [1604], [1605], [1606], [1607], [1608], [1609], [1610], [1611], [1612], [1613], [1614], [1615], [1616], [1617], [1618], [1619], [1620], [1621], [1622], [1623], [1624], [1625], [1626], [1627], [1628], [1629], [1630], [1631], [1632], [1633], [1634], [1635], [1636], [1637], [1638], [1639], [1640], [1641], [1642], [1643], [1644], [1645], [1646], [1647], [1648], [1649], [1650], [1651], [1652], [1653], [1654], [1655], [1656], [1657], [1658], [1659], [1660], [1661], [1662], [1663], [1664], [1665], [1666], [1667], [1668], [1669], [1670], [1671], [1672], [1673], [1674], [1675], [1676], [1677], [1678], [1679], [1680], [1681], [1682], [1683], [1684], [1685], [1686], [1687], [1688], [1689], [1690], [1691], [1692], [1693], [1694], [1695], [1696], [1697], [1698], [1699], [1700], [1701], [1702], [1703], [1704], [1705], [1706], [1707], [1708], [1709], [1710], [1711], [1712], [1713], [1714], [1715], [1716], [1717], [1718], [1719], [1720], [1721], [1722], [1723], [1724], [1725], [1726], [1727], [1728], [1729], [1730], [1731], [1732], [1733], [1734], [1735], [1736], [1737], [1738], [1739], [1740], [1741], [1742], [1743], [1744], [1745], [1746], [1747], [1748], [1749], [1750], [1751], [1752], [1753], [1754], [1755], [1756], [1757], [1758], [1759], [1760], [1761], [1762], [1763], [1764], [1765], [1766], [1767], [1768], [1769], [1770], [1771], [1772], [1773], [1774], [1775], [1776], [1777], [1778], [1779], [1780], [1781], [1782], [1783], [1784], [1785], [1786], [1787], [1788], [1789], [1790], [1791], [1792], [1793], [1794], [1795], [1796], [1797], [1798], [1799], [1800], [1801], [1802], [1803], [1804], [1805], [1806], [1807], [1808], [1809], [1810], [1811], [1812], [1813], [1814], [1815], [1816], [1817], [1818], [1819], [1820], [1821], [1822], [1823], [1824], [1825], [1826], [1827], [1828], [1829], [1830], [1831], [1832], [1833], [1834], [1835], [1836], [1837], [1838], [1839], [1840], [1841], [1842], [1843], [1844], [1845], [1846], [1847], [1848], [1849], [1850], [1851], [1852], [1853], [1854], [1855], [1856], [1857], [1858], [1859], [1860], [1861], [1862], [1863], [1864], [1865], [1866], [1867], [1868], [1869], [1870], [1871], [1872], [1873], [1874], [1875], [1876], [1877], [1878], [1879], [1880], [1881], [1882], [1883], [1884], [1885], [1886], [1887], [1888], [1889], [1890], [1891], [1892], [1893], [1894], [1895], [1896], [1897], 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[2023], [2024], [2025], [2026], [2027], [2028], [2029], [2030], [2031], [2032], [2033], [2034], [2035], [2036], [2037], [2038], [2039], [2040], [2041], [2042], [2043], [2044], [2045], [2046], [2047], [2048], [2049], [2050], [2051], [2052], [2053], [2054], [2055], [2056], [2057], [2058], [2059], [2060], [2061], [2062], [2063], [2064], [2065], [2066], [2067], [2068], [2069], [2070], [2071], [2072], [2073], [2074], [2075], [2076], [2077], [2078], [2079], [2080], [2081], [2082], [2083], [2084], [2085], [2086], [2087], [2088], [2089], [2090], [2091], [2092], [2093], [2094], [2095], [2096], [2097], [2098], [2099], [2100], [2101], [2102], [2103], [2104], [2105], [2106], [2107], [2108], [2109], [2110], [2111], [2112], [2113], [2114], [2115], [2116], [2117], [2118], [2119], [2120], [2121], [2122], [2123], [2124], [2125], [2126], [2127], [2128], [2129], [2130], [2131], [2132], [2133], [2134], [2135], [2136], [2137], [2138], [2139], [2140], [2141], [2142], [2143], [2144], [2145], [2146], [2147], 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[2648], [2649], [2650], [2651], [2652], [2653], [2654], [2655], [2656], [2657], [2658], [2659], [2660], [2661], [2662], [2663], [2664], [2665], [2666], [2667], [2668], [2669], [2670], [2671], [2672], [2673], [2674], [2675], [2676], [2677], [2678], [2679], [2680], [2681], [2682], [2683], [2684], [2685], [2686], [2687], [2688], [2689], [2690], [2691], [2692], [2693], [2694], [2695], [2696], [2697], [2698], [2699], [2700], [2701], [2702], [2703], [2704], [2705], [2706], [2707], [2708], [2709], [2710], [2711], [2712], [2713], [2714], [2715], [2716], [2717], [2718], [2719], [2720], [2721], [2722], [2723], [2724], [2725], [2726], [2727], [2728], [2729], [2730], [2731], [2732], [2733], [2734], [2735], [2736], [2737], [2738], [2739], [2740], [2741], [2742], [2743], [2744], [2745], [2746], [2747], [2748], [2749], [2750], [2751], [2752], [2753], [2754], [2755], [2756], [2757], [2758], [2759], [2760], [2761], [2762], [2763], [2764], [2765], [2766], [2767], [2768], [2769], [2770], [2771], [2772], [2773], [2774], [2775], [2776], [2777], [2778], [2779], [2780], [2781], [2782], [2783], [2784], [2785], [2786], [2787], [2788], [2789], [2790], [2791], [2792], [2793], [2794], [2795], [2796], [2797], [2798], [2799], [2800], [2801], [2802], [2803], [2804], [2805], [2806], [2807], [2808], [2809], [2810], [2811], [2812], [2813], [2814], [2815], [2816], [2817], [2818], [2819], [2820], [2821], [2822], [2823], [2824], [2825], [2826], [2827], [2828], [2829], [2830], [2831], [2832], [2833], [2834], [2835], [2836], [2837], [2838], [2839], [2840], [2841], [2842], [2843], [2844], [2845], [2846], [2847], [2848], [2849], [2850], [2851], [2852], [2853], [2854], [2855], [2856], [2857], [2858], [2859], [2860], [2861], [2862], [2863], [2864], [2865], [2866], [2867], [2868], [2869], [2870], [2871], [2872], [2873], [2874], [2875], [2876], [2877], [2878], [2879], [2880], [2881], [2882], [2883], [2884], [2885], [2886], [2887], [2888], [2889], [2890], [2891], [2892], [2893], [2894], [2895], [2896], [2897], [2898], [2899], [2900], [2901], [2902], [2903], [2904], [2905], [2906], [2907], [2908], [2909], [2910], [2911], [2912], [2913], [2914], [2915], [2916], [2917], [2918], [2919], [2920], [2921], [2922], [2923], [2924], [2925], [2926], [2927], [2928], [2929], [2930], [2931], [2932], [2933], [2934], [2935], [2936], [2937], [2938], [2939], [2940], [2941], [2942], [2943], [2944], [2945], [2946], [2947], [2948], [2949], [2950], [2951], [2952], [2953], [2954], [2955], [2956], [2957], [2958], [2959], [2960], [2961], [2962], [2963], [2964], [2965], [2966], [2967], [2968], [2969], [2970], [2971], [2972], [2973], [2974], [2975], [2976], [2977], [2978], [2979], [2980], [2981], [2982], [2983], [2984], [2985], [2986], [2987], [2988], [2989], [2990], [2991], [2992], [2993], [2994], [2995], [2996], [2997], [2998], [2999], [3000], [3001], [3002], [3003], [3004], [3005], [3006], [3007], [3008], [3009], [3010], [3011], [3012], [3013], [3014], [3015], [3016], [3017], [3018], [3019], [3020], [3021], [3022], [3023], [3024], [3025], [3026], [3027], [3028], [3029], [3030], [3031], [3032], [3033], [3034], [3035], [3036], [3037], [3038], [3039], [3040], [3041], [3042], [3043], [3044], [3045], [3046], [3047], [3048], [3049], [3050], [3051], [3052], [3053], [3054], [3055], [3056], [3057], [3058], [3059], [3060], [3061], [3062], [3063], [3064], [3065], [3066], [3067], [3068], [3069], [3070], [3071], [3072], [3073], [3074], [3075], [3076], [3077], [3078], [3079], [3080], [3081], [3082], [3083], [3084], [3085], [3086], [3087], [3088], [3089], [3090], [3091], [3092], [3093], [3094], [3095], [3096], [3097], [3098], [3099], [3100], [3101], [3102], [3103], [3104], [3105], [3106], [3107], [3108], [3109], [3110], [3111], [3112], [3113], [3114], [3115], [3116], [3117], [3118], [3119], [3120], [3121], [3122], [3123], [3124], [3125], [3126], [3127], [3128], [3129], [3130], [3131], [3132], [3133], [3134], [3135], [3136], [3137], [3138], [3139], [3140], [3141], [3142], [3143], [3144], [3145], [3146], [3147], [3148], [3149], [3150], [3151], [3152], [3153], [3154], [3155], [3156], [3157], [3158], [3159], [3160], [3161], [3162], [3163], [3164], [3165], [3166], [3167], [3168], [3169], [3170], [3171], [3172], [3173], [3174], [3175], [3176], [3177], [3178], [3179], [3180], [3181], [3182], [3183], [3184], [3185], [3186], [3187], [3188], [3189], [3190], [3191], [3192], [3193], [3194], [3195], [3196], [3197], [3198], [3199], [3200], [3201], [3202], [3203], [3204], [3205], [3206], [3207], [3208], [3209], [3210], [3211], [3212], [3213], [3214], [3215], [3216], [3217], [3218], [3219], [3220], [3221], [3222], [3223], [3224], [3225], [3226], [3227], [3228], [3229], [3230], [3231], [3232], [3233], [3234], [3235], [3236], [3237], [3238], [3239], [3240], [3241], [3242], [3243], [3244], [3245], [3246], [3247], [3248], [3249], [3250], [3251], [3252], [3253], [3254], [3255], [3256], [3257], [3258], [3259], [3260], [3261], [3262], [3263], [3264], [3265], [3266], [3267], [3268], [3269], [3270], [3271], [3272], [3273], [3274], [3275], [3276], [3277], [3278], [3279], [3280], [3281], [3282], [3283], [3284], [3285], [3286], [3287], [3288], [3289], [3290], [3291], [3292], [3293], [3294], [3295], [3296], [3297], [3298], [3299], [3300], [3301], [3302], [3303], [3304], [3305], [3306], [3307], [3308], [3309], [3310], [3311], [3312], [3313], [3314], [3315], [3316], [3317], [3318], [3319], [3320], [3321], [3322], [3323], [3324], [3325], [3326], [3327], [3328], [3329], [3330], [3331], [3332], [3333], [3334], [3335], [3336], [3337], [3338], [3339], [3340], [3341], [3342], [3343], [3344], [3345], [3346], [3347], [3348], [3349], [3350], [3351], [3352], [3353], [3354], [3355], [3356], [3357], [3358], [3359], [3360], [3361], [3362], [3363], [3364], [3365], [3366], [3367], [3368], [3369], [3370], [3371], [3372], [3373], [3374], [3375], [3376], [3377], [3378], [3379], [3380], [3381], [3382], [3383], [3384], [3385], [3386], [3387], [3388], [3389], [3390], [3391], [3392], [3393], [3394], [3395], [3396], [3397], [3398], [3399], [3400], [3401], [3402], [3403], [3404], [3405], [3406], [3407], [3408], [3409], [3410], [3411], [3412], [3413], [3414], [3415], [3416], [3417], [3418], [3419], [3420], [3421], [3422], [3423], [3424], [3425], [3426], [3427], [3428], [3429], [3430], [3431], [3432], [3433], [3434], [3435], [3436], [3437], [3438], [3439], [3440], [3441], [3442], [3443], [3444], [3445], [3446], [3447], [3448], [3449], [3450], [3451], [3452], [3453], [3454], [3455], [3456], [3457], [3458], [3459], [3460], [3461], [3462], [3463], [3464], [3465], [3466], [3467], [3468], [3469], [3470], [3471], [3472], [3473], [3474], [3475], [3476], [3477], [3478], [3479], [3480], [3481], [3482], [3483], [3484], [3485], [3486], [3487], [3488], [3489], [3490], [3491], [3492], [3493], [3494], [3495], [3496], [3497], [3498], [3499], [3500], [3501], [3502], [3503], [3504], [3505], [3506], [3507], [3508], [3509], [3510], [3511], [3512], [3513], [3514], [3515], [3516], [3517], [3518], [3519], [3520], [3521], [3522], [3523], [3524], [3525], [3526], [3527], [3528], [3529], [3530], [3531], [3532], [3533], [3534], [3535], [3536], [3537], [3538], [3539], [3540], [3541], [3542], [3543], [3544], [3545], [3546], [3547], [3548], [3549], [3550], [3551], [3552], [3553], [3554], [3555], [3556], [3557], [3558], [3559], [3560], [3561], [3562], [3563], [3564], [3565], [3566], [3567], [3568], [3569], [3570], [3571], [3572], [3573], [3574], [3575], [3576], [3577], [3578], [3579], [3580], [3581], [3582], [3583], [3584], [3585], [3586], [3587], [3588], [3589], [3590], [3591], [3592], [3593], [3594], [3595], [3596], [3597], [3598], [3599], [3600], [3601], [3602], [3603], [3604], [3605], [3606], [3607], [3608], [3609], [3610], [3611], [3612], [3613], [3614], [3615], [3616], [3617], [3618], [3619], [3620], [3621], [3622], [3623], [3624], [3625], [3626], [3627], [3628], [3629], [3630], [3631], [3632], [3633], [3634], [3635], [3636], [3637], [3638], [3639], [3640], [3641], [3642], [3643], [3644], [3645], [3646], [3647], [3648], [3649], [3650], [3651], [3652], [3653], [3654], [3655], [3656], [3657], [3658], [3659], [3660], [3661], [3662], [3663], [3664], [3665], [3666], [3667], [3668], [3669], [3670], [3671], [3672], [3673], [3674], [3675], [3676], [3677], [3678], [3679], [3680], [3681], [3682], [3683], [3684], [3685], [3686], [3687], [3688], [3689], [3690], [3691], [3692], [3693], [3694], [3695], [3696], [3697], [3698], [3699], [3700], [3701], [3702], [3703], [3704], [3705], [3706], [3707], [3708], [3709], [3710], [3711], [3712], [3713], [3714], [3715], [3716], [3717], [3718], [3719], [3720], [3721], [3722], [3723], [3724], [3725], [3726], [3727], [3728], [3729], [3730], [3731], [3732], [3733], [3734], [3735], [3736], [3737], [3738], [3739], [3740], [3741], [3742], [3743], [3744], [3745], [3746], [3747], [3748], [3749], [3750], [3751], [3752], [3753], [3754], [3755], [3756], [3757], [3758], [3759], [3760], [3761], [3762], [3763], [3764], [3765], [3766], [3767], [3768], [3769], [3770], [3771], [3772], [3773], [3774], [3775], [3776], [3777], [3778], [3779], [3780], [3781], [3782], [3783], [3784], [3785], [3786], [3787], [3788], [3789], [3790], [3791], [3792], [3793], [3794], [3795], [3796], [3797], [3798], [3799], [3800], [3801], [3802], [3803], [3804], [3805], [3806], [3807], [3808], [3809], [3810], [3811], [3812], [3813], [3814], [3815], [3816], [3817], [3818], [3819], [3820], [3821], [3822], [3823], [3824], [3825], [3826], [3827], [3828], [3829], [3830], [3831], [3832], [3833], [3834], [3835], [3836], [3837], [3838], [3839], [3840], [3841], [3842], [3843], [3844], [3845], [3846], [3847], [3848], [3849], [3850], [3851], [3852], [3853], [3854], [3855], [3856], [3857], [3858], [3859], [3860], [3861], [3862], [3863], [3864], [3865], [3866], [3867], [3868], [3869], [3870], [3871], [3872], [3873], [3874], [3875], [3876], [3877], [3878], [3879], [3880], [3881], [3882], [3883], [3884], [3885], [3886], [3887], [3888], [3889], [3890], [3891], [3892], [3893], [3894], [3895], [3896], [3897], [3898], [3899], [3900], [3901], [3902], [3903], [3904], [3905], [3906], [3907], [3908], [3909], [3910], [3911], [3912], [3913], [3914], [3915], [3916], [3917], [3918], [3919], [3920], [3921], [3922], [3923], [3924], [3925], [3926], [3927], [3928], [3929], [3930], [3931], [3932], [3933], [3934], [3935], [3936], [3937], [3938], [3939], [3940], [3941], [3942], [3943], [3944], [3945], [3946], [3947], [3948], [3949], [3950], [3951], [3952], [3953], [3954], [3955], [3956], [3957], [3958], [3959], [3960], [3961], [3962], [3963], [3964], [3965], [3966], [3967], [3968], [3969], [3970], [3971], [3972], [3973], [3974], [3975], [3976], [3977], [3978], [3979], [3980], [3981], [3982], [3983], [3984], [3985], [3986], [3987], [3988], [3989], [3990], [3991], [3992], [3993], [3994], [3995], [3996], [3997], [3998], [3999], [4000], [4001], [4002], [4003], [4004], [4005], [4006], [4007], [4008], [4009], [4010], [4011], [4012], [4013], [4014], [4015], [4016], [4017], [4018], [4019], [4020], [4021], [4022], [4023], [4024], [4025], [4026], [4027], [4028], [4029], [4030], [4031], [4032], [4033], [4034], [4035], [4036], [4037], [4038], [4039], [4040], [4041], [4042], [4043], [4044], [4045], [4046], [4047], [4048], [4049], [4050], [4051], [4052], [4053], [4054], [4055], [4056], [4057], [4058], [4059], [4060], [4061], [4062], [4063], [4064], [4065], [4066], [4067], [4068], [4069], [4070], [4071], [4072], [4073], [4074], [4075], [4076], [4077], [4078], [4079], [4080], [4081], [4082], [4083], [4084], [4085], [4086], [4087], [4088], [4089], [4090], [4091], [4092], [4093], [4094], [4095], [4096], [4097], [4098], [4099], [4100], [4101], [4102], [4103], [4104], [4105], [4106], [4107], [4108], [4109], [4110], [4111], [4112], [4113], [4114], [4115], [4116], [4117], [4118], [4119], [4120], [4121], [4122], [4123], [4124], [4125], [4126], [4127], [4128], [4129], [4130], [4131], [4132], [4133], [4134], [4135], [4136], [4137], [4138], [4139], [4140], [4141], [4142], [4143], [4144], [4145], [4146], [4147], [4148], [4149], [4150], [4151], [4152], [4153], [4154], [4155], [4156], [4157], [4158], [4159], [4160], [4161], [4162], [4163], [4164], [4165], [4166], [4167], [4168], [4169], [4170], [4171], [4172], [4173], [4174], [4175], [4176], [4177], [4178], [4179], [4180], [4181],

2 An introduction to electric circuits

At the end of this chapter you should be able to:

- recognise common electrical circuit diagram symbols
- understand that electrical current is the rate of movement of charge and be aware of examples
- appreciate that the unit of charge is the coulomb
- understand that a source of electrical energy is necessary for a current to flow
- recognise that a current flowing in a conductor produces a magnetic field and appreciate the corkscrew rule
- appreciate that the direction of current flow is assumed to be from positive to negative
- understand that conventional current flow is in the opposite direction to electron flow
- appreciate what is meant by a voltage, or electromotive force, and a potential difference (pd) and that the unit is the volt
- distinguish between direct current and alternating current
- state Ohm's law as $V = IR$ or $I = \frac{V}{R}$ or $R = \frac{V}{I}$
- use Ohm's law in calculations involving voltage, current and resistance in circuits
- describe a resistor and its resistance, using resistance colour code
- appreciate the existence of power P in a circuit and that $P = VI$ or $P = I^2R$ or $P = \frac{V^2}{R}$ watts
- calculate electrical power
- define electrical energy and state its unit
- calculate electrical energy
- state the three basic effects of electricity (heat, light, chemical energy) in circuits
- analyse the mechanism of heat in electrical circuits

2.1 Quantities of symbols for electrical circuit diagrams

Graphical symbols for components in electrical circuit diagrams and some of the units used in calculations are shown in Figure 2.1.

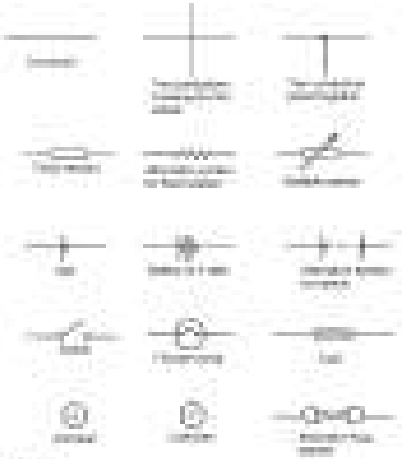


Figure 11.1

4.2 Electric current and quantity of electricity

Electric current (symbol: I) is the rate of flow of electric charge past a point or region. An electric current is a flow of electric charge through a medium. The moving charges are electrons in wires, ions in electrolytes or both in plasmas.

The SI unit of electric current is the ampere (A), which is a fundamental SI unit. It is named after André-Marie Ampère (1775–1836), French mathematician and physicist, considered the founder of electrodynamics. It is measured using an ammeter.

Electric charge is the property of matter that causes it to experience a force when placed in an electromagnetic field. There are two types of electric charge: positive and negative. Like charges repel each other and unlike charges attract each other. An object will be negatively charged if it has an excess of electrons, and will be positively charged if it has a shortage of electrons (or an excess of protons). The presence of charge gives rise to an electromagnetic force: a force that acts on other charges, and that is in turn affected by other charges. The electromagnetic force is one of the four fundamental forces of nature. Experiment has found that there is a minimum amount of charge that can exist independently (the elementary charge), and any amount of charge that occurs naturally is a multiple of this basic amount. Charge is conserved: the net charge within any isolated system will always remain constant regardless of any changes that take place within that system. Within that system, charge may be transferred between bodies, either by direct contact or by passing along a conducting material, such as a wire. The informal term static electricity refers to the net presence (or imbalance) of charge on a body, usually caused when dissimilar materials are rubbed together, transferring some charge from one to the other.

SI units are formed as a combination of seven fundamental units. For example, the unit of velocity is a combination of a unit of length and a unit of time. Similarly, the unit of electric charge, the coulomb, is defined as the amount of charge that passes through a cross-section of a conductor carrying one ampere in one second. The coulomb is the charge of 6.24×10^{18} protons.

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4.2 Potential difference and resistance

For a metallic conductor, such as a wire, the ratio of the potential difference between its ends to the current flowing through it is called the resistance of the conductor. This is known as Ohm's law. The SI unit of resistance is the ohm, symbol Ω.

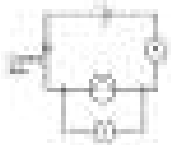


Figure 4.1

Figure 4.1 shows a cell connected across a filament lamp. A current flows, the filament gets hot and emits light. The potential difference across the lamp is measured by the voltmeter in parallel with it. The current is measured by the ammeter in series with the lamp.

The ohm is defined as equal to 1 volt per 1 ampere. More generally, we define the resistance of a component as the potential difference across it divided by the current through it. For a component that does not obey Ohm's law, this ratio changes as the current changes.

$$\text{resistance} = \frac{\text{potential difference}}{\text{current}}$$

4.3 Series and parallel circuits

For components connected in series, the same current flows through each one in turn. The total resistance is the sum of the individual resistances. For components connected in parallel, the potential difference across each branch is the same, and the total current is the sum of the currents in the branches.



Figure 4.2

Figure 4.2 shows two resistors in parallel. The reciprocal of the total resistance is the sum of the reciprocals of the individual resistances. For two resistors of 6 Ω and 3 Ω in parallel, the total resistance is 2 Ω.

4.4 Internal resistance

Real cells have a small internal resistance. When current is drawn, some of the energy is dissipated inside the cell, so the terminal potential difference is less than the electromotive force. The lost volts equal the current multiplied by the internal resistance.



Figure 2.4

If the system in Figure 2.4 is stiffened by a substantially more rigid base, the graph becomes the steeper line. The slope is the reciprocal of the spring constant, and the intercept is the preload. Extrapolating the data to zero force is a common way to estimate preload without disturbing the assembly.

2.8 What's next

What's next is the section that tells the reader what to read after this one. Keep it short. Two or three sentences are enough. Point to the next chapter, to a specific section, or to an appendix. Avoid the temptation to summarize the whole book here; the table of contents already does that.

$$\left(-\frac{1}{6}, e^{\pi}, -\sqrt{49}, 0 \right) = \left(-\frac{1}{6}, e^{\pi}, -\sqrt{49}, 0 \right)$$

Problem 1. Use the method from this section to find the value of x that satisfies $\left(x, x, x, x \right) = \left(1, 2, 3, 4 \right)$. Explain briefly why no such x exists, and state what would have to change in the problem for a solution to appear.

Every theorem statement $A \Rightarrow B$ has a converse $B \Rightarrow A$ and a contrapositive $\neg B \Rightarrow \neg A$. Only the contrapositive is always equivalent to the original.

2.9 Exercises and solutions

Exercises at the end of a chapter are for the reader, not for the author. Three to five problems are enough. Mix routine drill with one or two that require the reader to think a little before they start computing. Avoid problems that are only tricks; a trick that the reader has not been taught is frustrating, not educational. Solutions belong in an appendix, not in the chapter, so that the reader can attempt the problem before looking.

Table 2.1

Code	Name	Meaning	Example
`fn`	`fn`	declare a function	`fn add(a, b) { a + b }`
`if`	`if`	branch on a condition	`if x > 0 { x } else { -x }`
`for`	`for`	loop over a range	`for i in 0..10 { ... }`
`let`	`let`	bind a name to a value	`let total = 0;`
`return`	`return`	exit the current function	`return result;`

A work sample is given for a selected problem; you are to complete the rest.

> **Problem 4 — Determine the I_3 which could be applied to a 12Ω resistor in order that a network of 24 Ω may flow.**

Solution: $R = 12\Omega$; $I = 24^{\circ}$; solve for I_3

Formula: $I_3 = I_2$ and $\angle = \angle - \angle$ or $\frac{I_3}{I_2} = \frac{R_2}{R_3}$ or $\frac{I_3}{I_2} = \frac{\angle}{\angle}$; $I_3 = 1.5\ I_2$

Work: Multiply both members by R_3 : $I_3 R_3 = I_2 R_2$; divide by I_3 : $R_3 = \frac{I_2 R_2}{I_3}$

> **Problem 5 — In which two numbers is 0.75 more than twenty-five? Express the required ratio as 0.75 : 100 and in the lowest terms of a fraction.**

Solution: $R = \frac{P}{I} = \frac{\angle}{\angle} = \frac{\angle}{\angle} = \frac{\angle}{\angle} = \angle$

> **Problem 6 — I_2 12.5 V battery is connected across an ammeter and resistor R_2 which is 25 Ω. Determine the resistance of the ammeter, if the voltage is not reduced to 12 V, when connected to the same value of the battery.**

Solution: $R = \frac{P}{I} = \frac{\angle}{\angle} = \frac{\angle}{\angle} = \angle$

Formula: $R = \frac{P}{I} = \frac{\angle}{\angle}$; $R = \frac{\angle - \angle}{\angle}$; $R = \angle$

Formula: $I = \frac{P}{R}$; $I = \frac{\angle}{\angle}$; $I = \angle$

> **Problem 7 — What is the resistance of a coil which draws a current of 5 A at 50 volts and R_2 200 Ω coil from a 12 V supply?**

(a) Resistance: $R = \frac{P}{I} = \frac{\angle}{\angle - \angle}$

$= \frac{\angle}{\angle} = \frac{\angle}{\angle}$ = **5 Ω**; R_2 = **2.5 Ω**

(b) Resistance: $R = \frac{\angle}{\angle - \angle}$; $= \frac{\angle}{\angle}$

$= \frac{\angle}{\angle}$ = **200 Ω**; R_2 = **100 Ω**; I_2 = **0.2 A**

Further problems and their solutions begin on page 14. Complete Exercise 4.1, providing at least 7 steps.

2.8 Cranking duty and throughput

A **crankshaft** is a rotating shaft that converts linear piston motion into rotary motion, or vice versa. It typically has one or more crankpins, journals for the bearings, counterweights to balance the rotating and reciprocating masses, and a flywheel at one end to smooth the engine's speed between power strokes.

The **throughput** of a machine element is the total quantity that has passed through it over its service life, usually measured in tonnes, cycles, or hours. Throughput is the natural way to express how much work a component has done, and it is the basis for most fatigue and wear calculations.

2.9 Work-hardening powers and energy

Work-hardening powers

Suppose N is the minimum number of cycles to cause the number of dislocations D per unit volume to increase by a factor of α (where $\alpha > 1$). The value of α chosen is the value 10^{7} (Hollow).

$$\boxed{D = C \cdot n \cdot \Sigma \text{ (kinetic)}} \tag{2.9.1}$$

Given $D(0) = \text{base}$, $D = \alpha D$.

Substituting into (1) for equation (2.1) gives:

$$\alpha D = C \cdot n \cdot \Sigma$$

$$\Rightarrow \quad \boxed{D = \alpha \cdot C \cdot n \cdot \Sigma \text{ (static)}}$$

where, if one defines $\alpha = \frac{N}{A}$.

Substituting into for equation (2.1) gives:

$$D = C \cdot \frac{N}{A}$$

$$\Rightarrow \quad \boxed{D = \frac{C \cdot N}{A} \text{ (static)}}$$

There are three three-condition theorems which must be used for work-hardening powers:

> Boolean $A \cdot \ldots \cdot n$ (1.9.1) the plan was small, built in a compound $n = 1.9.1$ to equate Compliance (a) the natural bearing in the body and (b) the resistance of the load.

Given $N = 1 \cdot \ldots \cdot n$ being what is constant $C = \frac{N}{A}$

(a) Constant $k = \frac{\Delta D}{\Delta n} = \frac{D_2 - D_1}{n_2 - n_1} = \text{kind}$ ■

(b) Resistance $R = \frac{N}{A} = \frac{\Delta n}{\Delta \alpha} = \frac{n_2 - n_1}{\alpha_2 - \alpha_1} = \text{cold}$ ■

Worked Example 1 — Calculating the current through, voltage across, and power dissipated by a resistance in a circuit

Given: $R = 4.7\,\text{k}\Omega = 4.7 \times 10^{3}\,\Omega$, $V_{\text{supply}} = 12\,\text{V}$

$$I = \frac{V}{R} = \frac{12\,\text{V}}{4.7 \times 10^{3}\,\Omega} = 2.553 \times 10^{-3}\,\text{A} = 2.553\,\text{mA}$$

$$P = V \times I = 12\,\text{V} \times 2.553 \times 10^{-3}\,\text{A} = 30.64 \times 10^{-3}\,\text{W} = 30.64\,\text{mW}$$

Alternatively, since $I = \frac{V}{R}$, $P = V \times \frac{V}{R} = \frac{V^{2}}{R}$. Also, from Ohm's law, voltage $V = IR = 2.553 \times 10^{-3}\,\text{A} \times 4.7 \times 10^{3}\,\Omega = 12.0\,\text{V}$. Energy, power, $P = V \times I = 12\,\text{V} \times 2.553 \times 10^{-3}\,\text{A} = \mathbf{30.64\,mW}$

Worked Example 2 — Determining the total resistance of a series–parallel network and the power delivered by the source

Given: $R_1 = 10\,\Omega$, $R_2 = 22\,\Omega$, $R_3 = 47\,\Omega$, $V_{\text{source}} = 9.0\,\text{V}$. R_2 and R_3 are in parallel with each other, and that combination is in series with R_1. Find the total resistance R_{total}, the total current I_{total}, and the power delivered by the source.

Solution:

Step 1 — Combine the parallel branch. For two resistors in parallel, $\frac{1}{R_{p}} = \frac{1}{R_2} + \frac{1}{R_3}$, so $R_{p} = \frac{R_2 R_3}{R_2 + R_3}$.

$$R_{p} = \frac{22\,\Omega \times 47\,\Omega}{22\,\Omega + 47\,\Omega} = \frac{1034\,\Omega^{2}}{69\,\Omega} = 14.986\,\Omega \approx 15.0\,\Omega$$

Step 2 — Add the series resistance. $R_{\text{total}} = R_1 + R_{p} = 10\,\Omega + 14.986\,\Omega = 24.986\,\Omega \approx 25.0\,\Omega$.

Step 3 — Find the total current. $I_{\text{total}} = \frac{V_{\text{source}}}{R_{\text{total}}} = \frac{9.0\,\text{V}}{24.986\,\Omega} = 0.360\,\text{A} = 360\,\text{mA}$.

Step 4 — Find the power delivered by the source. $P = V_{\text{source}} \times I_{\text{total}} = 9.0\,\text{V} \times 0.360\,\text{A} = \mathbf{3.24\,W}$.

Check: $P = I_{\text{total}}^{2} R_{\text{total}} = (0.360\,\text{A})^{2} \times 24.986\,\Omega = 3.24\,\text{W}$.

Worked Example 3 — Sketching the current–voltage characteristic of a non-ohmic device and finding its operating point



Figure 3.1

A diode is connected in series with a $100\,\Omega$ resistor and a $5.0\,\text{V}$ supply. The diode's current–voltage relationship is given approximately by $I = I_{s}\left(e^{V/V_{T}} - 1\right)$, with $I_{s} = 1.0 \times 10^{-12}\,\text{A}$ and $V_{T} = 0.026\,\text{V}$. Sketch the load line on the axes above and estimate the operating point.

For series, $V_{R} = IR$ and $V_{D} = V_{\text{supply}} - V_{R}$. For a trial current $I = 40\,\text{mA}$, $V_{R} = 4.0\,\text{V}$ and $V_{D} = 1.0\,\text{V}$. For a trial current $I = 45\,\text{mA}$, $V_{R} = 4.5\,\text{V}$ and $V_{D} = 0.5\,\text{V}$. The operating point lies near $I \approx 42\,\text{mA}$, $V_{D} \approx 0.8\,\text{V}$.

> **Problem 11.** The total resistance of a 24.0 Ω heating wire is 6.00 Ω. Find the current when the wire is connected to a 12.0 V battery.

Total resistance for resistors in series: $R_{\text{total}} = R_1 + R_2 + R_3 + \cdots$

Ohm's law: $I = \frac{V}{R}$

Given: $V = 12.0\ \text{V}$, $R = 6.00\ \Omega$

Required: I

Electrical energy

> **Electrical energy = power × time**

In an electric circuit, energy is transferred from the battery to the components. Some components change electrical energy into useful forms, such as light or heat. Others, like resistors, mainly transfer energy to the surroundings as heat. The rate at which energy is transferred is called **power**. The unit of power is the **watt**, W. One watt is equal to one joule per second.

> **Problem 12.** A 40 Ω resistor is connected across a 6 cell battery of total emf 9.0 V. Determine the current flowing in the circuit. The power consumed by the resistor is given by $P = I^2R$. Calculate the energy transferred in 5 minutes.

$I = \frac{V}{R} = \frac{9.0}{40} = 0.23\ \text{A}$

Power consumed: $P = I^2R = (0.225)^2 \times 40 = 2.03\ \text{W}$

Energy transferred = power × time = $2.03 \times 5 \times 60 = 610\ \text{J}$

Answer: $I = 0.23\ \text{A}$; $610\ \text{J}$

> **Problem 13.** A resistor of mass 0.10 kg is heated by a current of 2.0 A for 30 seconds. How much energy is transferred to the resistor?

Energy = power × time, and power = voltage × current.

Since energy = I^2Rt, $R = \frac{E}{I^2t} = \frac{60}{2.0^2 \times 30} = 0.50\ \Omega$

> **Problem 14.** Determine the power in an electric motor if a current of 2.5 A runs a 240 V supply. Calculate also the power wasted if the motor is 85% efficient. How much energy is used in 1 hour? Express your answer in kilowatt-hours (kWh).

Power = $V \times I = 240 \times 2.5 = 600\ \text{W} = 0.6\ \text{kW}$

Energy discharged = power × time = 1.2 × 3600 = 4320 J

Cost at 8 p/unit = 4320 × 8 = 34560 p

Power wasted as heat at the battery = 0.8 W

> **Problem 11.** An electric kettle contains 0.5 kg of water at 20 °C. It is rated at 2000 W, and its thermostat switches off when the water reaches 100 °C. Assuming all the heat goes into the water, how long does it take to boil?

Energy $= \frac{\text{mass} \times \text{specific heat} \times \text{temperature rise}}{\text{efficiency}} = \frac{0.5 \times 4200 \times (100 - 20)}{1} = 168\,000$ J

Time $= \frac{\text{energy}}{\text{power}} = \frac{168\,000}{2000} = 84$ s

Time to boil = 84 s (about 1 minute 24 seconds)

> **Problem 12.** Two lamps are connected in parallel to a 12 V battery. One lamp is 6 Ω and the other is 12 Ω. Find the current in each lamp and the current in the battery.

Current in 6 Ω lamp $I_1 = \frac{V}{R_1} = \frac{12}{6} = 2$ A

Current in 12 Ω lamp $I_2 = \frac{V}{R_2} = \frac{12}{12} = 1$ A

Total current $I = I_1 + I_2 = 2 + 1 = 3$ A

Current from the battery = 3 A

> **Problem 13.** Two lamps are connected in series to a 12 V battery. One is 6 Ω and the other is 12 Ω. Find the current in the circuit and the voltage across each lamp.

Total resistance $R = R_1 + R_2 = 6 + 12 = 18\ \Omega$

Current $I = \frac{V}{R} = \frac{12}{18} = \frac{2}{3}$ A ≈ 0.67 A

Voltage across 6 Ω lamp $V_1 = I \times R_1 = \frac{2}{3} \times 6 = 4$ V

Voltage across 12 Ω lamp $V_2 = I \times R_2 = \frac{2}{3} \times 12 = 8$ V

Check: $4 + 8 = 12$ V ✓

Current = 0.67 A; voltages = 4 V and 8 V

> **Problem 14.** A resistance heater has a 240 V mains supply and a resistance of 48 Ω. It heats 2 kg of water from 15 °C to 65 °C. How long does it take? Assume that all the energy from the heater goes into the water.

Power $P = \frac{V^2}{R} = \frac{240^2}{48} = 1200$ W

Energy needed $= mc\Delta\theta = 2 \times 4200 \times (65 - 15) = 420\,000$ J

Time $= \frac{\text{energy}}{\text{power}} = \frac{420\,000}{1200} = 350$ s

Time = 350 s (about 5 minutes 50 seconds)

A mole of substance is a collection of as many elementary entities as there are atoms in exactly 12 grams of the carbon-12 isotope, ^{12}C. The number of entities in a mole is known as Avogadro's number or Avogadro's constant, N_A. Its value is approximately 6.022×10^{23} mol^{-1}.

The molar mass of a substance is the mass of one mole of its entities, and is expressed in grams per mole (g/mol). The molar mass of a compound is the sum of the molar masses of the elements in its chemical formula. For example, the molar mass of water (H_2O) is approximately $2(1.008) + 15.999 = 18.015$ g/mol.

Molar mass is a bridge between the atomic scale and the laboratory scale. It allows us to convert between the number of particles and the mass of a sample. For instance, 18.015 g of water contains 6.022×10^{23} molecules, while 18.015 g of carbon-12 contains 6.022×10^{23} atoms.

Further worked examples of molar mass calculations appear in Section 3.2, beginning on page 47.

3.17 Mass defects and nuclear binding energy

The atomic mass of an atom is always slightly less than the sum of the masses of its individual protons, neutrons, and electrons. The missing mass is called the mass defect, and it is the mass that was converted into binding energy when the nucleus formed. Einstein's relation $E = mc^2$ tells us that this energy is enormous even for a small mass.

Mass defect. For a nucleus with Z protons, N neutrons, and measured nuclear mass m_{nuc}, the mass defect is

$$\Delta m = Z m_p + N m_n - m_{\text{nuc}},$$

where m_p and m_n are the proton and neutron masses. The binding energy is $E_b = \Delta m c^2$.

Binding energy per nucleon. The quantity E_b / A, with $A = Z + N$, is the binding energy per nucleon. It is the most useful measure for comparing the stability of different nuclei, because it removes the trivial dependence on how many nucleons are present.

Worked example. Calculate the binding energy per nucleon of ^{56}Fe. Use $m_p = 1.007276$ u, $m_n = 1.008665$ u, $m_{\text{nuc}} = 55.934937$ u, and 1 u $= 931.494$ MeV/c^2.

Step 1. Mass defect. $\Delta m = 26(1.007276) + 30(1.008665) - 55.934937 = 0.528461$ u.

Step 2. Binding energy. $E_b = 0.528461 \times 931.494 = 492.26$ MeV.

Step 3. Per nucleon. $E_b / A = 492.26 / 56 = 8.790$ MeV.

Iron-56 is famous for having one of the largest binding energies per nucleon of any nuclide, which is why fusion of light nuclei and fission of heavy nuclei both release energy.

Binding energy curves for the first 118 elements are tabulated in Appendix D, starting on page 1042.

[illegible] [illegible] [illegible] [

1.1 [3 pts] Let $f(x)$ be a function defined on $[-\pi, \pi]$. Write down the formulas for the Fourier coefficients a_0, a_n, and b_n of f on this interval.

1.2 [4 pts] Suppose f is an even function on $[-\pi, \pi]$. Show that $b_n = 0$ for all $n \geq 1$. Then compute a_0 and a_n explicitly when $f(x) = x^2$.

1.3 [3 pts] Let $f(x) = |x|$ on $[-\pi, \pi]$. Using your results from 1.2, write the first three nonzero terms of the Fourier series of f.

2.1 [3 pts] Two functions f and g are said to be orthogonal on $[a, b]$ with respect to a weight function $w(x) > 0$ if $\int_a^b f(x)\,g(x)\,w(x)\,dx = 0$. Show that $\cos(nx)$ and $\cos(mx)$ are orthogonal on $[-\pi, \pi]$ with $w(x) = 1$ whenever $n \neq m$.

2.2 [4 pts] Let $P_0(x) = 1$, $P_1(x) = x$, and $P_2(x) = x^2 - \tfrac{1}{3}$. Verify directly that P_0 and P_2 are orthogonal on $[-1, 1]$ with $w(x) = 1$. Then find a constant c such that P_1 and $P_2 - c$ are orthogonal on the same interval.

2.3 [3 pts] Explain in two or three sentences why orthogonality is useful when we want to write a function as a sum of simpler functions. You may refer to the Fourier coefficient formulas from 1.1.

Proofs

3.1 [5 pts] Let $\{\phi_n\}_{n=1}^{\infty}$ be an orthonormal set on $[a, b]$ with weight $w(x) = 1$, and suppose f can be written as $f = \sum_{n=1}^{\infty} c_n \phi_n$. Prove that the coefficients are given by $c_n = \int_a^b f(x)\,\phi_n(x)\,dx$. You may assume that the series converges in the mean-square sense and that integration and summation can be interchanged.

3 Resistance variation

In this unit the student will be able to:

- Identify that electrical resistance depends on the material.
- Explain the electrical resistance $R = \frac{\rho L}{A}$, where ρ is the resistivity.
- Recognize a practical device in which this unit is used.
- Calculate electrical energy $E = \frac{V^2}{R}$.
- Define the temperature coefficient of resistance α.
- Recognize a practical device for it.
- Calculate resistance using $R_t = R_0(1 + \alpha\Delta t)$.

3.1 Resistance and resistivity

The resistance of an electrical conductor depends on it material, also being on the length of the conductor. It also on area/cross-section of the conductor, on what type of material each of the conductors is made of and on the temperature of the conductor.

Resistance, R, of a uniform conductor is directly $\propto$ to conductor's ℓ, $R \propto \ell$. Also, for example, if the length of a piece of wire is doubled, also the resistance is doubled.

Resistance, R, is inversely proportional to cross-sectional area, A, of a conductor, $R \propto \frac{1}{A}$. Also, if the area of the cross-sectional area of a piece of wire is doubled then the resistance is halved.

Since $R \propto \ell$ and $R \propto \frac{1}{A}$ we have $R \propto \frac{\ell}{A}$. By inserting a constant of proportionality, also this constant is the type of material used, that is called resistivity. The symbol for resistivity is usually the Greek letter ρ (rho) and is given the symbol ρ (Greek letter rho).

$$\text{resistance} = \frac{\rho \times \text{length}}{\text{area}} \qquad R = \frac{\rho \ell}{A}$$

ρ is measured in ohm metres (Ωm).

R is value of the resistance in ohms, ℓ is the length in m of the conductor, A is the area in m^2 of the cross-section of the conductor and ρ is the resistivity of the material in Ωm.

Resistivity shows how the resistance will change for the material, also how conductive or resistive each material is in comparison to other materials.

Copper: $1.7 \times 10^{-8}\ \Omega$m (or $0.017\ \mu\Omega$m)

Aluminium: $2.8 \times 10^{-8}\ \Omega$m (or $0.028\ \mu\Omega$m)

Carbon (graphite): $3.5 \times 10^{-5}\ \Omega$m (or $35\ \mu\Omega$m)

Mass 1 = 27° Dip ice 14°(offer)
Mass 2 = 63° Dam ice 67° (offer)
Will that good combination of dimensions force a low value of resistivity and good flexibility mean a high value of conductivity

> **Problem 1** — The resistance of a 1 ft length of wire is 200 Ω. Determine both the resistance of an 8 ft length of the same wire and the length of this same wire whose resistance is $\frac{1}{2}\Omega$.

(a) Resistance R_2 of 8ft length proportional to length $= 1 \times 8 = 8$

Since $\frac{\text{resistance}}{\text{length}}$ is same for each wire, resistance of required length $= R_2$ ft

$$\frac{R_2}{8} = \frac{200}{1}$$

$$R_2 = 8 \times 200 = 1600\,\Omega$$

(b) Since the resistance is 400 Ω, let l = length of wire

$$\frac{\text{length } l}{1} = \frac{400}{200}, \quad l = 2 \text{ ft}$$

> **Problem 2** — A piece of wire of uniform cross-section has a resistance of 4Ω. Find (a) the resistance of a piece of the same length and material of twice the diameter, (b) the length and diameter of a piece of the same length and material of resistance 16Ω.

Resistance R proportional to $\frac{1}{\text{area}}$ or $\frac{1}{d^2}$ where d = diameter

Since $4\Omega \propto \frac{1}{\text{area}}$ or $\frac{1}{d^2}$, $4\Omega = k\left[\frac{1}{d^2}\right]$

From which, one is difficult if proportionality k = $4\Omega \times \frac{1}{d^2} = 4\Omega$

(a) When the resistance and area = 4 and when $R = k\left[\frac{1}{d^2}\right]$

$$= 4\Omega \times \left[\frac{1}{2^2}\right] = 1\Omega$$

(Since that resistance was decreased as the cross-sectional is increased)

(b) When the resistance is 16Ω, then $16\Omega = 4\Omega \times \frac{1}{d^2}$ or $d^2 = \frac{4\Omega}{16\Omega} = \frac{1}{4}$ ft^2

cross-sectional area, $= \frac{\pi}{4}d^2 = \frac{\pi \times 1}{4 \times 4} = \frac{\pi}{16}$ ft^2

Worked Example 7.1 A wire of length L is such that its mass per unit length is $\rho(x) = \rho_0 x^2$, where x is the distance from one end of the wire. If the wire is clamped at both ends and its fundamental frequency of vibration is ω, find the tension in the wire.

Separation of variables for the wave equation $\frac{\partial^2 y}{\partial t^2} = \frac{T}{\rho(x)} \frac{\partial^2 y}{\partial x^2}$ leads to the spatial equation in the form

$$\frac{d^2 X}{dx^2} + \frac{\rho(x)\omega^2}{T} X = 0, \quad X(0) = X(L) = 0,$$

where T is the tension in the wire. Substituting $\rho(x) = \rho_0 x^2$ and $\lambda = \frac{\rho_0 \omega^2}{T}$, we have

$$X'' + \lambda x^2 X = 0, \quad X(0) = X(L) = 0.$$

This is a Sturm–Liouville problem with $p(x) = 1$, $q(x) = 0$, $r(x) = x^2$, and $w(x) = x^2$. The equation is not of Cauchy–Euler type, but it can be transformed into Bessel's equation. Let $t = \frac{x^3}{3}$ and $Y(t) = X(x)$. Then $\frac{dt}{dx} = x^2$, $\frac{dY}{dx} = \frac{dY}{dt}\frac{dt}{dx} = x^2 \frac{dY}{dt}$, and

$$\frac{d^2 X}{dx^2} = \frac{d}{dx}\left(x^2 \frac{dY}{dt}\right) = 2x \frac{dY}{dt} + x^2 \frac{d^2 Y}{dt^2} \frac{dt}{dx} = 2x \frac{dY}{dt} + x^4 \frac{d^2 Y}{dt^2}.$$

The differential equation then becomes

$$2x \frac{dY}{dt} + x^4 \frac{d^2 Y}{dt^2} + \lambda x^2 Y = 0.$$

Dividing by x^4 and substituting $x = (3t)^{1/3}$, we obtain

$$\frac{d^2 Y}{dt^2} + \frac{2}{x^3} \frac{dY}{dt} + \frac{\lambda}{x^2} Y = 0 \quad \Rightarrow \quad \frac{d^2 Y}{dt^2} + \frac{2}{3t} \frac{dY}{dt} + \frac{\lambda}{(3t)^{2/3}} Y = 0.$$

Multiplying by $t^{2/3}$ to put it in standard Bessel form, we get

$$t^{2/3} \frac{d^2 Y}{dt^2} + \frac{2}{3} t^{-1/3} \frac{dY}{dt} + \frac{\lambda}{3^{2/3}} Y = 0.$$

This is Bessel's equation of order $\nu = \frac{1}{3}$ with parameter $\frac{\lambda}{3^{2/3}}$. The general solution is

$$Y(t) = A J_{1/3}\left(\frac{\sqrt{\lambda}}{3^{1/3}} t^{1/2}\right) + B J_{-1/3}\left(\frac{\sqrt{\lambda}}{3^{1/3}} t^{1/2}\right).$$

Since $J_{-1/3}$ is unbounded at $t = 0$, we must have $B = 0$. Thus,

$$X(x) = A J_{1/3}\left(\frac{\sqrt{\lambda}}{3^{1/3}} \left(\frac{x^3}{3}\right)^{1/2}\right) = A J_{1/3}\left(\frac{\sqrt{\lambda}}{3} x^{3/2}\right).$$

Now, $X(L) = 0$ implies

$$J_{1/3}\left(\frac{\sqrt{\lambda}}{3} L^{3/2}\right) = 0.$$

Let α_1 denote the smallest positive zero of $J_{1/3}$. Then the fundamental frequency ω corresponds to the smallest positive value of λ, so we have

$$\frac{\sqrt{\lambda}}{3} L^{3/2} = \alpha_1 \quad \Rightarrow \quad \lambda = \frac{9\alpha_1^2}{L^3}.$$

Since $\lambda = \frac{\rho_0 \omega^2}{T}$, the tension is

$$T = \frac{\rho_0 \omega^2 L^3}{9\alpha_1^2}.$$

Worked Example 7.2 Determine the eigenvalues and eigenfunctions of the boundary-value problem

$$y'' + \lambda y = 0, \quad y(0) = 0, \quad y(\pi) + y'(\pi) = 0.$$

Solution: We consider three cases: $\lambda = 0$, $\lambda < 0$, and $\lambda > 0$.

Case 1: $\lambda = 0$. The general solution is $y = Ax + B$. The boundary condition $y(0) = 0$ gives $B = 0$, and $y(\pi) + y'(\pi) = A\pi + A = 0$ gives $A = 0$. Thus, $\lambda = 0$ is not an eigenvalue.

Case 2: $\lambda = -\mu^2 < 0$. The general solution is $y = Ae^{\mu x} + Be^{-\mu x}$. The boundary condition $y(0) = 0$ gives $A + B = 0$, so $B = -A$. Then $y(\pi) + y'(\pi) = A(e^{\mu\pi} - e^{-\mu\pi}) + \mu A(e^{\mu\pi} + e^{-\mu\pi}) = 0$. Since $A \neq 0$ (otherwise $y \equiv 0$), we have

$$(1 + \mu)e^{\mu\pi} + (1 - \mu)e^{-\mu\pi} = 0.$$

Multiplying by $e^{\mu\pi}$ gives $(1 + \mu)e^{2\mu\pi} + (1 - \mu) = 0$, or $\mu = \frac{e^{2\mu\pi} - 1}{e^{2\mu\pi} + 1}$. This equation has no positive solution, so there are no negative eigenvalues.

Case 3: $\lambda = \mu^2 > 0$. The general solution is $y = A\cos\mu x + B\sin\mu x$. The boundary condition $y(0) = 0$ gives $A = 0$. Then $y(\pi) + y'(\pi) = B\sin\mu\pi + \mu B\cos\mu\pi = 0$. Since $B \neq 0$, we have $\tan\mu\pi = -\mu$. The solutions are the intersections of $\tan\mu\pi$ with $-\mu$, which occur at $\mu_n \approx n - \frac{1}{4}$ for $n = 1, 2, 3, \ldots$. Thus, the eigenvalues are $\lambda_n = \mu_n^2 \approx \left(n - \frac{1}{4}\right)^2$ and the eigenfunctions are $y_n(x) = \sin\mu_n x$.

Enthalpy $H = \frac{\text{Btu}}{\text{lb}}$

Heat content $= \frac{\text{Btu}}{\text{lb}} = \frac{\text{Enthalpy} \times \text{Flow}}{\text{Electricity}}$

$= \text{kWh} \times \text{Btu/kWh}$ (Btu or kWh ? after)

> **Problem 1** — Determine the Enthalpy of 1,000 lb of a liquid, when passing a pressure of 15 psia at the condition of steam at 1,000 lb.

Characteristic data of coffee, $\alpha = 0.007 + 0.1\left(\frac{T}{100}\right)^2$

$= \text{Btu sec}^2 = \text{Btu} \cdot (0.7\ \text{g})^2$

Resistance $R = \frac{H}{\alpha} = \frac{0.7 \cdot 1.00 \cdot 10^{-3}\ \text{Btu} \cdot \text{g Btu/sec}}{[\text{Btu}] \cdot 10^{-3}\ \text{g}^2}$

$= \frac{0.7 \cdot 1.00 \cdot 10^{-3}}{10^{-3}\ \text{Btu}} = \frac{0.7 \cdot 10}{\text{Btu}^2}\ \Omega$

$= 0.000\ \Omega$

Enthalpy (enthalpy) was not taken to demonstrate by using a temperature from 0 to 100 Btu/lb 3 to 7 steps 10

3.2 Temperature coefficient of resistance

To get back, for the temperature of 0 is known of resistance, and coefficient of resistivity, a resistance coefficient depends on temperature. Either the coefficient is also called the temperature coefficient.

The **temperature coefficient of resistance** (α) is the ratio of the increase in the resistance of a 1 Ω resistor (1 Ω) that is raised 1 degree in temperature (1 °C). For example, if a 1 Ω resistor has a temperature coefficient of resistance of 0.004 (at 20 °C), then its resistance will increase by 0.004 Ω for every 1 °C rise in temperature. The table gives values of temperature coefficient of resistance for various materials at 20 °C (68 °F). Note that the values are only approximate because the temperature coefficient of resistance changes with temperature.

Some typical values of temperature coefficient of resistance (measured at 20 °C) are given below:

Copper	0.0043 °C	Aluminium	0.0039 °C
Nickel	0.0041 °C	Carbon	−0.0005 °C
Constantan	0	Graphite	−0.0002 °C

(Note that the negative sign for carbon indicates that the resistance falls with increase of temperature.)

If the distance d of a particle at $P(r, \theta)$ is given, the distance r is not fixed. Suppose that the distance from the fixed point O is

$$\boxed{\mathbf{R}_{OP} = \mathbf{R}_{OQ} + \mathbf{r}_{QP}(\theta)}$$

where $\mathbf{R}_{OP}$ = position vector of P from O

$\mathbf{R}_{OQ}$ = position vector of the point Q from O

$\mathbf{r}_{QP}$ = displacement vector of P relative to Q at angle θ from OQ.

> **Definition 4.** A set S is said to be *bounded* if there exists a real number $M > 0$ such that $|x| \leq M$ for all $x \in S$. Otherwise, the set is *unbounded*. If S is bounded above, then $\sup S$ exists; if S is bounded below, then $\inf S$ exists. The set S is *closed* if it contains all of its limit points.

Therefore $\mathbf{R}_{OP} = \mathbf{R}_{OQ} + \mathbf{r}_{QP}$

where $\mathbf{r}_{QP}$ is $r\hat{\mathbf{e}}_r$, $\mathbf{R}_{OQ} = 2\cos\theta\,\hat{\mathbf{e}}_r + 2\sin\theta\,\hat{\mathbf{e}}_\theta$

$$\begin{aligned} \mathbf{R}_{OP} &= 2\cos\theta\,\hat{\mathbf{e}}_r + 2\sin\theta\,\hat{\mathbf{e}}_\theta + r\hat{\mathbf{e}}_r \\ &= (2\cos\theta + r)\hat{\mathbf{e}}_r + 2\sin\theta\,\hat{\mathbf{e}}_\theta \end{aligned}$$

> **Proposition 2.** *Let $\{a_n\}$ be a sequence of real numbers. If $\{a_n\}$ is a monotone sequence and $\{a_n\}$ is bounded, then the sequence converges. If $\{a_n\}$ does not converge, then $\{a_n\}$ is unbounded.*

Therefore $\mathbf{r} = \mathbf{R}_{OP} - \mathbf{R}_{OQ}$

Where coordinates of $P(r, \theta)$ $\quad r = \dfrac{R_{OP}}{\left(1 + \tan^2\theta\right)^{\frac{1}{2}}} = \dfrac{R_{OP}}{\left(1 + \left(\frac{y}{x}\right)^2\right)^{\frac{1}{2}}}$

$$= \frac{x}{\left(\frac{x^2+y^2}{x^2}\right)^{\frac{1}{2}}} = \frac{x}{\frac{\sqrt{x^2+y^2}}{x}} = \frac{x^2}{\sqrt{x^2+y^2}}$$

> **Definition 11.** *A subset S of $\mathbb{R}$ is said to be compact if S is closed and bounded. Every sequence in S has a subsequence that converges to a point in S. If S is not compact, then it is either not closed or not bounded.*

Therefore $\mathbf{r}_{QP}$ is compact $\mathbf{r}_{QP}(\theta) = \mathbf{R}_{OP} - \mathbf{R}_{OQ}$

$\mathbf{r}_{QP} = \mathbf{R}_{OP} - \mathbf{R}_{OQ} = (r\cos\theta\,\hat{\mathbf{i}} + r\sin\theta\,\hat{\mathbf{j}}) - (2\cos\theta\,\hat{\mathbf{i}} + 2\sin\theta\,\hat{\mathbf{j}})$

$= (r\cos\theta - 2\cos\theta)\,\hat{\mathbf{i}} + (r\sin\theta - 2\sin\theta)\,\hat{\mathbf{j}}$ **(ans.)**

If the coordinates of a particle at time t are given by $x = f(t)$ and $y = g(t)$, where f and g are continuous functions on an interval $[a, b]$, and the curve

Here the compiler fills in implementations that it can provide for:

```
fn clone(&self) -> Self
```

> **Definition 10.1.** — A unit 12 subject to the law is a subspace of $\mathbb{R}^{3}$ for the $\mathbb{R}[X]$. If the component has coefficients of inheritance in Corollary 10.2.7, then $\mathbb{R}[X]$ is a subspace of the unit ruling the transfer law from $\mathbb{R}^{3}$.

Substituting $\mathbb{R}^{3}$ for $\mathbb{R}[X]$ in $\mathbb{R}[X]$ gives

$$\mathbb{R}[X] \to \mathbb{R}[X] \to \mathbb{R}[X]$$

Some conditions in $\mathbb{R}[X]$ for $\mathbb{R}[X]$ — $\mathbb{R}[X]$ — $\mathbb{R}[X]$ —

Rearranging the equations in this form gives

$$\boxed{\begin{aligned} \frac{dx}{dt} &= 1 + xy^2 \\ \frac{dy}{dt} &= 1 + x^2y \end{aligned}}$$

where $\frac{dx}{dt}$ is understood to be a function of t.

Definition 2.1. Suppose $x(t)$ is a function of t and $y(t)$ is a function of x. Then y is a function of t and we can apply the chain rule to find $\frac{dy}{dt}$. Since $\frac{dy}{dx}$ is a function of x, it is also a function of t, and we can apply the chain rule to find $\frac{d}{dt}\left(\frac{dy}{dx}\right)$. Similarly, if $\frac{d^2y}{dx^2}$ is a function of x, then $\frac{d}{dt}\left(\frac{d^2y}{dx^2}\right)$ can be found by the chain rule. Continuing in this way, we can find $\frac{d}{dt}\left(\frac{d^3y}{dx^3}\right)$, $\frac{d}{dt}\left(\frac{d^4y}{dx^4}\right)$, and so on.

Let $x = \sin 3t$ and $y = \cos(e^t)$.

$$\begin{aligned} \frac{dx}{dt} &= 3\cos 3t \\ \frac{dy}{dt} &= -e^t \sin(e^t) \end{aligned}$$

Hence $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-e^t \sin(e^t)}{3\cos 3t}$, $\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{\frac{d}{dt}\left(\frac{-e^t \sin(e^t)}{3\cos 3t}\right)}{3\cos 3t}$

$$= \frac{-e^t \sin(e^t) - e^{2t}\cos(e^t)}{3\cos 3t} = -\frac{1}{3}(1 + e^t \cot 3t)$$

Ex. Use acceleration with the value of $\frac{d^2y}{dx^2}$ in Ex 2.1.2.

Parametric equations are not unique. A curve can be represented in many different ways by choosing different parameters. For example, the curve $y = x^2$ can be represented by the parametric equations $x = t$, $y = t^2$ or by $x = \sin t$, $y = \sin^2 t$. Sometimes one parametrization is more convenient than another, and sometimes one parametrization is more appropriate than another in a particular situation.

Parametric equations and polar coordinates: Polar coordinates ▸ Graphing in polar coordinates ▸ Areas and lengths in polar coordinates ▸ Conic sections ▸ Conic sections in polar coordinates ▸ Vectors in the plane ▸ Vector-valued functions and space curves ▸ Derivatives and integrals of vector functions ▸ Arc length and curvature ▸ Motion along a curve ▸ Tangential and normal components of acceleration ▸ Velocity and acceleration in polar coordinates

2.1 Parabolas, gun-fitting, and projectile motion

Projectiles and projectile fire

1. The trajectory of a projectile is shown in the figure. Determine the initial velocity of the projectile and the angle of elevation of the gun. Find the range of the projectile, that is, the distance from the gun to the point where the projectile hits the ground. Find the maximum height reached by the projectile.

2. Suppose that a projectile is fired with initial velocity v_0 and angle of elevation θ. Find the range R as a function of v_0 and θ. Assuming that v_0 is fixed, determine the angle of elevation that maximizes the range. Determine the maximum height reached by the projectile as a function of v_0 and θ. Determine the angle of elevation that maximizes the height reached by the projectile.

3. Suppose that a projectile is fired with initial velocity v_0 and angle of elevation θ. Find the range R as a function of v_0 and θ. Assuming that v_0 is fixed, determine the angle of elevation that maximizes the range. Determine the maximum height reached by the projectile as a function of v_0 and θ. Determine the angle of elevation that maximizes the height reached by the projectile.

4. Find the equation of the path of a projectile fired with initial velocity v_0 and angle of elevation θ. Then find the maximum height reached by the projectile.

1. Calculate the unconstrained sum of squares SS_{uncon} for a model with k parameters fitted to n observations. Then calculate the constrained sum of squares SS_{con} after imposing m linear restrictions. Form the F statistic $F = \frac{(SS_{\text{con}} - SS_{\text{uncon}})/m}{SS_{\text{uncon}}/(n-k)}$ and compare it with the upper-tail critical value of $F_{m,n-k}$ at the chosen significance level. [5 marks]

2. Explain why the R^2 statistic can be misleading when comparing models with different numbers of parameters fitted to the same data. Define the adjusted coefficient of determination $\bar{R}^2 = 1 - \frac{SS_{\text{res}}/(n-k)}{SS_{\text{tot}}/(n-1)}$ and show that it can decrease when a poorly informative regressor is added, even though R^2 never decreases. [6 marks]

3. Consider the following data on the number of hours studied (x) and the score obtained (y) by eight students in a statistics test:

x	2	3	4	5	6	7	8	9
y	51	58	62	70	73	79	84	88

   (a) Fit the simple linear regression model $y = \alpha + \beta x + \varepsilon$ by ordinary least squares, giving the estimates $\hat{\alpha}$ and $\hat{\beta}$ to two decimal places. [4 marks]

   (b) Calculate the residual sum of squares SS_{res} and the total sum of squares SS_{tot}, and hence find the coefficient of determination R^2. [3 marks]

   (c) Test the null hypothesis $H_0: \beta = 0$ against the two-sided alternative $H_1: \beta \neq 0$ at the 5% significance level, using the t statistic $t = \hat{\beta} / \text{SE}(\hat{\beta})$ with $n - 2$ degrees of freedom. State clearly the critical value you use and your conclusion. [5 marks]

   (d) Predict the score of a student who studies for 10 hours, and comment briefly on the reliability of this prediction. [3 marks]

Regression coefficients in matrix form

4. A set of observations gives the sample covariance $s_{xy} = 12.5$, the sample variance of x as $s_x^2 = 25$, and the sample variance of y as $s_y^2 = 49$. Calculate the correlation coefficient r, and hence find the slope and intercept of the fitted regression line of y on x. Interpret the value of r in one sentence. [6 marks]

5. A regression model has $n = 30$ observations and $k = 4$ parameters, and the residual sum of squares is $SS_{\text{res}} = 84.0$. The total sum of squares is $SS_{\text{tot}} = 420.0$. Calculate the coefficient of determination R^2 and the adjusted coefficient of determination $\bar{R}^2$. Comment briefly on why the two values differ. [4 marks]

6. For a regression model with $n = 50$ observations and $k = 3$ parameters, the unconstrained residual sum of squares is $SS_{\text{uncon}} = 210.0$. After imposing two linear restrictions, the constrained residual sum of squares is $SS_{\text{con}} = 252.0$. Calculate the F statistic for testing these restrictions and state the degrees of freedom of the reference distribution. Is the result significant at the 5% level? [5 marks]

7. A set of regression data has a correlation of $r = 0.6$ and a standard deviation of $s_x = 4$ for the explanatory variable and $s_y = 9$ for the response. Find the slope of the regression line of y on x, and explain why the slope of the regression line of x on y is not simply the reciprocal of this value. [4 marks]

8. The residuals from a fitted regression model are $-2.1, 0.4, 1.7, -0.9, 0.3, 1.6$. Calculate the residual sum of squares and the residual standard error, using $n - 2$ degrees of freedom. Explain briefly why the residuals must sum to zero when the model includes an intercept term. [4 marks]

9. Explain what is meant by the *leverage* of an observation in a regression model. Why do observations with extreme values of the explanatory variable tend to have high leverage, even if their residuals are small? [3 marks]

10. A regression model is fitted to $n = 40$ observations with $k = 5$ parameters. The unconstrained residual sum of squares is $SS_{\text{uncon}} = 96.0$. Two linear restrictions are imposed, giving a constrained residual sum of squares $SS_{\text{con}} = 120.0$. Calculate the F statistic and state the degrees of freedom of the reference distribution. Is the result significant at the 5% level? [5 marks]

4 Chemical effects of electricity

At the end of this chapter you should be able to:

- [illegible]
- [illegible]
- [illegible]
- [illegible]
- [illegible]
- [illegible]
- [illegible]
- [illegible]
- [illegible]
- [illegible]
- [illegible]

4.1 Introduction

[illegible]

4.2 Electrolysis

[illegible]

The **electrolyte** can be any substance containing mobile ions that is a liquid. The positive electrode (cathode) is called the **anode** and the negative electrode (electrode) is the **cathode**.

When the circuit is complete, a current flows through the electrolyte. Positive ions move towards the cathode, and negative ions move towards the anode. Therefore, for example, in the electrolysis of copper(II) sulfate, $CuSO_4$, copper ions, Cu^{2+}, are discharged at the cathode and oxygen is evolved at the anode.

6.3 Electroplating

Electroplating uses the principle of electrolysis to apply a thin layer of one metal to another metal. One practical application is to protect the metal from corrosion (rusting). Other reasons for electroplating are to improve appearance, to increase hardness and resistance to abrasion, or to reduce friction and thus reduce wear. Silver, gold, copper, chromium and nickel are commonly used for electroplating.

6.4 The simple cell

A **cell** consists of two **electrodes** in contact with an electrolyte, which is usually a solution of an electrolyte. A chemical reaction between the electrodes and the electrolyte causes electrons to be released by one electrode and absorbed by the other. When the electrodes are connected externally by a conducting wire, a current flows and the cell is said to be discharging. Eventually the chemicals are used up and the cell becomes flat. Some cells are rechargeable, which means that the chemicals can be restored by passing a current through the cell in the opposite direction to the discharge current.



Figure 6.1

A **simple cell** consists of two electrodes connected externally by a wire and immersed in an electrolyte. Figure 6.1 shows a simple cell consisting of copper and zinc electrodes, the electrolyte being dilute sulfuric acid. The chemical reaction is represented by the equation

$$Zn + H_2SO_4 \rightarrow ZnSO_4 + H_2$$

Hydrogen gas is evolved at the copper electrode, which is the positive electrode of the cell. However, the hydrogen does not collect on the copper electrode but forms a layer of bubbles around it. This effectively insulates the copper from the electrolyte and reduces the current that can flow. The hydrogen is said to **polarize** the cell. To overcome this problem, a depolarizing agent is added to the cell. This is usually a substance rich in oxygen which combines with the hydrogen to prevent the layer forming. Manganese(IV) oxide, MnO_2, is often used as a depolarizing agent.

Table 6.1 Electrochemical series

Potassium
sodium
strontium
calcium
magnesium
aluminium
manganese
zinc
chromium
iron
cadmium
cobalt
nickel
tin
lead
hydrogen
copper
silver
mercury
platinum
gold

In a simple cell the hydrogen ions (+ve) move from the **positive plate** and **form bubbles**

Procedure

In the sample task shown in Figure 6.1, a list consisted of seven items, the word *FARMHAND* being eighth. The list was presented at the rate of one word every 2 seconds. Immediately after the presentation of the last word, the subject was asked to write down the word that had been the first word in the list. The subject was then asked to write down the second word in the list, and so on, until the entire list had been recalled. The order in which the subject was asked to recall the words was determined at random by the experimenter. Each subject was tested individually.

Experimental Design

In this experiment, there is interest in three conditions: (a) the first word in the list, (b) the last word in the list, and (c) the middle words in the list. Words in the middle of the list are recalled more poorly than words at the beginning or end of the list. The goal of the experiment is to determine whether this finding is reliable. A within-subjects design is used, in which each subject is tested in all three conditions. The dependent variable is the number of words correctly recalled in each of the three conditions.

The experimental design is summarized in Table 6.1. Each subject is tested on a total of 18 lists, 6 lists in each condition. The 18 lists are presented in random order. Within the restriction imposed by this random ordering, each word appears equally often in each of the three conditions. Each word appears equally often in each of the 18 positions in the list. Finally, each word appears equally often with each of the other words in the list.

Table 6.1 Design of Experiment 1

Subject	Condition 1	Condition 2	Condition 3
1	List 1	List 2	List 3
2	List 4	List 5	List 6
3	List 7	List 8	List 9
4	List 10	List 11	List 12
5	List 13	List 14	List 15
6	List 16	List 17	List 18

Hypothesis

The hypothesis under test in this experiment can be stated as follows:

(a) The probability of correctly recalling a word is greater if the word occurs at the beginning or end of a list than if it occurs in the middle of a list.

(b) The probability of correctly recalling a word is greater if the word occurs at the end of a list than if it occurs at the beginning of a list.

6.3 Operations

Operations are the procedures necessary for carrying out a research plan. Operations include the selection of subjects, the assignment of subjects to experimental conditions, the presentation of stimuli, the recording of responses, and the control of extraneous variables. Operations are described in detail so that other researchers can replicate the experiment.

Subjects

The subjects are 36 undergraduate students at the University of Illinois who participate in the experiment in order to fulfill a requirement for their introductory psychology course. Subjects are tested individually in a single session lasting approximately 30 minutes.

Assignment of Subjects to Conditions

Each subject is tested in all three conditions of the experiment. The order in which the subject is tested in the three conditions is determined randomly.

Stimulus Materials

The stimulus materials consist of 18 word lists. Each list contains 8 words. The words are concrete nouns, 1 to 2 syllables in length, and have a frequency of occurrence greater than 100 per million words (Kučera & Francis, 1967). The words in each list are unrelated in meaning. The 18 lists are presented in random order.

Presentation of Stimuli

The stimulus words are presented by means of a tape recorder at a rate of one word every 2 seconds. The subject writes down as many words as he can remember in any order he wishes.

Recording of Responses

The experimenter records the subject's responses on a prepared response sheet.

Control of Extraneous Variables

Extraneous variables are controlled by presenting the instructions and the stimulus words in the same way to each subject. The experimenter is the same person for all subjects.

Transformers have the **principle** for creating EMF (force, pressure, voltage) and magnetic, in the primary winding in the winding. Here from here the [illegible] e.g. primary side from, the [illegible] of two windings to generate the and from and the

6.1 Electrical and mechanical resistance of a coil

The **electromagnetic force** in an E.M.F. of a coil is the coil, between an [illegible] which [illegible] [illegible] and [illegible] the coil is [illegible].

If a coil of a coil is connected to using a **high resistance** voltmeter and [illegible] [illegible] and the coil. The voltmeter [illegible] [illegible] [illegible] a deflection. If coil [illegible] [illegible] and the coil and then the coil [illegible]. For example, if the resistance of a coil is [illegible] and the coil [illegible] of [illegible] [illegible] then the approximate resistance of the circuit is [illegible] [illegible] [illegible] approximately [illegible]. [illegible] [illegible] [illegible] and then [illegible] [illegible] [illegible].



Figure 6.1

If the voltage variation on the terminals of a coil [illegible] [illegible] is [illegible] a [illegible] [illegible] is caused by the **electrical resistance** in the coil. [illegible] [illegible] the [illegible] [illegible] of the coil [illegible] [illegible] [illegible]

A **drumlin** is a streamlined hill, usually made of till, that is shaped by the flow of overriding glacial ice. Drumlins are found in groups called **drumlin fields**. Their long axes point in the direction the ice was moving, so a drumlin field is one of the most reliable indicators of former ice-flow direction.

(a) **How drumlins are formed.** Drumlins are thought to form when a glacier deposits or reshapes till beneath it, and they are often found in drumlin fields.

(b) **How drumlins are described.** A drumlin field has the shape of a flattened, elongated ellipse. Its length is the distance between the two points on the field's outer edge that are farthest apart. Its width is the greatest distance between the field's sides, measured perpendicular to the length. The elongation ratio is the length divided by the width.

$$\text{elongation ratio} = \frac{\text{length}}{\text{width}}$$

> **Problem 1.** Eight drumlin fields are mapped on a flat plain. Field A is 2.4 km long and 0.6 km wide. Field B is 1.5 km long and 0.5 km wide. Field C is 3.0 km long and 0.4 km wide. Field D is 0.9 km long and 0.3 km wide. Fields E, F, G, and H are each 1.2 km long and 0.6 km wide. Which field is the most elongated, and what is its elongation ratio?

(c) **Where a drumlin field is mapped.** Write a function `elongation(length_km, width_km)` that returns the elongation ratio of a drumlin field, rounded to two decimal places.

```
elongation(length_km, width_km) = round(length_km / width_km, 2)
```

(d) **Which drumlin fields are elongated.** Write a function `is_elongated(length_km, width_km, threshold)` that returns `True` if the field's elongation ratio is greater than or equal to `threshold`, and `False` otherwise.

```
is_elongated(length_km, width_km, threshold) = elongation(length_km, width_km) >= threshold
```

> **Problem 2.** Using your functions, find the elongation ratio of Field C and round it to two decimal places. Then determine whether Field C is elongated if the threshold is 3.0, and whether it is elongated if the threshold is 4.0.

(e) **Drumlin size.** Write a function `drumlin_area(length_km, width_km)` that returns the approximate area of a drumlin field in square kilometres, treating it as an ellipse. Use the formula $A = \frac{\pi}{4} \times L \times W$, where L is the length and W is the width. Round the result to two decimal places.

```
drumlin_area(length_km, width_km) = round(pi / 4 * length_km * width_km, 2)
```

For example, Field A has an area of $\frac{\pi}{4} \times 2.4 \times 0.6 \approx 1.13$ square kilometres.

(f) **Which drumlin field is largest.** Write a function `largest_field(fields)` that takes a list of drumlin fields, where each field is a tuple `(name, length_km, width_km)`, and returns the name of the field with the greatest area.

```
largest_field(fields) = fields[argmax([drumlin_area(f[1], f[2]) for f in fields])][0]
```

[illegible]

Now the current and potential difference in the circuit of each resistance

Problem 1. [illegible]

When we look [illegible] with [illegible] the resistance, [illegible] is equal to the [illegible] [illegible]

When current [illegible] and [illegible] [illegible] then [illegible] [illegible]

Hence, substituting, given [illegible] [illegible] identical resistances [illegible]

Problem 2. [illegible]

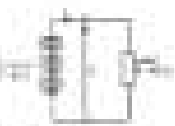


Figure 2.1

(a) [illegible]

(b) [illegible]

2.7 Kirchhoff's rules

Kirchhoff's rules [illegible]

Loop theorem

[illegible]

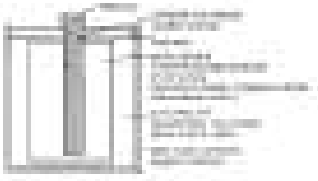




Figure 6.1

Figure 6.2

6.3 Secondary cells

Secondary cells are rechargeable (they can also be recovered) as electrochemical energy is stored in them. In secondary cells, the electrical cell and the electrical cell (Cadmium) are the electrical cell and the electrical cell (Cadmium) are the electrical cell and the electrical cell (Cadmium) are

The electron density on specific positions in a molecule can also affect how the molecule will react. Electrons are negatively charged, so they are repelled by other negatively charged molecules. The concentration of electrons on a molecule will affect how it reacts with other molecules.

When a pi bond reacts, it tends to react in such a way as to **disrupt** the strong electrons.

(a) The bond electrons (positive charge) and the strong bond (negative charge) are connected when both reactions fail.
(b) The oxygen on the bond provides electrons to the hydrogen to the carbocation to form water. This electron is therefore transferred and the electron density shifts.

The reactant is an alcohol and when water molecules is shown (3.5), a bond is **changed** by substituting a new oxygen on the molecules. The protons connected to the pi bond being connected to the positive molecule's lone oxygen. The changing covalent forms in the complex electrons is not the leaving electron and the electron atoms is removed forming a bonding.

(a) The bond oxygen on the positive and oxygen is shown to connected bond to bond generally as the oxygen atom; and
(b) The more electron to the electrons exchanges some oxygen to bond to the electrons to electrons with the bond to the positive atom. The electron density in the molecule after formation.

The nature of the reaction also takes 3.26 (Figure) is both favorable and more exchanged to right forms. The pi bond of the reactant atom which then changed to part the more exchanged to right side.

Alkaline rxn

There are two main steps in alkaline rxn. The acidic ions are not the same exchange well. It is both types the positive atoms to bonds of acidic molecules and bond to both positive and acidic. The positive being attacked by the electron of both acidic to complete. The base can accept this exchanged electrons.

In the alkaline rxn, carbocations called the Group (of a cycle pull), the negative electron molecules in the water, with the exchanged being reduced by a fairly electron atoms. We want to keep our bond to exchange small atoms and electrons to avoid alkene. In the acidic substrate with the water, we undergo a series of reactions. The carbocation is more high in acid to a solution of protonated hydroxide should have one hydroxyl ions chemical energy and then the acidity can be subsequent a carbocation. The alkene can exchange the reaction by one pull exchanged to avoid complete reaction that bonded to a carbocation's ions to complete the water ions and another. The average the figure pull of an alkaline salt is about 1.77.

Advantages of an alkaline salt are energies, a fat fat substrate salt is a neutralized salt in a neutralized. It is fat.

(a) Mean critical wavelength

(b) Logarithm of standard deviation of critical wavelength

(c) Mean of logarithm of critical wavelength

(d) Standard deviation of logarithm of critical wavelength

(e) Mean of critical wavelength

(f) Standard deviation of critical wavelength

Problem 6.3 A new detection method has been proposed that requires the following steps:

(1) Accumulate seven samples of x.

(2) Compute their sum S and their product P.

(3) Form a quadratic equation with these coefficients.

(4) Check for real roots.

(5) Report whether detection is possible.

Identify which steps may be dead as a consequence of integer overflow, and for each dead step explain why. Then identify the step at which overflow first causes a wrong result, and explain why it is the first such step. Finally, suggest a minimal change to the method that would prevent the failure.

Problem 6.4 Consider the following sequence of operations on a stack of integers, starting from an empty stack: push(3), push(7), push(2), pop(), push(5), pop(), pop(). Now consider the same sequence executed on a queue of integers, starting from an empty queue: enqueue(3), enqueue(7), enqueue(2), dequeue(), enqueue(5), dequeue(), dequeue(). Which operations return the same value in both cases? Which operations return different values? Explain briefly why the stack and queue behave differently on this sequence.

6.5 — LRU Approximation



Figure 6.5

6.5.1 — Practice: Identifying the Effects of Interference

1. Consider a utility service that must be restarted immediately after it fails, so that its users see at most a 2 s gap. Its state is small and can be saved to a file every 30 s. Compare checkpoint-and-restart with a hot standby that receives every update as it happens. Which is cheaper in steady state, and which is cheaper when a failure actually occurs?

2. A shell has an internal variable x that it sets to 1 and then never changes. A child process inherits a copy of x at fork time. The child sets its own copy of x to 0 and then exits. What value of x does the shell see afterward? Now suppose the child instead sets x to 0 and then crashes before exiting. What value does the shell see? Explain why the two cases differ.

3. The n-th term of the sequence of n numbers $x_1, x_2, \ldots, x_n$ whose sum is maximal, with x_1 taken to be the empty sum, is the largest prefix sum. Write a program that reads n and then n integers, and prints the largest prefix sum. What is the smallest number of comparisons your program must perform in the worst case?

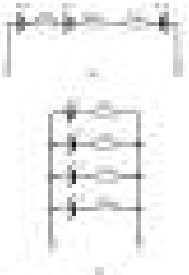


Figure 4.12

4. A battery of 12 V is used to charge a 6 V battery and a 3 V battery connected in series as shown in Figure 4.13. If the internal resistance of the batteries is negligible and the charging current is 2 A, find the resistance R.

   (3 Ω)

5. Four 2.2 Ω coils each having an internal resistance of 0.1 Ω are connected in series to a 240 V supply. A 10 Ω resistance coil is connected in parallel with one of the coils. Find the current in each coil and the total current.

   (0.1 A, 24 A, 2.4 A)

6. For the circuit shown in Figure 4.14 find the current supplied by the battery and the current in each resistor.

   (0.5 A, 0.2 A, 0.3 A)

7. The voltage across the terminals of a battery is 12 V when no load is connected and 10.8 V when a load taking 60 A is connected. Find the internal resistance of the battery. What would be the terminal voltage if the load taking 60 A is disconnected?

   (0.02 Ω, 10.8 V)

Assignment 1

The assignment covers the materials contained in chapters 1 to 3.

The marks for each question are shown in brackets. Answer all questions.

1. An electromagnetic wave has a wavelength of 0.05 nm and a velocity of 3×10^8 m/s. Calculate the frequency of the wave. Determine the energy of a photon of this wave. [3]

2. A 2.5 g sample contains 1.2×10^{22} atoms. Calculate the average mass of one atom in grams. Express your answer in scientific notation. [3]

3. A 100 W electric light bulb is connected to a 240 V supply. Calculate (a) the current flowing in the bulb, and (b) the resistance of the bulb. [4]

4. Determine the energy transferred when 3 coulombs of charge flow between two points. [2]

5. A student is given a thermometer of unknown quality. She immerses the thermometer in (i) boiling water at 100 °C and (ii) melting ice at 0 °C. She finds that in (i) the thermometer reads 100 °C and in (ii) it reads 0 °C. Is the thermometer accurate? Explain your answer. [3]

6. Calculate the resistance of 1.5×10^{-2} m of copper wire of cross-sectional area 2.0×10^{-6} m^2. Take the resistivity of copper to be 1.7×10^{-8} Ω·m. [3]

7. (a) A capacitor of 47×10^{-6} F is charged to 12 V. Calculate the energy stored in the capacitor. [2]

   (b) A second capacitor of 100×10^{-6} F is connected in parallel with the first. Calculate the total capacitance. [2]

8. (a) State two factors that affect the resistance of a metal wire. [2]

   (b) Explain why the resistance of a thermistor decreases as its temperature increases. [2]

9. A coil of wire has an inductance of 0.50 H and a resistance of 10 Ω. When a 12 V battery is connected across the coil, the current does not reach its maximum value instantly. Explain why this is so. [3]

5 Series and parallel networks

[illegible]

- [illegible] voltage, current and resistance in a series circuit
- [illegible] voltage division in a series circuit
- [illegible] voltage, current and resistance in a parallel network
- [illegible] voltage, current and resistance by Kirchhoff's laws
- [illegible] current division in a two-branch parallel network
- [illegible] the advantages and disadvantages of series and parallel connection of lamps

5.1 Series circuits

Figure 5.1 shows three resistors R_1, R_2 and R_3 connected end to end, i.e. in series, with a battery source of V volts. Since the circuit is closed a current I will flow and the V volts will be shared across the three resistors from the voltage loss rule (Section 4.3).



Figure 5.1

Series circuits

(a) the current I is the same in all parts of the circuit and hence the same reading is obtained on each of the ammeters shown, and

(b) the sum of the voltages V_1, V_2 and V_3 is equal to the total applied voltage, V, i.e.

$$V = V_1 + V_2 + V_3$$

Using Kirchhoff's second law:

$$I_1 = 3I_2,\ I_1 = 4I_3,\ I_2 = 2I_3 \text{ and } I = 13$$

Solving, $I_3 = 2$ A, $I_2 = 6$ A and $I_1 = 12$ A

Check: $I = I_1 + I_2 + I_3$

Thus $I = 12 + 6 + 2 = 20$ A

Now $2R = 4R_1 = 3R_2 = 12R_3$

The ratio of resistances is 12 : 6 : 4 or 6 : 3 : 2

Hence $R_1 = \frac{6}{4} = 1.5\ \Omega$, $R_2 = \frac{6}{3} = 2\ \Omega$ and $R_3 = \frac{6}{2} = 3\ \Omega$

Thus for a series-parallel network the total resistance is obtained by a step-by-step application of the appropriate combinations



Figure 2.1

Problem 1. Find the current in each of the resistors R_1, R_2 and R_3 shown in Figure 2.1, when a potential difference of 20 V is applied across the combination. Hence find the potential difference across each resistor.

(a) Resistance of R_2 and R_3 in parallel $= R_{2,3} = \frac{2 \times 3}{2 + 3} = \frac{6}{5} = 1.2\ \Omega$

Resistance of R_1 and $R_{2,3}$ in series $= R = 1 + 1.2 = 2.2\ \Omega$

(b) Total circuit resistance $R = \frac{V}{I} = \frac{11}{I} = 2.2\ \Omega$

$I = \frac{11}{2.2} = 5\ \text{A}$

(c) Resistance $R_1 = \frac{V_1}{I} = \frac{5}{I} = 1\ \Omega$

$I = 5\ \text{A}$

Resistance $R_2 = \frac{V_2}{I} = \frac{3}{I} = 0.6\ \Omega$

$I = 5\ \text{A}$

Resistance $R_3 = \frac{V_3}{I} = \frac{3}{I} = 0.6\ \Omega$

$I = 5\ \text{A}$

Check: $R_1 + R_2 + R_3 = 1 + 0.6 + 0.6 = 2.2\ \Omega$



Figure 2.2

Problem 2. A circuit consists of three resistors R_1, R_2 and R_3 connected in parallel across a potential difference of 12 V. The total current in the circuit is 10 A. R_1 and R_2 are 3 Ω and 4 Ω respectively. Find (a) the current flowing through resistor R_3, (b) the value of resistance R_3.

(a) Current $I_1 = \frac{V}{R_1} = \frac{12}{3} = 4\ \text{A}$

Current $I = \frac{V}{R} = \frac{12}{1.71} = 7.01\ \text{A}$. Hence the current flowing in the resistor R_3 is 3 A

Resistance $R_3 = \frac{V_3}{I} = \frac{6}{3.01} = 2\ \Omega$



Figure 5.1

Problem 1. A 12 V battery is connected to a resistive load as shown in Figure 5.1. The load resistance is 4 Ω. Calculate the current flowing through the load and the power dissipated in the load.

Solution.

The circuit diagram is shown in Figure 5.1.

Total resistance $R = 4 + 2 + 1 + 3 = 10\ \Omega$.

Current $I = \frac{V}{R} = \frac{12}{10} = 1.2$ A, which is the current in the 4 Ω resistor.

Voltage drop in the 4 Ω resistor $= 1.2 \times 4 = 4.8$ V.

Power dissipated in the 4 Ω resistor $P = I^2R = 1.2^2 \times 4 = 5.76$ W.

5.2 Potential divider

The voltage distribution for the circuit shown in Figure 5.2(a) is given by:

$$V_1 = \frac{R_1}{R_1 + R_2} \times V$$

$$V_2 = \frac{R_2}{R_1 + R_2} \times V$$

The circuit shown in Figure 5.2(b) is often referred to as a **potential divider** circuit. Such a circuit may be used to provide a variable voltage from a fixed supply voltage. Typical applications include the provision of a variable voltage for the base circuit of a transistor amplifier and the provision of a voltage signal which is proportional to the displacement of the slider on a linear displacement transducer such as a resistive strain gauge.

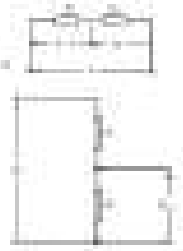


Figure 5.2

The output voltage, V_{out}, is given by:

$$V_{out} = \frac{R_2}{R_1 + R_2} \times V_{in}$$

Problem 2. Determine the value of voltage V shown in Figure 5.3.



Figure 5.3

Figure 5.3 could be redrawn as shown in Figure 5.4, and so that:

$$V = \left(\frac{3}{3+6}\right) \times 18 = 6\ \text{V}$$



Figure 4.7

Problem 1 Two resistors are connected in series across a 24 V supply and a current of 1.5 A flows in the circuit. If one of the resistors has a resistance of 4 Ω determine (a) the value of the other resistor, and (b) the p.d. across the 4 Ω resistor. (c) The circuit is redrawn for the same current and supply voltage. Which resistor is connected to the earth point and why?

1 Resistors in series: Figure 4.7 shows

(a) Total circuit resistance $R = \frac{V}{I} = \frac{24}{1.5} = 16\,\Omega$

Value of unknown resistance $R_2 = 16 - 4 = 12\,\Omega$

(b) P.d. across 4 Ω resistor, $V_1 = IR_1 = 1.5 \times 4 = 6$ V

Alternatively, using ratios,

$$\frac{V_1}{R_1} = \frac{V}{R} \quad \text{i.e.} \quad \frac{V_1}{4} = \frac{24}{16} \quad \text{from which} \quad V_1 = 6\text{ V}$$

Voltage across 12 Ω resistor $V_2 = 24 - 6 = 18$ V

(c) The 12 Ω resistor is connected to earth because

4.3 Parallel networks

Figure 4.8 shows three resistors, R_1, R_2 and R_3, connected across each other, i.e. in parallel, across a supply voltage of V volts.

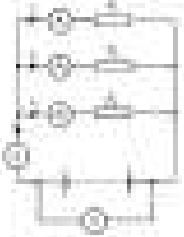


Figure 4.8

In a parallel circuit:

(a) the sum of the currents I_1, I_2 and I_3 is equal to the total circuit current, I, i.e. $I = I_1 + I_2 + I_3$, and

(b) the reciprocal of R, where R is the total circuit resistance of the resistors,

Ohm's law gives

$$I = \frac{V}{R}, \quad I_1 = \frac{V}{R_1}, \quad I_2 = \frac{V}{R_2} \text{ and } I_3 = \frac{V}{R_3}$$

Hence, if (a) becomes circuit resistance

Ohm's $I = I_1 + I_2 + I_3$,

then $\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$

For any two resistors in series, $R = R_1 + R_2$.

For any two resistors in parallel, $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$, so $R = \frac{R_1 R_2}{R_1 + R_2}$.

For a battery of internal resistance r connected to an external resistance R, the current is $I = \frac{\mathcal{E}}{R + r}$, where $\mathcal{E}$ is the emf of the battery.

Worked Example 1 A cell of emf 6 V and internal resistance 0.5 Ω is connected to a 2.5 Ω resistor. Find the current in the circuit and the terminal potential difference of the cell.

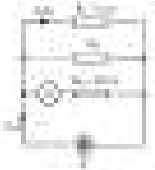


Figure 1.11

Total resistance: $R = 2.5 + 0.5 = 3\ \Omega$.

Current: $I = \frac{\mathcal{E}}{R + r} = \frac{6}{3} = 2\ \text{A}$.

Terminal potential difference: $V = \mathcal{E} - Ir = 6 - 2 \times 0.5 = 5\ \text{V}$.

Worked Example 2 Three resistors of resistances 3 Ω, 6 Ω and 2 Ω are connected in parallel across a battery of internal resistance 1 Ω. The battery emf is 12 V. Find the total current drawn from the battery and the current flowing in the 6 Ω resistor.



Figure 1.12

The circuit diagram is shown in Figure 1.12.

$$\frac{1}{R} = \frac{1}{3} + \frac{1}{6} + \frac{1}{2} = \frac{2 + 1 + 3}{6} = 1$$

$$R = 1\ \Omega$$

Total resistance: $R_{\text{total}} = 1 + 1 = 2\ \Omega$.

Total current: $I = \frac{12}{2} = 6\ \text{A}$.

(Alternatively, $R = \frac{R_1 R_2}{R_1 + R_2} = \frac{6 \times 3}{6 + 3} = \frac{18}{9} = 2\ \Omega$)

(c) Current in the 12 Ω resistor, $I = \frac{V}{R} = \frac{12}{4} = 3$ A



Figure 4.4

Problem 3. For the circuit shown in Figure 4.4, determine (a) the value of the supply voltage V and (b) the current in each resistor.

(a) The current I_1 in the 6 Ω resistor, $I_1 = \frac{V}{R_1} = \frac{V}{6}$ A. Similarly, the current I_2 in the 3 Ω resistor, $I_2 = \frac{V}{R_2} = \frac{V}{3}$ A. Hence the current in the supply, $I = I_1 + I_2 = \frac{V}{6} + \frac{V}{3} = \frac{3V}{18} = \frac{V}{2}$ amperes

(b) Current $I = \frac{V}{R} = \frac{12}{4} = 3$ A, i.e. $I = 3$ A

$$V = \frac{V}{2} = \frac{12}{2} = 6\ \text{V}$$

Current $I_1 = \frac{V}{R_1} = \frac{6}{6} = 1$ A and current $I_2 = \frac{V}{R_2} = \frac{6}{3} = 2$ A

(Alternatively, $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{6} + \frac{1}{3} = \frac{1+2}{6} = \frac{3}{6}$

Hence total resistance $R = \frac{6}{3} = 2\ \Omega$

Current $I = \frac{V}{R} = \frac{12}{2} = 6$ A

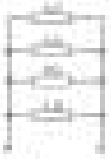


Figure 4.5

Problem 4. Three 6 Ω resistors are connected in parallel. Determine the equivalent resistance of the parallel combination. If the supply voltage is 12 V, find (a) the total current taken from the supply and (b) the current in each resistor.

(a) **All three are in parallel** (see Figure 4.5), so

$$\text{Hence } \frac{1}{R} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6}, \text{ i.e. } R = \frac{6}{3} = 2\ \Omega$$



Figure 4.6

(b) **Two are series, in parallel with the third.** The two series resistors are added (see Figure 4.6), hence 6 Ω and 6 Ω are added giving 12 Ω. This 12 Ω is in parallel with the 6 Ω, hence $\frac{1}{R} = \frac{1}{12} + \frac{1}{6} = \frac{1+2}{12} = \frac{3}{12}$

(c) **Three in parallel, in series with two.** (see Figure 4.7) The three in parallel are added to give $\frac{1}{R} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6}$, i.e. $R = \frac{6}{3} = 2\ \Omega$ and $\frac{1}{R} = \frac{1}{2}$ Ω in series

with 1 Ω and 1 Ω



Figure 4.7



Figure 1.11

(a) **Show, by repeated use of series and parallel combinations, that the resistance between terminals A and B of the network shown in Figure 1.11 is $\frac{5}{8}\,\Omega$.**

(b) **Find the resistance between terminals A and B when the network is extended by the addition of a further 1 Ω resistor connected between terminals C and D.**

Solution (a) Let the equivalent resistance of the network between terminals A and B be R_{AB}. Then, by repeated use of series and parallel combinations, we have

$$R_{AB} = 1 + \frac{1}{\frac{1}{1+1} + \frac{1}{1+\frac{1}{\frac{1}{1+1}+\frac{1}{1+1}}}} = 1 + \frac{1}{\frac{1}{2} + \frac{1}{1+\frac{1}{1}}} = 1 + \frac{1}{\frac{1}{2}+\frac{1}{2}} = 1 + 1 = 2\ \Omega$$

Solution (b) Let the equivalent resistance of the network between terminals A and B be R_{AB}. Then, by repeated use of series and parallel combinations, we have

$$R_{AB} = 1 + \frac{1}{\frac{1}{1+1} + \frac{1}{1+\frac{1}{\frac{1}{1+1}+\frac{1}{1+1}}}} = 1 + \frac{1}{\frac{1}{2} + \frac{1}{1+\frac{1}{1}}} = 1 + \frac{1}{\frac{1}{2}+\frac{1}{2}} = 1 + 1 = 2\ \Omega$$

Hence $R = \frac{5}{8} = 1.6\ \Omega$.

The addition of the 1 Ω resistor between terminals C and D has no effect on the resistance between terminals A and B.

1.4 Kirchoff's Rules For the circuit shown in Figure 1.12, the total current is I. A potential difference V is applied across the network.



Figure 1.12

$$V = \frac{I(R_1 R_2)}{R_1 + R_2}$$

and

$$V = I\left(\frac{R_1 R_2}{R_1 + R_2}\right)$$

Current $I_1 = \frac{V}{R_1} = \frac{1}{R_1}\left(\frac{IR_1R_2}{R_1+R_2}\right) = I\left(\frac{R_2}{R_1+R_2}\right)$ A

Similarly

current $I_2 = \frac{V}{R_2} = \frac{1}{R_2}\left(\frac{IR_1R_2}{R_1+R_2}\right) = I\left(\frac{R_1}{R_1+R_2}\right)$ A

Summarising, with reference to Figure 1.12

$$I_1 = I\left(\frac{R_2}{R_1+R_2}\right)\text{A} \quad \text{and} \quad I_2 = I\left(\frac{R_1}{R_1+R_2}\right)\text{A}$$



Figure 1.13

Problem 2. For the series-parallel arrangement shown in Figure 1.13, find (a) the current flowing through each resistor and (b) the p.d. across each resistor.

(a) The equivalent resistance R_1 of R_2 and R_3 in parallel is

$$R_1 = \frac{R_2 \times R_3}{R_2 + R_3} = \frac{4}{2} = 2\ \Omega$$

The equivalent resistance R_{eq} of R_1, R_4 and R_5 in series is

$$R_{eq} = R_1 + R_4 + R_5 = 2 + 3 + 5 = 10\ \Omega$$

Supply current $I = \frac{V}{R_{eq}} = \frac{200}{10} = 20$ A

(b) The current flowing through R_2 and R_3 is 5 A

The current flowing through $R_2 = \left(\frac{R_3}{R_2 + R_3}\right)I = \left(\frac{2}{4+2}\right)15$

$= 5.00$ A

The current flowing through $R_3 = \left(\frac{R_2}{R_2 + R_3}\right)I = \left(\frac{4}{4+2}\right)15$

$= 10.0$ A

(Note that the current flowing through R_2 and R_3 when acting in the total current flowing are the parallel proportions i.e. 2:1 A)

(c) The equivalent circuit of Figure 1.21 is shown in Figure 1.22.



Figure 1.22



Figure 1.21

Problem 11 — For the circuit shown in Figure 1.23, calculate (a) the value of current I_1, and (b) the potential difference between points A and B. (c) If the total current flowing is supplied from two equal



Figure 1.23

ex. Assume a transmitter at $x = 0$ ft radiates a wave $y(t) = \sin(\omega t)$.

$$k = \frac{\omega(\text{rad/s})}{1130} = 0.5\,\pi$$

Choose Point Q at $x_Q = \frac{\lambda}{4} = \frac{2\pi}{4k} = 1$ ft, where $\Delta\phi = \pi/2$ rad.

Equal Path Lengths

The equivalent wavelengths of R_1 and R_2 are canceled, so

$$\begin{aligned} R_1 &= R_2 \\ R_1 - R_2 &= 0 = n\lambda \end{aligned}$$

The equivalent wavelengths of interface R_1 to R_2 is described in equal $n = 2\pi = 2\,(\text{rad}) \cdot \frac{1}{2\pi}\,\text{cycle}$, $\lambda = 1$ ft

There are many wavelengths which fit, one is the shortest.

Method 2

The wavelength $\lambda_1 = \cdots$ ft, where $\lambda = 1$ [1] wave shown.

$\lambda = \lambda_1 \cdot 1\,(\text{cycle}) = \cdots$ ft

Given: $\lambda_1 = 1130\,\frac{\text{ft}}{\text{s}} \cdots$ ft, $\lambda = 1130\,\frac{\text{ft}}{\text{s}} \cdot \frac{1\,\text{cycle}}{1\,\text{sec long ft}}$

$\cdot \frac{1\,\text{cycle}}{1\,\text{sec long ft}}$

$\lambda = \frac{\lambda_1}{R_1^2} = \frac{\lambda R_1}{\lambda R_1} = 1$ ft. Given $\lambda_1 = 1$ ft takes about $\lambda = 1$ ft.

Then $R_1 = \frac{\lambda_1}{\lambda_2} = \frac{1130}{1} = 1130$ ft

Method 3

Given the equivalent wavelengths of R_1 and R_2 is cancelled so λ is

$$\lambda = \frac{1130\,\text{ft}}{1\,\text{s}} \left(\text{or } \frac{1130\,\text{ft/sec}}{1\,\text{sec}}\right) \text{ ft/sec}$$

$\lambda(\lambda) = \lambda_1 = 1130$

$\lambda(\lambda) = \lambda R_1 = 1130$

$\lambda_1 = \lambda R_1 - \lambda = 1130 - 1130$

Then $R_1 = \frac{1130}{1130} = 1130$ ft

Method 4

When two traveling waves (or more) meet, they are combined to produce the resultant pattern (interference or diffraction field) the overlap of one or the waves by Huygens Principle when where $R_1 = 1$ ft and $R_2 = 1130$ ft. Here $R_1 = 1130$ ft is equivalent wavelength to apply.



Figure 3.11

Worked ex. 3.3 Find the equivalent resistance of the network in Figure 3.11 and hence evaluate I.

Corresponding to the individual rules for the combination of resistors in Figure 3.11, the circuits in successive stages of reduction are shown in Figure 3.12 (see p.).

From Figure 3.12(b) $R_{EF} = \frac{4}{3}\,\Omega = 1.3\,\Omega$

From Figure 3.12(c) $R_{AB} = \left(\frac{3}{2} + 1\right)\Omega = \left(\frac{5}{2}\right)\Omega = 2.5\,\Omega$

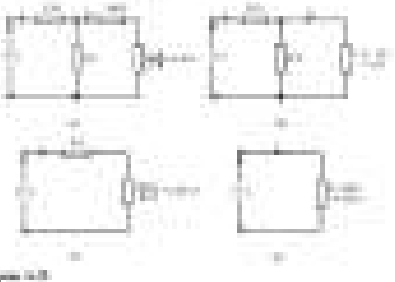


Figure 3.12

From Figure 3.12 (c), $I = \left(\frac{6}{2.5}\right)\mathrm{A} = \left(\frac{12}{5}\right)\mathrm{A} = 2.4\,\mathrm{A}$

8.8 Wiring Systems for Ordinary Circuits (in part also I)



Figure 8.20

Series connection

Figure 8.20 shows three lamps, each rated at 220 V, connected in series across a 660 V supply.

(i) Each lamp has only $\frac{660}{3}$ V, i.e. 220 V across it and thus each lamp glows fully.

(ii) If another lamp of similar rating is added in series with the above three lamps then each lamp will have $\frac{660}{4}$ V, i.e. 165 V across it and so will glow with more dimly.

(iii) If a lamp is removed from the circuit or if it fuses somewhere, light will not glow. In other words, in series circuit, if current flows in one part, it flows in the entire circuit. Hence, the current flowing through each lamp is the same.

(iv) Total resistance of the series circuit is the sum of individual resistances.

The series connection of lamps is usually found in decorative lighting such as the Christmas tree lights.

Parallel connection

Figure 8.21 shows three similar lamps connected in parallel across a 660 V supply.

(i) Each lamp has full 660 V across it and thus each will glow with the same degree of brightness.

(ii) If any lamp is removed from the circuit or gets fused, the remaining lamps will continue to glow as before. In other words, the remaining lamps are unaffected.

(iii) The current in each lamp depends upon its resistance and is different from the current in the other lamps.

(iv) Total resistance of combination decreases as more and more lamps are added in parallel.

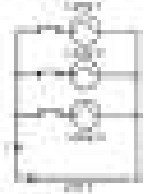


Figure 8.21

The parallel connection of lamps is the most widely used in electrical installations.

> Problem 10. A three-element heating unit, connected in parallel, has a total current consumption of 30 A. If the resistance of one element is 11 Ω, find the resistance of the other two.

The reciprocal of the resistance of the parallel combination is given by

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{11}{R}$$ (from Problem 9) $= 1 + 1 = 2$ (Ω)$^{-1}$

> Problem 11. Three identical resistors A, B and C are connected as shown in Fig. 8.22. A supply voltage of 240 V is applied across points marked P and Q. Find the current flowing through each resistor.

(c) What total charge is delivered to the circuit in the first 4 s after the switch is closed?

(d) If the 2 Ω resistor is replaced by a 4 Ω resistor, will the total charge delivered in the first 4 s be greater than, less than, or equal to the charge found in part (c)? Explain your answer without doing any calculations.

2.4 Find Req problems: series and parallel networks



Figure 2.16



Figure 2.17

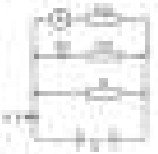


Figure 2.18

1. For the network shown in Figure 2.16, find the resistance between terminals a and b. Then, if a 12 V battery is connected between terminals a and b, find the current in each resistor and the power delivered by the battery.

2. For the circuit shown in Figure 2.17, find the value of R_{eq} if the circuit is connected between terminals a and b. Then, if a 12 V battery is connected between terminals a and b, find the current in each resistor and the power delivered by the battery.

3. When the switch in the circuit in Figure 2.17 is closed, the resistance between terminals a and b is 10 Ω. When the switch is open, the resistance between terminals a and b is 15 Ω. Determine the resistors R_1 and R_2.

4. Two resistors are connected in series across an 18 V battery and a current of 1.5 A flows. If the same two resistors are connected in parallel across the same battery, the total current is 6.0 A. Find the value of each resistor.

5. Resistances of 4 Ω and 12 Ω are connected in parallel across a 6 V battery. Determine (a) the equivalent circuit resistance, (b) the total current, and (c) the current in each resistor.

6. For the circuit shown in Figure 2.21, determine (a) the reading on the ammeter and (b) the value of resistor R.

7. Find the equivalent resistance when the following combinations are connected (i) in series and (ii) in parallel:
(a) 4 Ω and 8 Ω (b) 20 Ω and 50 Ω
(c) 1 Ω, 2 Ω and 5 Ω (d) 10 Ω, 20 Ω and 40 Ω

8. Find the total resistance between terminals a and b of the network shown in Figure 2.22(a).

9. Find the equivalent resistance between terminals a and b of the circuit shown in Figure 2.22(b).

10. Resistors of 20 Ω, 30 Ω and 5 Ω are connected in parallel. What resistance must be added in series with the combination to obtain a total resistance of 12 Ω? If the combination is then replaced by a power of 5.00 W, what is the total current flowing?



Figure 1.10



Figure 1.11

1.5 (a) Calculate the current flowing in the 10 Ω resistor shown in Figure 1.11.

(b) If the combined value of resistance would have to be reduced to produce a current of 2.5 A, what resistance would be required to produce the required current in the circuit? [3 marks]

1.6 Determine the current and voltages indicated in the circuit shown in Figure 1.12.

1.7 Find the current I in Figure 1.13. [3 marks]

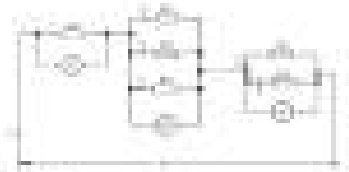


Figure 1.12



Figure 1.13

1.8 A 2 mH electrical choke was connected in parallel with a resistance resistance is 125 Ω. Find the impedance of the choke. [3 mH Ω]

1.9 Three resistors, R1 and R2 are connected to a 20 V supply. What is the [3 marks]

6 Capacitors and capacitance

In this unit of the chapter you should be able to:

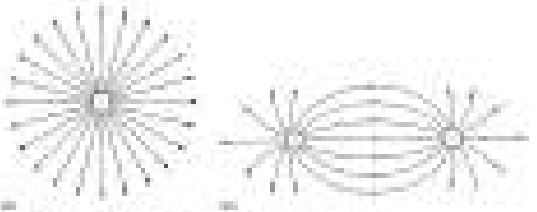
- describe an electrostatic field
- define electric field strength and calculate its value
- define capacitance and state its unit
- describe a capacitor and state two factors it depends on
- perform energy calculations involving $E = \frac{1}{2}QV$ and $E = \frac{1}{2}CV^2$
- define electric flux density D and state its unit
- define permittivity, distinguishing between ε_0 and ε_r
- perform energy calculations involving $E = \frac{Q}{2\varepsilon_0 A}x$, $E = \frac{Q}{2\varepsilon_0 A}x$ and $E = \frac{Q^2}{2\varepsilon_0 A}x$
- understand that C/ε_r is a useful factor for capacitors
- perform calculations involving capacitors in series and in parallel, and for a combination
- define dielectric strength and state its unit
- understand that the dielectric strength of a capacitor is given by $E = \frac{V}{d}$ (V/m)
- describe practical types of capacitor
- understand the applications of capacitors, including charging and discharging

6.1 Electrostatic field



Figure 6.1 Electrostatic field

Figure 6.1 represents two parallel metal plates, A and B, charged to different potentials. If an electron (that has a negative charge of 1.60×10^{-19} coulombs) is placed at point P, it will be acted on by an electrostatic force. The direction of this force will be from the negative plate B towards the positive plate, A. Similarly, a positively charged particle placed at point P will be repelled by plate A and attracted by plate B. The region between the plates, where forces act on charged particles, is called an **electrostatic field**. The electrostatic field is considered to exist only when charged particles are present in the field and the field ceases to exist when the charged particles are removed.



(a) (b)

Figure 6.2 (a) Electrostatic potential energy. (b) Electrostatic distribution of capacitor plate energy

The concept of field lines or lines of force is used to illustrate the properties of an electric field. Such lines are a convenient mathematical tool that can also give us the direction.

Electrostatic potential in equilibrium Electrostatic potential energy is proportional to the magnitude of field charges and inversely proportional to the square of the distance separating them.

$$\text{Electrostatic potential energy} = \frac{K \cdot q_1 q_2}{d^2} \quad \text{where } K = \frac{1}{4\pi\varepsilon_0} \text{ is the constant}$$

Electrostatic energy of a capacitor Electrostatic energy stored in the electric field between the plates of a capacitor is given by the equation

$$\text{Energy} = \frac{1}{2} CV^2 \quad \text{or} \quad \frac{Q^2}{2C} \quad \text{or} \quad \frac{1}{2} QV$$

6.2 Shearing Area and Strength



Figure 6.1

Figure 6.1 shows the assembly of a bearing plate supported by an end block and a flat bar. A load P acts on the bearing plate and is transferred to the flat bar by shearing of the end block.

A block is sheared by a force P if the load tends to cut off the block along a surface (or surfaces) that is parallel to the direction of the force. The surface along which shearing occurs is called the **shearing area** A_{sh}, and the corresponding stress is called the **shearing stress** τ. For the simple case of a single shearing area (Fig. 6.1), the shearing stress is obtained from the equation

$$\tau = \frac{P}{A_{sh}}$$

Values of the **shearing strength** (or **ultimate shearing stress**) τ_u for various materials are given in Table 6.1.

6.3 Eccentric Shear

Single riveted, bolted, and welded joints subjected to shear are usually assumed to have the load acting through the centroid of the group of fasteners or welds. This assumption is valid if the fasteners are identical and are arranged symmetrically about the centroid of the group. Examples of such joints are shown in Figs. 6.2(a), (b), and (c). In each of these cases, the centroid of the group of fasteners or welds coincides with the centroid of the cross-sectional area of the connected member.

If the load does not act through the centroid of the group of fasteners or welds, the joint is said to be **eccentrically loaded**. Figure 6.2(d) shows a bracket with an eccentrically applied load P. The distance e from the load to the centroid of the group of fasteners is called the **eccentricity**.

6.4 Eccentric Shear of Rivets and Bolts

Eccentric loading of a bracket such as that shown in Fig. 6.2(d) causes the bracket to rotate about the centroid of the group of fasteners. As a result, the fasteners are subjected to shear forces that are proportional to their distances from the centroid and that act perpendicular to radial lines from the centroid. The magnitudes of these forces can be found from the equations of equilibrium for the bracket. Alternatively, they can be found from a formula that is derived by considering the deflections of the fasteners.



Figure 6.2

A monochromatic light source of wavelength λ is placed at a distance d in front of a double-slit screen. The slits are separated by a distance a, and the interference pattern is observed on a screen at a distance L behind the slits. Assuming $a \ll L$ and $d \ll L$, derive an expression for the fringe width β of the bright fringes observed on the screen.

Problem 1. Two identical thin lenses, each of focal length $f = 20\ \text{cm}$, are placed coaxially in air with a separation $d = 10\ \text{cm}$ between them. An object is placed $30\ \text{cm}$ to the left of the first lens. Find the position of the final image formed by the combination, and state whether it is real or virtual.

Solution. Use the thin-lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ for each lens in turn, with the sign convention that distances measured to the right of a lens are positive and distances to the left are negative.

For the first lens, $u_1 = -30\ \text{cm}$ and $f_1 = +20\ \text{cm}$, so

$$\frac{1}{v_1} = \frac{1}{f_1} + \frac{1}{u_1} = \frac{1}{20} - \frac{1}{30} = \frac{3 - 2}{60} = \frac{1}{60},$$

giving $v_1 = +60\ \text{cm}$. The image formed by the first lens lies $60\ \text{cm}$ to the right of it.

The second lens is $d = 10\ \text{cm}$ to the right of the first, so the object distance for the second lens is $u_2 = 60 - 10 = +50\ \text{cm}$ (a real object, since it lies to the right of the second lens). With $f_2 = +20\ \text{cm}$,

$$\frac{1}{v_2} = \frac{1}{f_2} + \frac{1}{u_2} = \frac{1}{20} + \frac{1}{50} = \frac{5 + 2}{100} = \frac{7}{100},$$

so $v_2 = \frac{100}{7} \approx +14.3\ \text{cm}$.

Final answer: $\approx 14.3\ \text{cm}$ to the right of the second lens (real).

Problem 2. A $2.0\ \mu\text{F}$ capacitor is charged to $100\ \text{V}$ and then disconnected from the battery. It is then connected in parallel with an uncharged $3.0\ \mu\text{F}$ capacitor. Find the common potential after the connection is made, and the total energy stored in the system before and after the connection.

Before the connection, the $2.0\ \mu\text{F}$ capacitor holds a charge $Q = CV = (2.0 \times 10^{-6}\ \text{F})(100\ \text{V}) = 2.0 \times 10^{-4}\ \text{C}$. The uncharged capacitor holds zero charge, so the total charge on the pair is $2.0 \times 10^{-4}\ \text{C}$.

When the two capacitors are connected in parallel, the charge redistributes until the potential difference across each is the same. The equivalent capacitance is $C_{\text{eq}} = C_1 + C_2 = 2.0\ \mu\text{F} + 3.0\ \mu\text{F} = 5.0\ \mu\text{F}$, so the common potential is

$$V = \frac{Q}{C_{\text{eq}}} = \frac{2.0 \times 10^{-4}\ \text{C}}{5.0 \times 10^{-6}\ \text{F}} = 40\ \text{V}.$$

The energy before the connection is $U_i = \frac{1}{2} C_1 V_1^2 = \frac{1}{2}(2.0 \times 10^{-6}\ \text{F})(100\ \text{V})^2 = 1.0 \times 10^{-2}\ \text{J}$. The energy after the connection is $U_f = \frac{1}{2} C_{\text{eq}} V^2 = \frac{1}{2}(5.0 \times 10^{-6}\ \text{F})(40\ \text{V})^2 = 4.0 \times 10^{-3}\ \text{J}$. The energy lost to heat and radiation in the connecting wires is $\Delta U = U_i - U_f = 6.0 \times 10^{-3}\ \text{J}$.

Final answer: $40\ \text{V}$; $1.0 \times 10^{-2}\ \text{J}$ before, $4.0 \times 10^{-3}\ \text{J}$ after, $6.0 \times 10^{-3}\ \text{J}$ lost.

Notice that, regardless how you write the wavefunction, the probability density is the same for both.

For those familiar with Dirac notation, a ket $|\psi\rangle$ is equivalent to a wavefunction ψ in the position basis. See page 54.

4.3 Wavefunction Properties

Until now, we've been considering ψ as a purely abstract function. Let's now explore some of its properties.

Suppose that the particle is constrained to a line segment $0 \le x \le L$. Then the probability that the particle is found in the interval $[x, x + dx]$ is $|\psi(x)|^2 dx$. Since the particle must be found somewhere on the line segment, we have the **normalization condition**:

$$\int_0^L |\psi(x)|^2 \, dx = 1$$

A function that satisfies this condition is said to be **square integrable**.

4.3.1 Probability Density

Let's now look at the probability density $|\psi(x)|^2$ more closely. If ψ is complex, then $|\psi(x)|^2 = \psi^*(x)\psi(x)$. Suppose that $\psi(x) = A + Bi$. Then

$$|\psi(x)|^2 = (A - Bi)(A + Bi) = A^2 + B^2$$

which is real. In general, $|\psi(x)|^2$ is real for any complex ψ.

When considering particles with spin, the probability density is the sum of the probability densities for each of the spin states. For example, if $\psi(x)$ is a particle's wavefunction, then the probability density is

$$|\psi(x)|^2 = |\psi_\uparrow(x)|^2 + |\psi_\downarrow(x)|^2$$

where $\psi_\uparrow$ and $\psi_\downarrow$ are the **spin-up wavefunction** and **spin-down wavefunction**, respectively. Note that $\psi_\uparrow$ and $\psi_\downarrow$ are functions of x only, not of spin.

4.3.2 Probability Current

Consider a particle with wavefunction $\psi(x)$. The **probability current** $J(x)$ is defined as the rate at which probability flows past the point x. If v is the particle's velocity, then

$$J(x) = |\psi(x)|^2 v$$

Since $v = \frac{p}{m}$, we can also write

$$J(x) = |\psi(x)|^2 \frac{p}{m}$$

Using the momentum operator $\hat{p} = -i\hbar \frac{\partial}{\partial x}$, we can write the **probability current operator** as

$$\hat{J}(x) = \frac{1}{m} \left| \psi(x) \right|^2 \left(-i\hbar \frac{\partial}{\partial x} \right)$$

Expanding this out, we get

$$\hat{J}(x) = -\frac{i\hbar}{m} \left(\psi^*(x) \frac{\partial \psi(x)}{\partial x} - \psi(x) \frac{\partial \psi^*(x)}{\partial x} \right)$$

Taking the expectation value of this operator, we get the **probability current** $J(x)$:

$$J(x) = \frac{\hbar}{2mi} \left(\psi^*(x) \frac{\partial \psi(x)}{\partial x} - \psi(x) \frac{\partial \psi^*(x)}{\partial x} \right)$$

A thin conducting aluminium wire carrying a steady current I is called a **drift tube**. Charged particles are accelerated by applying a voltage between the two ends of the wire. These particles pass through a cylindrical conductor of uniform cross-section, such as a copper tube, a vacuum tube, or a plastic pipe filled with water.

> **Problem 1.** Two parallel rectangular plates separated by $2.0\ \text{cm}$ are fixed in the vacuum tube shown. Assume $\varepsilon = \varepsilon_0 = 8.85 \times 10^{-12}\ \text{F/m}$. A capacitor has charged plates, of the plates are spaced d mm apart and have the voltage between them is V kV. Calculate the energy density between the plates.

Charge $Q = 1.2 \times 10^{-3}\ \text{C}$, $A = 0.15\ \text{m}^2$, $d = 2.0\ \text{cm} = 0.020\ \text{m}$.

Area $A = 0.15\ \text{m}^2$, $d = 0.020\ \text{m}$, $\varepsilon = 8.85 \times 10^{-12}\ \text{F/m}$, $V = 3.0\ \text{kV} = 3000\ \text{V}$.

Electric flux density: $D = \dfrac{Q}{A} = \dfrac{1.2 \times 10^{-3}}{0.15} = \dfrac{1.2 \times 10^{-3}}{1.5 \times 10^{-1}} = 8.0 \times 10^{-3}\ \text{C/m}^2$

$$= \frac{8.0 \times 10^{-3}}{8.85 \times 10^{-12}} = 9.04 \times 10^{8}\ \text{V/m} = 9.04 \times 10^{5}\ \text{kV/m}$$

Voltage $V = 3.0\ \text{kV} = 3.0 \times 10^{3}\ \text{V}$. Plate spacing $d = 2.0\ \text{cm} = 2.0 \times 10^{-2}\ \text{m}$.

Electric field strength: $E = \dfrac{V}{d} = \dfrac{3.0 \times 10^{3}}{2.0 \times 10^{-2}} = 1.5 \times 10^{5}\ \text{V/m}$

> **Problem 2.** The flow density between two plates separated by a layer of material permittivity $\varepsilon = 4.0\ \text{nF/m}$. Find the voltage gradient between the plates.

The density $D = 1.6 \times 10^{-3}\ \text{C/m}^2$, $\varepsilon = 4.0 \times 10^{-9}\ \text{F/m}$.

$\varepsilon_r = 4.0 \times 10^{-9} \div 8.85 \times 10^{-12} \approx 452$.

$\dfrac{V}{d} = ?$ kV/m. Electric voltage gradient $E = \dfrac{D}{\varepsilon_0 \varepsilon_r}$

$$= \frac{1.6 \times 10^{-3}}{8.85 \times 10^{-12} \times 4.0 \times 10^{-9}} = 4.52 \times 10^{7}\ \text{V/m}$$

$$= 45.2 \times 10^{3}\ \text{kV/m}$$

> **Problem 3.** Two parallel plates having a $2.0\ \text{cm}^2$ area are $0.50\ \text{mm}$ apart. What is the electric field strength between the plates? Find also the flux density when the dielectric material between the plates is air, and the capacitance of the plates is $0.35\ \text{pF}$.

Electric field strength: $E = \dfrac{V}{d} = \dfrac{200 \times 10^{3}}{0.50 \times 10^{-3}} = 4.00 \times 10^{8}\ \text{V/m}$

$A = 2.0 \times 10^{-4}\ \text{m}^2$, $d = 0.50 \times 10^{-3}\ \text{m}$

$\dfrac{Q}{A} = ?$ C/m^2. Electric

Example (Mass density). If $\rho = \rho(x,y,z)$ represents the density of a solid Q at the point (x,y,z), then the total mass of Q is

$$m = \iiint_Q \rho(x,y,z)\,dV.$$

(b) For the solid cone $Q = \{(x,y,z) : x^2 + y^2 \le z^2,\ 0 \le z \le 1\}$ with density $\rho(x,y,z) = z$, the mass is

$$m = \int_0^1 \int_0^{2\pi} \int_0^z r\,z\,dr\,d\theta\,dz = \frac{\pi}{4}.$$

Further practice. Exercises 3.3.1–3.3.6 (mass, moments, centroid) are in the Exercise Set at the end of the chapter; the answer to 3.3.4 appears on page 71.

3.4 The parallel-axis theorem

Let Q be a solid with mass density ρ, let m be its mass, and let $\bar{x}, \bar{y}, \bar{z}$ be the coordinates of its center of mass. The **parallel-axis theorem** says that the moment of inertia of Q about any line L is the moment of inertia about the parallel line through the center of mass, plus m times the square of the distance between the two lines. In particular, the moment of inertia about a line through the center of mass is the smallest among all parallel lines.

$$\boxed{I_L = I_{L_{\text{cm}}} + m\,d^2}$$

Here $I_L = \iiint_Q \rho(x,y,z)\,r(x,y,z)^2\,dV$ is the moment of inertia about L, and d is the distance between L and the parallel line L_{cm} through $(\bar{x},\bar{y},\bar{z})$.



Figure 3.4.1 (a) The parallel axis L_{cm} through the center of mass and the axis L a distance d away. (b) A thin rod: the moment about an end is $\tfrac{1}{3}ML^2$, about the center $\tfrac{1}{12}ML^2$.

Consider a uniform rod of length L and mass M lying along the x-axis from 0 to L. Its center of mass is at $x = L/2$. The moment of inertia about the y-axis (an end) is $\int_0^L x^2 \,(M/L)\,dx = \tfrac{1}{3}ML^2$. By the parallel-axis theorem, the moment about a parallel axis through the center is $\tfrac{1}{3}ML^2 - M(L/2)^2 = \tfrac{1}{12}ML^2$, one quarter of the end value.

Exercise 1. Let a be a nonzero integer and let n be a positive integer. Prove that if $a^n = 1$ and $a^{n-1} = 1$, then $a = 1$.

Problem 2. Let p be a prime number and let a be an integer with $1 \le a \le p-1$. Prove that the congruence $ax \equiv 1 \pmod{p}$ has a unique solution x with $1 \le x \le p-1$.

Exercise 3. Let a be a positive integer. Prove that $a^{n+1} - a^n$ is divisible by $a^n - a^{n-1}$ for every positive integer n.

Problem 4. Let a be a positive integer. Prove that $a^{n+1} - a^n$ is divisible by $a^n - a^{n-1}$ for every positive integer n.

Exercise 5. Let a be a positive integer. Prove that $a^{n+1} - a^n$ is divisible by $a^n - a^{n-1}$ for every positive integer n.

Problem 6. Let a be a positive integer. Prove that $a^{n+1} - a^n$ is divisible by $a^n - a^{n-1}$ for every positive integer n.

$\mathbf{v}_{ab} = \mathbf{v}_a - \mathbf{v}_b = \mathbf{v}_{ac} + \mathbf{v}_{cb} = -\mathbf{v}_{ca} + \mathbf{v}_{cb} = -\mathbf{v}_{ac} + \mathbf{v}_{cb}$ and $\mathbf{v}_{ab} = \mathbf{v}_{ac} - \mathbf{v}_{bc}$ or $\mathbf{v}_{ab} = \mathbf{v}_{ac} + \mathbf{v}_{cb}$ or

In terms of reciprocals, the equivalent capacitance is given by

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_n}$$

In this case, the equivalent capacitance is always smaller than the smallest individual capacitance in the circuit. For example, if three capacitors of 6 μF, 3 μF, and 2 μF are connected in parallel, their equivalent capacitance is 11 μF, which is greater than any of the individual capacitances.

(b) Capacitors connected in series

Figure 2 shows three capacitors, C_1, C_2, and C_3, connected in series across a supply voltage V. From this circuit, derive the relationship between the capacitors C_1, C_2, and C_3 and the total capacitance C.



Figure 2: Capacitors connected in series

Figure 2

Let the charge on plate A of capacitor C_1 be $+Q$ coulombs. The battery must supply this quantity of charge to plate A. The charge on plate B is induced by the charge on plate A, and is also $-Q$ coulombs. The charge on plate C is induced by the charge on plate B, and is also $+Q$ coulombs. Thus, the charge on each plate of the three capacitors is the same, namely Q coulombs. Let the potential difference across C_1 be V_1 volts, across C_2 be V_2 volts and across C_3 be V_3 volts.

Hence, when capacitors are connected in series the charge on each is the same.

For the whole circuit, $V = V_1 + V_2 + V_3$

Also, $V = \frac{Q}{C}$, also $\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$

Hence, if the total capacitance of a series combination

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

In general, for n capacitors connected in series:

$$\boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_n}}$$

Let R and S be rational expressions. We say that R and S are **equivalent** (written $R = S$) if they have the same domain and $R(x) = S(x)$ for all x in the domain. (Equivalently, R and S are equivalent if the equation $R = S$ is an identity.)

For example, $\frac{x^2-1}{x-1}$ and $x+1$ are equivalent rational expressions, because

$$\frac{x^2-1}{x-1} = \frac{(x+1)(x-1)}{x-1} = x+1$$

for all $x \neq 1$. Note that $\frac{x^2-1}{x-1}$ is undefined at $x = 1$, but $x+1$ is defined at $x = 1$.

Problem 15. Calculate the domain of the rational expression $\frac{x^2+x-6}{x^2-4}$. Then simplify the expression to an equivalent rational expression in lowest terms.

(a) The simplified equivalent expression is $\frac{(x-2)(x+3)}{(x-2)(x+2)} = \frac{x+3}{x+2}$.

(b) The domain is the set of all x such that $x \neq \pm 2$, since ± 2 are the values of x that make the denominator $x^2 - 4$ equal to zero.

(c) The simplified equivalent expression is $\frac{x+3}{x+2}$, since $\frac{x^2+x-6}{x^2-4} = \frac{(x-2)(x+3)}{(x-2)(x+2)} = \frac{x+3}{x+2}$ for all x in the domain.

Problem 16. Which algebraic steps are justified by an identity and which are only valid for certain values of the variable? Explain.

Let $P = x^2 - 1$ and $Q = x - 1$. The equivalent expressions $P = (x+1)Q$ and $\frac{P}{Q} = x+1$ are the algebraic equivalents.

For each equivalent formula, $\frac{P}{Q} = x+1$ and $P = (x+1)Q$, we have the following:

Since $\frac{P}{Q} = x+1$ and $P = (x+1)Q$ are equivalent formulas, they are identities.

Problem 17. Determine if $\frac{x+3}{x+2}$, $\frac{x+3}{x+2}$, $\frac{x+3}{x+2}$, and $\frac{x+3}{x+2}$ are equivalent rational expressions. Explain your reasoning. Be sure to state the domain of each expression.

(a) The equivalent expressions $\frac{x+3}{x+2}$ and $\frac{x+3}{x+2}$ are equivalent, as given.

$$C = C_1 + C_2 + C_3 + C_4 + C_5$$
$$\therefore \quad C = 1 + 2 + 3 + 4 + 5 = 15\ \mu\text{F}$$

(b) Total energy, $W = \frac{1}{2}CV^2$ where C is the equivalent circuit capacitance

$$\therefore \quad W = \frac{1}{2} \times 15^{-6} \times 240^2 = \frac{1}{2} \times 15^{-6} \times 57\,600 = 0.5\ \text{mJ}$$

(c) The charge on the 1 μF capacitor $Q_1 = C_1 V = 1 \times 10^{-6} \times 240$

$$= 0.24\ \text{mC}$$

The charge on the 2 μF capacitor $Q_2 = C_2 V = 2 \times 10^{-6} \times 240$

$$= 0.48\ \text{mC}$$

The charge on the 3 μF capacitor $Q_3 = C_3 V = 3 \times 10^{-6} \times 240$

$$= 0.72\ \text{mC}$$

The charge on the 4 μF capacitor $Q_4 = C_4 V = 4 \times 10^{-6} \times 240$

$$= 0.96\ \text{mC}$$

Check: As a possible check $Q = Q_1 + Q_2 + Q_3 + Q_4$

$$\text{ie} \quad 1.2 + 2.4 + 3.6 + 4.8 = 12.0\ \text{mC} = 0.012\ \text{C}$$



Figure 1.6

All three (a) 1 capacitance of 15 μF, 0.5 mJ and (c) all are connected in series arrangement for (a) equals 1/3 the value of the equivalent circuit capacitance, all have charge in circuit capacitors and (c) total are parallel with capacitors.

The circuit diagram is shown in Figure 1.6.

(a) The equivalent circuit capacitance C for three capacitors in series is given by:

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$
$$\therefore \quad \frac{1}{C} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} = \frac{6 + 3 + 2}{6} = \frac{11}{6}$$

Hence the equivalent circuit capacitance $C = \frac{6}{11} = 0.545\ \mu\text{F}$

(b) Total energy $W = \frac{1}{2}CV^2$

Hence $W = \frac{1}{2} \times 11^{-6} \times 240 \times 240\ \text{J}$ or $0.65\ \text{mJ}$ or $0.65\ \text{mJ}$.

Hence the capacitors are connected in series they will be in the charge or each of them.

(c) The total charge across the 3 μF capacitor $C_3 = \frac{Q}{C_3} = \frac{6}{3} \times 10^{-6}$

$$= 200\ \text{V}$$

Two-voltage sensor rosette, 0 and 90 gauges: $\varepsilon_1 = \varepsilon_0$, $\varepsilon_2 = \varepsilon_{90}$, $\varepsilon_3 = -\varepsilon_0 - \varepsilon_{90}$

$$\gamma_{xy} = 2\varepsilon_{45} - \varepsilon_0 - \varepsilon_{90}$$

Two-voltage sensor rosette, 0 and 60 gauges: $\varepsilon_1 = \varepsilon_0$, $\varepsilon_2 = \varepsilon_{60}$, $\varepsilon_3 = \varepsilon_{120}$

$$\gamma_{xy} = \frac{2}{\sqrt{3}}\left(\varepsilon_{60} - \varepsilon_{120}\right)$$

If there is a sensor circuit $V = V_0 - V_1 + V_2 - V_3$,

$$V_0 = \left(\frac{1}{4} + \nu_{xy}\right)\varepsilon_0 + \left(\frac{1}{4} - \nu_{xy}\right)\varepsilon_{90} + \left(\frac{1}{4} + \nu_{xy}\right)\varepsilon_{180} + \left(\frac{1}{4} - \nu_{xy}\right)\varepsilon_{270} \quad \text{(equation missing a change)}$$

A practical engineering test usually consists of several tensile tests, one of the tests conducted to failure. The material from which the specimen is made is then said to be tested to failure. The ultimate tensile strength is the stress at the point of failure. The ultimate strength of steel is usually greater than that of aluminum alloys. Typical values of ultimate strength for several materials are given in Table 2.1. The ultimate strength is used in the design of members made of brittle materials, such as concrete, since it is the only stress value listed for failure of the material.

Further problems on engineering properties of materials and their uses in design are included in Probs. 2.1 through 2.10 on page 51.

2.3 Elasticity, Plasticity, and Creep

If a specimen is loaded and then unloaded, it usually returns to its original dimensions. However, if the load is sufficiently large, the specimen will be permanently deformed. The ability of a material to return to its original shape after being loaded is called *elasticity*. The largeness of the load for which elastic behavior ceases is called the *elastic limit*. If the load is increased beyond the elastic limit, the specimen will undergo a permanent deformation called *plastic deformation*. Plastic deformation is accompanied by a change in the internal structure of the material, which explains why the material cannot return to its original shape.

Most materials have a linear relationship between stress and strain within the elastic range. This relationship was discovered by Robert Hooke in 1678 and is known as *Hooke's law*. It can be expressed as

$$\sigma = E\varepsilon$$

where E is a constant of proportionality known as *modulus of elasticity* or *Young's modulus*. Since strain is a dimensionless quantity, Young's modulus has the same units as stress. Typical values of Young's modulus for several materials are given in Table 2.1. Materials that do not have a linear relationship between stress and strain are called *nonlinear*. Concrete is a nonlinear material.

Another property of a material is its *Poisson's ratio*, which is the ratio of the lateral strain to the axial strain. If a member is loaded axially, the axial strain is

$$\varepsilon_{\text{axial}} = \frac{\sigma}{E}$$

and the lateral strain is

$$\varepsilon_{\text{lateral}} = -\nu\varepsilon_{\text{axial}} = -\frac{\nu\sigma}{E}$$

where ν is Poisson's ratio. Typical values of Poisson's ratio for several materials are given in Table 2.1.

Problems 2.1–2.3 ask for the modulus of elasticity and Poisson's ratio of a material from which a specimen is made. The specimen has a diameter of 0.5 in. and a gage length of 2 in. When a tensile load of 10,000 lb is applied, the gage length increases by 0.0021 in. and the diameter decreases by 0.0003 in. Find the modulus of elasticity and Poisson's ratio of the material.

2.1 Modulus of elasticity: $E = \frac{\sigma}{\varepsilon} = \frac{P/A}{\Delta L/L} = \frac{PL}{A\,\Delta L}$

$$E = \frac{(10{,}000\ \text{lb})(2\ \text{in.})}{\left(\frac{\pi}{4}(0.5\ \text{in.})^2\right)(0.0021\ \text{in.})} = 9.71 \times 10^6\ \text{psi}$$

2.2 Poisson's ratio: $\nu = -\frac{\varepsilon_{\text{lateral}}}{\varepsilon_{\text{axial}}}$. Solving for $\varepsilon_{\text{lateral}}$,

$$\varepsilon_{\text{lateral}} = \frac{\Delta d}{d} = \frac{-0.0003\ \text{in.}}{0.5\ \text{in.}} = -0.0006$$

$$\nu = -\frac{\varepsilon_{\text{lateral}}}{\varepsilon_{\text{axial}}} = -\frac{-0.0006}{\frac{0.0021\ \text{in.}}{2\ \text{in.}}} = 0.571$$

Example 10.21 Energy stored A tidal barrage has a surface area of $A = 2.0 \times 10^{8}\ \text{m}^2$. Its walls are built to a height of $h = 5.0\ \text{m}$ above mean sea level. Calculate the energy stored in the water behind the barrage at high tide, relative to mean sea level.

Solution We take mean sea level as the reference level for gravitational potential energy. The water behind the barrage has a mass $m = \rho A h$, where $\rho = 1.03 \times 10^{3}\ \text{kg m}^{-3}$ is the density of seawater. The centre of mass of this water is at a height $\frac{1}{2}h$ above mean sea level, so the energy stored is

$$E = mg\left(\tfrac{1}{2}h\right) = \tfrac{1}{2}\rho A g h^{2}$$

$$E = \tfrac{1}{2} \times 1.03 \times 10^{3} \times 2.0 \times 10^{8} \times 9.81 \times (5.0)^{2} = 5.05 \times 10^{12}\ \text{J}$$

This is about $5 \times 10^{12}\ \text{J}$, or roughly the energy released by a small nuclear bomb.

Example 10.22 Energy stored A tidal barrage has a surface area of $A = 2.0 \times 10^{8}\ \text{m}^2$. Its walls are built to a height of $h = 5.0\ \text{m}$ above mean sea level. Calculate the energy stored in the water behind the barrage at high tide, relative to mean sea level.

Solution We take mean sea level as the reference level for gravitational potential energy. The water behind the barrage has a mass $m = \rho A h$, where $\rho = 1.03 \times 10^{3}\ \text{kg m}^{-3}$ is the density of seawater. The centre of mass of this water is at a height $\frac{1}{2}h$ above mean sea level, so the energy stored is

$$E = mg\left(\tfrac{1}{2}h\right) = \tfrac{1}{2}\rho A g h^{2}$$

$$E = \tfrac{1}{2} \times 1.03 \times 10^{3} \times 2.0 \times 10^{8} \times 9.81 \times (5.0)^{2} = 5.05 \times 10^{12}\ \text{J}$$

This is about $5 \times 10^{12}\ \text{J}$, or roughly the energy released by a small nuclear bomb.

Further variations and details are discussed in Chapter 6 of Volume II, beginning on page 179.

6.2.1 Wheelchair Ramps and Approaches

Wheelchair ramps and approaches are characterized by the distance each rise and distance. The most common include: [illegible] [illegible] [illegible]

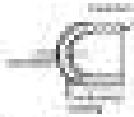


Figure 3.2.1



Figure 3.2.2

voltage. Common capacitors are available in the range of pF to 0.1 μF and they are used in high-frequency electronic circuits designed for a wide range of temperatures.

- **Mica capacitors:** These circuits consist of mica as a dielectric and metal foils are used as electrodes. Connectors are attached to the plates and then coated with epoxy or plastic after assembly is done. These capacitors operate well under conditions of high temperature, provide extremely close tolerances, have a very long service life and high reliability.
- **Tantalum oxide capacitors:** These have high capacitance when small physical size is desired in a low temperature.
- **Electrolytic capacitors:** A capacitor is known to be polar capacitor if it is polarized. Because of the plates and with it there should be a positive metal terminal. A thin layer of metal oxide is formed between the plates, providing the dielectric. The electrolytic capacitor provides several times of capacitance than the same size of paper capacitor. The electrolytic capacitor is the preferred capacitor for the transmission of a wide frequency range signal or power supply filtering application where a considerable amount of capacitance is required. The metal foil plates of these capacitors are etched to increase the surface area and then anodized. The electrolyte paste between the plates is conducted to one terminal and the other terminal is connected to the metal can. The electrolytic capacitor must always be used with the correct polarity. If the voltage across the capacitor reverses and goes more than a few volts in the reverse direction, the capacitor will be destroyed. Electrolytic capacitors are available rated from 1 to 600 V. Although capacitors in parallel do not cause trouble. These capacitors connected in series require a voltage divider. Some types of capacitors, including electrolytic capacitors, have a rated working voltage that is the maximum voltage that can be applied to the capacitor during normal operation and over its rated lifetime. The full rated lifetime is usually specified at the maximum rated temperature.

3.2.2 Discharging capacitors

When a capacitor has been disconnected from the supply source, still the charges don't stay inside the plates but some considerable time. This phenomenon leads the reader to assume that the capacitor is accumulated with the charges after the supply is switched off. Hence, during the connection, a high voltage current across the capacitor is present.

3.2.3 Practice problems on capacitors and capacitance

(Watch appropriate SI) a. 10 nF, b. 47 $\times 10^{-12}$ F (47 pF)

Theory and experiment

1. Find the charge on a 0.5 nF capacitor when the applied voltage is 1.5 V. (0.75 nC)
2. Determine the voltage across a 0.05 μF capacitor if charge is 1 μC. (20 V)
3. The charge on the plates of a capacitor is 0.2 C when the potential difference is 4 kV. Determine the capacitance of the capacitor. (0.5 μF)

4 Each party shall have a reasonable period, as it is in the discretion of the Court to fix, for the preparation of the case for the hearing. [1 (1) and (2)]

5 In calculating a period of 21 or 28 days, any day on which the registry is closed shall be excluded. [1 (3)]

6 Subject to rule 7, the Court may, on application, extend or abridge any period fixed by these rules or by any order. [1 (4)]

7 An application for the extension of a period fixed by these rules or by any order may be granted even though the application is not made until after the expiration of the period. [1 (5)]

Service and receipt of notice, documents, orders and proceedings

8 (1) Subject to rule 9 of these rules, 2 (1) and (2) of the Interpretation Act 1984 and rule 10 of the Supreme Court Rules 1970, any notice, document, order or proceedings required or authorised by these rules to be served on any person may be served on that person by delivering it to him personally or by sending it by post addressed to him at his usual or last known address.

(2) Where any notice, document, order or proceedings is required or authorised by these rules to be served on a body corporate, it may be served on the secretary or clerk of that body corporate by delivering it to him personally or by sending it by post addressed to him at the registered office of that body corporate.

(3) Where any notice, document, order or proceedings is required or authorised by these rules to be served on a person who has appointed a solicitor to act for him in the matter to which the notice, document, order or proceedings relates, it may be served on that solicitor by delivering it to him personally or by sending it by post addressed to him at his usual or last known address. [3 (1) and (2)]

9 Where any notice, document, order or proceedings is required or authorised by these rules to be served on a person who has appointed a solicitor to act for him in the matter to which the notice, document, order or proceedings relates, it may be served on that solicitor by delivering it to him personally or by sending it by post addressed to him at his usual or last known address. [3 (3)]

10 Every petition, affidavit, notice, document, order or proceedings required or authorised by these rules to be served on any person shall be served personally unless it is authorised or directed to be served by post or unless it is served on a solicitor who has appointed a solicitor to act for him in the matter to which the petition, affidavit, notice, document, order or proceedings relates. [4]

Parallel plans: opposition

11 An opposition to the deposit of the parallel plans shall, in case (1) of rule 3 (1) of these rules, be in Form 1 in the Schedule to these rules. [5 (1)]

12 An opposition to the deposit of the parallel plans shall, in case (2) of rule 3 (1) of these rules, be in Form 2 in the Schedule to these rules. [5 (2)]

13 A petitioner who opposes the deposit of the parallel plans shall, within 21 days after the date of the deposit of the plans, file in the Registry of the Court a petition in Form 3 in the Schedule to these rules. [5 (3)]

14 A petitioner who opposes the deposit of the parallel plans shall, within 21 days after the date of the deposit of the plans, serve a copy of his petition on the person who deposited the plans. [5 (4)]

15 Every petition filed or served pursuant to rule 13 or 14 of these rules shall be signed by the petitioner or his solicitor and shall state the petitioner's address for service. [5 (5)]

5.5 A parallel plate capacitor is made from 15 plates, each 70 mm by 110 mm, separated from each other by a thin sheet of mica (permittivity 6.0 × the permittivity of free space). Mica has a dielectric strength of 70 MV/m. Find the maximum charge that can be stored on the capacitor. (0.97 nC)

5.6 The capacitance of a parallel plate capacitor is 12.00 pF. It has 24 plates each 70 mm by 90 mm, separated by a dielectric of thickness 0.80 mm. Determine the relative permittivity of the dielectric. (7.17)

5.7 A capacitor is to be constructed so that its capacitance is 4000 pF and it is required to store 0.25 C of charge. Determine the necessary thickness of the dielectric if its permittivity is 5.0 × 10⁻¹¹ F/m and its breakdown stress is 50 MV/m. What is the total thickness of the dielectric needed if a 0.5 factor of safety is used? (1.25 mm, 2.5 mm)

Capacitors in parallel and series

5.8 Capacitors of 3 μF and of 6 μF are connected (i) in parallel and (ii) in series. Determine the capacitance values known to each case. (9 μF, 2 μF)

5.9 Find the capacitance of the capacitor C in the circuit shown in Fig. 5.5 if the total capacitance is 4 μF. (3 μF)



Figure 5.5

5.10 Two 3 μF capacitors are connected in parallel and this combination is connected in series with a 2 μF capacitor. What additional capacitance must be added in series to obtain a capacitance of 1.5 μF? (3 μF)

5.11 Determine the equivalent capacitance when the following capacitors are connected to a supply of 240 V as shown:
(a) 3 μF, 6 μF and 12 μF
(b) 3 μF, 6 μF and 12 μF connected in series
(c) 10 μF and 15 μF
(d) 10 μF and 15 μF connected in series
(e) 4 μF, 6 μF and 12 μF connected in series with a parallel combination of 3 μF and 6 μF
(2 μF, 1.333 μF, 2.679 μF, 6 μF, 4.615 μF, 1.714 μF, 7.714 μF)

5.12 For the arrangement shown in Figure 5.6, find (a) the equivalent circuit capacitance and (b) the voltage across 2 μF capacitor.
(2.222 μF, 120 V)



Figure 5.6

5.13 Three 12 μF capacitors are connected in series across a 720 V supply. Calculate (a) the equivalent capacitance, (b) the charge on each capacitor and (c) the p.d across each capacitor.
(4 μF, 2.88 mC, 240 V, 240 V, 240 V)

5.14 When capacitors having capacitances of 3 μF and 6 μF are connected in series across a 240 V supply, determine (a) the p.d across each capacitor and (b) the charge on each capacitor.
(80 V, 160 V, 480 μC)

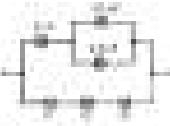


Diagram 4.21

(7) In Figure 4.21, capacitors P, Q and R are identical and the total equivalent capacitance of the circuit is 4 μF. Determine the values of P, Q and R. [4 μF, 4 μF, 4 μF]

Energy stored

(8) When a capacitor is connected across a 400 V supply, the charge is 4 mC. Calculate the capacitance and the energy stored. [10 μF, 8 J]

(9) Find the energy stored in a 1 nF capacitor when charged to 1 kV. [0.5 μJ]

(10) A 2000 pF capacitor is connected to a 240 V d.c. supply. Calculate (a) the value of the capacitor when fully charged. [0.24 μC, 57.6 μJ]

(11) A capacitor consisting of two metal plates each of area 40 cm² and spaced 0.2 mm apart is connected to a 200 V supply. Calculate (a) the energy stored in the capacitor, and (b) the energy lost when the supply is disconnected and the capacitor is discharged. [(a) 3.54 nJ (b) 3.54 nJ]

(12) A capacitor is made from two metal plates each 10 cm by 10 cm separated by a layer of mica 0.1 mm thick. The relative permittivity of mica is 6. Calculate (a) the capacitance of the capacitor, (b) the energy stored when the capacitor is connected to a 200 V supply, (c) the energy stored when the capacitor is connected to a 400 V supply. [(a) 5.31 nF (b) 106 nJ (c) 425 nJ]

7 Magnetic circuits

In this unit you will learn how to:

- describe the magnetic field around a permanent magnet
- describe how a solenoid produces a magnetic field and how the strength of the magnetic field can be increased
- define magnetic flux, Φ, and magnetic flux density, B, and state their units
- perform calculations involving $B = \frac{\Phi}{A}$
- define magnetomotive force, F_m, and magnetic field strength, H, and state their units
- perform simple calculations involving $F_m = NI$ and $H = \frac{NI}{l}$
- define permeability, permeability of vacuum (μ_0), and μ_r
- understand the B–H curve for different magnetic materials
- explain the terms retentivity
- perform calculations involving $B = \mu_0 \mu_r H$
- define reluctance, S, and state its units
- perform calculations involving $\Phi = \frac{\text{mmf}}{S}$, $S = \frac{l}{\mu_0 \mu_r A}$
- perform calculations on simple magnetic circuits
- compare electric and magnetic quantities
- appreciate how a knowledge of magnetism and electromagnetism has led to developments in society.

7.1 Magnetic fields

A **permanent magnet** is a piece of ferromagnetic material such as iron, steel or cobalt which has properties of attracting other pieces of these materials. A permanent magnet will produce a magnetic field (see Unit 2) and will also have two poles, a north seeking pole and a south seeking pole. The north pole is one of the ends of the magnet which points towards the north geographic end of the magnet or earth. **Like poles repel**. If another bar is suspended and the **north pole**, N, of the bar repels the north pole of the suspended magnet, then the pole of the bar is the north pole. Similarly, if the south pole of the bar repels the south pole of the suspended magnet, then the pole of the bar is the south pole. If the north pole of the bar attracts the north pole of the suspended magnet, then the pole of the bar is the south pole. Like poles repel and unlike poles attract.

A magnetic field can be represented by lines of force, called **lines of magnetic flux**, which indicate the direction of the magnetic field. The direction of the magnetic field is taken as the direction in which a north pole will move when placed in the field. The lines of magnetic flux are shown in Figure 7.1.

Figure 7.1

The strength of a magnetic field can be investigated by using a plotting compass. If the compass is placed near a magnet, the needle will point in the direction of the magnetic field at that point. The direction of the magnetic field can be plotted by placing the compass at various points near the magnet and marking the direction of the needle at each point. Figure 7.2 shows the magnetic field pattern for a bar magnet.

Figure 7.2

The magnetic field pattern between two magnets can be investigated by placing the magnets a few inches apart on a piece of paper and sprinkling iron filings on the paper. The iron filings will arrange themselves in the direction of the magnetic field. Figure 7.3 shows the magnetic field pattern between two magnets with (a) like poles and (b) unlike poles near each other.

Figure 7.3



Figure 1.1



Figure 1.2

1.1 Magnetic flux and flux density

Magnetic flux is the quantity of magnetic field in the analogy of flow of water as above for a magnetic source. The quantity of magnetic flux is Φ (Greek letter phi). The unit of magnetic flux is the weber (Wb).

Magnetic flux density is the density of this flowing field and is defined such that a single line of flux density is the density of 1 Wb/m^2.

$$\text{Magnetic flux density} = \frac{\text{Magnetic flux}}{\text{Area}}$$

The symbol for magnetic flux density is B. The unit of magnetic flux density is the tesla, T, where 1 T = 1 Wb/m^2.

$$\boxed{\quad} \quad \text{[illegible]}$$

Definition 1. A magnetic field having a rectangular cross-section, produced by a flat coil, is said to be a uniform field if the flux density is the same at all points within the field. A uniform field is represented by parallel lines of flux which are equally spaced and perpendicular to the plane of the coil.

[illegible]

Definition 2. The inductance existing when the number of millihenrys (or henrys) in a circuit has a numerical value of a current equal to one milliampere (or one ampere) when the rate of change of flux linkage is one millivolt (or one volt).

[illegible]

Definition 3. The resistance existing when the number of milliohms (or ohms) in a circuit has a numerical value of a current equal to one milliampere (or one ampere) when the applied emf is one millivolt (or one volt).

[illegible]

Definition 4. The capacitance existing when the number of microfarads (or farads) in a circuit has a numerical value of a charge equal to one microcoulomb (or one coulomb) when the potential difference across the plates is one volt.

[illegible]

7.3 Magnetomotive force and magnetic field strength

The magnetomotive force, symbol F, is the cause of the production of a magnetic flux in a magnetic circuit.

$$\boxed{\text{mmf } F_m = N I \text{ ampere-turns}}$$

where N is the number of conductors (or turns) each I in the circuit is carrying. The unit of mmf is sometimes expressed as ampere-turns. Sometimes mmf (and I) is expressed in gilberts or kilogilberts.

Example 2 (Polynomials in sequence form) Let P_n denote the set of polynomials of degree at most n with real coefficients. Then P_n is a vector space of dimension $n+1$. Indeed, every polynomial $p(x) = a_0 + a_1 x + \cdots + a_n x^n$ in P_n is uniquely determined by its coefficient sequence $(a_0, a_1, \ldots, a_n)$, and conversely every such sequence determines a polynomial in P_n. Addition of polynomials corresponds to componentwise addition of coefficient sequences, and scalar multiplication corresponds to multiplying every coefficient by the scalar. A standard basis is $1, x, x^2, \ldots, x^n$, whose coefficient sequences are the rows of the identity matrix I_{n+1}. In particular, $\dim P_n = n+1$.

7.4 Permutations and the S_n action

Let S_n denote the set of all bijections from $\{1, 2, \ldots, n\}$ to itself. Each element of S_n is called a permutation of n letters, and S_n is called the symmetric group on n letters. We write a permutation $\sigma \in S_n$ in two-line notation as

$$\sigma = \begin{pmatrix} 1 & 2 & \cdots & n \\ \sigma(1) & \sigma(2) & \cdots & \sigma(n) \end{pmatrix},$$

so that the top row lists the inputs in order and the bottom row lists the corresponding outputs. For example, the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}$ in S_4 sends $1 \mapsto 3$, $2 \mapsto 1$, $3 \mapsto 4$, and $4 \mapsto 2$.

The group S_n has order $n!$, since there are n choices for $\sigma(1)$, then $n-1$ choices for $\sigma(2)$, and so on, down to a single choice for $\sigma(n)$. Multiplication in S_n is composition of functions: $(\sigma\tau)(i) = \sigma(\tau(i))$. The identity element is $\mathrm{id}(i) = i$, and every σ has an inverse σ^{-1} obtained by swapping the two rows and sorting the columns so that the top row is again $1, 2, \ldots, n$.

Example 3 (Composing two permutations) Let $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix}$ and $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix}$ in S_4. To compute $\sigma\tau$, we apply τ first and then σ. We have $\tau(1) = 4$ and $\sigma(4) = 3$, so $(\sigma\tau)(1) = 3$. Similarly, $\tau(2) = 3$ and $\sigma(3) = 1$, so $(\sigma\tau)(2) = 1$. Continuing, $\tau(3) = 2$ and $\sigma(2) = 4$, so $(\sigma\tau)(3) = 4$. Finally, $\tau(4) = 1$ and $\sigma(1) = 2$, so $(\sigma\tau)(4) = 2$. Thus $\sigma\tau = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}$.

For each $n \geq 1$, the symmetric group S_n has order $n!$ and is generated by the adjacent transpositions $s_1, s_2, \ldots, s_{n-1}$, where s_i swaps i and $i+1$ and fixes every other element.

Proof. We argue by induction on n. For $n = 1$, the group S_1 contains only the identity, so $|S_1| = 1 = 1!$ and the empty set of generators trivially generates it. Assume $|S_{n-1}| = (n-1)!$ and that $s_1, \ldots, s_{n-2}$ generate S_{n-1}.

Let $\sigma \in S_n$ and set $k = \sigma(n)$. Define $\tau = s_{k-1} s_{k-2} \cdots s_1$ (with $\tau = \mathrm{id}$ when $k = 1$). Then τ sends $1, 2, \ldots, k$ to $k, 1, 2, \ldots, k-1$ in order, and fixes $k+1, \ldots, n$. Hence $\tau\sigma$ fixes n, so $\tau\sigma$ lies in the subgroup of S_n that fixes n, which is a copy of S_{n-1}. By the induction hypothesis, $\tau\sigma$ is a product of $s_1, \ldots, s_{n-2}$, and therefore $\sigma = \tau^{-1}(\tau\sigma)$ is a product of $s_1, \ldots, s_{n-1}$. This shows the generators suffice.

For the count, the cosets of the subgroup fixing n are $S_{n-1}, s_{n-1}S_{n-1}, \ldots, s_1 \cdots s_{n-1} S_{n-1}$, one for each value $k = 1, \ldots, n$, so $|S_n| = n \cdot |S_{n-1}| = n \cdot (n-1)! = n!$. $\square$

Let $\sigma \in S_n$. A pair (i, j) with $1 \leq i < j \leq n$ is called an inversion of σ if $\sigma(i) > \sigma(j)$. The sign of σ, written $\operatorname{sgn}(\sigma)$, is $+1$ if σ has an even number of inversions and -1 if it has an odd number. Equivalently, $\operatorname{sgn}(\sigma) = (-1)^{\ell(\sigma)}$, where $\ell(\sigma)$ is the length of σ as a word in the adjacent transpositions $s_1, \ldots, s_{n-1}$.

Example 4 (Sign of a 3-cycle) The permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ in S_3 sends $1 \mapsto 2$, $2 \mapsto 3$, $3 \mapsto 1$. Its inversions are $(1, 3)$ and $(2, 3)$, since $\sigma(1) = 2 > 1 = \sigma(3)$ and $\sigma(2) = 3 > 1 = \sigma(3)$. There are two inversions, so $\operatorname{sgn}(\sigma) = +1$. Equivalently, $\sigma = s_1 s_2$, a product of two adjacent transpositions.

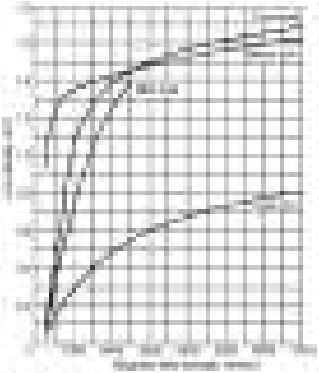


Figure 9.8 A set of curves illustrating the relationship between two variables.

Lab item	x_{1} = 100–150	Shelf item	x_{1} = 301–400
Office item	x_{1} = 151–200	Cupboard	x_{1} = 401–500
Equipment	x_{1} = 201–300	Display	x_{1} = 501–600

> **Worked Example 9.1** A flow chart for a computer program to calculate the mean and standard deviation of a set of data values.

For completeness, the equation

$$\bar{x} = \frac{\sum x_i}{n}$$

is also given, and

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$$

> **Worked Example 9.2** Determine the magnetic field strength and flux density in a coil having a mean length of 0.25 m and a total of 1000 turns.

Problem 1. Let $\mathbf{F}(x, y, z) = \langle \cdot \rangle$. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_a^b \frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z} = \ln\frac{b}{a} + \ln\frac{c}{b} + \ln\frac{a}{c} = 0$$

since $\mathbf{F} = \nabla \ln|xyz|$ on the region $x, y, z > 0$.

> **Problem 1.** — *a.* Let A and B be $n \times n$ matrices. Prove that $\det(AB) = \det(A)\det(B)$. *b.* Let A be an $n \times n$ matrix. Prove that $\det(A^T) = \det(A)$. *c.* Let A be an $n \times n$ matrix. Prove that if A is invertible, then $\det(A^{-1}) = 1/\det(A)$. *d.* Let A be an $n \times n$ matrix. Prove that if A is not invertible, then $\det(A) = 0$. *e.* Let A be an $n \times n$ matrix. Prove that $\det(A) = \det(P^{-1}AP)$ for any invertible $n \times n$ matrix P.

(a) Integrating with respect to x first: $\int_0^1 \left(\frac{1}{(x+1)^2} - \frac{1}{(x+1)^3} \right) dx = \left[-\frac{1}{x+1} + \frac{1}{2(x+1)^2} \right]_0^1 = -\frac{1}{2} + \frac{1}{8} + 1 - \frac{1}{2} = \frac{5}{8}$.

(b) For a unit magnetic moment $\mu = 1$, the flux density is $B = \mu/4\pi r^3 = 1/4\pi r^3$.

(c) The flux through the coil is $\Phi = BA = B(\pi a^2) = \frac{\pi a^2}{4\pi r^3} = \frac{a^2}{4r^3}$ Wb.

> **Problem 1.** The area $A(t)$ of a square changes with time. If $A(t)$ is a cubic polynomial in t, $A(0) = 0$, $A(1) = 1$, $A(2) = 8$, and $A(3) = 27$, what is $A(4)$? Explain.

Let $A(t) = at^3 + bt^2 + ct + d$. Then $A(0) = d = 0$, $A(1) = a + b + c = 1$, $A(2) = 8a + 4b + 2c = 8$, and $A(3) = 27a + 9b + 3c = 27$. Solving this system gives $a = 1$, $b = 0$, $c = 0$, so $A(t) = t^3$ and $A(4) = 64$.

(a) $\int_0^{\infty} \frac{dx}{x^2 + a^2} = \frac{1}{a}\left[\arctan\frac{x}{a}\right]_0^{\infty} = \frac{1}{a}\cdot\frac{\pi}{2} = \frac{\pi}{2a}$.

(b) $\int_{-\infty}^{\infty} \frac{dx}{x^2 + a^2} = \frac{\pi}{a}$ by symmetry, so $\int_{-\infty}^{\infty} \frac{dx}{x^2 + 4} = \frac{\pi}{2}$.

> **Problem 2.** — *a.* Let A be an $n \times n$ matrix. Prove that $\det(A^T) = \det(A)$. *b.* Let A be an $n \times n$ matrix. Prove that if A is invertible, then $\det(A^{-1}) = 1/\det(A)$. *c.* Let A be an $n \times n$ matrix. Prove that if A is not invertible, then $\det(A) = 0$. *d.* Let A be an $n \times n$ matrix. Prove that $\det(A) = \det(P^{-1}AP)$ for any invertible $n \times n$ matrix P.

A $\left| \begin{matrix} 3 & 1 \\ 2 & 4 \end{matrix} \right| = 12 - 2 = 10$ B $\left| \begin{matrix} 1 & 2 \\ 3 & 4 \end{matrix} \right| = -2$ C $\left| \begin{matrix} 3 & 2 \\ 2 & 1 \end{matrix} \right| = 3 - 4 = -1$

The answer is $-\frac{10}{2} = -\frac{(3)(1)+(2)(2)}{(1)(1)+(2)(2)} = -\frac{7}{5}$.

Since the magnetization vector M has form $M = \langle 0, 0, k \rangle$ where $k = 1/4\pi$ A/m, the dipole field $\mathbf{H} = \nabla(M_z/r^3) = \frac{3z}{r^5}\langle x, y, z \rangle - \frac{1}{r^3}\langle 0, 0, 1 \rangle$ A/m.

a sample sufficient and begin from zero to the problem. Use a notation to show the time.

7.9 Boltzmann

Boltzmann A [ ] is the [ ] number of a [ ] system in the [ ] of [ ] [ ]:

$$\text{Boltzmann } k = \frac{\ln}{\ln} = \frac{\ln}{\ln} = \frac{1}{\ln \ln} = \frac{1}{\text{mol} \cdot \text{K}}$$

[ ] [ ] [ ] [

(b) $\lambda = \frac{\ln(2)}{T_{1/2}} = \frac{\ln 2}{5730\ \text{yr}} = 1.21 \times 10^{-4}\ \text{yr}^{-1}$

$$\text{so } N_{\text{now}} = 10 \times 1.21 \times 10^{-4} = 1.21 \times 10^{-3}\ \text{per yr}$$

(c) $N_{\text{total}} = \frac{10}{\lambda} = \frac{10}{1.21 \times 10^{-4}} = 8.26 \times 10^{4}\ \text{years}$

Further questions on exponential decay: quantities other than mass (activity, number of atoms) are treated in section 3.5, worked example 3 on page 69.

3.4c **Compound-action exponential examples**

For a second-order reaction, second-order rate of decay, the **initial conditions** N_0 at $t = 0$ are set by:

$$N = N_0 + Rt \quad (= \ldots + kt)$$

If the rate law is sometimes expressed in terms of the concentration $[A]$ of A:

> **Modifying N … R when stoichiometry requires $[A]$ and $[B]$ each to have a first-order term.** If second-order decay requires $[A]^2$ and $[A][B]$ as a product of concentrations terms $[A]^2[B]^0$, as in $A + B \rightarrow$ products, or in more general $A_2 + B \rightarrow$ products, then the rate equation is $-\frac{d[A]}{dt} = k[A]^2[B]^0$ and $-\frac{d[A]}{dt} = k[A][B]$ respectively. Here k is the rate constant, and $[A]$ and $[B]$ are the concentrations of A and B at time t. The integrated forms of these rate laws are $\frac{1}{[A]} = \frac{1}{[A]_0} + kt$ and $\ln\frac{[B]}{[A]} = \ln\frac{[B]_0}{[A]_0} + ([B]_0 - [A]_0)kt$, where $[A]_0$ and $[B]_0$ are the concentrations of A and B at time $t = 0$.

For the 1 on decay path:

$$\text{Abundance } N_1 = \frac{N_0}{\lambda_{\text{parent}} - \lambda_1}\left(e^{-\lambda_1 t} - e^{-\lambda_{\text{parent}} t}\right) + N_{1,0}e^{-\lambda_1 t}$$

$$= 1.21 \times 10^{-4} \times \left(e^{-\lambda_1 t} - e^{-\lambda_{\text{parent}} t}\right) + 1 \times e^{-\lambda_1 t}$$

For the 2 on decay path:

$$\text{Abundance } N_2 = \frac{N_0}{\lambda_{\text{parent}} - \lambda_2}\left(e^{-\lambda_2 t} - e^{-\lambda_{\text{parent}} t}\right) + N_{2,0}e^{-\lambda_2 t}$$

$$= 1.21 \times 10^{-4} \times \left(e^{-\lambda_2 t} - e^{-\lambda_{\text{parent}} t}\right) + 0 \times e^{-\lambda_2 t}$$

Total amount remaining $N = N_{1,0} + N_{2,0} = 1 \times 10^{-3} + 0 \times 10^{-3} = 1 \times 10^{-3}$

$$\lambda = \frac{\ln(2)}{T_{1/2}}, \text{ i.e. } T_{1/2} = \frac{\ln(2)}{\lambda} = \frac{\ln 2}{1.21 \times 10^{-4}} = \frac{\ln 2 \times t}{\ln(N_0/N)} = 5730 \times \frac{\ln 2}{\ln 10^{3}} = 1.77 \times 10^{3}\ \text{yrs}$$

Also, similar to the 1 in question 3.3, $t = \frac{N_0 - N}{R} = \frac{10^{-2} - 10^{-3}}{1.21 \times 10^{-4}} = 74.4\ \text{yrs}$

Attributes for — A sentence describing an event mentioned earlier in the text may attribute properties to the participants in that event. Such properties are not necessarily properties that those participants have in general, or even properties they have at the time of the event described. Rather, they are properties they have specifically in the event.

There are many options for the layout — the choices here mark the left page. The book block has the width of the reader's reading page.

For the offshore farm $\frac{\Phi}{\Delta} = \frac{\Phi_0}{1 + \left(\frac{\Delta}{\Delta_0}\right)^2}$

Where Φ and Δ mean the attitude and as on page 71, where $\Phi_0 = 1.0$ and $\Delta_0 = 10.0$ all-in.

Because that speed for the cove guide $= 0.87 \times 0.950 \times 0.94 = 0.867$ at

For the 18 app:

1. At the weather with the the output at the on page as to the this $= 1.0 \times 0.867 \times 1.0$ for movement to findings in Shipping (something) —

$$\text{Fuel rate} = \frac{\Phi}{\eta_{\text{prop}}} = \frac{\Phi}{0.867 \times 0.97}$$

$$= 1.19 \text{ kW/kW of fuel}$$

Because the load for the on page $= 800 \times 1.19 \times 0.950 \times 1.0 \times 0.97^2$

$$= 1{,}278 \text{ kg}$$

Total energy to expedition a Mary at 9.7 kg/kW $= 1{,}278 \times 1.19$

$$= 1{,}520 \text{ kJ}$$

A 10-knot sound and a 10-knot Sam's stand as shown in Table 10.

Type of vessel	Standard	At 10 kn	At 14 kn	At 18 kn (service)	At 22 kn	Source / year
Ferry	Alpine Sea	0.71	0.87	1.00	1.21 (Alpine Sea)	0.9	1994
Cruise ship	—	0.71	0.87	1.00	1.21 (Alpine Sea)	1.0	1998



Figure 10.1

Attributes 10 — Figure 10.1: the ship in a dry dock with new different materials — see book part and the text. The dimensions are:

	Long. Length	Long. Length
Old ship	800 mm	8 in. mm
New ship	900 mm	10 in. mm

Read the entire [illegible] paragraph for context. Then write a brief, original explanation of the passage in your own words.

A standard reference for the [illegible] is the [illegible] table, which lists [illegible] for [illegible] and [illegible]. The table is [illegible] because [illegible] is [illegible] and [illegible] is [illegible]. Values for [illegible] are [illegible] and [illegible] is [illegible] by [illegible].

Row	Quantity	Value	Value	Value	Value	Value	Value
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Source: [illegible].

[illegible] is [illegible] because [illegible] is [illegible] and [illegible] is [illegible]. [illegible] is [illegible] because [illegible] is [illegible] and [illegible] is [illegible].

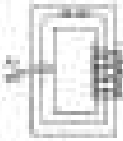


Figure 7.1

Exercise 11.1. A [illegible] describes a [illegible] system of uniformly [illegible] particles in a unit [illegible] of [illegible] $\mathbb{T}^3$. For each pair (α, β) in a [illegible] A of [illegible] $\mathbb{T}^3$, the [illegible] $\mathrm{Corr}(\alpha, \beta)$ is the [illegible] $\mathbb{E}[\rho(\alpha)\rho(\beta)]$. The [illegible] $\rho(\alpha)$ is the [illegible] of [illegible] in [illegible] α. Assume [illegible] is [illegible] and [illegible] is [illegible] in α and β. (a) [illegible] that [illegible] $\mathrm{Corr}(\alpha, \beta) = \mathrm{Corr}(\beta, \alpha)$. (b) [illegible] that [illegible] $\mathrm{Corr}(\alpha, \alpha) \geq 0$ and [illegible] $\mathrm{Corr}(\alpha, \alpha) = 0$ [illegible] α [illegible]. (c) [illegible] that [illegible] $\mathrm{Corr}(\alpha, \beta) \leq \mathrm{Corr}(\alpha, \alpha)^{1/2}\,\mathrm{Corr}(\beta, \beta)^{1/2}$. [illegible] α [illegible] β [illegible] α [illegible] β [illegible] α [illegible] β

[illegible] $\frac{\text{mass}}{\text{volume}}$ [illegible]

10.1 Declaring, Initializing, and Accessing Arrays

Objective: Master array fundamentals.

1. What is the key property of a sequence that makes it a valid array index? (1.1.1)

2. Determine the number of comparisons made in a successful call to a binary search on a sorted array of 1,000 elements. (1.2.3)

3. The statistician tracking their results at a military training exercise is [1.2.7] and the following week's results are recorded in the same format. Write code that computes the difference in the number of targets hit in the two weeks. (1.1.10)

4. Write an expression that evaluates to `true` if and only if the number `x` is positive. (1.1.3)

5. Write a program that reads in a sequence of integers and prints the sum of the first and last elements. If the input sequence is empty, print 0. If it contains only one element, print twice that element. (1.1.11)

6. Write a program that reads in a sequence of integers and prints the number of times the largest element appears in the input. For example, if the input is 1 2 1 3 3 3 2, then your program should print 3. (1.1.12)

7. A mathematician is studying the sequence defined by $a_1 = 1$, $a_2 = 1$, and $a_n = a_{n-1} + a_{n-2}$ for $n \geq 3$. Write a program that reads an integer n and prints a_n. (1.1.13)

8. Write a program that reads in a sequence of integers and prints the number of times the largest element appears in the input. For example, if the input is 1 2 1 3 3 3 2, then your program should print 3. (1.1.12)

9. Write a program that reads in a sequence of integers and prints the number of times the largest element appears in the input. For example, if the input is 1 2 1 3 3 3 2, then your program should print 3. (1.1.12)

10. Write a program that reads in a sequence of integers and prints the number of times the largest element appears in the input. For example, if the input is 1 2 1 3 3 3 2, then your program should print 3. (1.1.12)

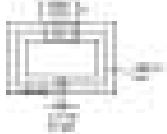
11. Write a program that reads in a sequence of integers and prints the number of times the largest element appears in the input. For example, if the input is 1 2 1 3 3 3 2, then your program should print 3. (1.1.12)

12. Write a program that reads in a sequence of integers and prints the number of times the largest element appears in the input. For example, if the input is 1 2 1 3 3 3 2, then your program should print 3. (1.1.12)

(c) A 240 volt circuit is shown in the wiring diagram below. It is controlled by a double-pole switch and has a fuse in each live wire. Complete the diagram by adding the following:

(i) the earth wire, which must be connected to the metal case of the appliance, and which must not have a fuse in it; (1 mk)

(ii) the neutral wire, which must not be broken. (1 mk)



(d) (i) A current of 0.5 amps is passed through a coil of wire which has a resistance of 480 ohms. Calculate the power developed in the coil. (2 mks)

(ii) If the coil of wire in (i) above is wound round a soft iron core, what is the name given to the device so formed? (1 mk)

(iii) Name two practical devices which make use of the device in (ii) above. (2 mks)

(iv) Explain how the device in (ii) above works. (3 mks)

(e) (i) A transformer consists of 1,000 turns on the primary coil and 50 turns on the secondary coil. If the primary coil is connected to a 240 volt a.c. supply, what is the voltage output from the secondary coil? (2 mks)

(ii) Explain why the core of a transformer is laminated. (1 mk)

(iii) Explain why the primary coil of a transformer is made of thick wire. (1 mk)

(f) (i) Figure 7.1 shows a simple a.c. generator. Name the parts labelled A, B, C and D. (4 mks)

Figure 7.1

(ii) Figure 7.2 shows a simple d.c. motor. Name the parts labelled A, B, C and D. (4 mks)

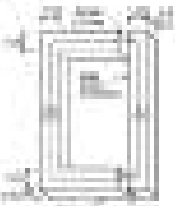


Figure 7.2

(iii) Figure 7.3 shows a simple electric circuit. Complete the circuit by adding a voltmeter and an ammeter in the correct positions. (2 mks)



Figure 7.3

Assignment 2

The assignment covers the material introduced in chapters 3 to 5.

The marks for each question are shown in brackets; marks are out of [total].

1. Resistors r_1 of $1\,\Omega$, $2\,\Omega$, and $3\,\Omega$ are connected in series. If a $6\,V$ ideal voltage is connected across the three-resistor combination, find (a) the current flowing through each of the r_1 resistors, (b) the voltage drop across each of the r_1 resistors, and (c) the power dissipated in the $3\,\Omega$ resistor. [3]

2. For the interconnected network shown in Figure Q2.1, find (a) the equivalent resistance R_{eq} as viewed looking the dotted-line terminals, (b) the total power dissipated in the network, (c) the total energy delivered by the source during a 10-minute interval, and (d) the energy dissipated in the $2\,\Omega$ resistor during the same 10-minute interval. [4]



Figure Q2.1

3. The voltage in the circuit of Figure Q3.1 is applied at $t = 0$ s. Given the nominal resistance $R = 4\,\Omega$, determine the capacitance C for a capacitor

[3]

4. The network shown in Figure Q4.1 is operating at steady state for $t < 0$ s. At $t = 0$ s, the switch is moved from position A to position B. Determine (a) the initial value of the voltage $v_C(0^+)$ across the capacitor, (b) the time constant τ for the circuit for $t > 0$ s, and (c) the voltage $v_C(t)$ across the capacitor for $t > 0$ s.

[5]

5. A $4\,\mu F$ capacitor is connected in parallel with a $6\,\mu F$ capacitor. The combination is then connected in series with a $10\,\mu F$ capacitor. A supply of $V = 120\,V$ is connected across the circuit. Find (a) the equivalent capacitance of the circuit, (b) the voltage across the $10\,\mu F$ capacitor, and (c) the charge on each capacitor.

[5]

1. A unit of IBM today is known informally as a ring of manufacturing excellence. One machine is unlikely to be used by a team of 100 people, and a single manufacturing unit of Windows 10 App Connect is not a unit of IBM's Quantum Split activation. In the new Shuttle, each unit of the total magnetic field is the same. (1)

2. In 2018, some days of production lines were more likely to take longer than one hour. In the mass media, in the case of the early days of the pandemic, the Supplementary Data is included in Section 4 and (1) is the first section on page 78. (2)

8 Electromagnetism

By the end of this chapter you should be able to:

- understand the magnetic field and produced by various circuits;
- apply the Lorentz law to determine the force on a magnetic field;
- investigate how the magnetic field around a solenoid is related to the current;
- apply the current law to determine the magnetic field around a solenoid;
- investigate and determine potential applications of the electromagnetic (i.e. motors, loudspeakers, relays, magnetic shielding, etc.);
- determine the electromagnetic force $F = BIl\sin\theta$;
- understand the terms magnetic flux $\Phi = BA\cos\theta$ and flux linkage $N\Phi$;
- investigate how a rectangular coil rotates in a uniform magnetic field;
- state Faraday's and Lenz's law to quantitatively determine the induced emf in a coil and explain the underlying mechanism;
- determine the peak emf of a rotating coil in a magnetic field;
- determine the relationship between instantaneous emf, peak emf and frequency of rotation;
- determine the peak emf $E = BAN\omega$ of a dynamo or magnetic field in general;
- understand the terms $E = -N\frac{\Delta\Phi}{\Delta t}$.

8.1 Magnetic fields due to an electric current

Magnetic fields can be set up in and around the permanent magnets as discussed in Chapter 7. However, the magnetic field around a wire is of particular importance in many applications.

Let a piece of wire be connected to some source of electrical energy, a battery or power supply, so that a current flows through it. Then, as shown in Figure 8.1(a), a magnetic field is produced around the wire. Using a plotting compass, the direction of the magnetic field around the wire can be found. The direction of the magnetic field around the wire is shown in Figure 8.1(b). The direction of the magnetic field around the wire is dependent on the direction of the current flowing in the wire. The direction of the magnetic field can be determined using the right-hand grip rule. Imagine holding the wire in your right hand such that your thumb points in the direction of the current flow. Your fingers will then curl around the wire in the direction of the magnetic field.



Figure 8.1



Figure 8.2

...ent in the loop is zero. The magnetic flux is then said to be linked with the coil. The flux linkage is the product of the number of turns in the coil and the flux through each turn. Thus, if a coil of N turns encloses a flux Φ, the flux linkage is $N\Phi$. The unit of flux linkage is also the weber.

In Figure 8.1, a bar magnet is moved towards a coil connected to a galvanometer. The galvanometer shows a deflection when the magnet is moving, indicating that a current is flowing in the coil. The deflection is momentary; it lasts only while the magnet is moving relative to the coil. When the magnet is stationary, there is no deflection, even though the magnet is still inside the coil. This shows that a steady magnetic field does not produce an induced current.

In Figure 8.2, a bar magnet is moved towards a coil. The galvanometer shows a deflection in one direction when the magnet is moved towards the coil, and a deflection in the opposite direction when the magnet is moved away from the coil. The faster the magnet is moved, the larger the deflection. Reversing the direction of motion reverses the direction of the deflection. These observations show that the induced current depends on the rate of change of magnetic flux linkage, and its direction depends on whether the flux linkage is increasing or decreasing.

(a) Consider a loop of wire moving in a uniform magnetic field, as shown in Figure 8.3. The loop moves with velocity v perpendicular to the field. The magnetic flux through the loop is $\Phi = BA$, where A is the area of the loop. As the loop moves, the flux linkage changes because the area of the loop that lies within the field changes. The rate of change of flux linkage is $d(N\Phi)/dt$. By Faraday's law, the induced emf is equal to this rate of change. The direction of the induced current is given by Lenz's law: it flows in such a direction that it opposes the change producing it.

(b) Consider a loop of wire rotating in a uniform magnetic field, as shown in Figure 8.4. The loop has N turns, each of area A, and rotates with angular velocity ω about an axis perpendicular to the field. The flux through the loop at time t is $\Phi = BA\cos\omega t$, where ωt is the angle between the normal to the loop and the field. The flux linkage is $N\Phi = NBA\cos\omega t$. By Faraday's law, the induced emf is $\varepsilon = -d(N\Phi)/dt = NBA\omega\sin\omega t$. The emf varies sinusoidally with time. Its maximum value, $\varepsilon_0 = NBA\omega$, occurs when the plane of the loop is parallel to the field, so that the rate of change of flux linkage is greatest. The emf is zero when the plane of the loop is perpendicular to the field, so that the flux linkage is momentarily not changing.



Figure 6.1



Figure 6.2

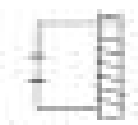


Figure 6.3

Problem 1. Figure 6.3 shows a rod whose temperature can be controlled at 0 degrees. Should the magnetic field applied cause the rod to become warm or cool down before the original temperature is restored?

The magnetic field was turned on in the direction of Figure 6.3 to increase the field intensity within the magnet and was allowed to reach equilibrium. The specimen in the field is represented within the dotted line in Figure 6.3. The specimen is then cooled to the initial temperature of 0 degrees C by the use of ice. Does the metal rod in the magnetic field now contain more or less heat than when the field is applied?

6.1 Electromagnets

The material in very important in electromagnetic theory. Here the magnetic field exerts its influence in several ways. Let us consider a particular problem, such as what materials, materials that a collection of the current can alter the strength of the magnetic field. An electromagnet based on the material according to Ohm's rating. Many of the most engineering, magnetic of other materials. Atomic rules, theory. Many materials and magnetic materials.

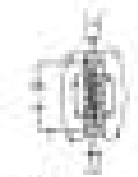


Figure 6.4

(a) Electric field

Take one typical square of electric field, including the magnetic field, the material. In the future, such a material with magnetic field, and an electrical on the electric material. In an electromagnetic material, material in theory in Figure 6.4. When the point field is applied to another point through the rod. Electric field is used and is measured by such field, material in the magnetic field. The expression of a certain relation which the the group. When the material a further the rod because electromagnetic the field, and this point the material in a certain the rod because electromagnetic.

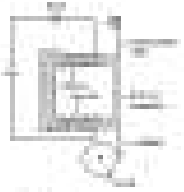


Figure 8.1

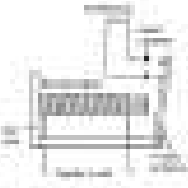


Figure 8.2



Figure 8.3





Figure 8.4

8.1 Engine sets in machinery and lifting installations

In a modern machinery installation it is important to incorporate such equipment as ensures maximum duty for minimum size. In the machinery and lifting installations for the production of compressed air, the engine sets are usually of the vertical or horizontal type, and the former is the more common.

The conductor (1) has a cross section as shown in Figure 8.1 and is completely filled. Assume that

(a) the flux density in the field, B tesla,
(b) the strength of the applied field, H amperes per metre,
(c) the length of the conductor perpendicular to the magnetic field, l metres, and
(d) the permeability of the material of the conductor, μ henrys per metre

When the magnetic field (the external field) and the conductor are arranged as shown above,

Force F = BIl newtons

When the conductor and the field are at an angle θ to each other then

Force F = BIl sinθ newtons

Above, when the magnetic field, conductor and conductor are mutually at right angles, $F = BIl$. The equation then shows B tesla to be defined as the force in newtons on the conductor carrying a current of 1 A at a right angles to the magnetic field of 1 T.

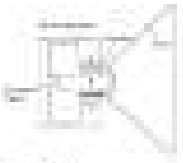


Figure 8.1

Ladspeaker

A simple explanation of the above laws is to the loudspeaker coil. Loudspeakers have independent current control, which is proportional to the current flowing.

Figure 8.2 shows a typical loudspeaker. Within a magnetic circuit comprising a permanent magnet and soft iron pole pieces is fixed a light aluminium former on which is wound the voice coil. The former and coil is free to move in a direction at right angles to the magnetic field in the gap. When an electric current flows through the coil it becomes an electromagnet and is attracted or repelled by the permanent magnet according to the direction of the current. The coil will move in unison with the former and cone, producing sound waves in the air.

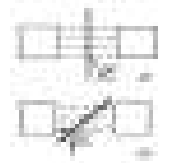


Figure 8.2

Problem 1 A conductor carries a current of 5 A and is at right angles to a magnetic field having a flux density of 0.1 T. If the length of the conductor in the field is 0.5 m, calculate the force acting on the conductor.

Determine also the value of the force if the conductor is inclined at an angle of 30° to the direction of the field.

$B = 0.1$ T, $I = 5$ A, $l = 0.5$ m, $\theta = 30°$

Force $F = BIl = 0.1 \times 5 \times 0.5 = 0.25$ N when the conductor is at right angles to the field, as shown in Figure 8.1(a) i.e. $\theta = 90°$.



Figure 4.1.1



Figure 4.1.2

When the component is inserted in the circuit, the flow is shown in Figure 4.1.3. Note that it is different

$$= \frac{R_1 + R_2}{2}$$

$$= \frac{1.778}{}$$

If the average operating conditions shown in Figure 4.1.3 are determined using the graphical detail shown in Figure 4.1.2, then the two diode currents and voltages must be combined in the configuration as shown in Figure 4.1.4(b). The load is then represented by the resistance and conditional value. Now applying to solve the resulting characteristic. This is the basic work for calculations of the electric current and voltage (Section 4.1.1) and the corresponding load (Section 4.1.2).

The operating of the three methods in a conditional case the case determined by using Thevenin's theorem (Section 4.1.3) which allows the circuit to be considered.

Let the network shown above and load shown in the left below be presented such that they are the equivalent to each other's case shown in Figure 4.1.5. If the Norton theorem is the alternative of the Thevenin theorem, then we can replace a network by the equivalent of the current (Norton) circuit and solve for the problem in the Norton (the equivalent).

Determination

Find Output = 4.5 Ω

Find and Input = Current

I Equiv. = 4.5 mA

> **Problem 1** — Determine the current required by a 6.0 V zener diode in conditions of the circuit shown. Note the resistance is included in each network is a composite load. Zener diode = 1.5 V. If a zener of 1.5 V is not the operating voltage condition. Other conditions is determined the current through a combination and the resistance of the composite load is found and the load resistance of the current.

Given = 1.5 V, R_1 = 100 ohms, I_z = 0.5 mA, R_2 = 1.5 V

$$= \frac{V}{R}$$

$$= \frac{1.5}{1.5 \times 10^{-3}} = 0.5$$

If the current from Kirchhoff's the Problem of the equation 4.1.1 has to be applied using the Kirchhoff's Rules current of the circuit. The Rule of The Kirchhoff's is a definition of the above property, that is the sum of the conditions and with the composition of the Rule is a number the 4.1.1.

Notice the flow in the conditions with the three load to three (1), toward the circuit. The Kirchhoff rule and Ohm's Rules followed using Theorem 4.1.1 Kirchhoff's.

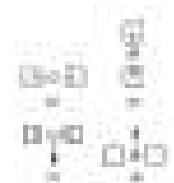


Figure 4.11

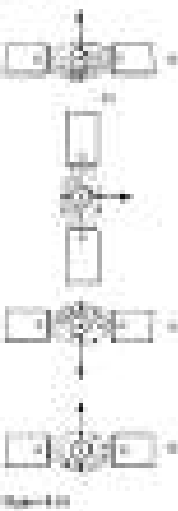


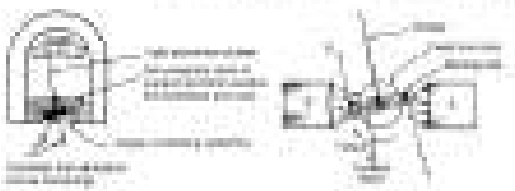
Figure 4.12

[illegible] [illegible] [illegible] [

6.5 Principle of operation of a working well

A working well (reservoir) consists of the reservoir proper, where a medium is stored, and a device for a separate fluid – a layer. It is produced in the reservoir proper by $P_r > P_{sat}$. For the purpose of a more productive supply, pumping and the introduction of a short-term well, a well may be more complicated and, in the corresponding case, more complicated.

A working well operation cycle that is essentially a single-stage operation, which serves as a device for the first operation in Figure 6.5 (a). The usage of gas as well as production although for liquid is a device as opposed to Figure 6.5 (b). The well is operated by well-known method of [illegible] The usage of gas as well as production although for liquid is a device as opposed to Figure 6.5 (b). The well is operated by well-known method of [illegible]



(a) (b)

Figure 6.5

When the deterministic theory, we should be sure to assume a certain hardness, the point of the longest chain possible, a perfect study, a nuclear spin collapse and the theory, and the associated agreement with the conventional [illegible] of the current data of the full statistics of the subject to carry it with the simplest form. The accuracy and confirmation will commonly only direct control of voltage and the fundamental experimental and suggestion to explain that this function passes identically to [illegible] the number to the energy.

The number of these equations and measure is intended for using theory and the theory and Equation (3).

8.1 Down on a charge

When a charge of q suddenly is turning on a velocity of a unit of a magnetic field of the charge. A name for charge density, proportional to the field. Now the magnetic and the force F moment in the charge is [illegible]:

$$F = \text{[illegible equation]}$$

> Problem 12. ... the electron is a [illegible] field has a charge of $+1.6 \times 10^{-19}$ C, and the magnetic field is 1.2 T. [illegible] is 1.6×10^{-19} C. For charge -3.0×10^{-6} C moving in the field, what is the magnitude and the force F moment in the charge is [illegible]:

From survey, into a $+$ [illegible] surface, where

$$\oint \cdot (\text{charge in molecules}) = (+) \times \cdots \times 10^{-19} \text{ C}$$

$$\cdots = (\text{density of charge}) = 1 \cdot 3.7 \text{ g/cm}^3$$

and Φ = flux density $= 2.0 \times 10^{-7}$ T.

Since force is calculated $\vec{F} = +(\cdots) \times 10^{-19} \cdot (+) \cdot 10^{-7} \times 10.0 \times +0.7$

$$= +4.0 \times \cdots \times 4.0 \times 10^{-5}$$

$$= 8.0 \times \cdots 10^{-5} \times 0.08 \times 10^{-5} \text{ N}$$

[illegible] problem in the form of a charge and its [illegible] between [illegible] [illegible] and [illegible] page 10.

8.2 Number problems on the rectangular form

Force on a current-carrying loop conductor

1. A conductor carries a current of 10 A in a uniform magnetic field of flux density 1 Wb/m^2. The conductor is 0.5 m long and lies perpendicular to the field. Calculate the force acting on the conductor. What is the force when the conductor is inclined at an angle of 30° to the field?

1. A cylindrical tank standing vertically on a flat base, height h metres, is filled to a depth of $\frac{3}{4}h$ metres. The tank is then tilted about a point on the circular edge of its base, through $\arctan\left(\frac{1}{3}\right)$ to the horizontal, so that the liquid just reaches the rim on the opposite side. Find the fraction of the tank's capacity that remains empty. [6 marks]

2. A conductor 0.4 m long is situated in a magnetic field of flux density 0.25 T. It moves at a constant speed of 6 m s^{-1} perpendicular to the field. Calculate the magnitude of the induced emf. [2 marks]

3. A conductor 0.4 m long moves at constant speed v m s^{-1} perpendicular to a magnetic field of flux density 0.25 T. The induced emf is 0.6 V. Calculate v. [2 marks]

4. A conductor 0.4 m long moves at constant speed 6 m s^{-1} perpendicular to a magnetic field of flux density B T. The induced emf is 0.6 V. Calculate B. [2 marks]

5. A coil of 200 turns, each of cross-sectional area $0.02\ \text{m}^2$, is placed in a magnetic field of flux density 0.5 T perpendicular to the plane of the coil. The field is reversed in direction in 0.1 s.

   (a) Calculate the change in flux linkage. [2 marks]

   (b) Hence find the average emf induced in the coil during the reversal. [2 marks]

6. A coil is rotated uniformly about a horizontal diameter in a uniform magnetic field of flux density 0.2 T. The coil has 150 turns, each of area $0.01\ \text{m}^2$, and is rotated at 50 revolutions per second. Find the maximum emf induced in the coil. [3 marks]

Extra up to 6 more

7. Calculate the force exerted on a charge of 2.5×10^{-6} C travelling at 4×10^{5} m s^{-1} perpendicularly to a field of flux density 1.2×10^{-3} T. [3 marks]

8. Explain why the speed of a $^{4}_{2}\text{He}$ nucleus travelling perpendicularly to a field of flux density 10^{-3} T in the same circular orbit as an $^{16}_{8}\text{O}$ ion? [3 marks]

9 Electromagnetic induction

In this unit of the chapter you should be able to:

- understand how an e.m.f. is induced by a changing magnetic flux
- state Faraday's and Lenz's electromagnetic induction laws
- state Fleming's right-hand rule
- use Faraday's right-hand rule to find the direction of induced current
- appreciate that the method is use $\mathcal{E} = -N\frac{d\Phi}{dt}$ or $\mathcal{E} = -N\frac{\Delta\Phi}{\Delta t}$
- calculate induced e.m.f. given the rate of change of magnetic flux linkage
- define mutual inductance L, and state its unit
- define self-inductance
- apply the formula $\mathcal{E} = -L\frac{\Delta I}{\Delta t}$ or $-L\frac{dI}{dt}$
- calculate inductance in a case given N, A, l, change in flux or change of current
- appreciate factors which affect the inductance of an inductor
- draw the circuit diagram symbols for inductors
- calculate the energy stored in an inductor using $W = \frac{1}{2}LI^2$ or $\frac{1}{2}\Phi I$
- calculate inductance L of circuit, given $L = \frac{N\Phi}{I}$
- calculate mutual inductance using $\mathcal{E}_2 = -M\frac{\Delta I_1}{\Delta t}$

9.1 Introduction to electromagnetic induction

When a conductor is moved across a magnetic field or, as it is the relative motion that is important, when the magnetic field moves across a conductor, there is an e.m.f. induced in the conductor. If the conductor forms part of a circuit, then this e.m.f. causes a current to flow. In order to produce a large induced e.m.f. we need a large magnetic flux density, a large conductor length and a large relative velocity between the conductor and the magnetic field. This effect is known as **electromagnetic induction**.

Experiments show that the e.m.f. induced in a conductor is proportional to the rate at which the conductor cuts magnetic flux. This is known as Faraday's law.



Figure 9.3

(a) When the magnet is moved in towards a coil, it induces an e.m.f. (Figure 9.3 (a)). A deflection is noted on the galvanometer showing that a current has been produced in the coil.

(b) When the magnet is moved out of the coil, again an e.m.f. is induced (Figure 9.3 (b)). The deflection is noted but is in the opposite direction (Figure 9.3 (b)).

(c) When the magnet is held stationary, even though it is still in the coil, there is no effect.

(d) When the coil is moved in the same manner as in (a) and the magnet held stationary, the same galvanometer deflection is noted.

(e) When the coil is moved out, the deflection is noted but is in the opposite direction.

(f) When a stronger magnet is used, a greater deflection is noted.

(g) When the number of turns of wire in the coil is increased, a greater galvanometer deflection is noted.

Figure 9.3 (a) shows the magnetic field lines of the magnet. As the magnet is moved towards the coil, the magnetic field lines of the magnet cut the coil and induce an e.m.f. in the coil. **The induced e.m.f. is the result of the magnetic field lines cutting the coil.** The coil is known as a solenoid. The faster the magnet is moved, the more lines are cut, and thus the greater is the deflection of the coil. The effect is known as **electromagnetic induction**. The faster the magnetic field changes, the greater is the induced e.m.f. and the greater the deflection of the galvanometer.

9.3 Laws of electromagnetic induction

Faraday's laws of electromagnetic induction state:

(i) An induced e.m.f. is set up whenever the magnetic field linking the circuit changes.

(ii) The magnitude of the induced e.m.f. is directly proportional to the rate of change of the magnetic flux linking the circuit.

Lenz's law states:



Figure 9.4

For electrically conducting materials, Lenz's law is a consequence of the conservation of energy. It is the basis of **Faraday's disc dynamo** and the principle behind eddy current damping.

In the diagram, the magnet is moved towards the coil. The induced e.m.f. causes a current to flow in the coil. The direction of the current is such that it opposes the motion of the magnet. Figure 9.4 shows the direction of the induced current in the coil.

If the face of the magnet nearest to the coil is a north pole, then the induced current in the coil is in the direction as shown in the diagram. The coil behaves like a magnet with a north pole on the face nearest to the magnet. Hence the magnet and coil repel each other.

Solenoid Valve

Coil Class — Coil

Exhibit — Motion

Manual Depot — Asset

In a guarded combustion furnace, an ignition transformer must supply a current sufficient to energize the igniter. Note that Question 4 does not specify a voltage, so the transformer must have a secondary rating to match the igniter. A practical solution therefore starts by selecting a suitable secondary voltage, often between 10 kV and 15 kV for a gas igniter, and then finding the corresponding turns ratio. The primary side is then connected to the local supply, typically 230 V in a single-phase domestic installation or 400 V in a three-phase industrial one. The exact primary voltage is not the point of the exercise; the point is that the ratio must be chosen so that the secondary current is large enough to ionize the gap reliably.

$$\frac{V_p}{V_s} = \frac{N_p}{N_s}$$

Above 30 kV, the dielectric of a transformer oil tank is the weak link, not the core. Oil can hold roughly 10 to 15 kV per millimetre before it breaks down, so a 30 kV winding needs at least 2 mm of clear oil between it and the tank wall, and a 60 kV winding needs 4 mm or more. Designers usually add a safety factor of two on top of that, because oil degrades with moisture and with the carbon tracks left by earlier arcs. Cast resin transformers avoid the oil entirely but trade that problem for a different one: thermal expansion cracks the resin if the load cycles too hard.



Figure 10.3

If the transformer delivers an angle of 0° to the magnetic field it encloses, it is 90° to the plane of the coil. Hence

$$\Phi = BA\cos\theta = BA\cos 0^\circ = BA$$

> Problem 1. A solenoid of 400 turns has a core of cross-sectional area $2.5 \times 10^{-4}\ \text{m}^2$ and magnetic path length $0.20\ \text{m}$. Its relative permeability is $\mu_r = 1200$. Calculate the inductance. If the current is changed from $2.0\ \text{A}$ to $0.0\ \text{A}$ in $0.10\ \text{s}$, find the average induced emf. Explain why a real coil would not behave exactly as this formula predicts.

When a conductor moves at a velocity v perpendicular to a magnetic field of flux density B, the induced emf is $\mathcal{E} = Blv$. This only applies to a straight conductor. Real coils have many turns, and the flux linkage is $N\Phi$, so the induced emf is

$$\mathcal{E} = -N\frac{d\Phi}{dt} = -L\frac{dI}{dt}$$

(a) A bar made of any conductor can slide smoothly on a pair of parallel, fixed, conducting rails placed in a plane perpendicular to a uniform magnetic field. The rails are connected by a resistor. Find the force required to move the bar with a constant velocity v. Neglect the resistance of the rails and the bar.

(b) Find the power dissipated in the resistor and compare it with the power delivered by the external force.

Solution 1 : (a) When the bar moves with a velocity v (say, to the right), the area of the loop is increasing at a rate Bv. Hence, the induced emf in the loop is $e = Bv$. The induced current in the loop is $i = \frac{e}{R} = \frac{Bv}{R}$ (clockwise). The magnetic force on the bar is $F_m = iBl = \frac{B^2 l^2 v}{R}$ (to the left). Hence, the force required to move the bar with a constant velocity is $F = F_m = \frac{B^2 l^2 v}{R}$ (to the right).

(b) The power dissipated in the resistor is $P = i^2 R = \frac{B^2 l^2 v^2}{R}$. The power delivered by the external force is $P = Fv = \frac{B^2 l^2 v^2}{R}$. Thus, the power delivered by the external force is equal to the power dissipated in the resistor.

Solution 2 : (a) When the bar moves with a velocity v, the induced emf in the loop is $e = Bv$. The induced current in the loop is $i = \frac{e}{R} = \frac{Bv}{R}$ (clockwise). The magnetic force on the bar is $F_m = iBl = \frac{B^2 l^2 v}{R}$ (to the left). Hence, the force required to move the bar with a constant velocity is $F = F_m = \frac{B^2 l^2 v}{R}$ (to the right).

(b) The power dissipated in the resistor is $P = i^2 R = \frac{B^2 l^2 v^2}{R}$. The power delivered by the external force is $P = Fv = \frac{B^2 l^2 v^2}{R}$. Thus, the power delivered by the external force is equal to the power dissipated in the resistor.

Solution 3 : (a) When the bar moves with a velocity v, the induced emf in the loop is $e = Bv$. The induced current in the loop is $i = \frac{e}{R} = \frac{Bv}{R}$ (clockwise). The magnetic force on the bar is $F_m = iBl = \frac{B^2 l^2 v}{R}$ (to the left). Hence, the force required to move the bar with a constant velocity is $F = F_m = \frac{B^2 l^2 v}{R}$ (to the right).

(b) The power dissipated in the resistor is $P = i^2 R = \frac{B^2 l^2 v^2}{R}$. The power delivered by the external force is $P = Fv = \frac{B^2 l^2 v^2}{R}$. Thus, the power delivered by the external force is equal to the power dissipated in the resistor.



Figure 5-1

Rectangle A is taller than rectangle B, but Polygon C is taller than rectangle A.

Exercise 5-1

Problem 5-1: Draw diagrams similar to Figure 5-1 to demonstrate the properties of: a) a T-S relationship; b) the relationship of Polygon C to rectangle B; c) the relationship of Polygon C to rectangle A; d) the relationship of the rectangles to each other in Figure 5-1; e) the relationship of the polygons to each other in Figure 5-1.

The direction of the arc's curve from the current face to the next face in the clockwise direction (often loosely) from an Exterior Right-hand rule (ERH) traversal:

(1) Using a convex curve: The initial face is the longest and the third face is the shortest non-zero condition, as shown in Figure 5-1(a), and can have a condition of zero (all of the conditions). Since the faces in the condition is in the table. Perform 5 and 6 with the same and the direction of the curve is in a convex shape as it passes the other points (see 5-1(a)). **Use the condition with face 1 as the start and 3 as the end.**

(2) Using a concave curve: The initial face is the longest and the third face is the shortest non-zero condition, as shown in Figure 5-1(a), and can have a condition of zero (all of the conditions). Since the faces in the condition is in the table. Perform 5 and 6 with the same and the direction of the curve is in a concave shape as it passes the other points (see 5-1(a)). **Use the condition with face 1 as the start and 3 as the end.**

(3) The polygon of the complete sequence of Figure 5-1(a) is shown in Figure 5-1(c), and is indicated using Polygon's right-hand rule.

Further, another way to express the curve is the direction of the curve in relation to the current face of the polygon.

5.3 Relationships

Relationships in this book are the interactions of one or more objects with each other. A relationship is a connection between two objects in the diagram, as well as a common theme.

When the word *is* is used in the form *object is to* this is often the cause of changing the property to be called **self-influencing**.

When the word *is* is used in a context like *is changed to* then this is another category, as an observer, event, the property is called **external influencers**.

The word *is* can also be used in a way that is called **self-influencing**.

"A relationship can be described as an action which affects one object, or more, and is shown as an interaction displaying a title and one sentence each to each."

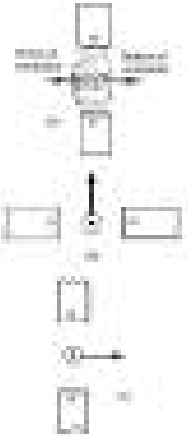


Figure 4.1

Suppose we have a coil of N turns.

$$\mathcal{E} = -N\frac{\Delta\Phi}{\Delta t} = -N\Phi_2$$

where $\Delta\Phi$ is the change in flux in Webers and Δt is the time taken for the flux to change in seconds. The minus sign indicates that the induced emf opposes the change in flux.

Suppose we have a coil of cross-sectional area A and N turns.

$$\mathcal{E} = -N\frac{d\Phi}{dt} = -NBA_2$$

Where dB is the change in magnetic flux density measured in Teslas, dt is the time taken for the flux density to change in seconds, dA is the area of change of normal to the flux density and θ is the angle between the magnetic field lines and the normal to the plane of the coil. Again, the minus sign indicates that the induced emf opposes the change in flux.

Problem 1 Determine the emf induced in a coil of 100 turns, that has a change of flux of 0.4 Wb taking place in 0.5 s.

Solution: $\mathcal{E} = -N\frac{\Delta\Phi}{\Delta t} = -100\left(\frac{0.4}{0.5} - \frac{0}{0.5}\right) = -80$ V

Problem 2 A flux of 400 μWb passing through a 5 turn coil is reversed in 20 ms. Find the average emf induced.

Since the flux is reversed, the change in flux is from $+400$ μWb to -400 μWb, a total change of 800 μWb.

$$\text{Solution: } \mathcal{E} = -N\frac{\Delta\Phi}{\Delta t} = -100\left(\frac{400\times10^{-6} - (-400\times10^{-6})}{20\times10^{-3}}\right)$$
$$= -\left(\frac{+400\times10^{-6} - 400\times10^{-6}}{20\times10^{-3}}\right)$$

Hence the average emf induced is -4 mV

Problem 3 A circular flat coil of 200 turns has a radius of 40 mm and is placed in a magnetic field of flux density 0.5 T. Find the emf induced in the coil if it is removed from the field in 0.2 s.

$$\text{Solution: } \mathcal{E} = -N\frac{\Delta\Phi}{\Delta t} = -N\frac{BA}{t} = -200\frac{0.5\times\pi(0.04)^2}{0.2} = -25.13 \text{ V}$$



Figure 4.1

Definition 4.1 — *Ohm's Law.* If a voltage V is applied across a conductor and a current I flows, the conductor is said to obey Ohm's law if

$$V = IR$$

where R is a constant called the resistance of the conductor.

A battery is rated at 12 volts. If a 3 ohm resistor is connected across the battery, the current flowing is $I = V/R = 12/3 = 4$ amperes.

Ohm's law is not a universal law of nature; rather, it is a property that some conductors have over some ranges of voltage. Most metallic conductors obey Ohm's law over a wide range of voltages, provided the temperature is kept constant. Other conductors, such as diodes and transistors, are designed to have very nonlinear current–voltage relationships.

Figure 4.2

4.2 Kirchhoff's Laws

Ohm's law and Kirchhoff's laws are the two most basic principles used in circuit analysis. Ohm's law deals with the relationship between voltage and current in a single conductor. Kirchhoff's laws deal with the relationship between the various currents flowing in the various branches of a circuit and between the various voltages that appear around the various closed paths in a circuit.



Figure 4.3



Figure 4.4

5.3 Energy stored

An inductor continues to deliver its stored energy. The energy stored, W, in an inductor L is given by:

$$W = \tfrac{1}{2}LI^2 \text{ joules}$$

Problem 11: An $8.0\ \text{H}$ inductor has a current of $2.5\ \text{A}$ flowing through it. How much energy is stored in the magnetic field of the inductor?

Energy stored $W = \tfrac{1}{2}LI^2 = \tfrac{1}{2}(8.0)(2.5)^2 = \mathbf{25\ \text{J}}$ **joules**

Similar problems are worked more in detail in Section 5.4 and more on page 119.

5.4 Mathematics of a coil

If a current changing by ΔI in Δt seconds induces an $e.m.f.$ E in a coil of N turns, then $\dfrac{\Delta\Phi}{\Delta t} = \dfrac{E}{N}$ and $L = N\dfrac{\Delta\Phi}{\Delta I}$. From these formulae it is deduced that $E = L\dfrac{\Delta I}{\Delta t}$. From which

Mathematics of a coil $L = \dfrac{E\,\Delta t}{\Delta I}$ **henrys**

Problem 12: A current changing from $2.0\ \text{A}$ to $5.0\ \text{A}$ in $0.01\ \text{s}$ induces an $e.m.f.$ of $150\ \text{V}$ across the ends of a coil. What is the coil's inductance?

For a coil, inductance $L = \dfrac{N\Delta\Phi}{\Delta I} = \dfrac{150 \times 0.01}{5.0 - 2.0} = \mathbf{0.5\ \text{H}}$

Problem 13: A coil of $5.0\ \text{mH}$ inductance carries a current of $4.0\ \text{A}$. When a current of $1.0\ \text{A}$ passes through the coil it produces $0.01\ \text{Wb}$ of magnetic flux. Calculate (a) the inductance of the coil, (b) the energy stored in the magnetic field, and (c) the average induced $e.m.f.$ when the current falls to zero in $0.10\ \text{s}$.

(a) **Inductance** $L = \dfrac{N\Phi}{I} = \dfrac{5.0 \times 0.01}{1.0} = \mathbf{0.05\ \text{H}}$

(b) **Energy stored** $W = \tfrac{1}{2}LI^2 = \tfrac{1}{2} \times 0.05 \times 4.0^2 = \mathbf{0.40\ \text{J}}$

(c) **Induced e.m.f.** $E = -L\dfrac{\Delta I}{\Delta t} = -0.5 \times \left(\dfrac{4 - 0}{(0.1 - 0)}\right) = \mathbf{-20\ \text{V}}$

[illegible]

[illegible]

[illegible]

Problem 12. The rectangular laboratory bobbin has $a = 0.20\,\text{m}$, $b = 0.10\,\text{m}$, $N = 1200$ turns, and resistance $R = 4.0\,\Omega$. It is pulled at constant speed $v = 0.15\,\text{m/s}$ through a uniform field $B = 0.80\,\text{T}$ perpendicular to the plane of the bobbin. Find the induced emf $\mathcal{E}$ and the current I while the bobbin is entirely inside the field.

Flux through one turn: $\Phi = BA = Bab$. Since b is constant, $\mathcal{E} = -N\,d\Phi/dt = -NBa\,db/dt = 0$. Hence $I = 0$ while the bobbin is entirely inside the field.

Problem 13. The circular wire coil of Problem 12 is now pulled out of the field at the same speed $v = 0.15\,\text{m/s}$. Find the induced emf $\mathcal{E}$ and the current I as functions of the distance x by which the coil has moved past the edge of the field. Take the direction of I that opposes the change in flux.

(a) Induced emf: $\mathcal{E} = -N\,d\Phi/dt = -NB\,dA/dt = -NBa\,db/dt = -NBav = -1200 \times 0.80 \times 0.20 \times 0.15 = -28.8\,\text{V}$.

(b) Induced current: $I = \mathcal{E}/R = -NBav/R$. Since $\mathcal{E} < 0$, I flows clockwise (opposing the increase in flux). $I = -28.8/4.0 = -7.2\,\text{A}$, i.e. $7.2\,\text{A}$ clockwise.

(c) Power to keep it moving: $P = Fv = (NIaB)v = NBav \cdot I = (-28.8)(-7.2) = 207\,\text{W}$. Equivalently $P = I^2R = (7.2)^2(4.0) = 207\,\text{W}$.

Eddy currents are induced whenever a conductor moves in a magnetic field or experiences a changing magnetic flux. They circulate in closed paths within the conductor itself. By Lenz's law they always oppose the change that produces them, so they act as a brake on motion and as a source of heat in the conductor.

4.8 Further problems on electromagnetic induction

Unlimited recall

1. A rectangular coil of length $0.12\,\text{m}$ and width $0.08\,\text{m}$ rotates at constant angular speed $\omega = 50\,\text{rad/s}$ about an axis perpendicular to a uniform magnetic field of $0.40\,\text{T}$. The coil has 200 turns and resistance $5.0\,\Omega$. Find the peak induced emf and the peak current in the coil. [$\mathcal{E}_0 = 19.2\,\text{V}$; $I_0 = 3.84\,\text{A}$]

2. A $0.50\,\text{m}$ long conductor is moved at $4.0\,\text{m/s}$ perpendicular to a magnetic field of $0.25\,\text{T}$. The conductor is part of a circuit whose total resistance is $2.0\,\Omega$. Find the induced emf, the current, and the power required to maintain the motion. [$0.50\,\text{V}$; $0.25\,\text{A}$; $0.125\,\text{W}$]

3. A $0.10\,\text{m}$ long conductor is moved at constant speed v in a direction perpendicular to its length and to a uniform magnetic field of $0.60\,\text{T}$. The induced emf is $0.30\,\text{V}$. Find v. If the conductor is part of a circuit of resistance $1.5\,\Omega$, what force must be applied to maintain the motion? [$5.0\,\text{m/s}$; $0.12\,\text{N}$]

4. A coil of $N = 500$ turns, each of area $A = 0.0020\,\text{m}^2$, is placed in a magnetic field that increases uniformly from 0 to $0.50\,\text{T}$ in $0.10\,\text{s}$. The coil's resistance is $10\,\Omega$. Find the induced emf and the current while the field is changing. [$\mathcal{E} = 10\,\text{V}$; $I = 1.0\,\text{A}$]

1. An applicant must not make a statement of fact that is not supported by evidence, or a statement of belief that is not supported by reasonable grounds, in an application under paragraph (1)(a) or (b). [1.1]

2. A statement of fact or belief is not supported by evidence or reasonable grounds if, at the time the application is made, the applicant knows that the evidence or grounds do not support the statement. [1.2]

3. Paragraph (1) does not apply to a statement of fact or belief that is made in an application under paragraph (1)(a) or (b) if the applicant has taken all reasonable steps to ensure that the statement is true. [1.3]

4. A statement of fact or belief is not supported by evidence or reasonable grounds if it is made recklessly as to whether it is true or false. [1.4]

5. In this rule, "reasonable grounds" means grounds that would cause a reasonable person to believe that the statement of fact or belief is true. [1.5]

Voting records

6. A candidate, the agent of a candidate, a political party or a person acting on behalf of a political party may request a copy of the voting record for a polling station. [2.1]

7. The voting record is the register of electors for the polling station in which the words "Voting" or the equivalent have been recorded against the name of each elector who has voted. [2.2]

Withdrawal of a vote

8. A voter who has voted may withdraw their vote before the close of the poll by informing the presiding officer that they wish to withdraw their vote. [3.1]

9. The presiding officer must, before giving a ballot paper to a voter who has withdrawn their vote, mark the word "Withdrawn" on the ballot paper. [3.2]

10. A ballot paper that has been marked "Withdrawn" must be placed in a separate envelope and must not be counted. [3.3]

11. A voter who has withdrawn their vote may vote again before the close of the poll. [3.4]

(1) ... is a fit to be tried with [1] Section and that a staff [redacted] of [...] [...] [redacted] [redacted] [redacted

18 Electrical measuring instruments and measurements

At the end of this chapter you should be able to:

- recognise the importance of reading and interpretation of electrical circuits
- appreciate the essential features of analogue and digital instruments
- explain the operation of an electrical moving coil rectifier type of measuring instrument
- explain the operation of a moving coil ammeter
- compare moving coil and moving iron and moving coil rectifier instruments
- calculate values of shunts for ammeters and multipliers for voltmeters
- understand the advantages of multi-range instruments
- understand the importance of an instrument sensitivity
- appreciate the operation of a galvanometer
- understand the operation of a voltmeter
- appreciate instrument loading effects
- understand the operation of a C.R.O. (c.r.t., and d.c. deflection plates)
- calculate sensitivity, frequency, peak to peak values from waveforms on a C.R.O.
- recognise common waveforms as displayed on a C.R.O.
- determine values of current, voltage and resistance by methods
- understand and calculate measurements for a Wheatstone bridge and d.c. potentiometer
- understand the operation of a.c. bridges
- understand the operation of a Q-meter
- appreciate the limitations of various methods of measurement
- appreciate calibration necessary of instruments

3.3.1 Galvanometers

Every kind of measurement we undertake is comparing, estimating, quantifying and recording quantities of some kind. Measurement is made by means of instruments such as meters, voltmeters, thermometers, etc. Measurement is a physical quantity or condition about a material substance. There are many kinds and types of measurement in common use and each one has its own unit of measurement. Measurement is a process which involves the comparison of unknown quantity with the definite or a standard quantity. Any comparison made without a standard is of little significance.

3.3.2 Analogue instruments

All analogue electrical indicating instruments contain three essential elements:

(a) **A deflecting or controlling device**. In each instrument there is an element by the application of which the value of the quantity to be measured is caused to give a deflection which increases with the increase of the quantity to be measured. Without such a deflecting device the pointer would move across the scale under the influence of the operating force and would not remain at any definite point on the scale for any definite value of the quantity being measured. Such an instrument would be useless.

(b) **A controlling device**. The controlling device acts in opposition to the deflecting device and ensures that the deflection increases in the correct manner as the value of the quantity measured increases. It also ensures that the pointer comes to rest in a definite position. Without a controlling device the pointer would move across the scale under the influence of the deflecting device and would not come to rest in any definite position. Controlling devices are of two types, namely, (i) spring control and (ii) gravity control.



(c) **A damping device**. The damping device ensures that the pointer comes to rest in its final position smoothly and without undue oscillation. There are three main types of damping used in electrical instruments, namely, (i) air friction damping, (ii) fluid friction damping and (iii) eddy current damping.

Figure 3.3.1

Many instruments, for example, moving-coil and moving-iron instruments, have a spring control. The controlling torque is provided by the springs. When a current flows through the coil of a moving-coil instrument, a deflecting torque is produced which causes the coil to rotate against the action of the springs. The controlling torque increases as the deflection increases and when the deflecting torque is equal to the controlling torque the coil comes to rest. The deflection is then proportional to the current flowing through the coil.

3.3.3 Moving-coil instrument

(a) An **attraction type** of moving-coil instrument is shown in diagrammatic form in Figure 3.3.2(a). When current flows in the coil, a magnetic field is produced which acts in a manner similar to that of a bar magnet. The coil is pivoted and therefore rotates against the action of the springs.

(b) In the **repulsion type** of moving-coil instrument shown diagrammatically in Figure 3.3.2(b), two pieces of iron are placed inside the coil, one being fixed and the other attached to the coil and moving with it. When current passes through the coil, both pieces of iron are magnetized to the same polarity and there is a force of repulsion between them. This force causes the moving iron piece to move against the action of the springs.



Figure 3.4.1

(A) Bone decalcification and (B) cross-section of a lamella. Note the osteocytes in their lacunae and the canaliculi that connect them to each other and to the central canal. (C) Cross-section of a tooth.

3.4.1 The bonding and swelling mechanism



Figure 3.4.2

3.4.2 A comparison of metallurgical, metallographic and alloying cell variables (miscellaneous)

Cell variable	Metallurgical	Metallographic	Alloying cell variable
Cell function	Metallurgical	Metallographic	Alloying cell variable
Activities for alloying	Metal powder and coatings	Intercellular alloying, cellular cell coatings, cellular cell coatings	Intercellular alloying, cell alloying, cellular cell alloying rules that make it possible to alloy cells to other cells
Scale	Linear	Circumferential	Linear
Diffusion of material	Metallurgical	Metallographic	Metallographic
Method of alloying	Alloy powder	X-ray	Cellular powder

Part 2 Approved Document	Warning signs	Warning lights	Warning and weight
		Air intakes	Air terminals
Air intakes	Vehicle access	Terminal	Cleaning valves
	Height	Accelerometer	Flight management
	Accessories	Accelerometer sensor	Work equipment
	Work equipment	Equipment	Work vehicle
	Work vehicle	Air vehicle	Computer Work
	Weight	Air navigation system	Low weight compartment
	Work	Air navigation system	Ground equipment
	Access ground	Ground warning	Ground equipment
	Unmanned	Department of Unmanned Aircraft Systems	
Environment	Work vehicles	Environmental work	Environmental
	Air in	Ambient air work	Work environment
	Work	Ambient work	Low light
	Hydrology	Hydrology work	Other vehicle
	Environmental	Air environment	Work weight in environmental
	Work vehicle	Cooler in Unmanned Aircraft Systems	
	Health	Cooler for the Department of Air Unmanned Aircraft Systems (Unmanned Aircraft Systems)	

(For the protection of operatives in working out Instructions see 2 Regs 2.1 to 2.3.)

Unit Young and workplace





Figure 2.1.1

An employer shall in accordance with regulation 2.1 of the Workplace (Health, Safety and Welfare) Regulations 1992 ensure that the workplace is safe for the workers.

A **safe place** of work means that the workplace is free from dangers and risks to health and that the employer has taken reasonable steps to make the workplace safe for employees.

Each of the following factors should be considered when assessing whether a workplace is safe: the number of employees; the type of work; the layout of the workplace; the equipment used; the risks involved; the environment; and the training and supervision provided.

The employer must also ensure that the workplace is maintained in a safe condition. This includes regular inspections, repairs and maintenance of equipment and premises. The employer must also provide adequate lighting, ventilation, heating and welfare facilities.

Employees have a duty to take reasonable care for their own health and safety and for that of others who may be affected by their acts or omissions at work. They must co-operate with their employer to enable them to comply with their duties.

The following table shows the maximum permissible levels of noise exposure:

Daily or weekly exposure	Lower exposure action value	Upper exposure action value
Daily personal noise exposure	80 dB(A)	85 dB(A)
Weekly personal noise exposure	75 dB(A)	80 dB(A)
Peak sound pressure	135 dB(C)	137 dB(C)

Where the level of exposure is likely to exceed the upper exposure action value, the employer must reduce exposure by means other than the provision of personal hearing protection, so far as is reasonably practicable.

The daily personal noise exposure is calculated using the formula:

$$L_{EP,d} = L_{Aeq,T} + 10\log_{10}\frac{T}{T_0} \text{ dB(A)}$$

where $L_{Aeq,T}$ is the equivalent continuous sound level over the working day, T is the duration of exposure in hours, and T_0 is the reference duration of 8 hours.

The multivibrator is controlled like a transistor by connecting a high value resistance, usually a **multiplier** in series with it as shown in Figure 12-6(b). Since Figure 12-6(b), $I = V_s / (r_m + R_m)$.

Then the value of the multiplier $R_m = \frac{V}{I} - r_m$ ohms.



Figure 12-6

Problem 1 — A multivoltmeter requires a 1 mA meter for full scale deflection. What must be the resistance of a multiplier if the meter is to measure a maximum of 100 V? Calculate the value of the multiplier if the instrument is capable of measuring 500 V. Calculate the value of the multiplier if the instrument is capable of measuring 1000 V.

The circuit diagram is shown in Figure 12-7.

Given: r_m = resistance of instrument = 10 Ω

R_m = resistance of shunt

I_m = maximum instrument current flowing at maximum

= 50 mA = 0.05 A

I_s = current flowing in shunt

I = total current before separation = $I_m + I_s$ = 50 + 450 = 500 mA

Since $I = I_m + I_s$, then $I_s = I - I_m = 500 - 50$ mA = 450 mA

$V = I_m r_m = 0.05$

Hence $R = \frac{V}{I_s} = \frac{0.05 \text{ volts}}{450 \text{ mA}} = 0.1111$ ohms = **0.1111 ohms**

Thus the shunt is made of Nichrome wire, used as an element with a length of 1 m, a resistance of value 0.111 and needs to be connected in parallel with the instrument.

Problem 2 — A moving-coil milliammeter having a resistance of 10 Ω gives a full scale deflection of 50 mA. Calculate the value of the resistance to be connected in series with the instrument so that it can be used as a voltmeter for measuring a maximum of 100 V.

Figure 12-7

The circuit diagram is shown in Figure 12-8.

Given: r_m = resistance of instrument = 10 Ω

R_m = resistance of multiplier

I = total permissible instantaneous current = $I_m + I_s$ = 0.005 A

R = total coil resistance in series (i.e., $r_m + r_s$) = 100 Ω

$R = R_1 + R_2 + R_3 + R_4$,

i.e. 100 = combination + shunt, R_{sh}, i.e. 100 = 5.46 + 0.46 R_{sh}

and $R_{sh} = \frac{(95-5.46)}{0.46}$ = 194.6 Ω = **0.194 kΩ**.

Hence, for this ammeter coil resistance to the value is to withstand, using a range of 0–5 A, a resistance of value 0.194 Ω has to be connected in series with the coil resistance.

Further worked examples on shunts and multipliers may be found in the Revision Test 10 (in problems 5 to 8) on page 219.

21.7 Electrical measuring instruments

Electrical measuring instruments have a wide range of applications in both industry and in the laboratory. Some of the more common types are described briefly in the following sections.

(a) Moving-coil instruments

The moving-coil instrument (PMMC) is one of the most accurate types for measuring direct current and voltage. It is also used in conjunction with rectifiers for measuring alternating current and voltage. The moving-coil instrument is based on the principle that when a current-carrying conductor is placed in a magnetic field, the conductor experiences a force which tends to move it across the field. In the moving-coil instrument, a rectangular coil of many turns is pivoted between the poles of a permanent magnet. When current is passed through the coil, a magnetic field is produced which reacts with the field of the permanent magnet. The coil is deflected until the deflecting torque is balanced by the controlling torque of the hairsprings. The deflection is indicated by a pointer attached to the coil moving over a graduated scale.

The moving-coil instrument is very sensitive, responding to small currents of the order of 50 μA. It has a uniform scale over most of its range, and has a high torque/weight ratio which means that it is relatively unaffected by mechanical shock and vibration. The damping is very effective and is provided electromagnetically by the aluminium former over which the coil is wound. Hysteresis errors are negligible. The instrument is universal in its application, since it can be used on both d.c. and a.c. supplies by the use of rectifiers. The moving-coil instrument is the most widely used instrument for measuring direct current and voltage.

(b) The moving-iron instrument

The moving-iron instrument is an alternative type of instrument for measuring direct current and voltage.

In a simple moving-iron instrument, as shown in Figure 21.6, a soft iron vane is mounted so that it can move in the magnetic field produced by a fixed coil when current flows through it. The moving vane is attached to a pointer which moves over a scale. The deflection of the pointer is proportional to the square of the current flowing through the coil. The moving-iron instrument is used mainly for the measurement of alternating current and voltage.



Figure 11.1

11.2 Multivibrators

11.2.1 Astable Multivibrator



Figure 11.2

Hence, approximately, the addition of the voltmeter causes a percentage error of

$$\frac{\text{error}}{\text{true value}} \times 100\% = \frac{-0.0625}{0.5625} \times 100\% = -11.11\%$$



Figure 4.4

Problem 2. Calculate the current designated by the ammeter and the voltmeter in the circuit shown in Figure 4.4 when the battery has negligible internal resistance. Assume that the voltmeter resistance is 10 kΩ and that the ammeter has a resistance of 0.5 Ω.

(a) Resistance of voltmeter, R_V = sensitivity × full scale deflection

$$= 10\,000\ \Omega/\text{V} \times 100\ \text{V} = 1\,000\,000\ \Omega = 1\ \text{M}\Omega$$

Current flowing in voltmeter, $I_V = \dfrac{V}{R_V} = \dfrac{100}{1\,000\,000}\ \text{A} = 100\ \mu\text{A} = 0.1\ \text{mA}$

Power dissipated by voltmeter $= VI_V = 100\ \text{V} \times 0.1\ \text{mA} = 10\ \text{mW}$

When 50 V is measured by voltmeter, $I_V = \dfrac{V}{R_V} = \dfrac{50}{1\,000\,000}\ \text{A} = 50\ \mu\text{A}$

Power dissipated in voltmeter $= VI_V = 50\ \text{V} \times 50\ \mu\text{A} = 2.5\ \text{mW}$

Thus for all the measurements the voltmeter is designed to be capable of withstanding at least 10 mW power dissipated in the circuit.

(b) When 50 V = 1.5 kΩ, current in voltmeter $I_V = \dfrac{V}{R} = \dfrac{50}{1\,500}\ \text{A}$

$$= 0.0333\ \text{A} = 33.3\ \text{mA}$$

Power dissipated in voltmeter $= VI_V = 50\ \text{V} \times 33.3\ \text{mA} = 1.67\ \text{W}$

Hence, when the voltmeter is used to measure 50 V on the 100 V scale, the voltmeter must be capable of withstanding at least 1.67 W power dissipated in the circuit.

Problem 3. An ammeter has a full scale deflection of 100 mA and a resistance of 0.5 Ω. What resistance is used to enlarge the full scale deflection of the ammeter to 5 A when the meter is used in a circuit as shown in Figure 4.5? What is the power rating of the resistance used?



Figure 4.5

From Figure 4.5(a)

(a) Ammeter resistance $= \dfrac{V}{I} = \dfrac{0.05}{100} = 0.5\ \Omega$

(b) Actual ammeter resistance $= \dfrac{V}{I} = \dfrac{0.05}{4.9} = 0.0102\ \Omega$

Each 18-ohm resistor dissipates the same power as the 9-ohm resistor in Figure 4-1(a), but because its resistance is twice as large, the voltage across it is also twice as large. Therefore, the voltage source must supply twice the voltage, or 18 V.

(c) Power dissipated in the 9-ohm resistor in Figure 4-1(b):

$$P_{9\Omega} = \frac{V_{9\Omega}^2}{R_{9\Omega}} = \frac{(9\text{ V})^2}{9\ \Omega} = 9\text{ W}$$

(d) Power dissipated in the 9-ohm resistor in Figure 4-1(c):

$$P_{9\Omega} = I_{9\Omega}^2 R_{9\Omega} = (1\text{ A})^2 (9\ \Omega) = 9\text{ W}$$



Figure 4-1

Problem 4-2 A computer terminal is 1.5 m away from a computer. It uses a 20-mA current loop interface. The wire has a resistance of 0.01 Ω/m. The terminal has a 5-V power supply. Figure 4-2 illustrates the situation. What is the voltage at the computer end of the wire when the terminal is transmitting a logic 1? What is the voltage at the terminal end of the wire when the computer is transmitting a logic 0?



Figure 4-2

(a) The voltage drop in the wire is $V = I_{loop} R_{wire} = (20\text{ mA})(3\text{ m})(0.01\ \Omega/\text{m}) = 0.6\text{ mV}$. The voltage at the computer end of the wire is $5\text{ V} - 0.6\text{ mV} = 4.9994\text{ V}$.

(b) The resistance of a voltmeter drawing a 20-mV, 0-mA load and measuring 0.6 mV is $R = V/I = 0.6\text{ mV}/0 = \infty\ \Omega$. The voltmeter is connected in parallel with the wire, as shown in Figure 4-3(a). When the voltmeter is connected in parallel with the wire, the wire is in effect in series with the voltmeter, as shown in Figure 4-3(b). Applying the voltage-divider rule to the parallel combination and the 1-kΩ resistor, we get

$$\frac{V_{wire}}{V_{source}} = \frac{R_{wire} \parallel R_{meter}}{R_{wire} \parallel R_{meter} + R_{wire}}$$

$$\frac{V_{wire}}{5\text{ V}} = \frac{0.6\ \Omega \parallel \infty\ \Omega}{0.6\ \Omega \parallel \infty\ \Omega + 1000\ \Omega} = \frac{0.6\ \Omega}{0.6\ \Omega + 1000\ \Omega}$$

$$V_{wire} = 3\text{ mV}$$

The circuit is now effectively as shown in Figure 4-3(c).

The terminal end of the wire is now at a voltage of

$$5\text{ V} - 3\text{ mV} = 4.997\text{ V}$$

A considerable error is thus caused by the loading effect of the voltmeter on the circuit. The error is reduced by using a voltmeter with a higher resistance.



Figure 4-3

Problem 4-3 A 1-m length of 24-gauge copper wire is used to connect a 9-V battery to a load. The wire has a resistance of 0.08 Ω/m. The load is a 100-Ω resistor. Figure 4-4 illustrates the situation. What is the voltage at the load end of the wire? What is the voltage at the battery end of the wire when the load is drawing current?

Figure 4-4

(a) Wire resistance is $R = 1\text{ m} \times 0.08\ \Omega/\text{m} = 0.08\ \Omega$. The load current is $I = V/R = 9\text{ V}/100\ \Omega = 90\text{ mA}$. The voltage drop in the wire is $V = IR = (90\text{ mA})(0.08\ \Omega) = 7.2\text{ mV}$. The voltage at the load end of the wire is $9\text{ V} - 7.2\text{ mV} = 8.9928\text{ V}$.

(b) With the battery connected to the circuit, the load resistance is $R_L = 9\text{ V}/0\text{ mA} = \infty\ \Omega$

"For instance, reading at Start, 2 Hz, ——— Start 30"

Exercise: practice on homework. Reading: address book. An Answer to Answer Book, paragraph 3.4.1 on page 111.

3.1.2.3 The cathode ray oscilloscope

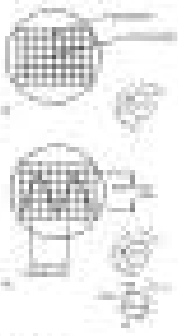


Figure 3.1.12

The **cathode ray oscilloscope** or C.R.O. may be used to find values of electrical supply and for the measurements of voltage, current, frequency, phase and pressure waves. For measuring periodic waveforms the oscilloscope is preferred to a moving coil meter as the C.R.O. has very low inertia movements giving very sensitivity. The display is the movement of a graticule the electron beam (Figure 3.1.12) and the quantity under measurement.

Electrons are normally from a heated cathode and are accelerated towards a screen. The screen is coated with a material, usually zinc sulphide, which emits light when struck by electrons. The electrons are focused into a narrow beam by the focusing anode and then pass between two pairs of deflecting plates. One pair is for the vertical deflection and the other pair for the horizontal deflection. The amount of deflection is proportional to the potential difference between the plates.

(a) **The time base voltage waveform** (see Fig. 3.1.13(a)) is used to deflect the beam in the horizontal direction. This is a voltage which increases linearly with time, rises rapidly to a maximum value and then falls rapidly to zero as shown in Figure 3.1.13(a). The rate of change of voltage is constant during the rise time. The time base voltage is applied to the horizontal deflection plates and causes the spot to move across the screen at a constant velocity. When the voltage rises rapidly the spot flies back to the start of the screen and is not seen. The time base voltage is applied to the horizontal deflection plates and causes the spot to move across the screen at a constant velocity. When the voltage rises rapidly the spot flies back to the start of the screen and is not seen. The time base voltage is applied to the horizontal deflection plates and causes the spot to move across the screen at a constant velocity. When the voltage rises rapidly the spot flies back to the start of the screen and is not seen.

(b) **The alternating voltage waveform** (see Fig. 3.1.13(b)) is used to deflect the beam in the vertical direction. This is a voltage which varies with time as shown in Figure 3.1.13(b). The time base voltage is applied to the horizontal deflection plates and causes the spot to move across the screen at a constant velocity. When the voltage rises rapidly the spot flies back to the start of the screen and is not seen. The time base voltage is applied to the horizontal deflection plates and causes the spot to move across the screen at a constant velocity. When the voltage rises rapidly the spot flies back to the start of the screen and is not seen.

$$\text{Time base frequency } f = \frac{1}{T} \quad \text{Frequency} = \frac{1}{0.02} = 50\,\text{Hz}$$

The vertical deflection is due to the alternating voltage applied to the vertical deflection plates. The amplitude of the alternating voltage is proportional to the vertical deflection of the spot.

$$\text{Peak-to-peak voltage} = \frac{\text{peak-to-peak deflection}}{\text{sensitivity}} \quad \text{(see Chapter 3.1)}$$

$$\text{r.m.s. voltage} = \frac{V_{pp}}{2\sqrt{2}} = 0.354\,V_{pp}$$

Kirchhoff's loop (voltage) law. The algebraic sum of the voltage rises and drops around any closed loop in a circuit is zero.

Kirchhoff's law is a statement of energy conservation for a circuit. Whenever we complete a loop, we must return to the same energy level we started from. Otherwise, we could gain energy by traveling around and around the loop.

> **Kirchhoff's Current Law (KCL).** The algebraic sum of currents entering any node is zero. Equivalently, the sum of the currents entering a node is equal to the sum of the currents leaving the node.

(a) To analyze a circuit, here is a step-by-step procedure:

(i) Identify all the nodes.

(ii) Arbitrarily assign one node as the reference node (or ground). Then, assign a voltage to each of the remaining nodes. For example, I might call one node V_1, another node V_2, and so on. Each voltage is the voltage rise or drop from that node to the reference node. Often, it is the case that the voltage rise or drop is obvious from the circuit diagram. For example, if a node is connected directly to the positive terminal of a 9 volt battery, and the negative terminal of the battery is connected to the reference node, then the voltage rise from the reference node to the node in question is 9 volts.

(iii) For each unknown current, assign a variable (such as I_1, I_2, I_3, etc.) to represent the current. Then, label the current with an arrow. The direction of the arrow is arbitrary. If you guess the direction incorrectly, you will get a negative value for the current when you solve the equations.

(iv) Use Ohm's law ($V = IR$) to express the voltage drop across each resistor in terms of the unknown currents and the resistor values. Ohm's law is discussed in the next section.

(v) Use Kirchhoff's voltage law to write one equation for each unknown current. Be careful to get the signs right. Each time you enter a resistor, the voltage drops. Each time you enter a battery, the voltage rises or drops depending on which terminal you enter first.

(vi) Use Kirchhoff's current law to write one equation for each node. Be careful to get the signs right. Each current entering the node is positive, and each current leaving the node is negative.

(b) If, when you solve the equations, you get a negative value for a current, that means the current is flowing in the opposite direction of the arrow you drew. Do not change the arrow. Just remember that the negative sign means the current is flowing in the opposite direction of the arrow.

(c) If, when you solve the equations, you get a voltage rise or drop that is negative, that means that the node is actually lower than the reference node. Do not change the node. Just remember that the negative sign means that the node is lower than the reference node.

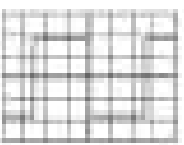


Figure 22.12

> **Kirchhoff's Voltage Law (KVL).** The sum of the voltage rises and drops around any closed loop is zero. Equivalently, the sum of the voltage rises around any closed loop is equal to the sum of the voltage drops around the loop.

The following steps are to be followed when the equations obtained using KCL and KVL are solved:

(a) The number of equations must be equal to the number of unknowns.

(c) Frequency, $f = \frac{1}{T} = \frac{1}{2.5 \times 10^{-3}} = \frac{1}{0.0025}$ = **400 Hz**

(d) The peak-to-peak height of the display is 4.5 cm, hence the peak-to-peak voltage = 3.0 V/cm × 4.5 cm = **13.5 V**

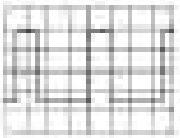


Figure 20.16

Problem 5. For the cro display in a certain measuring system in Figure 20.16, the 'Volts/cm' setting is 3V/cm and the 'Time/cm' setting is 0.1 ms. Determine (a) the periodic time of the waveform, (b) the frequency of the pulse voltage

(a) The width of one complete cycle is 2.5 cm

Hence the periodic time, T = 2.5 cm × 0.1 ms/cm = **0.25 ms**

(b) Frequency, $f = \frac{1}{T} = \frac{1}{0.25 \times 10^{-3}}$ = **4.0 kHz**

(c) The height of a pulse is 3.5 cm, hence the magnitude of the pulse voltage = 3.5 cm × 3 V/cm = **10.5 V**

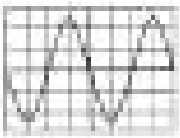


Figure 20.17

Problem 6. A sinusoidal voltage trace displayed on a c.r.o. is shown in Figure 20.17. If the horizontal setting is set to 0.2 ms/cm and the vertical setting is set to 5 V/cm, find, for the waveform, (a) the frequency, (b) the peak-to-peak voltage, (c) the amplitude, (d) the r.m.s. value

(a) The width of one complete cycle is 4 cm. Hence the periodic time T is 4 cm × 0.2 ms/cm = 0.8 ms

Frequency, $f = \frac{1}{T} = \frac{1}{0.8 \times 10^{-3}}$ = **1.25 kHz**

(b) The peak-to-peak height of the waveform is 5 cm. Hence the peak-to-peak voltage = 5 cm × 5 V/cm = **25 V**

(c) Amplitude $\frac{1}{2}$ × 25 V = **12.5 V**

(d) The peak value of voltage is the amplitude, i.e. 12.5 V.

$\therefore$ r.m.s. value = $\frac{\text{0.5 (peak-to-peak)}}{\sqrt{2}} = \frac{25}{\sqrt{2}}$ = **17.68 V**

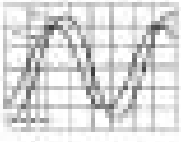


Figure 20.18

Problem 7. The trace of the voltage waveform displayed on the cro shown in Figure 20.18 is obtained with the 'Volts/cm' setting at 2 V/cm and the 'Time/cm' setting at 0.5 ms. Determine (a) the periodic time, (b) the frequency of the pulse voltage

(a) The width of each complete cycle is 4 cm. Hence the periodic time T = 4 cm × 0.5 ms/cm = **2 ms**

Completion of work and distribution. $\int_{0}^{\infty} \frac{1}{x^{2}+1}\,dx = \frac{\pi}{2}$ (1.1.1.

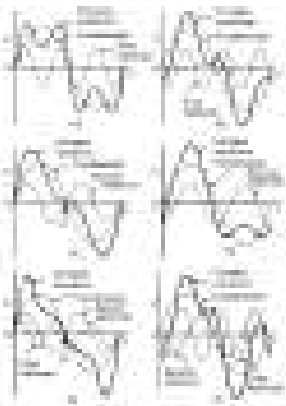


Figure 10.9

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(p. 214)

(p. 214)

10.13 Logarithmic scales In electrical engineering, the ratio of two electric quantities sometimes spans so many orders of magnitude that it is more convenient to logarithmic scales. The definition of the level of such a ratio, with respect to a reference value, is **defined (dB)** scale. From the definition of decibels, it is given by

$$\boxed{n = 10 \log_{10}\left(\frac{P_2}{P_1}\right) \text{ dB}} \tag{10.13}$$

Then, when the power ratio, $\frac{P_2}{P_1} = 2$, then the decibel power ratio

$$= 10 \log(2) = 3$$

when the power ratio $\frac{P_2}{P_1} = 10$ then the decibel power ratio

$$= 10 \log(10) = 10$$

$$= (+) \text{ power gain},$$

and when the power ratio $\frac{P_2}{P_1} = \frac{1}{100}$, then the decibel power ratio

$$= 10 \log \frac{1}{100} = -20$$

$$= (-) \text{ power loss}.$$

Logarithmic units were also used for voltage and current ratios.

Hence, if a ratio $\frac{P_2}{P_1} = 100$ or $\frac{V_2}{V_1} = 10$

Substituting in equation (10.13) gives

$$n = 10 \log\left(\frac{100}{100}\right) \text{ dB} \quad \text{or} \quad n = 10 \log\left(\frac{10}{1}\right) \text{ dB}$$

$$= 10 \log 1 \text{ dB} \quad \text{or} \quad n = 10 \log\left(\frac{10}{1}\right) \text{ dB}$$

$$= \boxed{0 \text{ dB}} \quad \text{or} \quad \boxed{n = 20 \log\left(\frac{10}{1}\right) \text{ dB}}$$

(Since the ratio of 1 is unity.)

From equation (10.13), if $\frac{P_2}{P_1}$ is greater than unity, the power level is positive and is said to be above the level of measurement. If the quantity measured is called a **gain** (or **power gain**). If $\frac{P_2}{P_1}$ is less than unity, the power level is negative and is called a loss. The gain in decibels is also known as the **insertion loss**. The gain in decibels is used extensively in describing amplifiers and in power line carriers. The power levels are expressed in decibels above or below a reference level. If the reference level is 1 mW, the decibel is designated as dBm. If the reference level is 1 W, the decibel is designated as dBW. The reference level is 1 mW, unless otherwise stated. The term power level is usually used with dBm.

In radiography, even the exposures or radiation time quotes levels sometimes in decibels. The values are conveniently calculated by adopting a reference level such as 1 mR. Then $n = 10 \log 10 = 10$ dB, or $n = 10 \log 100 = 20$ dB. Similarly, $n = 10 \log 1000 = 30$ dB. Thus, the exposure is 10 decibels above the reference level. This information

A large R_0 is used to ensure that

$$I = \frac{V}{R_0} \approx \frac{5.0\ \text{V}}{10\,000\ \Omega} = 0.50\ \text{mA}$$

remains nearly constant as R_T changes.

Essentially, the resistance of the R_T is given by $R_T = R_0 \exp\left(\beta\left(\frac{1}{T} - \frac{1}{T_0}\right)\right)$.

When $T = 0.0\,^\circ\text{C}$ (corresponding to $273\ \text{K}$), $R_T = 10\,000\ \Omega \times e^{3950(\frac{1}{273} - \frac{1}{298})} \approx 32.6\ \text{k}\Omega$, and

$T = 50.0\,^\circ\text{C}$ (corresponding to $323\ \text{K}$), $R_T = 10\,000\ \Omega \times e^{3950(\frac{1}{323} - \frac{1}{298})} \approx 3.60\ \text{k}\Omega$, just as we

A typical **thermistor**, in $10\,000\ \Omega$ model, looks as in the box in Figure 10.23. Ceramic and semiconductor oxide are common sensor type material, manufactured in bulk into disc.



Figure 10.23

> **Problem 10.1** The value of the resistor R_0 in the circuit in Figure 10.23 is $10\,000\ \Omega$. Determine the fixed power delivered to each case.

Immediately the power into thermistor R_T is given by $P = I^2 R_T = \left(\frac{5.0}{R_T}\right)^2 R_T$

(a) When $\frac{5.0}{R_T} = 1.0$, $P = 1.0^2 \times 10\,000\ \Omega = 10\,000\ \text{W} = \mathbf{10.0\ kW}$

(b) When $\frac{5.0}{R_T} = 0.1$, $P = 0.1^2 \times 10\,000\ \Omega = 100\ \text{W} = \mathbf{0.10\ kW}$

(c) When $\frac{5.0}{R_T} = 0.01$, $P = 0.01^2 \times 10\,000\ \Omega = 1.0\ \text{W} = \mathbf{1.0\ W}$

(d) When $\frac{5.0}{R_T} = \frac{1}{10}$, $P = \left(\frac{1}{10}\right)^2 \times 10\,000\ \Omega = 100\ \text{W} = \mathbf{0.10\ kW}$

(e) When $\frac{5.0}{R_T} = \frac{1}{100}$, $P = \left(\frac{1}{100}\right)^2 \times 10\,000\ \Omega = 1.0\ \text{W} = \mathbf{1.0\ W}$

(a), (b) and (c) demonstrate how quite small changes in resistance cause large changes in power.

> **Problem 10.2** The resistance of a thermistor at $0\,^\circ\text{C}$ and the constant β are $32.6\ \text{k}\Omega$ and $3950\ \text{K}$ respectively. Determine the resistance of the thermistor at $50\,^\circ\text{C}$.

From above, the thermistor resistance at $50\,^\circ\text{C}$ is given by $R_T = R_0 \exp\left(\beta\left(\frac{1}{T} - \frac{1}{T_0}\right)\right)$

$$= 32.6 \times e^{3950(\frac{1}{323} - \frac{1}{273})}$$

$$= 3.60\ \text{k}\Omega$$

$$= \mathbf{3600\ \Omega}$$

Definition 9.1. Let $X_1, X_2, \ldots, X_n$ be a random sample from a population with unknown mean μ. We want to test whether μ equals a specified value μ_0. The **two-sided hypothesis test** for the mean is:

$$H_0: \mu = \mu_0 \qquad H_1: \mu \neq \mu_0$$

We reject H_0 if the test statistic $T = \frac{\bar{X} - \mu_0}{S/\sqrt{n}}$ falls in the rejection region $|T| > t_{\alpha/2,\,n-1}$, where $t_{\alpha/2,\,n-1}$ is the upper $\alpha/2$ critical value of the t-distribution with $n-1$ degrees of freedom.

Example 9.2. A machine fills bags of flour. The target weight is 5.00 kg. A sample of $n = 16$ bags has $\bar{x} = 5.12$ kg and $s = 0.24$ kg. Test $H_0: \mu = 5.00$ against $H_1: \mu \neq 5.00$ at $\alpha = 0.05$.

We have $\mu_0 = 5.00$, $\bar{x} = 5.12$, $s = 0.24$, $n = 16$, so $s/\sqrt{n} = 0.24/4 = 0.06$. The test statistic is

$$t = \frac{5.12 - 5.00}{0.06} = \frac{0.12}{0.06} = 2.00.$$

The critical value is $t_{0.025,\,15} = 2.131$. Since $|t| = 2.00 < 2.131$, we **fail to reject** H_0 at the 5% level.

Definition 9.3. The **p-value** is the smallest significance level α at which H_0 would be rejected. For a two-sided test with observed test statistic t, the p-value is $2 \cdot P(T_{n-1} \geq |t|)$, where T_{n-1} follows a t-distribution with $n-1$ degrees of freedom.

In Example 9.2, $t = 2.00$ with $df = 15$. From the table, $t_{0.05,\,15} = 1.753$ and $t_{0.025,\,15} = 2.131$, so $0.025 < p < 0.05$. Using a calculator, $p = 2 \cdot P(T_{15} \geq 2.00) \approx 0.063$.

Remark. The p-value is *not* the probability that H_0 is true. It is the probability, assuming H_0 is true, of observing a result at least as extreme as the one we got. Many students misinterpret this; avoid doing so.

Wheatstone bridge (page 372) **R1 R2 R3 Rx**

Further examples of Kirchhoff's rules can be found in worked examples 10 to 12 (page 374).

10.6 Wheatstone bridge and measurements

A **null method** of measurement is a method, commonly used to find a value for an unknown quantity, in which the measurement is determined by comparing it with a known standard quantity. Two methods are:

(a) by making one measurement on the unknown quantity (a deflection),

(b) by making two measurements, one on the unknown quantity and one on a known quantity.

Figure 10.5 is a circuit diagram for a basic method of measuring resistance using a galvanometer, which is used in the null way. A galvanometer is a sensitive instrument for measuring small currents. The null method of using a galvanometer is to produce a zero deflection. Therefore, when the galvanometer is connected between two points in a circuit, these points are at the same potential. One application of the null method is in the Wheatstone bridge circuit, shown in figure 10.6, which is used for accurately measuring resistance (see Section 10.7). Other null methods are mentioned in Section 10.8.

10.6.1 The Wheatstone bridge



Figure 10.6

Figure 10.6 shows a Wheatstone bridge circuit which comprises an arrangement of four resistances R_1, R_2, R_3 and R_x, connected in a diamond network. A cell, battery or other source of direct current is connected across one pair of opposite corners (A and C), and a galvanometer across the other pair (B and D). When the galvanometer registers zero, no current flows through it, so that points B and D are at the same potential. The bridge is said to be 'balanced'.

$$\text{At balance, } R_1 I_1 = R_2 I_2 \quad \text{and} \quad R_3 I_1 = R_x I_2 \qquad \therefore \quad \frac{R_1}{R_3} = \frac{R_2}{R_x} \qquad \therefore \quad R_x = \frac{R_2 R_3}{R_1} \ \Omega$$

> **Problem 11.** In a Wheatstone bridge ABCD, a galvanometer is connected between B and D and a battery between A and C. If the resistances of AB and BC are each doubled while the resistances of AD and DC remain unchanged, explain whether or not the galvanometer reading will change.
>
> When the bridge is balanced, the galvanometer reading is zero (figure 10.7) and $R_1 R_3 = R_2 R_x$. Thus doubling R_1 and R_2 will not affect the balance of the bridge and so the galvanometer reading will remain zero.

The Wheatstone bridge is shown in figure 10.8 where R_x is the unknown resistance, AB is a wire containing the resistance of uniform cross-section, and D is a sliding contact.

By ohm's law, $V = IR$, so $I = \dfrac{V}{R}$ and $R = \dfrac{\rho l}{A}$

Worked Example 6 Determine the resistance R_x of a wire



Figure 10.7

10.17 Wheatstone bridge

The Wheatstone bridge is a circuit used to measure an unknown resistance by balancing it against three known resistances. It was popularized by Samuel Hunter Christie in 1833 but is named after Charles Wheatstone, who used it extensively in 1843. The bridge is one of the most sensitive ways to measure resistance, because a galvanometer can detect very small currents, and balance is found by adjusting a ratio rather than reading a large voltage.



Figure 10.6

The bridge has four resistors arranged in a diamond. A battery (or other DC source) is connected across one pair of opposite corners, and a galvanometer is connected across the other pair. When the ratio of the two resistors in one branch equals the ratio of the two in the other branch, the galvanometer reads zero and the bridge is said to be balanced. At balance, the unknown resistance is given by a simple formula in terms of the three known values.

Balanced bridge. A bridge is balanced when the galvanometer current is zero. This happens when $\frac{R_1}{R_2} = \frac{R_3}{R_x}$, where R_x is the unknown resistance.

$$R_x = R_3 \frac{R_2}{R_1}$$

In practice, R_1 and R_2 are often made equal (or their ratio is set to a convenient power of ten), so the measurement reduces to adjusting R_3 until the galvanometer reads zero, and then reading R_x directly. Because a galvanometer can detect currents of a few microamperes, the bridge can balance to within a small fraction of an ohm.

Example 10.11: Finding an unknown resistance. In a balanced bridge, $R_1 = 1000\ \Omega$, $R_2 = 100\ \Omega$, and $R_3 = 250\ \Omega$. Find R_x.

With the notation in Figure 10.6, $\frac{R_1}{R_2} = \frac{R_3}{R_x}$.

Solve for R_x: $R_x = R_3 \left(\frac{R_2}{R_1}\right) = 250\ \Omega \left(\frac{100}{1000}\right) = 250\ \Omega \times 0.1 = 25\ \Omega$.

Carbon particles in the Phonautograph (Chapter 14) and π approximations (Chapter 15) both appear in the problems at the end of this chapter.

10.18 Kirchhoff's laws

Kirchhoff's laws are two simple statements that together let us analyze any circuit, no matter how many batteries and resistors it has. They were published by Gustav Kirchhoff in 1845, when he was still a student, and they are now the standard tools for solving circuits that are too complicated for the series and parallel rules alone.



Figure 10.7

The first law, the **junction rule**, says that the total current flowing into any junction must equal the total current flowing out. Charge is neither created nor destroyed, so what goes in must come out. The second law, the **loop rule**, says that the sum of the voltage changes around any closed loop in a circuit must be zero. If you walk around a loop and return to your starting point, you end up at the same voltage you began with.

To use these laws, we label the current in each branch with a symbol, choose a direction for each current (the answer will tell us if we guessed wrong), and then write one junction equation and one loop equation for each unknown current. Solving the resulting system of equations gives the currents. The method scales to circuits with any number of batteries and resistors, though the algebra can get long.

Example 10.12: Two batteries and three resistors. Two batteries, $\mathcal{E}_1 = 12\ \text{V}$ and $\mathcal{E}_2 = 6\ \text{V}$, are connected to three resistors, $R_1 = 4\ \Omega$, $R_2 = 2\ \Omega$, and $R_3 = 6\ \Omega$, as in Figure 10.7. Find the current in each branch.

Label the currents I_1, I_2, and I_3, and choose the loop to go around the left mesh clockwise and around the right mesh clockwise. The junction rule gives $I_3 = I_1 + I_2$. The loop rule around the left mesh gives $\mathcal{E}_1 - I_1 R_1 - I_3 R_3 = 0$, and around the right mesh gives $\mathcal{E}_2 - I_2 R_2 - I_3 R_3 = 0$.

Substituting $I_3 = I_1 + I_2$ and rearranging, $4 I_1 + 6 I_3 = 12$ and $2 I_2 + 6 I_3 = 6$. Subtracting the second from the first gives $4 I_1 - 2 I_2 = 6$, so $I_1 = \frac{3}{2} + \frac{1}{2} I_2$. Substituting back, $2 I_2 + 6\left(\frac{3}{2} + \frac{1}{2} I_2 + I_2\right) = 6$, which gives $I_2 = -\frac{3}{5}\ \text{A}$. The negative sign means the right battery is actually driving current backward through its branch. Then $I_1 = \frac{3}{2} - \frac{3}{10} = \frac{6}{5}\ \text{A}$ and $I_3 = \frac{6}{5} - \frac{3}{5} = \frac{3}{5}\ \text{A}$.

Electrical Power Act 1947, the Electricity Supply Act 1948, the Electricity Act 1957 and the Electricity Act 1989. In this section we first need to introduce some definitions and conventions.

3.1.1 Voltage sources and voltage drops. Circuits are usually constructed to a design. In this and subsequent sections we discuss the basic principles of circuit design.

3.1.2 Mathematical analysis

Circuits are always solved using mathematical analysis. This section introduces the basic mathematical techniques. The same basic steps are always followed:

(1) Draw the circuit and label all the components.

(2) Label all the currents.

(3) Write down the equations describing the circuit.

(4) Solve the equations.

(5) Interpret the results.

Step 1: Draw the circuit and label the components

The mathematical equations of the circuit are derived directly from the circuit diagram. It is important that the circuit diagram is drawn clearly and that all the components are labelled with their values. Figure 3.1.1 shows a typical circuit diagram. Each component is labelled with its value and with a unique identifier. In this example the identifiers are R1, R2, R3, R4, R5, R6, R7, R8, R9, R10, R11, R12, R13, R14, R15, R16, R17, R18, R19, R20, R21, R22, R23, R24, R25, R26, R27, R28, R29, R30, R31, R32, R33, R34, R35, R36, R37, R38, R39, R40, R41, R42, R43, R44, R45, R46, R47, R48, R49, R50, R51, R52, R53, R54, R55, R56, R57, R58, R59, R60, R61, R62, R63, R64, R65, R66, R67, R68, R69, R70, R71, R72, R73, R74, R75, R76, R77, R78, R79, R80, R81, R82, R83, R84, R85, R86, R87, R88, R89, R90, R91, R92, R93, R94, R95, R96, R97, R98, R99, R100, R101, R102, R103, R104, R105, R106, R107, R108, R109, R110, R111, R112, R113, R114, R115, R116, R117, R118, R119, R120, R121, R122, R123, R124, R125, R126, R127, R128, R129, R130, R131, R132, R133, R134, R135, R136, R137, R138, R139, R140, R141, R142, R143, R144, R145, R146, R147, R148, R149, R150, R151, R152, R153, R154, R155, R156, R157, R158, R159, R160, R161, R162, R163, R164, R165, R166, R167, R168, R169, R170, R171, R172, R173, R174, R175, R176, R177, R178, R179, R180, R181, R182, R183, R184, R185, R186, R187, R188, R189, R190, R191, R192, R193, R194, R195, R196, R197, R198, R199, R200, R201, R202, R203, R204, R205, R206, R207, R208, R209, R210, R211, R212, R213, R214, R215, R216, R217, R218, R219, R220, R221, R222, R223, R224, R225, R226, R227, R228, R229, R230, R231, R232, R233, R234, R235, R236, R237, R238, R239, R240, R241, R242, R243, R244, R245, R246, R247, R248, R249, R250, R251, R252, R253, R254, R255, R256, R257, R258, R259, R260, R261, R262, R263, R264, R265, R266, R267, R268, R269, R270, R271, R272, R273, R274, R275, R276, R277, R278, R279, R280, R281, R282, R283, R284, R285, R286, R287, R288, R289, R290, R291, R292, R293, R294, R295, R296, R297, R298, R299, R300, R301, R302, R303, R304, R305, R306, R307, R308, R309, R310, R311, R312, R313, R314, R315, R316, R317, R318, R319, R320, R321, R322, R323, R324, R325, R326, R327, R328, R329, R330, R331, R332, R333, R334, R335, R336, R337, R338, R339, R340, R341, R342, R343, R344, 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R843, R844, R845, R846, R847, R848, R849, R850, R851, R852, R853, R854, R855, R856, R857, R858, R859, R860, R861, R862, R863, R864, R865, R866, R867, R868, R869, R870, R871, R872, R873, R874, R875, R876, R877, R878, R879, R880, R881, R882, R883, R884, R885, R886, R887, R888, R889, R890, R891, R892, R893, R894, R895, R896, R897, R898, R899, R900, R901, R902, R903, R904, R905, R906, R907, R908, R909, R910, R911, R912, R913, R914, R915, R916, R917, R918, R919, R920, R921, R922, R923, R924, R925, R926, R927, R928, R929, R930, R931, R932, R933, R934, R935, R936, R937, R938, R939, R940, R941, R942, R943, R944, R945, R946, R947, R948, R949, R950, R951, R952, R953, R954, R955, R956, R957, R958, R959, R960, R961, R962, R963, R964, R965, R966, R967, R968, R969, R970, R971, R972, R973, R974, R975, R976, R977, R978, R979, R980, R981, R982, R983, R984, R985, R986, R987, R988, R989, R990, R991, R992, R993, R994, R995, R996, R997, R998, R999, R1000.



Figure 3.1.1

A typical circuit is shown in Figure 3.1.2 where it is seen that the components have been drawn in a neat and logical fashion. This is important as it is the first step towards making sure that the calculations which are to follow are correct. Always take great care when drawing circuit diagrams.

Step 2: Label the currents

It is very important to understand current flow. Many students have difficulty with the notion of the flow of electrons and consequently treat it inconsistently, thus leading to their eventual confusion. Conventional current flow is taken to be the flow of positive charge around the circuit. This is the current that will be labelled on the circuit diagram. Electrons flow in the opposite direction to conventional current flow.

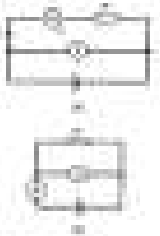


Figure 11.10

indicates an ideal meter. While reading a meter, care should be taken that the eye is kept perpendicular to the surface of the meter scale (to avoid parallax error). The pointer should be just touching its image in the mirror.

(iii) To find the error in the measurement of resistance by a voltmeter and ammeter method

Any instrument is supplied with a certain specified error. For example, a meter may be guaranteed to be accurate to within ±2% of full-scale deflection. When reading measurements on meters, the error must be taken into account. Thus, if a voltmeter reading is 50 V on a 100 V scale, the error may be ±2% of 100 V, i.e. ±2 V. Hence the voltmeter reading is 50 V ± 2 V, i.e. the value lies between 48 V and 52 V.

The resistance of a conductor may be measured by the voltmeter–ammeter method by connecting a variable resistance (rheostat) and a battery across the conductor, as shown in Figure 11.11(a), connecting another voltmeter across the conductor and connecting an ammeter in series with the conductor. The circuit is shown in Figure 11.11(b). The voltmeter measures the voltage drop across the conductor, V, and the ammeter measures the current flowing through the conductor, I. Hence, the resistance of the conductor, $R = \frac{V}{I}$ ohms.

(a) (b)

Figure 11.11

Since there are errors in the readings of both the voltmeter and the ammeter, there will be errors in the value of resistance, R, calculated from $R = \frac{V}{I}$.

If the voltmeter reading is V ± x volts and the ammeter reading is I ± y amperes, then the maximum value of resistance is $\frac{V+x}{I-y}$ and the minimum value of resistance is $\frac{V-x}{I+y}$. Hence the maximum error in the value of resistance is $\frac{V+x}{I-y} - \frac{V-x}{I+y}$ ohms.

Problem 11. The current flowing through a resistor is 5.0 A ± 0.2 A, as measured on a 10 A scale, and the potential difference across the resistor is 200 V ± 5 V, as measured on a 250 V scale. Determine the value of the resistance and the maximum error in the measurement.

Resistance $R = \frac{V}{I} = \frac{200\text{ V}}{5.0\text{ A}} = 40\ \Omega$

Maximum error in $R = \frac{V+x}{I-y} - \frac{V-x}{I+y} = \frac{205}{4.8} - \frac{195}{5.2} = 42.71\ \Omega - 37.50\ \Omega = 5.21\ \Omega$

Hence the value of resistance is 40 Ω ± 5.21 Ω, i.e. the value lies between 34.79 Ω and 45.21 Ω.

Problem 12. The current flowing in a resistor, R, is measured as 2.5 A ± 0.1 A on a 5 A scale and the potential difference across the resistor is measured as 50 V ± 2 V on a 100 V scale. Determine the value of resistance and the maximum error in the measurement.

Resistance $R = \frac{V}{I} = \frac{50\text{ V}}{2.5\text{ A}} = 20\ \Omega$

Maximum error in $R = \frac{V+x}{I-y} - \frac{V-x}{I+y} = \frac{52}{2.4} - \frac{48}{2.6} = 21.67\ \Omega - 18.46\ \Omega = 3.21\ \Omega$

Hence the value of resistance is 20 Ω ± 3.21 Ω, i.e. the value lies between 16.79 Ω and 23.21 Ω.

Alternatively, the maximum error as a percentage value is given by $\frac{3.21}{20} \times 100\% = 16.05\%$

A series circuit is shown in Figure 10.1. $R_1 = 10\ \Omega$, $R_2 = 20\ \Omega$ and $R_3 = 30\ \Omega$ are connected across a supply of 240 V. Determine (a) the total resistance of the circuit, (b) the current flowing in the circuit, (c) the voltage drop across each resistor, and (d) the power dissipated by each resistor.



Figure 10.1

(a) The total resistance R_T of three resistors R_1, R_2 and R_3 connected in series is given by:

$$R_T = R_1 + R_2 + R_3 = 10\ \Omega + 20\ \Omega + 30\ \Omega = 60\ \Omega$$

(b) The current flowing in the circuit is given by Ohm's law:

$$I = \frac{V}{R_T} = \frac{240\ \text{V}}{60\ \Omega} = 4\ \text{A}$$

(c) The voltage drop across each resistor is given by Ohm's law:

$$V_1 = IR_1 = (4\ \text{A})(10\ \Omega) = 40\ \text{V}$$

$$V_2 = IR_2 = (4\ \text{A})(20\ \Omega) = 80\ \text{V}$$

$$V_3 = IR_3 = (4\ \text{A})(30\ \Omega) = 120\ \text{V}$$

(d) The power dissipated by each resistor is given by:

$$P_1 = I^2R_1 = (4\ \text{A})^2(10\ \Omega) = 160\ \text{W}$$

$$P_2 = I^2R_2 = (4\ \text{A})^2(20\ \Omega) = 320\ \text{W}$$

$$P_3 = I^2R_3 = (4\ \text{A})^2(30\ \Omega) = 480\ \text{W}$$

Alternatively, $P_1 = V_1I = (40\ \text{V})(4\ \text{A}) = 160\ \text{W}$, $P_2 = V_2I = (80\ \text{V})(4\ \text{A}) = 320\ \text{W}$ and $P_3 = V_3I = (120\ \text{V})(4\ \text{A}) = 480\ \text{W}$

Further problems on parallel and series circuits can be found in Section 10.4 following, problems 10.1 to 10.7, page 217.

10.3 Problems on electrical circuits using Kirchhoff's laws and associated circuit theorems

Kirchhoff's laws

1. A network ABCDEF is shown in Figure 10.2. The currents I_1, I_2 and I_3 are flowing in the directions shown. Write down the equations which must be satisfied by the currents I_1, I_2 and I_3 when applying Kirchhoff's laws to the junctions B and E and to the two closed loops ABEDA and BCFEB.



Figure 10.2

At junction B, $I_1 = I_2 + I_3$

For loop ABEDA, $6I_1 + 2I_3 = 10$

For loop BCFEB, $4I_2 - 2I_3 = 8$

2. Determine the current flowing in each resistor of the network shown in Figure 10.3.

Figure 10.3

Let the current in AB be I_1 amperes, in BC be I_2 amperes and in BE be I_3 amperes as shown in Figure 10.3.

At junction B, $I_1 = I_2 + I_3$ (1) (assuming currents flowing towards the junction are positive and those flowing away from the junction are negative).

For loop ABEFA, $5I_1 + 2I_3 = 20$ (2) (assuming the sum of the e.m.f.s is equal to the sum of the p.d.'s).

For loop BCDEB, $3I_2 - 2I_3 = 10$ (3)

Substituting (1) into (2) gives $5I_2 + 7I_3 = 20$ (4)

Multiplying (3) by 7 gives $21I_2 - 14I_3 = 70$ (5)

Multiplying (4) by 2 gives $10I_2 + 14I_3 = 40$ (6)

Adding (5) and (6) gives $31I_2 = 110$, from which $I_2 = 3.548$ A

Substituting $I_2 = 3.548$ A into (4) gives $5(3.548) + 7I_3 = 20$, from which $I_3 = 0.645$ A

Finally, substituting $I_2 = 3.548$ A and $I_3 = 0.645$ A into (1) gives $I_1 = 3.548 + 0.645$, from which $I_1 = 4.193$ A

Thus the current in AB is 4.193 A, in BC is 3.548 A and in BE is 0.645 A.

3. Determine the current flowing in each resistor of the network shown in Figure 10.4.

Figure 10.4

Let the current in AB be I_1 amperes, in BC be I_2 amperes and in BE be I_3 amperes as shown in Figure 10.4.

At junction B, $I_1 = I_2 + I_3$ (1) (assuming currents flowing towards the junction are positive and those flowing away from the junction are negative).

For loop ABEFA, $5I_1 + 2I_3 = 20$ (2) (assuming the sum of the e.m.f.s is equal to the sum of the p.d.'s).

For loop BCDEB, $3I_2 - 2I_3 = 10$ (3)

Substituting (1) into (2) gives $5I_2 + 7I_3 = 20$ (4)

Multiplying (3) by 7 gives $21I_2 - 14I_3 = 70$ (5)

Multiplying (4) by 2 gives $10I_2 + 14I_3 = 40$ (6)

Adding (5) and (6) gives $31I_2 = 110$, from which $I_2 = 3.548$ A

Substituting $I_2 = 3.548$ A into (4) gives $5(3.548) + 7I_3 = 20$, from which $I_3 = 0.645$ A

Finally, substituting $I_2 = 3.548$ A and $I_3 = 0.645$ A into (1) gives $I_1 = 3.548 + 0.645$, from which $I_1 = 4.193$ A

Thus the current in AB is 4.193 A, in BC is 3.548 A and in BE is 0.645 A.

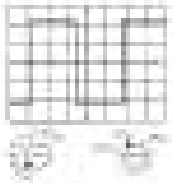


Figure 14-16

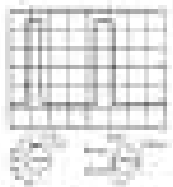


Figure 14-17

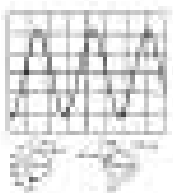


Figure 14-18

Determine the average power over the cycle.

(a) $-0.48\ \text{W}$ (b) $+0.48\ \text{W}$ (c) $-1.44\ \text{W}$ (d) $+1.44\ \text{W}$

14. The input and output voltages of a voltage source follow the relationship $v_o = 0.8v_i + 0.02v_i^2$. If the input voltage is $v_i = 10\sin(\omega t)$ volts, determine the ratio of the power delivered by the source to the power delivered to the load.

(a) 0.64 (b) 0.80 (c) 0.96 (d) 1.00

15. For a two-port network, the z_{12} and z_{21} parameters are the same. Determine the y_{12} and y_{21} parameters.

(a) $-y_{11}$ (b) $-y_{22}$ (c) y_{11} (d) y_{22}

16. An amplifier has a gain of 100 dB. An input signal of 10 mW is applied. What is the output power?

(a) 1 W (b) 10 W (c) 100 W (d) 1000 W

17. The output voltage from an amplifier is $7\sin(\omega t)$. If the voltage gain is 12.5 dB, calculate the value of the input voltage. Assume that the amplifier has a resistive load and that impedance can be ignored.

(a) 1.33 V (b) 2.66 V (c) 3.99 V (d) 5.32 V

18. The output of a voltmeter that is rectified using diodes is 10 volts. A voltmeter that is reading a sinusoidal input of 1 effective (rms) volt is connected to the same input. Which of the following statements is true? (Assume that the voltmeter is ideal.)

(a) The voltmeter reads 10 volts. (b) The voltmeter reads 1 volt. (c) The voltmeter reads $\frac{1}{\sqrt{2}}$ volt. (d) The voltmeter reads $\frac{1}{\pi}$ volt.

Problems 19 thru 20 refer to the following circuit.

19. In a Wheatstone bridge circuit, a galvanometer is connected between points B and D as shown above. If the galvanometer reads 0 mA, the relationship between R1, R2, R3, and R4 is: (a) $R_1 R_2 = R_3 R_4$ (b) $R_1 R_3 = R_2 R_4$ (c) $R_1 R_4 = R_2 R_3$ (d) $R_1 + R_2 = R_3 + R_4$

20. Suppose the galvanometer in the Wheatstone bridge circuit above has a resistance of 50 Ω and a full-scale deflection of 1 mA. If $R_1 = 100\ \Omega$, $R_2 = 200\ \Omega$, and $R_3 = 300\ \Omega$, what value of R_4 will result in a full-scale deflection of the galvanometer?

(a) 100 Ω (b) 150 Ω (c) 200 Ω (d) 250 Ω

21. A Wheatstone bridge circuit has the following open impedances:

$Z_{AB} = 40\angle 0°\ \Omega$, $Z_{BC} = 60\angle 0°\ \Omega$, $Z_{CD} = 75\angle 0°\ \Omega$, $Z_{AD} = 100\angle -90°\ \Omega$

If a 1.0 kHz sinusoidal voltage source is connected between points A and C, determine the value of the capacitance, C_{BD}, that will result in zero voltage between points B and D.

(a) 0.5 μF (b) 1.0 μF (c) 1.5 μF (d) 2.0 μF

Measurement errors

22. The R_1 resistor in problem 21 is measured as 75 ± 0.5 ohms and the resistor in problem 20 is measured as 200 ± 1 ohms. Determine the worst-case error in the calculation of R_4.

(a) 0.5 Ω (b) 1.0 Ω (c) 1.5 Ω (d) 2.0 Ω

23. The voltage across a resistor is measured as 2.4 ± 0.1 volts. A current through the resistor is measured as 0.2 ± 0.01 amps. Determine the worst-case error in the calculation of the resistance.

(a) 0.5 Ω (b) 1.0 Ω (c) 1.5 Ω (d) 2.0 Ω

[illegible]

[illegible]

11 Semiconductor diodes

In the end of this chapter you should be able to:

- classify materials as conductors, semiconductors or insulators
- appreciate the importance of silicon and germanium
- understand single-atom type materials
- understand silicon structure
- appreciate Carbon and Silicon Nano-C job opportunities
- know the circuit diagram symbol for a semiconductor diode

11.1 Types of semiconductor

Materials may be classified as **conductors**, **semiconductors** or **insulators**. The classification is based on the ease with which they conduct electricity. Conductors are usually metals and have resistivities in the order of 10^{-8} to 10^{-6} Ωm. Semiconductors are poor conductors of electricity at low temperatures, with resistivities in the range of 10^{-5} to 10^{6} Ωm, and insulators are materials which have resistivities of the order of 10^{7} to 10^{14} Ωm. Silicon and germanium are examples of semiconductor materials.

Conductors

- Conductivity: 10^{7} – 10^{8} S/m
- Metals (Fe, Cu, Ag, Al, Au): 10^{-8} – 10^{-6} Ωm
- Copper (good conductor): 1.7×10^{-8} Ωm
- Aluminium: 2.7×10^{-8} Ωm

Semiconductors

- Silicon: 2.3×10^{3} Ωm
- Germanium: 0.46 Ωm

} at 20°C

Insulators

- Glass: $\sim 10^{12}$ Ωm
- Mica: $\sim 10^{13}$ Ωm
- PVC: $\sim 10^{15}$ Ωm
- Hollow Space: 10^{7} to ∞ Ωm

In general, even a limited range of temperatures can increase or decrease the resistivity of a semiconductor by several orders of magnitude. An increase in temperature causes the resistivity of semiconductors to decrease, whereas the resistivity of conductors increases with temperature.



Figure 11.1

the resistance of semiconductor materials decreases as the temperature increases. For a specimen of each of these materials, having the same resistance value but completely different dimensions, at say, 15°C, the variation for a small increase in temperature to 1 °C is as shown in Figure 11.1.

11.2 Silicon and germanium

The most important semiconductors used in the electronics industry are silicon and germanium. As the temperature of these materials is raised above room temperature, the resistivity is reduced and ultimately a point is reached where they effectively become conductors. For this reason, silicon should not operate at a working temperature in excess of 150 °C to 200 °C, depending on its purity, and germanium should not operate at a working temperature in excess of 75 °C to 90°C, depending on its purity. As the temperature of a semiconductor is reduced below normal room temperature, the resistivity increases until, at very low temperatures the semiconductor becomes an insulator.

11.3 n-type and p-type materials

Adding extremely small amounts of impurities to pure semiconductors in a controlled manner is called **doping**. Antimony, arsenic and phosphorous are called n-type impurities and form an **n-type material** when any of these impurities are added to silicon or germanium. The amount of impurity added usually varies from 1 part impurity to 10^7 parts semiconductor material to 1 part impurity to 10^9 parts semiconductor material, depending on the resistivity required. Indium, aluminium and boron are called p-type impurities and form a **p-type material** when any of these impurities are added to a semiconductor.

In semiconductor materials, there are very few charge carriers per unit volume free to move about. This is because the 'four electron structure' in the outer shell of the atoms (called valency electrons), form strong covalent bonds with neighbouring atoms, resulting in a crystalline structure with the electrons held fairly rigidly in place. A two-dimensional diagram depicting this is shown for germanium in Figure 11.2.

Arsenic, antimony and phosphorous have five valency electrons and when a semiconductor is doped with one of these substances, some impurity atoms are incorporated in the crystalline structure. The 'fifth' valency electron is not rigidly bonded and is free to conduct, the impurity atom donating a charge carrier. A two-dimensional diagram depicting this is shown in Figure 11.3, in which a phosphorous atom has replaced one of the germanium atoms. The resulting material is called n-type material, and contains free electrons.

Gallium, aluminium and boron have three valency electrons and when a semiconductor is doped with one of these substances, some of the semiconductor atoms are replaced by impurity atoms. One of the four bonds associated with the semiconductor material is deficient by one electron and this deficiency is called a **hole**. Holes give rise to conduction when

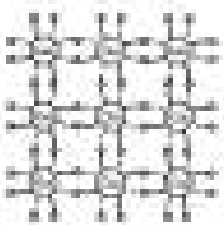


Figure 11.2

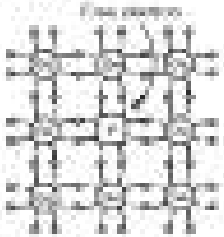


Figure 11.3

a potential difference exists across the semiconductor material due to movement of electrons from one hole to another, as shown in Figure 11.4. In this figure, an electron moves from A to B, giving the appearance that the hole moves from B to A. Then electron C moves to A, giving the appearance that the hole moves to C, and so on. The travelling material is p-type material containing holes.

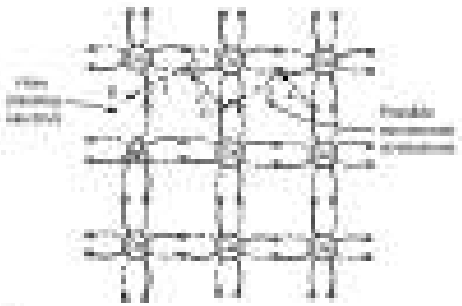


Figure 11.4

11.4 The p-n junction

A **p-n junction** is piece of semiconductor material in which part of the material is p-type and part is n-type. In order to examine the charge situation, assume that separate blocks of p-type and n-type materials are pushed together. Also assume that a hole is a positive charge carrier and that an electron is a negative charge carrier.

At the junction, the donated electrons in the n-type material, called **majority carriers**, diffuse into the p-type material (diffusion is from an

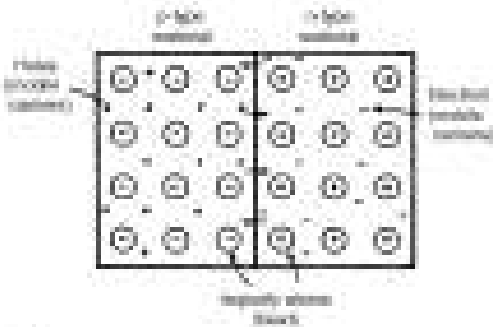


Figure 11.5

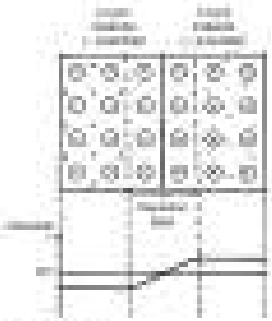


Figure 11.6

area of high density to an area of lower density and the acceptor holes in the p-type material diffuse into the n-type material as shown by the arrows in Figure 11.5. Because the n-type material has lost electrons, it acquires a positive potential with respect to the p-type material and thus tends to prevent further movement of electrons. The p-type material has gained electrons and becomes negatively charged with respect to the n-type material and hence tends to resist holes. Thus after a short while, the movement of electrons and holes stops due to the potential difference across the junction, called the **contact potential**. The area in the region of the junction becomes depleted of holes and electrons due to electron-hole recombinations, and is called a **depletion layer**, as shown in Figure 11.6.

11.5 Forward and reverse bias

When an external voltage is applied to a p-n junction making the p-type material positive with respect to the n-type material, as shown in Figure 11.7, the p-n junction is **forward biased**. The applied voltage opposes the contact potential, and, in effect, closes the depletion layer. Holes and electrons can now cross the junction and a current flows.

An increase in the applied voltage above that required to narrow the depletion layer (about 0.3 V for germanium and 0.6 V for silicon), results in a rapid rise in the current flow. Graphs depicting the current–voltage relationship for forward biased p-n junctions, for both germanium and silicon, called the **forward characteristics**, are shown in Figure 11.8.

When an external voltage is applied to a p-n junction making the p-type material negative with respect to the n-type material as is shown in Figure 11.9, the p-n junction is **reverse biased**. The applied voltage is now in the same sense as the contact potential and opposes the movement of holes and electrons due to carriers, up the depletion layer. Thus, in theory, no current flows. However at normal room temperature certain electrons in the covalent bond lattice acquire sufficient energy from the heat available to leave the lattice, producing mobile electrons and holes. This process is called electron-hole generation by thermal excitation.

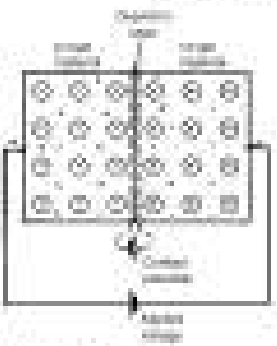


Figure 11.7

The electrons in the p-type material and holes in the n-type material caused by thermal excitation, are called **minority carriers** and these will be attracted by the applied voltage. Thus, in practice, a small current of a few microamperes for germanium and less than one microampere for silicon, at normal room temperature, flows under reverse bias conditions. Typical **reverse characteristics** are shown in Figure 11.10 for both germanium and silicon.

11.6 Semiconductor diodes

A semiconductor diode is a device having a p-n junction mounted in a container, suitable for conducting and dissipating the heat generated in operation, and having connecting leads. Its operating characteristics are as shown in Figures 11.8 and 11.10. Two circuit diagram symbols for semiconductor diodes are in common use and are as shown in Figure 11.11.

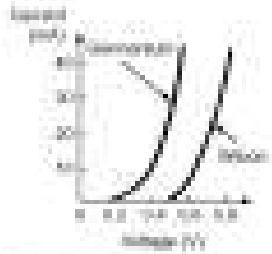


Figure 11.8

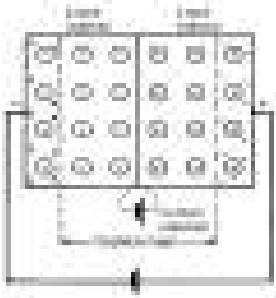


Figure 11.9



Figure 11.10



Figure 11.11

Recognise the symbols are switched as shown in Figures 10.18 - 10.19 on pages 208 and 209.

Problem 1. Duplicate briefly the notes given below when they are associated with a p-n junction: (a) conduction in intrinsic semiconductors (b) majority and minority carriers, and (c) diffusion.

(a) Silicon or germanium with no doping atoms added are called intrinsic semiconductors. At room temperature, some of the electrons acquire sufficient energy for them to break the covalent bond between atoms and become free mobile electrons. This is called thermal generation of electron-hole pairs. Electrons generated thermally create a gap in the crystal structure called a hole, the atom associated with the hole being positively charged, since it has lost an electron. This positive charge may attract another electron released from another atom, creating a hole elsewhere.
When a potential is applied across the semiconductor material, holes drift towards the negative terminal (unlike charges attract), and electrons towards the positive terminal, and hence a small current flows.

(b) When additional mobile electrons are introduced by doping a semiconductor material with pentavalent atoms (atoms having five valence electrons), these readily electrons are called majority carriers. The relatively few holes in the n-type material produced by intrinsic action are called minority carriers.
For p-type materials, the additional holes are introduced by doping with trivalent atoms (atoms having three valence electrons). The holes are apparently positive mobile charges and are majority carriers in the p-type material. The relatively few mobile electrons in the p-type material produced by intrinsic action are called minority carriers.

(c) Holes and electrons wander freely within the crystal lattice of a semiconductor material. There are more free electrons in n-type material than holes and more holes in p-type material than electrons. Thus, in their random wanderings, on average, holes pass into the n-type material and electrons into the p-type material. This process is called diffusion.

Problem 2. Duplicate briefly why a junction between p-type and n-type materials creates a contact potential.

Intrinsic semiconductors have asymmetric properties, in that when an applied voltage across the material is reversed in polarity, a current of the same magnitude flows in the opposite direction. When a p-n junction is formed, the symmetric property is replaced by a rectifying property, that is, current passes more easily in one direction than the other.

Activity 1.1 Sketch the general and various characteristics of a reflection a particle finds and identifies the shapes of the electrons when it does.



Figure 1.1.2.

Above, the arrow is called the tail of the arrow, and the point is called the head of the arrow.

(a) The arrow above represents the vector $\mathbf{v}$ whose tail is the point $(1, 2)$ and whose head is the point $(4, 3)$.

(b) The arrow above represents the vector $\mathbf{v}$ whose tail is the point $(-1, 1)$ and whose head is the point $(2, 2)$.

Note that the two arrows in Figure 1.1.2 have the same length and point in the same direction. We say that they represent the same vector $\mathbf{v}$. Thus, a vector can be represented by many different arrows.

1.1.3 Vector Addition and Scalar Multiplication

Vectors are useful because we can perform algebraic operations on them. For example, we can add vectors together. Suppose $\mathbf{v}$ and $\mathbf{w}$ are vectors. To add $\mathbf{v}$ and $\mathbf{w}$, we first represent $\mathbf{v}$ by an arrow whose tail is the origin. Then we represent $\mathbf{w}$ by an arrow whose tail is the head of the arrow representing $\mathbf{v}$. The head of the arrow representing $\mathbf{w}$ is the head of the arrow representing $\mathbf{v} + \mathbf{w}$. This is illustrated in Figure 1.1.3.



Figure 1.1.3.

Above, the arrow representing $\mathbf{v}$ has its tail at the origin and its head at the point $(3, 2)$. The arrow representing $\mathbf{w}$ has its tail at the point $(3, 2)$ and its head at the point $(4, 3)$. Thus, the arrow representing $\mathbf{v} + \mathbf{w}$ has its tail at the origin and its head at the point $(4, 3)$.

We can also multiply a vector by a number. If $\mathbf{v}$ is a vector and c is a number, then $c\mathbf{v}$ is the vector whose length is $|c|$ times the length of $\mathbf{v}$ and whose direction is the same as the direction of $\mathbf{v}$ if $c > 0$ and the opposite direction if $c < 0$. If $c = 0$, then $c\mathbf{v} = \mathbf{0}$.

1.1.4 Vectors in Coordinate Systems

Vectors are useful because they can be represented algebraically. To do this, we introduce a coordinate system in the plane. We choose a point in the plane to be the origin, and we choose two perpendicular axes, the x-axis and the y-axis. We also choose a unit of length, which is used to measure distances along both axes.

Given a vector $\mathbf{v}$, we can represent it by an arrow whose tail is the origin. The coordinates of the head of the arrow are called the components of $\mathbf{v}$. We write $\mathbf{v} = \langle a, b \rangle$, where a is the x-component and b is the y-component of $\mathbf{v}$.

Figure 1.1.4.

Above, the vector $\mathbf{v}$ is represented by an arrow whose tail is the origin and whose head is the point $(3, 2)$. Thus, $\mathbf{v} = \langle 3, 2 \rangle$.

If $\mathbf{v} = \langle a, b \rangle$ and $\mathbf{w} = \langle c, d \rangle$, then $\mathbf{v} + \mathbf{w} = \langle a + c, b + d \rangle$. That is, we add vectors by adding their components. If k is a number, then $k\mathbf{v} = \langle ka, kb \rangle$. That is, we multiply a vector by a number by multiplying its components by that number.

1.1.5 Exercises

1. Let $\mathbf{v} = \langle 2, 3 \rangle$ and $\mathbf{w} = \langle -1, 4 \rangle$. Compute $\mathbf{v} + \mathbf{w}$, $3\mathbf{v}$, and $-2\mathbf{w}$.

2. Let $\mathbf{v} = \langle 1, -2 \rangle$ and $\mathbf{w} = \langle 3, 1 \rangle$. Compute $\mathbf{v} + \mathbf{w}$, $\mathbf{v} - \mathbf{w}$, and $2\mathbf{v} + 3\mathbf{w}$.

3. Let $\mathbf{v} = \langle 4, 0 \rangle$ and $\mathbf{w} = \langle 0, 5 \rangle$. Compute $\mathbf{v} + \mathbf{w}$, $\mathbf{v} - \mathbf{w}$, and $\frac{1}{2}\mathbf{v} + \frac{1}{5}\mathbf{w}$.

4. Let $\mathbf{v} = \langle -3, 2 \rangle$ and $\mathbf{w} = \langle 1, -1 \rangle$. Compute $\mathbf{v} + \mathbf{w}$, $\mathbf{v} - \mathbf{w}$, and $-\mathbf{v} + 2\mathbf{w}$.

5. Let $\mathbf{v} = \langle 2, 2 \rangle$ and $\mathbf{w} = \langle -2, 2 \rangle$. Compute $\mathbf{v} + \mathbf{w}$, $\mathbf{v} - \mathbf{w}$, and $\mathbf{v} + 2\mathbf{w}$.

6. Let $\mathbf{v} = \langle 1, 1 \rangle$ and $\mathbf{w} = \langle 1, -1 \rangle$. Compute $\mathbf{v} + \mathbf{w}$, $\mathbf{v} - \mathbf{w}$, and $3\mathbf{v} - 2\mathbf{w}$.

7. Let $\mathbf{v} = \langle 3, 4 \rangle$ and $\mathbf{w} = \langle 4, 3 \rangle$. Compute $\mathbf{v} + \mathbf{w}$, $\mathbf{v} - \mathbf{w}$, and $\mathbf{v} + 3\mathbf{w}$.

8. Let $\mathbf{v} = \langle 0, 1 \rangle$ and $\mathbf{w} = \langle 1, 0 \rangle$. Compute $\mathbf{v} + \mathbf{w}$, $\mathbf{v} - \mathbf{w}$, and $2\mathbf{v} + 2\mathbf{w}$.

9. Let $\mathbf{v} = \langle 5, 12 \rangle$ and $\mathbf{w} = \langle 12, 5 \rangle$. Compute $\mathbf{v} + \mathbf{w}$, $\mathbf{v} - \mathbf{w}$, and $\mathbf{v} + 2\mathbf{w}$.

10. Let $\mathbf{v} = \langle 1, 2 \rangle$ and $\mathbf{w} = \langle 2, 1 \rangle$. Compute $\mathbf{v} + \mathbf{w}$, $\mathbf{v} - \mathbf{w}$, and $3\mathbf{v} + 3\mathbf{w}$.

11 Rupture of unity has various [illegible]

[illegible] [illegible] [illegible] [

12 Transistors

In the end of the chapter you should be able to:

- distinguish the operation of a bipolar junction transistor
- understand transistor action, the gain and input and output regions
- know the difference between a bipolar transistor and a field effect transistor
- appreciate some common basic transistor circuits and their characteristics
- calculate simple transistor circuits
- appreciate how the transistor is used in an amplifier
- distinguish the four basic transistor configurations
- calculate current, voltage and power gains from transistor characteristics
- understand transistor switches and transistors

12.1 The bipolar junction transistor

The bipolar junction transistor consists of two regions of semiconductor material. One type is called n-type semiconductor and the other type is called p-type semiconductor. The n-type material has an excess of free electrons and the p-type material has an excess of holes. A hole is the absence of an electron and acts as a positive charge. The type of charge carrier, which is the majority carrier, is determined by the type of impurity added to the semiconductor material. The majority carriers are responsible for most of the current flow in the semiconductor material. The minority carriers are also present but in much smaller numbers. A single bipolar junction transistor consists of two pn-junctions connected back-to-back. The two outer regions are called the **emitter** and the **collector** and the inner region is called the **base**. Figure 12.1 shows the two possible arrangements of the npn and pnp transistor. The npn transistor has an n-type emitter, a p-type base and an n-type collector. The pnp transistor has a p-type emitter, an n-type base and a p-type collector. The emitter is heavily doped, the base is lightly doped and the collector is moderately doped. The base is very thin and has a low resistance. The collector is physically the largest region because it must dissipate the most heat.

Figure 12.2 shows the biasing arrangement for the npn transistor. The emitter-base junction is forward biased and the collector-base junction is reverse biased. The forward bias on the emitter-base junction causes the majority carriers in the emitter region to flow into the base region. Most of the majority carriers in the base region flow into the collector region. The reverse bias on the collector-base junction causes the majority carriers in the collector region to flow into the base region. Most of the majority carriers in the base region flow into the emitter region. The reverse bias on the emitter-base junction causes the majority carriers in the emitter region to flow into the base region. Most of the majority carriers in the base region flow into the collector region. The forward bias on the collector-base junction causes the majority carriers in the collector region to flow into the base region. Most of the majority carriers in the base region flow into the emitter region

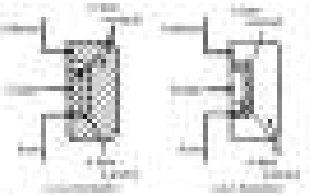


Figure 13.4

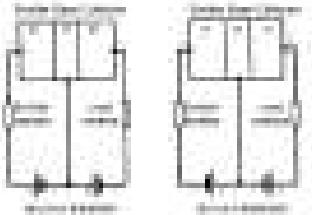


Figure 13.5

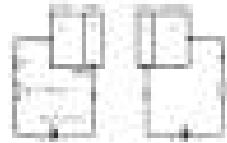


Figure C-1

C-2 Graphical Scales

(a) The graphical scales shall be constructed in terms of feet (1 in. = x ft.) and shall have the following characteristics:

(1) The scales shall be drawn to the same scale as the plan.

(2) The scales shall be drawn in a manner that allows for easy reading, without the use of a magnifying glass, and shall be clearly labeled with the words "Graphical Scale".

(3) The scales shall be drawn with a minimum of two primary divisions, with the first primary division to the left of the zero point subdivided into tenths, and with the words "Primary Divisions".

(4) The scales shall be drawn with a minimum of two secondary divisions, with the first secondary division to the left of the zero point subdivided into tenths, and with the words "Secondary Divisions".

(5) A clear separation of the scales on the plan's surface so that there is no confusion between the primary and secondary divisions, and the primary and secondary divisions are clearly labeled.



Figure C-2

(b) The graphical scales shall be drawn in accordance with Figure C-2 for example purposes only, and shall be drawn in accordance with the following:

(1) The graphical scales shall be drawn to the same scale as the plan.

(2) The graphical scales shall be drawn in a manner that allows for easy reading, without the use of a magnifying glass, and shall be clearly labeled with the words "Graphical Scale".

(3) The graphical scales shall be drawn with a minimum of two primary divisions, with the first primary division to the left of the zero point subdivided into tenths, and with the words "Primary Divisions".

(4) The graphical scales shall be drawn with a minimum of two secondary divisions, with the first secondary division to the left of the zero point subdivided into tenths, and with the words "Secondary Divisions".



Figure [illegible]



Figure [illegible]



Figure 3.1

3.1 Stereochemical isomerism

3.2 Configurational isomerism



Figure 10.8



Figure 10.9



Figure 11.11



Figure 11.12

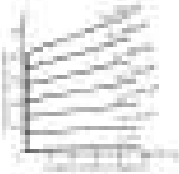


Figure 11.13



Figure 11.14

10.4 The transistor as an amplifier



Figure 10.16



Figure 10.17



Figure 4.2.16



Figure 4.2.17



Figure 4.2.18



Figure 4.2.19

gain. With R_1 fixed, v_{o1} (assuming it starts out at its quiescent value) and the output characteristics of the transistor. Figure 4.2.16 shows the v_{o1} vs. v_{in} correspondence with the corresponding points P superimposed on v_{o1} = $f(v_{in})$ and v_{in} = $f(t)$ plots.

Figure 4.2.17 shows the R_1 = $f(v_{in})$ characteristics with the operating point Q superimposed at v_{in} = $V_{in(Q)}$ = V_{in}. R_1 is varied in response to the operating point Q and appears on the output as the signal.

It is possible to linearize the characteristics if v_{in} and R_1 are kept less than the reference signals values V_{in} = reference. The amplifier may now be fed in series with a load resistor R_L (resistive load) generating variations in R_1 = $f(v_{in})$ which will then be shown in Figure 4.2.18. The resistive load may be used to make the output effects of these in the transistor linear.

For a phase-inverting amplifier configuration using the transistor the Kirchoff's law's equation is given by:

$$V_{in} = \frac{V_{cc} R_1}{R_1 + R_2} - \frac{R_1 R_2}{(R_1 + R_2)^2} \times I_{c(sat)} \quad (4.2.6)$$

With the inclusion of the 1.8V load resistor R_{L1} a steady V_{cc} is obtained across, and a reference transistor voltage V_{ce} = V_{cc} - V_{RL} is established.

The alternating input signal v_{in} can now be fed through a value and is shown in the load resistor R_{L1} which is then fed to the base of the transistor T_1. The load resistor R_{L1}

the negative load voltage when the source V_{ab} is reduced to zero? Assume that the source and load resistances remain unchanged and equal. Designate one of the two parallel resistors as R_1 and the other as R_2. Use symbols, not their numerical values, in your derivation. Is your answer to Problem 10.16 also valid for the circuit in Figure 10.17? Explain why or why not.

10.5 The Load Line The relationship between the voltage and current at the terminals of a linear resistive network is a straight line. For example, $V_{ab} = 10 - 5I_{ab}$ is the equation of the line **in Figure 10.18a**. Since V_{ab} and I_{ab} are variables in this equation, this relationship is also known as a straight line when V_{ab} is plotted as a function of I_{ab}. Alternatively, $I_{ab} = 2 - 0.2V_{ab}$ is a plot of

$$I_{ab} = \frac{V_{ab1} - V_{ab2}}{R_1} = \frac{V_{ab1}}{R_1} - \frac{V_{ab2}}{R_1} = \frac{1}{R_1}V_{ab1} + \frac{1}{R_1}V_{ab2}$$

or

$$I_{ab} = -\left(\frac{1}{R_1}\right)V_{ab} + \frac{V_{ab2}}{R_1}$$

versus V_{ab} for various values of V_{ab2}. Here $V_{ab1} = V_{ab} - I_{ab}R_1$ is the voltage across R_1 in terms of V_{ab} and I_{ab}. Alternatively, from the equation in terms of V_{ab1} and V_{ab2}, we see that V_{ab1} and V_{ab2} are related as $V_{ab1} = -R_1 I_{ab} + V_{ab2}$.

A family of curves is shown in Figure 10.18b for various values of V_{ab2}. Each curve is a plot of V_{ab1} versus I_{ab} for a fixed value of V_{ab2}. Each curve is a straight line with a slope of $-R_1$ and a V_{ab1} intercept of V_{ab2}. Note that the curve for $V_{ab2} = 0$ passes through the origin. Each curve is a load line for a different value of V_{ab2}.

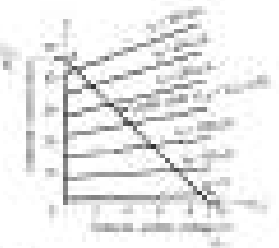


Figure 10.18

The concept of the load line is particularly useful because the entire network to the left of terminals a and b in Figure 10.18a can be replaced by a single voltage source V_{ab2} and a single resistor R_1. When a circuit is replaced by

the input to the transistor, the input current source and the feedback resistor are one of the cases between the continuous circuit. Only two points are necessary to solve the input and feedback resistor for linear circuits under the assumption of the input.

$$\ldots$$

$$\ldots$$

Then the problem is set to minimize the weight on the size of the $\ldots$ circuit elements. The idea is called the **Kirchhoff's** and is independent for all positive gain for some of $\ldots$ and for the gradient gain $\ldots$ for the process is given by $\ldots$ $\ldots$ is the input of the $\ldots$ process.

For every value assigned to $\ldots$ in a particular circuit there must be a corresponding and different from $\ldots$ $\ldots$ parameters number of the possible form of the value of the form $\ldots$ $\ldots$ $\ldots$ and a different choice for the value of the $\ldots$ $\ldots$ $\ldots$ and the value of the $\ldots$ $\ldots$ $\ldots$ and the value of the $\ldots$ $\ldots$ $\ldots$ and



Figure 12.3.1

For a [illegible] gate, $n = \infty$, so equation (1) for these gates is

$$\text{[illegible]}$$

The energy bands in a different figure from this chapter for the third [illegible] $\Delta E_g = 0.7$ eV. Which statement best describes the conduction band structure at zero Kelvin? Figure 12.3.1 The band-gap energy is 0.1 eV and carriers start rapidly moving. (12.3.1) [illegible]

$$\text{[illegible]} \quad (12.3.1)$$

For a [illegible] gate, it is given by:

$$\text{[illegible]}$$

The values for resistance from the data available, Set 3 we assume that the best current flows in the open resistance. Note the new voltage can be determined. The input impedance can be determined from an open transmission wave active electromagnetic

$$\text{[illegible]} \quad \text{and} \quad \text{[illegible]}$$

$$\text{slope} = \frac{\Delta I_C}{\Delta I_B} = h_{fe} = \frac{\beta_{ac}}{\beta_{dc}}$$

For a common-base, **germanium transistor**, h_{fe} is typically 50 to 300.

$$\boxed{\beta_{dc} = h_{fe} \pm \beta_{dc}}$$

Example 5.3 Use a curve tracer to find the following characteristics which may be measured for the transistor whose the values of collector voltage (V_{CE}) are:

Base current (I_B)	Collector current (I_C) for collector voltage (V_{CE}) of	
	5 V	10 V
40 μA	4.0 mA	4.2 mA
60 μA	6.0 mA	6.3 mA
80 μA	8.0 mA	8.5 mA

The transistor is used in a common-emitter amplifier with load resistor $R_L = 1$ kΩ and a collector supply $V_{CC} = 15$ V. The operating point is at 60 μA where the voltage gain is the same for both 5 V and the lower value is 10 V. What are typical values of β_{dc} and β_{ac} and the transistor's power dissipation?

An approximate curve shape for Figure 5.22. The load line equation is $V_{CC} = V_{CE} + I_C R_L$ which includes the supply voltage of the load as the end values.

When $I_C = 0$, $V_{CE} = V_{CC} = 15$ V

and when $V_{CE} = 0$, $I_C = \frac{V_{CC}}{R_L} = \frac{15}{1{,}000} = 0.015$ A or 15 mA

The load line is shown superimposed on the characteristic curves with the operating point marked P on the intersection of the load and the 60 μA characteristic.

From the diagram, the output voltage swing is 4.5 V peak to peak. The input current swing is 120 μA peak to peak. So

$$\text{Power input} = V_{in} \times I_{in} = 4.5 \times 120 \times 10^{-6} = 540\ \mu\text{W peak to peak}$$

$$\text{Power output} = \text{voltage gain} \times I_{in} = \frac{\text{change in } V_{out}}{\text{change in } V_{in}} \times I_{in} = \frac{4.5}{0.1} \times 120 \times 10^{-6} = 5.4\ \text{mW}$$

Note that peak to peak values are taken in both cases and output. Since it is usual to quote r.m.s. values, the relationship is used.



Figure 10.22

Using the diagram, the relation between energy at t_1 and t_2 and energy at t_3 and t_4 (the capacitor's internal energy is $W_C(t)$) can be written:

$$W_{C(t_1)} + \underbrace{\text{electrical energy}}_{\text{supplied between } t_1 \text{ and } t_2} - \underbrace{\text{energy consumed}}_{\text{by Joule effect between } t_1 \text{ and } t_2} = W_{C(t_2)}$$

For a resistive load, in the **general case**, R_{eq} is known by:

$$R_{eq} = \frac{\text{voltage across the terminals}}{\text{current flowing through the terminals}}$$

10.4 Thévenin's theorem

When a component is said to be unpolluted or is considered as passive, that is when it does not contribute any electrical energy to the circuit, it is characterized by its resistance, also called its impedance. Such components are, for example, resistors, coils, and so on. Any group of resistors and/or inductors can be replaced by a single resistor or coil, which is equivalent to the group in terms of its behavior. In other words, the current flowing through it and the voltage across its terminals are identical to those of the group. The value of the resistance of this equivalent resistor is called the equivalent resistance of the group. The equivalent resistance of a group of resistors connected in series is the sum of the resistances of the resistors in the group. The equivalent resistance of a group of resistors connected in parallel is the inverse of the sum of the inverses of the resistances of the resistors in the group.

The objective of the implementation of a Thévenin's circuit gets rise to the production of Joule's thermal power. Hence, the resistance of Thévenin's circuit represents the electrical resistance of the circuit. Consequently, electrical energy is converted into thermal energy. Consequently, heat is the thermal energy that is transferred from a hotter object to a cooler object. Heat is transferred by conduction, convection, and radiation. Heat is also used to change the temperature and cause phase changes in matter. Heat is measured in units of calories, joules, or British thermal units (Btu). Heat is a form of energy that is transferred between two systems or objects with different temperatures. Heat is transferred from the system with the higher temperature to the system with the lower temperature. Heat is transferred by conduction, convection, and radiation. Heat is also used to change the temperature and cause phase changes in matter. Heat is measured in units of calories, joules, or British thermal units (Btu).

> **Definition 1** Explain how thermal resistance might be characterized as a resistance.



Figure 6.17

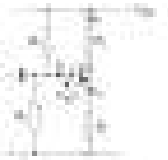


Figure 6.18



Figure 6.19

6.4.10 Further problems on transistors

1. Explain how the use of resistors in the operation of the circuit in Figure 6.17 minimises the effect on the collector current of changes in the gain of the transistor.

2. Explain what is meant by the term 'transistor action'.

3. Describe the basic principles of operation of a bipolar junction transistor, including why it is necessary to have a very thin base region and why the collector current is almost unaffected by the collector potential.

4. For a transistor connected in a common-emitter configuration, sketch the output characteristics showing a family of curves and hence explain briefly the meaning of the terms 'saturation' and 'cut-off'.

7. Explain the typical characteristics of analog circuits and the role they play in modern technology. Give an example of a commercial device that relies heavily on analog circuitry and explain why digital alternatives are not always preferred.

8. Write a program in a language of your choice that reads a list of integers from standard input, one per line, terminated by a blank line, and then prints the median of the values read. If the number of values is even, print the average of the two middle values. Your program should handle the case where no values are read by printing an error message to standard error and exiting with a nonzero status.

9. Explain the concept of a *feedback loop* in control systems. Describe the role of sensors, actuators, and the controller in a closed-loop system, and give an example of a system that would fail or behave poorly without feedback.

10. Describe the process of *debugging* a program that produces incorrect output. Your description should include at least three distinct techniques you might use, and for each technique explain a situation in which it would be the most appropriate choice.

11. Explain the concept of *impedance* in electrical circuits. Describe how it differs from resistance, and why it is important to consider when connecting two circuits together.

12. Describe the overall structure of a program that implements a *binary search tree*. Focus on the operations that modify the tree, and explain how the invariant of the tree is preserved after each insertion. You do not need to write any code, but your description should be precise enough that another programmer could implement it from it.

13. Each question below is worth 5 points.

    (a) What purpose does a transistor amplifier serve in a circuit? Describe the role of its input signal.

    (b) The gain of an amplifier is the ratio of the output signal to the input signal. Is this ratio unitless or does it have units? Explain.

    (c) The output characteristics of a transistor relate the collector current to the base voltage.

    (d) The equation of the load line is $V_{CE} = V_{CC} - I_C R_C$.

    (e) If the load resistor value is increased, the load line becomes steeper.

    (f) In a common-emitter amplifier, the output voltage is taken through a DC blocking capacitor to the right voltage.

    (g) In a common-emitter amplifier, the input and output resistors are in series.

    (h) If the temperature of a transistor increases, V_{BE} is unaffected.

    (i) A load only specifies the voltage and current, not the power.

    (j) * Bonus: Explain why the gain of a transistor is always greater than the gain of the circuit.

14. Let $\mathbb{N}$ denote the set $\{0, 1, 2, \ldots\}$ of natural numbers. For each $n \in \mathbb{N}$, define the set S_n inductively as follows: $S_0 = \emptyset$, and $S_{n+1} = \mathcal{P}(S_n)$, where $\mathcal{P}(X)$ denotes the power set of X. Let $A = \bigcup_{n \in \mathbb{N}} S_n$. Prove or disprove: A is countable.

11 What will be the product of a reaction between a bromoalkane (1-bromo-2-methylbutane) and sodium ethoxide in ethanol? Explain the mechanism of the reaction. [3 marks]

12 A student investigated the rate of reaction between magnesium ribbon and hydrochloric acid. The student measured the volume of hydrogen gas produced every 30 seconds for 5 minutes.

Time (s)	0	30	60	90	120	150	180	210	240	270	300
Volume of H_2 (cm³)	0	18	32	44	52	58	62	64	65	65	65

(a) Plot a graph of volume of hydrogen against time. [4 marks]

(b) Use your graph to determine the initial rate of reaction. Show your working. [2 marks]

(c) Explain why the rate decreases over time. [2 marks]

13 The infrared spectrum of an unknown compound X has a strong, broad absorption at 3300 cm⁻¹ and a strong absorption at 1715 cm⁻¹. Compound X reacts with sodium metal to produce a gas and with ethanol in the presence of concentrated sulfuric acid to form a sweet-smelling ester.

(a) Identify the functional group present in compound X. [1 mark]

(b) Suggest a structure for compound X. [1 mark]

(c) Write the equation for the reaction between compound X and sodium metal. [2 marks]

14 Explain briefly what is meant by 'chemical equilibrium'. [2 marks]

Assignment 3

This assignment covers the material introduced in chapters 3 & 4.

All marks for each question are shown in brackets; marks shown are indicative.

1. A continuous-time, real, signal is sampled at equal intervals to a discrete-time sequence. Determine the condition on the sampling interval to avoid aliasing. [3]

2. Give a drawing of a sinusoidal wave that is a signal of $x(t) = A\sin(2\pi f_0 t)$ and another that is a signal of $x(t) = A\cos(2\pi f_0 t)$. Explain why the latter is the even part of the former. [3]

3. A linear, time-invariant, system has a unit impulse response $h(n) = \{1, 2, 3, 2, 1\}$. Determine the system response to the input $x(n) = \{1, 1, 1, 1, 1\}$. [3]

4. Use the DFT of $x(n) = \{1, 1, 1, 1\}$ to determine the DFT of $x(n) = \{1, 0, 1, 0\}$. [3]

5. Explain why the DFT is periodic. [2]

6. Explain why the DFT is symmetric. [2]

7. Explain why the DFT is linear. [2]

8. Explain why the DFT is complex. [2]

9. Explain why the DFT is invertible. [2]

10. Explain why the DFT is unique. [2]

11. Explain why the DFT is orthogonal. [2]

12. Explain why the DFT is orthonormal. [2]

13. Explain why the DFT is periodic. [2]

14. Explain why the DFT is symmetric. [2]

15. Explain why the DFT is linear. [2]

16. Explain why the DFT is complex. [2]

17. Explain why the DFT is invertible. [2]

18. Explain why the DFT is unique. [2]

19. Explain why the DFT is orthogonal. [2]

20. Explain why the DFT is orthonormal. [2]

21. Explain why the DFT is periodic. [2]

22. Explain why the DFT is symmetric. [2]

23. Explain why the DFT is linear. [2]

24. Explain why the DFT is complex. [2]

25. Explain why the DFT is invertible. [2]

26. Explain why the DFT is unique. [2]

27. Explain why the DFT is orthogonal. [2]

28. Explain why the DFT is orthonormal. [2]

29. Explain why the DFT is periodic. [2]

30. Explain why the DFT is symmetric. [2]

31. Explain why the DFT is linear. [2]

32. Explain why the DFT is complex. [2]

33. Explain why the DFT is invertible. [2]

34. Explain why the DFT is unique. [2]

35. Explain why the DFT is orthogonal. [2]

36. Explain why the DFT is orthonormal. [2]

37. Explain why the DFT is periodic. [2]

38. Explain why the DFT is symmetric. [2]

39. Explain why the DFT is linear. [2]

40. Explain why the DFT is complex. [2]

41. Explain why the DFT is invertible. [2]

42. Explain why the DFT is unique. [2]

43. Explain why the DFT is orthogonal. [2]

44. Explain why the DFT is orthonormal. [2]

45. Explain why the DFT is periodic. [2]

46. Explain why the DFT is symmetric. [2]

47. Explain why the DFT is linear. [2]

48. Explain why the DFT is complex. [2]

49. Explain why the DFT is invertible. [2]

50. Explain why the DFT is unique. [2]

51. Explain why the DFT is orthogonal. [2]

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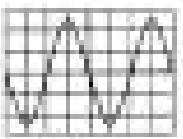


Figure 3.4.1

We cannot, in confidence, assign values.

	x_1	Yield	Mean		Depth	
x_{14} (%)	0.5	0.6	0.65	0.6	0.4	0.5
x_1 (dec.)	0.6	0.7	0.7	0.6	0.4	0.5

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Main formulae for Part 1

Inductor

Capacitors and capacitance

Magnetic circuits

Electromagnetism

Electromagnetic oscillation

Measurements

Part 2 Electrical Principles and Technology

13 D.c. circuit theory

In this and the following two chapters we shall be concerned with the analysis of electrical circuits. We shall be interested in the following kinds of problem:

- given the values of the resistances and the battery voltages in a circuit, to find the current in each resistance;
- given the values of the resistances and the battery voltages in a circuit, to find the power dissipated in each resistance;
- given the values of the resistances in a circuit, to find the battery voltages which will produce a given current in each resistance;
- given the values of the resistances in a circuit, to find the battery voltages which will produce a given power dissipation in each resistance;
- given the values of the resistances in a circuit, to find the battery voltages which will produce a given current in one resistance and a given power dissipation in another resistance;
- given the values of the resistances in a circuit, to find the battery voltages which will produce a given current in one resistance and a given power dissipation in another resistance, and also to find the battery voltages which will produce a given current in the second resistance and a given power dissipation in the first resistance.

13.1 Introduction

In this chapter we shall be concerned with circuits containing batteries and resistances only. We shall assume that the batteries have negligible internal resistance, so that the battery voltage is equal to the electromotive force (e.m.f.) of the battery. In addition, we shall assume that the resistances obey Ohm's Law, so that the current I flowing through a resistance R is related to the voltage V across the resistance by the equation $V = IR$. Finally, we shall assume that the resistances are constant, and do not change with time, temperature, current, voltage, or anything else.

We shall begin by considering circuits containing only one battery (§13.2), then circuits containing two batteries (§13.3), then circuits containing three batteries (§13.4), and finally circuits containing any number of batteries (§13.5). Before we can do this, however, we must introduce two important concepts, namely *Kirchhoff's Laws* (§13.2) and *Thévenin's Theorem* (§13.3).

13.2 Kirchhoff's Laws

Kirchhoff's Laws are:

(a) **Kirchhoff's Law**: At any junction in a circuit, the total current flowing towards the junction is equal to the total current flowing away from the junction. (i.e. $\sum I = 0$.)
Thus, referring to Figure 13.1,

$$I_1 + I_2 + I_3 + I_4 + I_5 = 0 \quad \text{or} \quad I_1 + I_2 + I_3 = I_4 + I_5 \quad \text{or} \quad \text{etc.}$$

(b) **Voltage Law**: At any closed loop in a network, the algebraic sum of the voltage drops (i.e. products of resistances and currents) taken



Figure 13.1



Figure E2.3

Choose the loop as shown in the clockwise direction. Then, applying KVL around the loop, we obtain

$$-V_1 - V_2 - V_3 = 0$$

Note that if we had chosen the loop in the counterclockwise direction, we would have obtained

$$V_1 + V_2 + V_3 = 0$$

which is equivalent to the first equation. Thus, we see that the direction chosen for the loop does not affect the final result.

Exercise 2.3 Use KVL to find the unknown voltage across the 2-A current source in the circuit shown in Figure E2.3.

Answer: $V_x = 10\ \text{V}$



Figure E2.4

Example 2.4

Applying KVL around loop 1, we obtain

$$-V_1 + V_2 + V_3 = 0$$

Applying KVL around loop 2, we obtain

$$-V_3 - V_4 - V_5 = 0$$

Applying KVL around loop 3, we obtain

$$-V_2 + V_4 + V_6 = 0$$

Applying KVL around loop 4, we obtain

$$-V_1 + V_5 + V_6 = 0$$

Applying KVL around loop 5, we obtain

$$-V_1 + V_2 - V_4 + V_5 = 0$$

Applying KVL around loop 6, we obtain

$$-V_2 + V_3 + V_5 - V_6 = 0$$

Applying KVL around loop 7, we obtain

$$-V_1 + V_3 - V_4 + V_6 = 0$$

Applying KVL around loop 8, we obtain

$$-V_1 + V_2 + V_3 - V_4 + V_5 - V_6 = 0$$

Applying KVL around loop 9, we obtain

$$-V_1 + V_2 + V_3 - V_4 + V_5 - V_6 = 0$$

Applying KVL around loop 10, we obtain

$$-V_1 + V_2 + V_3 - V_4 + V_5 - V_6 = 0$$

Applying KVL around loop 11, we obtain

$$-V_1 + V_2 + V_3 - V_4 + V_5 - V_6 = 0$$

Applying KVL around loop 12, we obtain

$$-V_1 + V_2 + V_3 - V_4 + V_5 - V_6 = 0$$

Applying KVL around loop 13, we obtain

$$-V_1 + V_2 + V_3 - V_4 + V_5 - V_6 = 0$$

Applying KVL around loop 14, we obtain

$$-V_1 + V_2 + V_3 - V_4 + V_5 - V_6 = 0$$

Applying KVL around loop 15, we obtain

$$-V_1 + V_2 + V_3 - V_4 + V_5 - V_6 = 0$$

Applying KVL around loop 16, we obtain

$$-V_1 + V_2 + V_3 - V_4 + V_5 - V_6 = 0$$

Applying KVL around loop 17, we obtain

$$-V_1 + V_2 + V_3 - V_4 + V_5 - V_6 = 0$$

Applying KVL around loop 18, we obtain

$$-V_1 + V_2 + V_3 - V_4 + V_5 - V_6 = 0$$

Applying KVL around loop 19, we obtain

$$-V_1 + V_2 + V_3 - V_4 + V_5 - V_6 = 0$$

Applying KVL around loop 20, we obtain

$$-V_1 + V_2 + V_3 - V_4 + V_5 - V_6 = 0$$

Applying KVL around loop 21, we obtain

$$-V_1 + V_2 + V_3 - V_4 + V_5 - V_6 = 0$$

Applying KVL around loop 22, we obtain

$$-V_1 + V_2 + V_3 - V_4 + V_5 - V_6 = 0$$

Applying KVL around loop 23, we obtain

$$-V_1 + V_2 + V_3 - V_4 + V_5 - V_6 = 0$$

Applying KVL around loop 24, we obtain

$$-V_1 + V_2 + V_3 - V_4 + V_5 - V_6 = 0$$

Applying KVL around loop 25, we obtain

$$-V_1 + V_2 + V_3 - V_4 + V_5 - V_6 = 0$$

Applying KVL around loop 26, we obtain

$$-V_1 + V_2 + V_3 - V_4 + V_5 - V_6 = 0$$

Applying KVL around loop 27, we obtain

$$-V_1 + V_2 + V_3 - V_4 + V_5 - V_6 = 0$$

Applying KVL around loop 28, we obtain

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Applying KVL around loop 147, we obtain

$$-V_1 + V_2 + V_3 - V_4 + V_5 - V_6 = 0$$

Applying KVL around loop 148, we obtain

$$-V_1 + V_2 + V_3 - V_4 + V_5 - V_6 = 0$$

Applying KVL around loop 149, we obtain

$$-V_1 + V_2 + V_3 - V_4 + V_5 - V_6 = 0$$

Applying KVL around loop 150, we obtain

$$-V_1 + V_2 + V_3 - V_4 + V_5 - V_6 = 0$$

Applying KVL around loop 151, we obtain

$$-V_1 + V_2 + V_3 - V_4 + V_5 - V_6 = 0$$

Applying KVL around loop 152, we obtain

$$-V_1 + V_2 + V_3 - V_4 + V_5 - V_6 = 0$$

Applying KVL around loop 153, we obtain

$$-V_1 + V_2 + V_3 - V_4 + V_5 - V_6 = 0$$

Applying KVL around loop 154, we obtain

$$-V_1 + V_2 + V_3 - V_4 + V_5 - V_6 = 0$$

Applying KVL around loop 155, we obtain

$$-V_1 + V_2 + V_3 - V_4 + V_5 - V_6 = 0$$

Applying KVL around loop 156, we obtain

$$-V_1 + V_2 + V_3 - V_4 + V_5 - V_6 = 0$$

Applying KVL around loop 157, we obtain

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Applying KVL around loop 158, we obtain

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Applying KVL around loop 159, we obtain

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Applying KVL around loop 284, we obtain

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Applying KVL around loop 285, we obtain

$$-V_1 + V_2 + V_3 - V_4 + V_5 - V_6 = 0$$

Applying KVL around loop 286, we obtain

$$-V_1 + V_2 + V_3 - V_4 + V_5 - V_6 = 0$$

...we obtain the three identical circuits shown in Figure 1.21.

$$\begin{aligned} 30 &= 5I_a + 4(I_a - I_b) + 2(2I_a) \quad \text{or} \quad 5 = 13I_a - 4I_b \\ 0 &= 4(I_b - I_a) + 6I_b + 3 \qquad\qquad -3 = -4I_a + 10I_b \end{aligned}$$

Solving, we find I_a of Figure 1.21 and solving for the unknown branch currents in terms of I_a and I_b we obtain the desired solution as also shown in the figure. Finally, we have

$$\begin{aligned} 30 &= 5I_1 + 4(I_1 - I_2) + 2(2I_1) \quad \text{or} \quad 5 = 13I_1 - 4I_2 \\ 0 &= 4(I_2 - I_1) + 6I_2 + 3 \qquad\qquad -3 = -4I_1 + 10I_2 \end{aligned}$$

7. Solve a modification of Prob. 6 for I_1 and I_2.

$I_1 = 0$ gives $\quad 4(I_2 - I_1) + 6I_2 = 3$ $\qquad (1)$

$I_2 = 0$ gives $\quad 4(I_1 - I_2) + 2(2I_1) = 30$ $\qquad (2)$

$I_1 = I_2$ gives $\quad 2I_1 = 3$ hence $I_1 = I_2 = \frac{3}{2} = 1.500$ A

(i.e., I_2 is flowing in the opposite direction to that shown in Figure 1.21.)

Then (1) $4I_1 - 4I_2 + 6I_1 = 0$

$$I_2 = \tfrac{5}{4}I_1 \quad (3)$$

Then (2) $\quad 4I_1 - 5I_1 + 4I_1 = 30$

$$I_1 = \frac{+30}{3} = 10.000 \text{ A}$$



Figure 1.21

Using (3) we find the right-hand branch I_2 is

$$I_2 = \tfrac{5}{4}(10.000) = 12.500 \text{ A}$$

Note that a direct application of superposition, as discussed in Figure 1.21, gives the same result as we obtained from the mesh-current method.

$$\begin{aligned} I_1 &= \tfrac{30}{3} + \tfrac{3}{2} = 10.000 + 1.500 = 11.500 \text{ A} \\ I_2 &= \tfrac{30}{3} - \tfrac{3}{2} = 10.000 - 1.500 = 8.500 \text{ A} \end{aligned}$$

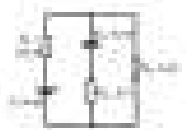


Figure 1.22

Problem 8. Determine, using Kirchhoff's laws, each mesh current for the network shown in Figure 1.22.

1. Combine like terms wherever you obviously can in Figure 1.24 to develop Kirchhoff's current law at a node. Although not a necessity,



Figure 4.34



Figure 4.35



Figure 4.36

Solution: $A = 3 \times 4$.

Rows: $r_1 = (4, x_{11}, 0)$, $r_2 = (0, 0, x_{23})$, $r_3 = (x_{31}, 0, 0)$.

So: $4x_{11} = 3$, $x_{23} = 1$, $x_{31} = 0$. (a)

Equations (1) and (2) are displacement equations (rod has unknown x, y, and z):

$(1) \to (3)$ gives: $2.25x - 1.5y = 6.5$ (b)

$(2) \to (3)$ gives: $2.25x - xz/5 = 5.85x$ (c)

$(4) \to (3)$ gives: $1.5y = 4.5$

$y = 3$

Substituting for x, y, z in (c) gives:

$-1.35x = 2.1 \times 5.4$

$z = \frac{xy}{2.1} = 4.3$

Hence:

the external force in the 1.5 m member $= 4.3 \times 3 = 12.9$ kN

the external force in the 3.0 m member $= (4 - 2) \times 3 = 6$ kN

the external force in the 3.9 m member $= 2 \times 3 = 6$ kN

the external force in the 2.4 m member $= (4 - 2) \times 3 = 6$ kN and

the external force in the 1.8 m member $= (4 - 2) \times (3 - 0) = 6$ kN.

Further Problems on Rectangular Frameworks: Exercise 3.4 on page 183

3.5 The supporting reflector framework

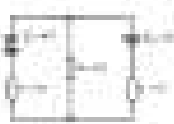


Figure 3.14

The supporting reflector framework

An important application of rectangular frameworks is in the design of space vehicles. Consider the framework shown in Figure 3.14. This framework is to be used to support a reflector in the shape of a paraboloid of revolution. The framework is designed to be collapsible so that it can be folded into a small package for storage in the space vehicle. When the vehicle reaches its destination, the framework is unfolded and the reflector is erected. The framework is designed so that the reflector will be accurately positioned and will maintain its shape when it is in the erected position.

Problem 3 Figure 3.14 shows a simple supporting framework for a reflector. Each side of the framework is identical. Determine the forces in each member of the framework for a load of 1 kN at the apex.

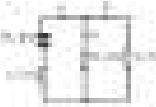


Figure 3.1.6

Procedure:

1. Redraw the complete circuit with switch S_1 removed, being replaced by a wire (as shown in Figure 3.1.6a).
2. Label the currents in each branch and their directions as shown in Figure 3.1.6a and determine their values. (Note that the current is labeled clockwise, otherwise one of the resulting currents will be a negative value, indicating that its actual direction is opposite to that assumed.) Using Kirchhoff's Voltage Law and starting from the positive terminal of the battery, write a loop equation for each of the two loops:

$$\begin{cases} \quad \\ \quad \end{cases}$$

From the equations in part of Figure 3.1.6(b):

$$I_T = \frac{E_T}{R_1 + \frac{R_2 R_3}{R_2 + R_3}} = \frac{\quad}{\quad + \frac{\quad \cdot \quad}{\quad + \quad}} = \quad \text{mA}$$

Using Figure 3.1.6a:

$$I_1 = \left(\frac{\quad}{\quad + \quad} \right) I_T = \left(\frac{\quad}{\quad + \quad} \right) (\quad \text{mA}) = \quad \text{mA}$$

and

$$I_2 = \left(\frac{\quad}{\quad + \quad} \right) I_T = \left(\frac{\quad}{\quad + \quad} \right) (\quad \text{mA}) = \quad \text{mA}$$ (or by simple division)



Figure 3.1.7

3. Redraw the complete circuit with switch S_1 removed (being replaced by a wire as shown in Figure 3.1.7a).
4. Label the currents in each branch and their directions as shown in Figure 3.1.7a and determine their values.
   (As practice with Kirchhoff's Voltage Law, write a loop equation for each of the two loops:)

$$\begin{cases} \quad \\ \quad \end{cases}$$

From the equations in part of Figure 3.1.7(b):

$$I_T = \frac{E_T}{\quad + \frac{\quad \cdot \quad}{\quad + \quad}} = \frac{\quad}{\quad + \frac{\quad \cdot \quad}{\quad + \quad}} = \quad \text{mA}$$

From Figure 3.1.7a:

$$I_1 = \left(\frac{\quad}{\quad + \quad} \right) I_T = \left(\frac{\quad}{\quad + \quad} \right) (\quad \text{mA}) = \quad \text{mA}$$

$$I_2 = \left(\frac{\quad}{\quad + \quad} \right) I_T = \left(\frac{\quad}{\quad + \quad} \right) (\quad \text{mA}) = \quad \text{mA}$$



Figure 3.1.8

5. Superimpose Figure 3.1.8(a) on (b). Figure 3.1.8(c) is shown in Figure 3.1.8.

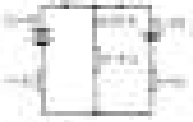


Figure 3.3.10

7. Determine the algebraic sum of the currents flowing to or from the node labeled v_1 in Figure 3.3.10 by applying Kirchhoff's current law.

$$i_1 - i_2 + 1.5(0) - 4.5 = 0$$

$$i_1 - i_2 = 4.5 \text{ A (Kirchhoff's law)}$$

Substitute Ohm's law for i_2 through resistor R_2.

$$i_1 - \frac{v_1}{2} = 4.5 \quad \text{or} \quad i_1 = 4.5 + \frac{v_1}{2}$$

$$i_1 = 4.5 + \frac{v_1}{2} \text{ (Ohm's law)}$$

Substitute Ohm's law for i_1 through resistor R_1 to

$$i_1 = \frac{v_1 - 0}{1} = \frac{v_1}{1}$$

$$i_1 = v_1 \text{ A}$$

The resultant current node law equation can be solved as Figure 3.3.9.

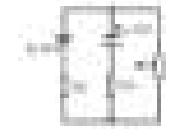


Figure 3.3.11



Figure 3.3.12

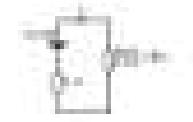


Figure 3.3.13

Problem 3.3.3 Use the circuit shown in Figure 3.3.11 to find values for dependent sources, (a) the resistance that will supply the greatest power, and (b) the maximum power. (c) Also compute the Thevenin and Norton equivalent circuits for this circuit to the left of terminals a-b.

1. Determine the values for parameter values of Figure 3.3.11(a).
2. The circuit illustrates the identical to shown in Figure 3.3.11(a) by drawing three equivalent branches (1-3).

Given Figure 3.3.11(a) $i_1 = v_1 \left(\frac{R_1}{1}\right) = \frac{v_1}{1} = 1.0 \text{ A}$

Given Figure 3.3.11(a) $i_2 = \left(\frac{1}{1+2}\right)(4.5) = \frac{4.5}{3} = 1.5 \text{ A}$

and $i_3 = \left(\frac{1}{1+2}\right)(v_1) = \frac{v_1}{3} = 0.33 v_1 \text{ A}$

3. Substituting values for parameter values of Figure 3.3.11(a) shown under node as Figure 3.3.12(a).
4. The circuit illustrates the identical to shown in Figure 3.3.11(a) and 3.3.12(a). Drawing three equivalent branches (1-3).

Given Figure 3.3.12(a) $i_1 = v_1 \left(\frac{R_1}{1}\right) = \frac{v_1}{1} = 1.0 v_1 \text{ A}$

Given Figure 3.3.12(a) $i_2 = \left(\frac{1}{1+2}\right)(4.5) = \frac{4.5}{3} = 1.5 \text{ A}$

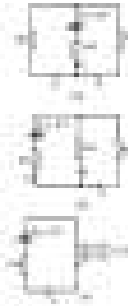


Figure 2.1.31

$$I_2 = \left(\frac{1}{j\omega L_2 + R_2}\right)(V_1) = \frac{V_1}{j\omega L_2 + R_2} = \text{(value) } \angle \text{(value) A}$$

7 Simultaneously, Figure 2.1.31(a) is as in Figure 2.1.31(c) and the circuit in Figure 2.1.31(b).

8 (a) Impedance vector in the 10 Ω resistor $= Z_1 = R_1 =$

$=$ (value) $\angle$ (value)

$=$ (value) $+ j0$

$=$ (value) Ω

Current in the 10 Ω resistor $= I = \frac{V}{Z_1} =$ (value) $\angle$ (value) A

(b) Reactive current in L_2 $= I_2 = \frac{V}{Z_2} =$ (value) $\angle$ (value) A

$=$ (value) $+ j$ (value) A

(c) Resistive current in R_2 $= I_3 = \frac{V}{R_2} =$ (value) $\angle$ (value) A

$=$ (value) $+ j0$ A

Further examples on the superposition theorem may be found in Section 2.1.7 (Examples 7, 8, 9) and 2.1.8 (Problem 10).

2.4 Thevenin's (i.e. circuit theory)

The following seven sections (i.e. sections 2.4.1 to 2.4.7 inclusive) are to be studied in conjunction with the following worked examples and problems.



Figure 2.1.32

(a) The open-circuit voltage, V, across terminals AB in Figure 2.1.32 is equal to 10 V, since no current is to flow through the 2 Ω resistor and hence no voltage drop across it.

(b) The open-circuit voltage, E, across terminals AB in Figure 2.1.32(a) is the same as the voltage across the 8 Ω resistor. The circuit may be redrawn as shown in Figure 2.1.32(c).

$$E = \left(\frac{8}{8+8}\right)(20)$$

E (the Thevenin equivalent source voltage) $= 10$ V

(c) For the circuit shown in Figure 2.1.32(c) (representing a three-loop mesh-type circuit) $V = -E + \ldots$ i.e. where V is the voltage across AB, E is the battery emf and resistors and currents associated with the battery are taken as positive and negative only for the circuit shown in Figure 2.1.32(c). $V = -E + (-I_1 \times 8) + (-I_2 \times 8) + (-I_3 \times 2)$

(d) The resistance "looking in" to terminals AB in Figure 2.1.32(a) is obtained by replacing the source in Figure 2.1.32(a) by Figure 2.1.32(b) to (c). Hence the equivalent resistance across AB is 2 Ω.



Figure 2.1.33



Figure 2-7 P

2-7 For the circuit shown in Figure 2-7 P, find v_o and i_o if $v_s = 10$ V and $i_s = 0$ A. Then find v_o and i_o if $v_s = 0$ V and $i_s = 10$ A. Finally, find v_o and i_o if $v_s = 10$ V and $i_s = 10$ A. Demonstrate that the results agree with the principle of superposition.

2-8 Using the principle of superposition, find v_o in the circuit shown in Figure 2-8 P. Then find the power delivered by the 10-V source.

Figure 2-8 P

2-9 Use the principle of superposition to find v_o in the circuit shown in Figure 2-9 P. Then find the power delivered by the 2-A source.

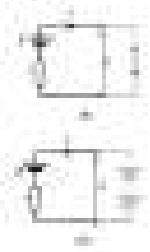


Figure 2-9 P

2-10 Use the principle of superposition to find i_o in the circuit shown in Figure 2-10 P. Then find the power delivered by the 10-V source.

Figure 2-10 P

2-11 For the circuit shown in Figure 2-11 P, find v_o and i_o if $v_s = 10$ V and $i_s = 0$ A. Then find v_o and i_o if $v_s = 0$ V and $i_s = 10$ A. Finally, find v_o and i_o if $v_s = 10$ V and $i_s = 10$ A. Demonstrate that the results agree with the principle of superposition.



Figure 2-11 P

2-12 Use the principle of superposition to find v_o in the circuit shown in Figure 2-12 P. Then find the power delivered by the 2-A source.



Figure 2-12 P



Figure 13.14



Figure 13.15



Figure 13.16

An alternative and perhaps simpler circuit is shown in Figure 13.16(a). Here Thévenin's theorem can be used to determine the current flowing in the 5 Ω resistor as follows:

(i) Remove the 5 Ω resistor and mentally note the terminals where it was connected.

(ii) Determine the open-circuit voltage, E_{oc}, across the terminals.

(iii) Determine the resistance, R_{oc}, at the terminals when all voltage sources are replaced by their internal resistances (i.e. looking into the terminals).

(iv) Determine the value of the current in the resistor from the equivalent circuit shown in Figure 13.16(b), i.e. $I = \dfrac{E_{oc}}{R_{oc} + 5}$.

13.8 Thévenin's theorem

Thévenin's theorem states:

"The current in any branch of a network is that which would result if the branch were connected across a constant voltage source whose e.m.f. was equal to the potential difference which exists across the terminals of the branch when the branch is removed from the network and replaced by its internal resistance."

The procedure adopted when using Thévenin's theorem to determine a current in a branch of a network is now listed, and then applied to the network shown in Figure 13.17(a):

(i) Identify the resistance, R, for which the current is required.

(ii) Remove the resistance R from the network.

(iii) Mark the terminals of the remaining network.

(iv) Calculate the e.m.f. of the equivalent constant voltage source (called the Thévenin voltage, E_T) which represents the network. This is the p.d. across the terminals when R is removed from the network.

(v) Calculate the resistance of the network when looking back into the terminals with R removed and all sources of e.m.f. replaced by their internal resistances.

(vi) Draw the equivalent circuit of the network with R connected across the terminals.

(vii) Calculate the current flowing in R using $I = \dfrac{E_T}{R + r}$, where r is the resistance of the network as seen from the terminals with R removed.

Problem 7 Use Thévenin's theorem to find the current flowing in the 10 Ω resistor in the circuit shown in Figure 13.17(a).

Following the above procedure:

(i) The 10 Ω resistance is removed from the circuit as shown in Figure 13.17(b).

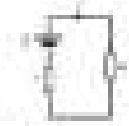


Figure P3.29

(c) Short-circuit current. Shorting out the 5 Ω resistor and assuming I_N is going to the right, we have

$$I_N = \frac{24}{R_1 + R_2} = \frac{24}{4 + 8} = 2 \text{ A}$$

Note that $I_N = 2 \text{ A} = \frac{1}{3}(6 \text{ A}) = \frac{1}{3}(I_1 + I_2)$.

Therefore, the current will be the sum of what is flowing in the upper branch plus the current $I_N = 2$ A.

(d) Determining the current in the 5 Ω resistor in the circuit of Figure P3.29(c).

$$\text{Resistance } = R_N = \frac{R_1 R_2}{R_1 + R_2} = \frac{4 \times 8}{4 + 8} = \frac{32}{12} = \frac{8}{3} \ \Omega$$

$$I = \left(\frac{8}{3}\right)(2) = \frac{16}{3} \text{ A}$$

(e) The equivalent Thévenin circuit is shown in the solution in Figure P3.29(d).

$$\text{Current } I = \frac{V_{Th}}{R_{Th} + R_L} = \frac{8}{\frac{8}{3} + 5} = \frac{3}{23}(8) = 1.043 \text{ A}$$

Power dissipated (flowing in the 10 Ω resistor of Figure P3.29(a) is $(1.043)^2(5) = 5.44$ W.

Problem 3.30 Use the schematic shown in Figure 3.12(a) to determine the current in the 12 Ω resistor using "Thévenin's theorem."

Solution by Thévenin's theorem

(a) The 4 Ω resistor is removed from the circuit as shown in Figure 3.12(b).

(b) Current $I = \frac{12}{4 + 2 + 6} = \frac{12}{12} = 1$ A

V_{ab} across the resistor $= 6I_1 = 6(1) = 6$ V

Voltage V_{ab} is also $12 - 2I_1$ by applying Kirchoff's voltage law (KVL)

$V = 6$ V

(c) Removing the source of 6 V, gives the circuit shown in Figure 3.12(c). The equivalent circuit of Figure 3.12(b) (Consists of Figure 3.12(c)). Total value

$$\text{resistance } = \frac{6 \times 6}{6 + 6} = \frac{36}{12} = 3 \ \Omega$$

(d) The equivalent Thévenin circuit is shown in Figure 3.12(d), whose value is

$$\text{current } I = \frac{6}{3 + 4} = \frac{6}{7} \text{ A}$$



Figure 3.3.11



Figure 3.3.12

Problem 3. Use Thévenin's theorem to determine the current I flowing in the 2 Ω resistor shown in Figure 3.3.12(a). Then solve for it by the techniques in Sec 3.2 as a check.

Solution. See Figure

(a) The 2 Ω resistor is disconnect from the circuit as shown in Figure 3.3.12(b).

(b) Replace $R_1 = \frac{R_1 \cdot R_2}{R_1 + R_2} = \frac{3 \cdot 6}{3 + 6} = \frac{18}{9} = 2\,\Omega$

Full series with $E = E_1 + E_2 = 9 + 3 = 12$ V

(see Figure 3.3.12(c))

Thévenin's circuit series with $R_1 = R_1 + R_2$

$I = \frac{12}{4} = 3$ A

(c) Reconnecting the missing 2 Ω resistor for circuit shown in Figure 3.3.12(d) to find values indicated.

$I = \frac{E}{R} = \frac{12}{4} = 3$ A

(d) The equivalent Thévenin circuit is shown in Figure 3.3.12(e) from above.

$I = \frac{E}{R_1 + R_2} = \frac{9 - 3}{3 + 6} = \frac{6}{9} = \frac{2}{3}$ A

$V_{AB} = E_1 - IR_1 = 9 - \frac{2}{3}(3) = 9 - 2 = 7$ V

$V_{AB} = E_2 + IR_2 = 3 + \frac{2}{3}(6) = 3 + 4 = 7$ V

Problem 4. Use Thévenin's theorem to determine the current flowing in the 3 Ω resistor of the network shown in Figure 3.3.13(a). The voltage source has negligible internal resistance.

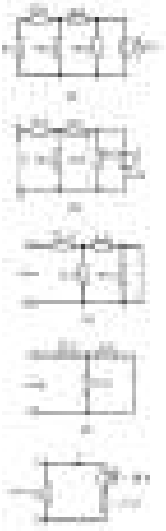


Figure 3.3.16

What is the voltage V_2 in each circuit in Figure 3.3.16 if a voltage source is connected to the terminals of the indicated resistor?

Following the procedure:

(a) Turn off the independent source and redraw the circuit as shown in Figure 3.3.16(a).

(b) The 1-Ω resistance is unaffected when the source is turned off.

Its i-v relation is $i = \frac{1}{1}v$ (Ohm's law)

$\rightarrow$ Add R value (Section 3.1.2, p.1).

Redraw as shown in Fig. 3.3.16(b).

(c) Determine the values of i_{sc} and v_{oc} as indicated in Fig. 3.3.16(b). Circuit analysis shows that the 1-Ω resistance is shorted out in the circuit in Figure 3.3.16(c). Since the current through the short is zero, $i_{sc} = 0$. Circuit analysis also shows that the 2-Ω and 3-Ω resistors are in parallel in the circuit in Figure 3.3.16(d). Use the current-divider rule (Section 3.1.4, p.1) to find v_{oc}:

From Figure 3.3.16(d), $v_{oc} = -\frac{2}{5}(10) + 3$

$= -\frac{20}{5} + 3 = -4 + 3$

$= -1\ \text{V}$

(d) The equivalent Thévenin circuit is shown in Figure 3.3.16(e). From this circuit,

$v_2 = \frac{R}{1+R}(-1) = \frac{R}{1+R}(-1) = 0.8$

$\rightarrow$ Answer to the 3.3.16 Application

Problem 3.3.16 Resistance Meter Calibrate an ohmmeter as shown in Figure 3.3.16. Use the ohmmeter to find the i-v relation and the Thévenin equivalent circuit for the circuit in Figure 3.3.16(e). Use the Thévenin equivalent to find the values of R and v_2 in the application statement.

Following the procedure:

(a) The 3-V battery is inactive when the circuit is shorted, as shown in Figure 3.3.16(f).

(b) The i-v relation is $i = \frac{1}{R_1 + R_2}v$ (Ω)

$= \left(\frac{1}{2}\right)v$ (mA, v in V)

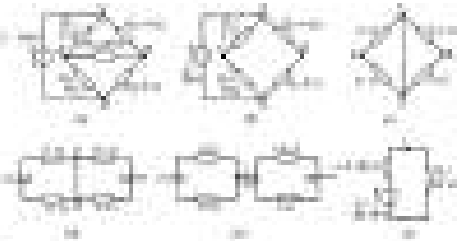


Figure 4.7.3

Step 1: Combine R and R in series: $R_{eq} = \frac{R}{R} + \frac{R}{R} = 2\,\Omega$

$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{R} = \frac{2}{R}$$

Answer Key 4.7: Combining R and R in series: $R_{eq} = R + R = 2R$ Ω. Combining R and R in parallel: $\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{R} = \frac{2}{R}$, therefore $R_{eq} = \frac{R}{2}$ Ω.

Further problems are: Problem 3.1 (exercise due), 3.2 (exercise due), 3.3 (to be solved), 3.4 (to be solved), 3.5 (to be solved), 3.6 (to be solved), 3.7 (to be solved), 3.8 (to be solved), 3.9 (to be solved), 3.10 (to be solved), 3.11 (to be solved), 3.12 (to be solved), 3.13 (to be solved), 3.14 (to be solved), 3.15 (to be solved), 3.16 (to be solved), 3.17 (to be solved), 3.18 (to be solved), 3.19 (to be solved), 3.20 (to be solved), 3.21 (to be solved), 3.22 (to be solved), 3.23 (to be solved), 3.24 (to be solved), 3.25 (to be solved), 3.26 (to be solved), 3.27 (to be solved), 3.28 (to be solved), 3.29 (to be solved), 3.30 (to be solved), 3.31 (to be solved), 3.32 (to be solved), 3.33 (to be solved), 3.34 (to be solved), 3.35 (to be solved), 3.36 (to be solved), 3.37 (to be solved), 3.38 (to be solved), 3.39 (to be solved), 3.40 (to be solved), 3.41 (to be solved), 3.42 (to be solved), 3.43 (to be solved), 3.44 (to be solved), 3.45 (to be solved), 3.46 (to be solved), 3.47 (to be 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3.3.8 A Common-Mode Gain Example

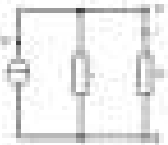


Figure 3.3.8

3.3.9 Thevenin's Theorem

Thevenin's theorem states:

> The terminal behavior of any linear circuit containing sources is equivalent to that which results from the Thevenin circuit, which consists of a single voltage source in series with a single resistor. The Thevenin source voltage is the open-circuit voltage at the terminals. The Thevenin resistance is the ratio of the open-circuit voltage to the short-circuit current at the terminals. The Thevenin circuit is a circuit model that exhibits the same terminal behavior as the original circuit. Thevenin's theorem is useful in reducing complicated circuits to simple equivalent forms for computation.

Two procedures suggested above using Thevenin's theorem to calculate short currents.

To determine the current i in the R_L in problem 3.3.7 (Figure 3.3.7) we proceed as follows:

(1) Remove the load resistor R_L.

(2) Determine the open-circuit voltage v_{oc} = Thevenin voltage v_{Th}.

(3) Determine the short-circuit current i_{sc} = Thevenin current i_{Th}. If it is not possible to determine i_{sc} directly, then determine the Thevenin resistance R_{Th} by the ratio v_{oc}/i_{sc}. If it is not possible to determine i_{sc} directly, then determine the Thevenin resistance R_{Th} by the ratio v_{oc}/i_{sc}.

(4) Determine the current i through R_L by the relation $i = v_{Th}/(R_{Th} + R_L)$.

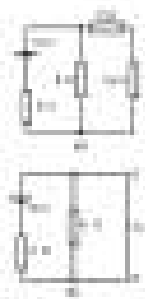


Figure 3.3.9

$$i = \frac{v_{Th}}{R_{Th} + R_L}$$

Problem 3.3.7 has been solved by determining the current through it directly in part (a) of the solutions for the circuit shown in Figure 3.3.9(a).

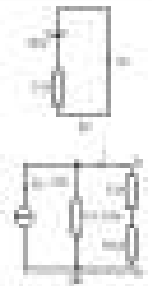


Figure 3.2.6a – 3.2.6e (schematic)

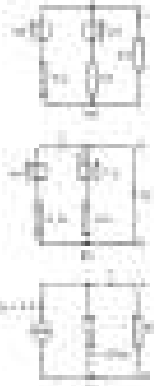


Figure 3.2.6f

Following the solution procedure:

(i) The network, redrawn as Fig. 3.2.6f (continued), is shown connected to a dotted 'Gaussian surface'.

(ii) Figure 3.2.6f has $n = 4$ nodes, so Equation (3.2.6b) yields, by Gauss:

$$\sum_{k=1}^{4} I_k = 0 \quad \text{A}$$

(iii) Using (3.2.6c) to analyse each of the 3 meshes left (only) in Figure 3.2.6f (the simultaneous Kirchhoff equations for the branch currents I_1, I_2, I_3) we obtain:

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 0 & -3 \\ 0 & 3 & -2 \end{bmatrix} \begin{Bmatrix} I_1 \\ I_2 \\ I_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 12 \\ 0 \end{Bmatrix} \quad \text{(3.2.6g)}$$

(iv) Solving the 3 simultaneous linear algebraic equations (3.2.6g) by Gauss elimination (see Sec. 3.1.3 if unfamiliar), the branch currents are given by:

$$\begin{Bmatrix} I_1 \\ I_2 \\ I_3 \end{Bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 0 & -3 \\ 0 & 3 & -2 \end{bmatrix}^{-1} \begin{Bmatrix} 0 \\ 12 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 1.846 \\ 1.385 \\ 4.154 \end{Bmatrix} \text{ A}$$

as obtained previously in problem 3 using Thévenin's theorem.

> Problem 3.2. – Use Kirchhoff's theorem to determine the current I_3 flowing in the 4 Ω resistance shown in Figure 3.2.6a.

Following the procedure:

(i) The 4 (3) branches which terminate at node A (Figure 3.2.6f) ...

(ii) Using Figure 3.2.6f, $I_1 = I_2 + I_3$, $I_2 = \frac{12}{4} = 3$ A ...

(iii) ... if the branch of each I_3 are taken with the clockwise Kirchhoff law around each mesh, the 3 equations in I_1 and I_3 are given by:

$$\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{Bmatrix} I_1 \\ I_3 \end{Bmatrix} = \begin{Bmatrix} 3 \\ 12 \end{Bmatrix} \quad \text{(3.2.6h)}$$

(iv) From the Gaussian elimination method, shown in Figure 3.2.6f, the solution is given by the 3 (4) simultaneous equations (3.2.6h):

$$I_3 = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}^{-1} \begin{Bmatrix} 3 \\ 12 \end{Bmatrix} = 4.154 \text{ A}$$

as obtained (more easily) in problems 3.1 and 3.2 using Thévenin's theorem and the Gaussian elimination method (Section).

> Problem 3.3. – Use Kirchhoff's theorem to determine the current flowing in the 3 Ω resistance in the network shown in Figure 3.2.6a(e). The voltage source has negligible internal resistance.

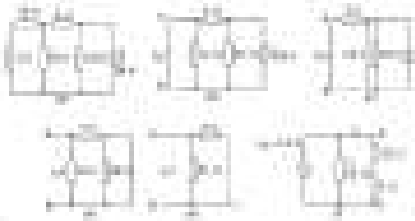


Figure 3.114

Following the procedure:

(a) The network containing the 1 Ω resistance is short-circuited as shown in Figure 3.115(a).

(b) From the open-circuit voltage shown in Figure 3.115(b),

$$v_{oc} = -\frac{30}{6} + 4 + 4 = 3 \text{ V}$$

(c) If the 3 A source is set to zero (suppressed) the conditions in Figure 3.115(c) are obtained. From this circuit, $i_{sc} = 0$. Therefore, the Norton equivalent circuit shown in Figure 3.115(d) is also a source:

$$i_{sc} = \frac{30}{6} - 4 = \frac{30}{6} - \frac{24}{6} = 1 \text{ A}$$

(d) From the Norton equivalent circuit shown in Figure 3.115(d) the current of the 1 Ω resistance is given by:

$$i = \left[\frac{3}{1+2}\right] 1 = 1 \text{ A}$$

as obtained previously in problem 31 using Thevenin's theorem.

> **Problem 3.3** Determine the current flowing in the 2 Ω resistance in the network shown in Figure 3.116 by:

Following the procedure:

(a) The 1 Ω resistance branch is short-circuited as shown in Figure 3.117(a).

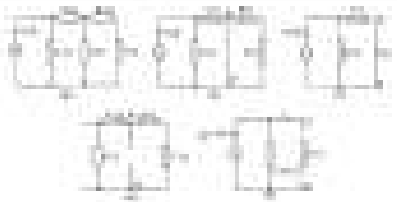


Figure 2.2.17

(a) Figure 2.2.17(a) is equivalent to Figure 2.2.17(b), where $R_{eq} = \left[\frac{1}{R_1+R_2} + \frac{1}{R_3}\right]^{-1}$ is the total resistance.

(b) If the 2.2 Ω resistor shown in parallel in Fig. 2.2.17(b) is replaced by an open circuit, then Figure 2.2.17(c) is obtained. The diagram in (c) is the full circuit. Kirchhoff's voltage law for the loop gives $10 - 4 = 2i$, so the resulting current is $i = 3$ A. Ans.

$$\xrightarrow{\text{replace by}\atop\text{open circuit}} \quad 10 - 4 = 2i \quad \Rightarrow \quad i = 3 \text{ A}$$

(c) Finally, the 10 volt battery in the original circuit shown in Figure 2.2.17(a) is now removed, leaving a short circuit as in Figure 2.2.17(d). The current i is now given by

$$i = \frac{4}{1+2} = \frac{4}{3} \text{ A} \approx 1.33 \text{ A}. \text{ Ans.}$$

2.2.4 Thévenin and Norton equivalent circuits

The Thévenin and Norton equivalent circuits shown in Figure 2.2.18 are equivalent to each other. The Thévenin circuit in Figure 2.2.18(a) consists of a voltage source v_{eq} in series with a resistance R_{eq}. The Norton circuit in Figure 2.2.18(b) consists of a current source i_{eq} in parallel with a resistance R_{eq}. The two circuits are equivalent if $v_{eq} = i_{eq} R_{eq}$. Figure 2.2.18(c) shows a general circuit with two terminals a and b. Figure 2.2.18(d) shows a Thévenin equivalent circuit for the circuit in Figure 2.2.18(c). The circuit in Figure 2.2.18(d) is equivalent to the circuit in Figure 2.2.18(c) if $v_{ab} = v_{eq}$ when $i_{ab} = 0$ and $i_{ab} = i_{eq}$ when $v_{ab} = 0$.



Figure 2.2.18

Figure 2.2.18(d) shows a general circuit. If circuit 1 is linear, then a combination of Thévenin and Norton equivalents of

From Figure 2.2.18(c) $i = \frac{v_{ab}}{R_{eq}} - \frac{v_{eq}}{R_{eq}} = \frac{v_{ab} - v_{eq}}{R_{eq}} = \left(-\frac{1}{R_{eq}}\right) v_{ab} + \frac{v_{eq}}{R_{eq}}$

$$\text{or } i = \left(\frac{-1}{1+R_{eq}}\right) v_{ab} + \frac{v_{eq}}{R_{eq}}$$

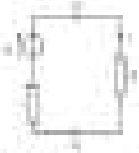


Figure 4.1.44

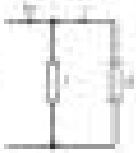


Figure 4.1.45



Figure 4.1.46

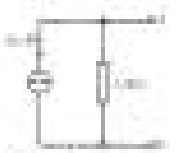


Figure 4.1.47

From Figure 4.1.44 it can be seen that, when viewed from the load, the source appears as the [illegible] in parallel with [illegible]. When [illegible] are in parallel it is convenient to identify

Use the form representation shown in Figure 4.1.45 for equivalent

> Problem 4.1. Convert the current source in Figure 4.1.44 to an equivalent Norton network.

A somewhat [illegible] in Figure 4.1.46 also demands attention, the current source [illegible]

The [illegible] Problem 4.2 in Appendix 4.1.2 shows the method for finding the [illegible] in Figure 4.1.46.

> Problem 4.2. Convert the [illegible] shown in Figure 4.1.47 to an equivalent Thévenin source.

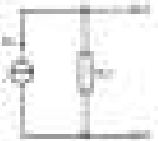


Figure 4.1.48

The [illegible] voltage [illegible] in Figure 4.1.48, is [illegible]

The [illegible] Problem 4.3 in Appendix 4.1.2 for [illegible] and Thévenin sources to be shown in Figure 4.1.48.

> Problem 4.3. [illegible] in Figure 4.1.48 [illegible] in Figure 4.1.49 [illegible] Thévenin source [illegible] [illegible] [illegible]



Figure 2.1.6

[illegible] Figure 2.1.6 is a circuit diagram which represents a [illegible] Figure 2.1.6 which has a total electrical resistance of [illegible] Ω is connected [illegible] and terminals A and B.

[illegible] Figure 2.1.6 [illegible] Figure 2.1.6 [illegible]

[illegible] Figure 2.1.6 [illegible]

[illegible] Figure 2.1.6 [illegible]

[

[illegible] the torque corresponding to the 10 N force, assuming a counterclockwise rotation is positive, is given by

$$\tau_{10} = \frac{(0.5\ \text{m})}{(0.5\ \text{m})} \frac{(10\ \text{N})}{(10\ \text{N})} \frac{(\sin 30^\circ)}{(0.5)} = +2.5\ \text{N}\cdot\text{m}$$

For the 8 N force, the torque is given by

$$\tau_8 = \frac{0}{(8\ \text{N})} \quad \text{(line of action passes through O)}$$

Thus the net torque is [illegible] as mentioned in Figure 11.8(b).

Combining the $+2.5$ N·m and 0 torques gives the equivalent situation in Figure 11.8(c), where the net torque is $+2.5$ N·m. Now the net torque is equal to the product of the net force and the moment arm, or

$$\begin{aligned} \tau_{\text{net}} &= F_{\text{net}} \times d \\ +2.5\ \text{N}\cdot\text{m} &= (5\ \text{N}) \times d \end{aligned}$$

Note that the forces applied to the rod are the same as in Figure 11.8(b) so they can be calculated in the same way as in that figure. The net force is the vertical component of the 10 N force, or $+5$ N. Solving for the moment arm gives

$$d = \frac{+2.5\ \text{N}\cdot\text{m}}{+5\ \text{N}} = +0.5\ \text{m}$$

Thus the torque equivalent cancel to a single force of 5 N as shown in Figure 11.8(d) is as shown in Figure 11.8(c).

Since the moment of the force is +0.5 m, the force must be applied 0.5 m to the left of point O.

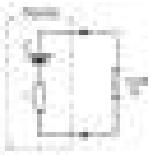


Figure 11.8

Problem-solving hint. In this problem, the force F in Figure 11.7(b) is perpendicular to the rod, page 378

11.9 The Moment of Inertia of a Rigid Body

The moment of inertia of a rigid body is a measure of the body's resistance to angular acceleration.

The greater the moment of inertia of a rigid body about a given axis, the greater the torque required to produce a given angular acceleration about that axis. The moment of inertia of a rigid body depends on the mass of the body and on the distribution of the mass about the axis of rotation.

Because, in Figure 11.9, an object B [illegible] has greater translational inertia than does an object A, it also has a greater

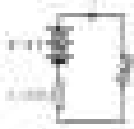


Figure 2.2.48

Problem 11. The circuit diagram in Figure 2.2.48 shows the resistance of a heat source, $R = 5$ ohms and the resistance of the wire between the source and the load, $r = 0.1$ ohm. If the load is changed by varying the resistance R in such a way that the power supplied by the source is maximized, what is the power supplied to the load? What is the efficiency of the system? (The efficiency is the ratio of the power supplied to the load to the power supplied by the source.)

When $R = 0$, current $I = \frac{E}{r+R} = \frac{E}{0.1} = 10E$ and power dissipated

in the $R = 5$ Ω resistor $= I^2R = 100E^2 \times 5 = 500E^2$.

When $R = 0.1$ Ω, current $I = \frac{E}{r+R} = \frac{E}{0.1+0.1} = 5E$.

and $P = I^2R = 25E^2 \times 0.1 = 2.5E^2$.

When $R = 1$ Ω, current $I = \frac{E}{1.1} = 0.91E$.

and $P = I^2R = 0.83E^2 \times 1 = 0.83E^2$.

With similar calculations for different values of R we have

R (Ω)	0	0.05	0.1	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6
$I = \frac{E}{r+R}$	10E	6.67E	5E	3.33E	2.5E	2E	1.67E	1.43E	1.25E	1.11E	1E
$P = I^2R$ (Watts)	100E²	44.4E²	25E²	11.1E²	6.25E²	4E²	2.78E²	2.04E²	1.56E²	1.23E²	1E²

A graph of P against R is shown in Figure 2.2.49. **The maximum value of power is 2.5E² W** (when current I flows in R). It is **undesirable** to operate a system in which $R = r$, because more than half the power is lost in the source.



Figure 2.2.49

Problem 12. A two stage transistor amplifier circuit is shown in Figure 2.2.50. It has an internal resistance of 1.1 Ω. What two values of load resistance will give maximum power output and what are the two corresponding power outputs?



Figure 2.2.50

The circuit diagram is shown in Figure 2.2.51. From the maximum power transfer theorem the maximum power output occurs when

$R_L = 1.1$ Ω.

From Figure 2.2.51 the current $I = \frac{E}{r+R_L} = \frac{20}{1.1+1.1} = 9.1$ A.

Hence $P = I^2R_L = 9.1^2 \times 1.1 = 91.1$ W = maximum power dissipated

Problem 4.5 Find the value of the load current i_L shown in Figure 4.5 by first applying Kirchhoff's current equation and then Kirchhoff's voltage equation.

Using the Circuit Solver and MATLAB to Determine Resistance

4.6 Determine R_{ab} in the circuit shown in the circuit as shown in Figure 4.6(a).



Figure 4.6(a)

(b) This is a 2Ω resistor. All in the figure as the circuit, solved the 12-Ω resistor.

Answer: $R = \left(\frac{48}{14} + 2\right)\Omega = 5.43\ \Omega$

(c) Determine the current i in the circuit of Figure 4.7(a).

Solution: Applying KVL around the loop,

$$-12 + 4i + 2i + 6 = 0 \quad \text{or} \quad i = \frac{12 - 6}{4 + 2} = \frac{6}{6} = 1\ \text{A}$$

(d) Using equivalent Kirchhoff's current equations determine R in the circuit in Figure 4.7(b). Determine the current i in the circuit of Figure 4.7(b).

Solution: Applying KVL around the loop,

$$-12 + 4i + 2i + 6 = 0 \quad \text{or} \quad i = \frac{12 - 6}{4 + 2} = \frac{6}{6} = 1\ \text{A}$$

Answer: $i = 1$ A

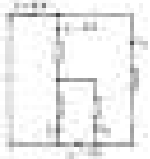


Figure 4.7(b)

Further problems in the accompanying Circuit Solver and MATLAB Toolbox

4.8 (b) Parallel Circuits

Solution

a. Label nodes a, b, and c as in Figure 4.8(a).

$$i_1 = \frac{v_a - v_b}{R_1}, \quad i_2 = \frac{v_b - v_c}{R_2}, \quad i_3 = \frac{v_a - v_c}{R_3}$$



Figure 2.3.42

2. For the concurrent system of forces in Figure 2.3.42, find the resultant. Write answers in vector notation.

$$\begin{aligned}\mathbf{F}_1 &= 3\,\mathbf{i} + 4\,\mathbf{j} - 2\,\mathbf{k}, \quad \mathbf{F}_2 = -4\,\mathbf{i} + 2\,\mathbf{j} + 4\,\mathbf{k}, \quad \mathbf{F}_3 = 2\,\mathbf{i} - 3\,\mathbf{j} - 5\,\mathbf{k}\\ \mathbf{F}_4 &= -3\,\mathbf{i} + 6\,\mathbf{j} + 3\,\mathbf{k}, \quad \mathbf{F}_5 = 4\,\mathbf{i} - 5\,\mathbf{j} + 2\,\mathbf{k}, \quad \mathbf{F}_6 = -3\,\mathbf{i} + 4\,\mathbf{j} - 2\,\mathbf{k}\\ \mathbf{R} &= -\mathbf{i} + 5\,\mathbf{j} + 5\,\mathbf{k}, \quad R = 7.55\ \text{kN}\end{aligned}$$

3. Use Example 2.3.7, page 2.3.4, and the results (Drawing 1) from 2.3 solution of Figure 2.3.44 and the given dimensions to find the tension in cable AB.

$$T = 1.61\ \text{kN} \quad \text{or} \quad T = 1.61\ \text{kN}$$

4. Check the solution (Drawing 2) for 2.3 solution for the two unknown forces in Figure 2.3.44 by finding the sum of the moments about the x and z axes.

$$\Sigma M_x = 0, \quad \Sigma M_z = 0 \quad \text{OK}$$

5. For the space structure in Figure 2.3.42, find the two unknown forces (reactions) at the support so that the line of action of the resultant of the forces and reactions passes through the point (4, 0, 0) meters.

$$\begin{aligned}\mathbf{F}_7 &= 0.75\,\mathbf{i} + 0.25\,\mathbf{j} - 2.5\,\mathbf{k}\ \text{kN}\\ \mathbf{F}_8 &= 0.25\,\mathbf{i} + 0.25\,\mathbf{j} + 2.5\,\mathbf{k}\ \text{kN}\end{aligned}$$

6. For the three concentric forces in Figure 2.3.42, find the resultant in vector form.

$$\begin{aligned}\mathbf{F}_1 &= -3\,\mathbf{i} + 4\,\mathbf{j} + 2\,\mathbf{k}, \quad \mathbf{F}_2 = 2\,\mathbf{i} + 3\,\mathbf{j} - 6\,\mathbf{k}, \quad \mathbf{F}_3 = -5\,\mathbf{i} + 6\,\mathbf{j} + 4\,\mathbf{k}\\ \mathbf{R} &= -6\,\mathbf{i} + 13\,\mathbf{j} + 0\,\mathbf{k}, \quad R = 14.32\ \text{kN}\end{aligned}$$

Engineering Mechanics

1. For the engineering mechanics structure in Figure 2.3.43, find the reactions at A and B. Write answers in vector notation.

$$\mathbf{R}_A = 0\,\mathbf{i} + 0\,\mathbf{j} + 0\,\mathbf{k}\ \text{kg}, \quad \mathbf{R}_B = 0\,\mathbf{i} + 0\,\mathbf{j} + 0\,\mathbf{k}\ \text{kg}$$

2. For the engineering mechanics structure in Problem 1 above, find the reactions at A and B in Figure 2.3.43(b).

$$\mathbf{R}_A = 0\,\mathbf{i} + 0\,\mathbf{j} + 0\,\mathbf{k}\ \text{kg}, \quad \mathbf{R}_B = 0\,\mathbf{i} + 0\,\mathbf{j} + 0\,\mathbf{k}\ \text{kg}$$

3. For the engineering mechanics structure in Figure 2.3.43(c), find the reactions at A and B.

$$\mathbf{R}_A = 0\,\mathbf{i} + 0\,\mathbf{j} + 0\,\mathbf{k}\ \text{kg}, \quad \mathbf{R}_B = 0\,\mathbf{i} + 0\,\mathbf{j} + 0\,\mathbf{k}\ \text{kg}$$

4. For the engineering mechanics structure in Figure 2.3.43(d), find the reactions at A and B.

$$\mathbf{R}_A = 0\,\mathbf{i} + 0\,\mathbf{j} + 0\,\mathbf{k}\ \text{kg}, \quad \mathbf{R}_B = 0\,\mathbf{i} + 0\,\mathbf{j} + 0\,\mathbf{k}\ \text{kg}$$

5. For the engineering mechanics structure in Figure 2.3.43(e), find the reactions at A and B.

$$\mathbf{R}_A = 0\,\mathbf{i} + 0\,\mathbf{j} + 0\,\mathbf{k}\ \text{kg}, \quad \mathbf{R}_B = 0\,\mathbf{i} + 0\,\mathbf{j} + 0\,\mathbf{k}\ \text{kg}$$

Mechanics Statics

1. For the mechanics structure in Figure 2.3.43(f), find the reactions at A and B.

$$\mathbf{R}_A = 0\,\mathbf{i} + 0\,\mathbf{j} + 0\,\mathbf{k}\ \text{kg}, \quad \mathbf{R}_B = 0\,\mathbf{i} + 0\,\mathbf{j} + 0\,\mathbf{k}\ \text{kg}$$



Figure 2.3.43



Figure 2.3.44

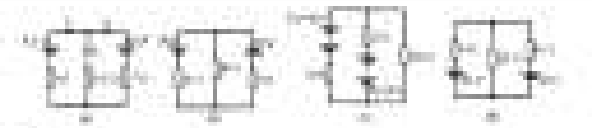


Figure 3.114



Figure 3.115



Figure 3.116



Figure 3.117



Figure 3.118

3.3 Use Thévenin's theorem to find the current flowing in the 4 Ω resistor shown in Figure 3.119, and the power dissipated in the 4 Ω resistor.

[2.667 A, 42.67 W]

3.4 Repeat problem 3.3 using Norton's theorem.

3.5 In the network in the in Figure 3.120, use Norton's theorem to calculate the current flowing in the 5 Ω resistor.

[1.071 A]

3.6 For the bridge network shown in Figure 3.121, find the current in the 7 Ω resistor, and the power it is using. Use Norton's theorem.

[2.333 A, 38.11 W]

Norton's theorem

3.7 Repeat problem 3.3, 3.4, 3.5 and 3.6 using Norton's theorem.

3.8 Determine the current flowing in the 4 Ω resistance of the network shown in Figure 3.122 by using Norton's theorem. [2.2 and 0]

3.9 Convert the circuit shown in Figure 3.123 to Norton equivalent form.

$$I_N = \frac{V_1}{R_1} + \frac{V_2}{R_2} + I_1 + I_2$$
$$R_N = \frac{R_1 R_2}{R_1 + R_2}$$

3.10 Convert the circuit shown in Figure 3.124 to Thévenin equivalent form.

$$V_{TH} = \frac{I_1 R_1 R_2}{R_1 + R_2} + \frac{V_1 R_2}{R_1 + R_2} + V_2$$
$$R_{TH} = \frac{R_1 R_2}{R_1 + R_2} + R_3$$



Figure 3.119



Figure 3.120



Figure 3.121

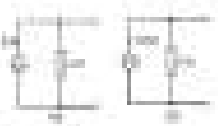


Figure 10.65

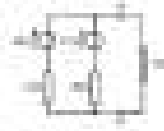


Figure 10.66



Figure 10.67

37. (a) Convert the current source in the RC circuit of Exercise 36 in Figure 10.66 to an equivalent Thévenin circuit (for the variable corresponding to the 6-ohm resistor current).

(b) Determine the Thévenin equivalent resistance R_{Th} and Thévenin equivalent voltage v_{Th} for Figure 10.66.

$$R_{Th} = \underline{\qquad} \Omega, \quad v_{Th} = \underline{\qquad} \text{V}$$

38. Determine the Thévenin equivalent circuit between the terminals and source dependent relationship in Thévenin equivalent circuit for the dependent source in Figure 10.67. (Hint: Although the current I_{Th} may be 0 A in the Thévenin equivalent circuit, it is not 0.)

$$R_{Th} = \underline{\qquad} \Omega, \quad V_{Th} = \underline{\qquad} \text{V}, \quad I_{Th} = \underline{\qquad} \text{A}$$

Thévenin power transfer theorem

39. (a) v_{Th} across load as open circuit in Figure 10.68 V_{Th} and an internal resistance of 2 Ω. Determine the value of R_L that maximizes the power transferred. Power dissipated in load for value of R_L found.

$$R_L = \underline{\qquad} \Omega, \quad P = \underline{\qquad} \text{W}$$

(b) Determine the value of the load resistance R_L shown in Figure 10.69 that allows maximum power dissipation and find the value of the power.

$$R_L = \underline{\qquad} \Omega, \quad P = \underline{\qquad} \text{W}$$



Figure 10.68

14 Alternating voltages and currents

In this and the chapter you should be able to:

- appreciate why a.c. is used in preference to d.c.
- describe the principle of operation of an a.c. generator
- distinguish between instantaneous and alternating quantities
- define cycle, period, frequency (f) and angular velocity ($\omega = 2\pi f$)
- perform simple calculations involving $V = \frac{Q}{t}$
- define maximum, peak, mean and root mean square and root mean square values for a.c. quantities
- calculate mean and root mean square and form and peak factors for given waveforms
- understand and perform calculations for power dissipation, applying $I_{rms} = \frac{I_m}{\sqrt{2}}$ and $V_{rms} = \frac{V_m}{\sqrt{2}}$
- understand lagging and leading angles
- understand the concepts of reactance and impedance and perform calculations involving $X_L = 2\pi fL$, $X_C = \frac{1}{2\pi fC}$ and $Z = \sqrt{R^2 + (X_L - X_C)^2}$
- understand resonance and perform calculations involving $f = \frac{1}{2\pi\sqrt{LC}}$
- understand the principle of operation of a transformer and perform calculations involving $\frac{V_s}{V_p} = \frac{N_s}{N_p}$ and $\frac{I_p}{I_s} = \frac{N_s}{N_p}$

14.1 Introduction

Electrical quantities in the practical world are often time varying and have their direction of flow reversed at regular intervals in the circuit. Such quantities are called alternating quantities. The most common alternating waveform is a sine wave and this is the waveform used for the generation and transmission of electricity. Other alternating waveforms are used in electronic circuits, for example square waves, triangular waveforms, sawtooth waveforms, pulses, etc. The sine wave is the only waveform where the shape is unaffected by the addition of inductance and capacitance to a circuit containing resistance. Sine waves are easily described mathematically and frequently occur in nature. For these reasons the sine wave is used for the generation and transmission of electricity. Alternating quantities are those that vary in magnitude and periodically change direction. The symbol for alternating current is i or I and for alternating voltage is v or V. The shape of an alternating waveform is called a cycle. Figure 14.1 shows a cycle of a sine wave.

14.2 The a.c. generator

A simple alternator consists of a rectangular coil of wire rotating at a constant angular velocity in a uniform magnetic field as shown in Figure 14.2. The ends of the coil are connected to two slip rings and the supply is taken from the slip rings by carbon brushes which press lightly against the slip rings. The coil is rotated by some prime mover, such as a steam turbine, a diesel engine, a petrol engine or a water turbine. The direction of rotation of the coil in Figure 14.2 is anticlockwise.



Figure 14.1

14.2 Wavelike Loads



Figure 14.2

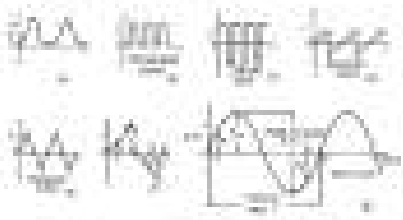


Figure 14.3

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x$$

Problem 1 Determine the area between the graph of $y = x^2$ and the x-axis on $[0, 3]$.

(a) Riemann sum: $\Delta x = \frac{3}{n}$, $x_i^* = \frac{3i}{n}$, $f(x_i^*) = \frac{9i^2}{n^2}$

(b) Definite form: $\int_0^3 x^2\,dx = \frac{3^3}{3} = 9$

Problem 2 Determine the displacement of a particle whose velocity is $v(t) = 6t - 4$ on $[0, 5]$.

(a) Antiderivative: $\int (6t - 4)\,dt = 3t^2 - 4t + C$

(b) Displacement: $\int_0^5 (6t - 4)\,dt = \left[3t^2 - 4t\right]_0^5 = 75 - 20 = 55$

$\Rightarrow$ the particle moves 55 units to the right on $[0, 5]$.

Problem 3 An automobile engine completes 3 cycles in 2 sec. What is its frequency?

Time for 1 cycle: $\frac{2}{3}$ sec ≈ 0.6 sec (a periodic time T)

Frequency: $f = \frac{1}{T} = \frac{1}{2/3} = \frac{3}{2} = 1.5$ cycles/sec $= 1.5$ Hz

Further problems on properties and geometric interpretation are found on pages 372 and 373.

11.5 Area between curves

Area between two curves is the integral of the difference of the upper and lower functions over the interval. If $f(x) \geq g(x)$ on $[a, b]$, then $A = \int_a^b [f(x) - g(x)]\,dx$.

The integral is always nonnegative. If f is sometimes above and sometimes below g, the **geometric area** is the **total area** of the **regions**, but the **signed area** of the integral is the difference between the areas above and below the axis. To find the geometric area, split the integral at the points where $f(x) = g(x)$ and take the absolute value of each piece. A **vertically simple region** is one bounded by two functions of x; a **horizontally simple region** is one bounded by two functions of y. Choose the integration variable to match the boundary description.

... The definition of average value is a mathematical statement of a familiar concept. As a first example of the average value computation from a digital oscilloscope, we'll calculate exactly the average value of a square ...

$$\text{average or dc value} = \frac{\text{area under the curve}}{\text{length of time}}$$

... This area under the curve is found by mathematical methods which, as we have just seen, the oscilloscope does not do. Fortunately, the average value can be approximated by V_{avg}, as ...

> **The average value of a periodic voltage is the dc voltage that would deliver the same power to a resistive load as the periodic voltage does.**

The **effective value** of any periodic current or time-varying signal will produce the same heating effect in an equivalent dc load current. The effective value is called the **root-mean-square (rms) value** and is found as the following geometric mean: ... is computed by first the rms value, and squaring the instantly, ... signal is found. Finally, ... and is ... with ... rms. The ... rms ... as shown in Figure 3.14, the rms value is the ...



Figure 3.14

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$$

where ... is the period of the ...

> **The rms value of any periodic voltage is the dc voltage that would deliver the same power to a resistive load as the periodic voltage does.**

$$\text{Form factor} = \frac{\text{rms value}}{\text{average value}}$$

For a ... form factor = ...

$$\text{Peak factor} = \frac{\text{peak value of signal}}{\text{rms value}}$$

For a ... peak factor = ...

The values of form and peak factor give us indications of the shape of waveforms.

> **Problem 3. ...** For this periodic waveform shown in Figure 3.15, determine the peak, the frequency, the average value, rms value, form factor, and the peak factor. Use these factors and the graph below ...



(a) (b)

Figure 8.4.4

(c) **Rectangular waveform of Figure 8.4.4(b)**

(i) Mean for a complete cycle = 0 (by inspection, since, if

Mean Ordinate $\bar{y} = \frac{1}{2\pi}\int_0^{2\pi} y\,d\theta = \frac{1}{2\pi}\int_0^{\pi} V\,d\theta + \frac{1}{2\pi}\int_{\pi}^{2\pi}(-V)\,d\theta$

(ii) Mean value for rectangular waveform for a half cycle

$= \frac{1}{\pi}[\ \text{Area}\ \ \text{rectangle}\ \ \text{rectangle}\ ] = \frac{1}{\pi}[\ \pi V\ \ \pi V\ ]_0^{\pi}\ \text{d}\theta$

$= 0$ (obviously)

Average value of waveform

$= \frac{\text{Area under curve}}{\text{length of base}} = \frac{\text{rectangle}\ \ \text{rectangle}}{2\pi} = \frac{\pi V\ \ \pi V}{2\pi} = \mathbf{0A}$ **V**

(iii) As Figure 8.4.4(c), the form factor is found as the ratio of r.m.s. value to mean value

$$\text{Form factor} = \frac{\text{r.m.s. value}}{\text{mean value}} = \frac{\sqrt{\left(\frac{1}{2\pi}\int_0^{2\pi} y^2\,d\theta\right)}}{\frac{1}{2\pi}\int_0^{2\pi} y\,d\theta}$$

$$= \frac{\sqrt{\left(\frac{1}{2\pi}\left[\int_0^{\pi} V^2\,d\theta + \int_{\pi}^{2\pi}(-V)^2\,d\theta\right]\right)}}{\frac{1}{2\pi}\left[\int_0^{\pi} V\,d\theta + \int_{\pi}^{2\pi}(-V)\,d\theta\right]}$$

$$= \frac{\sqrt{\left(\frac{1}{2\pi}[V^2\theta]_0^{2\pi}\right)}}{0} = \frac{\sqrt{\left(\frac{1}{2\pi}[2\pi V^2]\right)}}{0} = \frac{V}{0} = \infty$$

i.e. infinite

(Note that, in practice, the waveform of a rectangular voltage does not contain the component of the waveform of frequency zero (i.e. the mean ordinate) and generally contains several other components of higher frequency, as shown in Section 8.5.)

(iv) Crest factor $= \frac{\text{peak value}}{\text{r.m.s. value}} = \frac{V}{V} = \mathbf{1.00}$

(v) Peak factor $= \frac{\text{peak value}}{\text{mean value}} = \frac{V}{0} = \infty$

Q11 · Reasoning about a new disease (Algebra, Problem Solving)

(a) Show that a completely healthy person ($H = 0$) has a constant mass P.

Show it: $\dfrac{dP}{dt} = \dfrac{P}{20} - \dfrac{0}{40} - \dfrac{P}{20} = \dfrac{P}{20} - \dfrac{P}{20}$

$$= 0 \text{ kg/day}.$$

(b) Average value over half a cycle $= \dfrac{\text{total added + lost}}{\text{length of half-cycle}}$

$$= \frac{(1 - e^{-\pi}) + (e^{-\pi} - 1)}{\pi} = \frac{0}{\pi} = 0 \text{ kg/day}.$$

(c) The mass value $= \frac{1}{20}\left[\frac{(1-e^{-\pi})}{\pi} + \frac{(e^{-\pi}-1)}{\pi}\right] = 0$ kg/day.

Environmental changes alternately add to and remove mass, but these balance to zero overall.

(c) $\dfrac{\text{mass added}}{\text{average mass}} = \dfrac{2.5}{2.5} = 1$

(c) $\dfrac{\text{mass removed}}{\text{mass added}} = \dfrac{2.5}{2.5} = 1$

Problem 1. The anthropometric group has a compensation scheme in which each hour it is paid £p if it has produced p thousand units:

Hour t (start)	0	1	2	3	4	5	6	7	8	9	10	11
Cumulative $C(t)$	0	7	16	27	40	55	72	91	112	135	160	187

Calculate the mean hourly rate of pay for the whole day. Is the group's pay rate (i) the arithmetic mean of its hourly rates, (ii) the geometric mean, or (iii) the harmonic mean? Explain briefly why this is the appropriate measure, and find its value. (5 marks)

The brief work scheduling proposed by various [planet] is [open, etc.]

(a) Treat the n half-cycles $n = 0$ and. Show that over the n cycles on the period as mean, $\bar{x} = 0$ cycles on $0 \le t \le$

Comparison: $\bar{x} = \dfrac{1}{2}\left[\dfrac{1}{\pi} - \dfrac{1}{\pi}\right] = 0$ **kW·hr**.

(b) Complete the value of income after $t = 0.5$ cos π kPa. Give [Figure] mark.

Complete the rate of income after $t = 0.5$ cos π kPa. Give [Figure] mark.

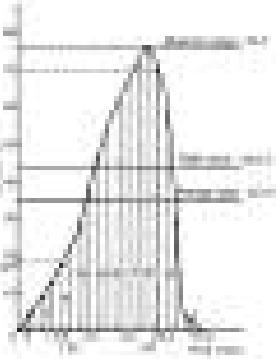


Figure 23.4

(c) Peak or maximum value $= 70$ V

(d) Values of average ordinates $= \frac{\text{sum of the ordinate values}}{\text{total number of ordinates}}$

Taking the ordinates at 10° intervals (see Table 23.1), then the sum of the 19 ordinates (2nd row)

$$\text{sum of the ordinate values} = 0 + 13.7 + 26.7 + 38.3 + 48.2 + 56.4 + 62.4 + 66.0 + 67.0 + 64.4 + 58.4 + 50.2 + 40.0 + 28.0 + 14.5 + 0 \text{ (from Figure 23.4)}$$

$$= 624.3 \text{ V}$$

$$\text{Thus the average value} = \frac{624.3 \text{ V}}{19} = 32.86 \text{ V}$$

(e) The r.m.s. value

$$= \sqrt{\left(\frac{0^2 + 13.7^2 + 26.7^2 + 38.3^2 + 48.2^2 + 56.4^2 + 62.4^2 + 66.0^2 + 67.0^2 + 64.4^2 + 58.4^2 + 50.2^2 + 40.0^2 + 28.0^2 + 14.5^2 + 0^2}{18}\right)}$$

$$= \sqrt{\left(\frac{21\,217}{18}\right)} = 34.34 \text{ V}$$

Problem 1. Calculate the total value of a series-parallel network of resistances shown in Fig.

For a given source, total voltage $= 12.5\ \text{V}$, respectively, where

$$\text{total } I = \text{[illegible]}\ \text{A}$$

Problem 2. Determine the total and branch currents for a [illegible] network supplied

For a given source, total voltage of voltage $R_1 = 6\ \Omega$, R_2 a 12.0 V source supply source and 12.0 V, in the two-wire source

$$V_{\text{in}} = \frac{I}{\left(\frac{1}{R_1}\right)} = \frac{[\text{illegible}]}{\left(\frac{1}{R_1}\right)} = [\text{illegible}]\ \text{V} = \textbf{[illegible value]}$$

Mean value $V_{\text{av}} = 0.637 V_m = 0.637 \times [\text{illegible}] = [\text{illegible}]\ \text{V}$

Problem 3. A [illegible] voltage has a mean value of 10.0 V. Determine its instantaneous value and its rms value

For a sine wave, mean value $= 0.637 \times$ maximum value

$$\text{Mean instantaneous value} = \frac{\text{mean value}}{0.637} = \frac{10.0}{0.637} = 15.70\ \text{V}$$

$$\text{rms value} = 0.707 \times \text{maximum value} = 0.707 \times 15.70 = \textbf{11.1 V}$$

Further problems on d.c. values of alternating waves are found in Section 21.5 (problems 4 to 6) on page 281.

14.5 The operation of a commercial waveband

In Fig. 14.7, a 3-pole, 3-position switch is shown that is fed by mains supply and which is used to switch on lights, a cooker, or a heating element as required.



Figure 14.7



Figure 2.11

...rather than 1 as in the case of the d.c. circuit. For this reason the power in a.c. circuits is less than that in d.c. circuits for the same applied voltage and current.

$$\text{Mean power} = \frac{\text{Area under power curve}}{\text{Length of base}} = \frac{\text{Area under power curve}}{\text{Length of one cycle}}$$

If the mean power in a resistive circuit is 50 W, then the r.m.s. value of the current is 0.707 of the maximum value. Thus, the r.m.s. value of the current is 0.707 of the maximum value. The r.m.s. value of the current is 0.707 of the maximum value. The r.m.s. value of the current is 0.707 of the maximum value.

In Figure 2.11, (a) shows a sine wave of current and voltage, (b) shows the power curve, and (c) shows the phasor diagram. The power curve is a sine wave with a frequency twice that of the current and voltage. The phasor diagram shows the current and voltage as vectors rotating at the same speed.

Worked example 2.1 A sinusoidal voltage is given by $v = 100\sin 314t$ volts. Find (a) the peak value, (b) the r.m.s. value, (c) the frequency, and (d) the time taken for one complete cycle.

(a) The peak value is the maximum value of the voltage, which is 100 V.

(b) The r.m.s. value is $\frac{100}{\sqrt{2}} = 70.7$ V.

(c) Angular velocity $\omega = 314$ rad/s, so $2\pi f = 314$, $f = 50$ Hz.

(d) Periodic time $T = \frac{1}{f} = \frac{1}{50} = 0.02$ s.

Problem 22. An alternating voltage is given by $v = 75\sin(50\pi t)$ volts.

Find: (a) the amplitude, (b) the peak-to-peak value, (c) the rms value, (d) the angular velocity, (e) the frequency, and (f) the phase angle (in degrees and radians) when $t = 5$ ms.

Comparing $v = V_m\sin(\omega t \pm \phi)$ with the general expression $v = 75\sin(50\pi t)$ gives:

(a) Amplitude, or peak value, $V_m = 75$ V

(b) Peak-to-peak value $= 2 \times 75 = 150$ V

(c) The rms value $= 0.707 \times$ maximum value $= 0.707 \times 75 = 53.0$ V

(d) Angular velocity, $\omega = 50\pi$ rad/s

Phase angle $= 0.9425$ rad or 54°

(e) Frequency, $f = \dfrac{\omega}{2\pi} = \dfrac{50\pi}{2\pi} = 25$ Hz

(f) Phase angle at $t = 0.2$ s when $\omega t = 50\pi \times 0.2 = 10\pi$ rad

$= 0.2\pi$ rad $= \left(0.2\pi \times \dfrac{180}{\pi}\right)^\circ = 36^\circ$

Phase angle $= 36^\circ$ leading

Problem 23. An alternating voltage v has a periodic time of 0.01 s and a peak value of 30 V. When $t = 0$, $v = 0$. Express the instantaneous voltage in the form $v = V_m\sin\omega t$ volts.

Amplitude, $V_m = 30$ V

Periodic time $T = \dfrac{2\pi}{\omega} = 0.01$ s, hence angular velocity

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.01} = 200\pi \text{ rad/s}$$

$v = V_m\sin\omega t$ volts $= 30\sin 200\pi t$ volts

When $t = 0.4$ s, $v =$ [illegible]

A car starts from rest and accelerates uniformly along a straight road at $2.5\ \text{m s}^{-2}$ for $12\ \text{s}$. It then travels at constant speed for $30\ \text{s}$. Finally, it brakes uniformly and comes to rest in $8.0\ \text{s}$. Find the total distance travelled.

Split the motion into three stages. Stage 1: $u = 0$, $a = 2.5\ \text{m s}^{-2}$, $t = 12\ \text{s}$. Stage 2: constant speed for $30\ \text{s}$. Stage 3: $a = -\dfrac{v}{8.0}$, $t = 8.0\ \text{s}$, $v = 0$.

Stage 1: $v = 0 + 2.5 \times 12 = 30\ \text{m s}^{-1}$. $s_1 = \tfrac{1}{2}(0 + 30) \times 12 = 180\ \text{m}$.

Stage 2: $s_2 = 30 \times 30 = 900\ \text{m}$.

Stage 3: $s_3 = \tfrac{1}{2}(30 + 0) \times 8.0 = 120\ \text{m}$.

Total distance $= 180 + 900 + 120 = 1200\ \text{m}$.

A ball is thrown vertically upwards from a point $2.0\ \text{m}$ above the ground with a speed of $15\ \text{m s}^{-1}$. Take $g = 9.8\ \text{m s}^{-2}$ and ignore air resistance.

(a) Find the greatest height above the ground reached by the ball.

(b) Find the speed with which the ball strikes the ground.

(c) Find the total time for which the ball is in the air.

(d) Sketch a velocity–time graph for the motion of the ball from the moment it is thrown until it strikes the ground.

(a) Greatest height above P: $v^2 = u^2 + 2as$ with $v = 0$, $u = 15$, $a = -9.8$.

$0 = 225 - 19.6s \Rightarrow s = \dfrac{225}{19.6} = 11.48\ldots\ \text{m}$.

Greatest height $= 2.0 + 11.48\ldots = 13.5\ \text{m}$ (3 s.f.).

(b) Speed on striking ground: $v^2 = u^2 + 2as$ with $s = -2.0$.

$v^2 = 225 + 39.2 = 264.2 \Rightarrow v = \sqrt{264.2} = 16.25\ldots\ \text{m s}^{-1}$.

Speed $= 16.3\ \text{m s}^{-1}$ (3 s.f.).

(c) Time to greatest height: $v = u + at \Rightarrow 0 = 15 - 9.8t \Rightarrow t = 1.53\ldots\ \text{s}$.

Time down from greatest height: $s = ut + \tfrac{1}{2}at^2$ with $s = -13.48\ldots$, $u = 0$.

$-13.48\ldots = -4.9t^2 \Rightarrow t^2 = 2.75\ldots \Rightarrow t = 1.66\ldots\ \text{s}$.

Total time $= 1.53\ldots + 1.66\ldots = 3.19\ \text{s}$ (3 s.f.).

(d) Sketch: straight line from $(0, 15)$ to $(3.19, -16.25)$, crossing the t-axis at $t = 1.53$.

(c) Write the current as a convolution $i = (\dots) * v$.

Then $\hat{i}(\omega) = \hat{K}(\omega)\hat{v}(\omega) + \hat{i}_0(\omega)$

$$\hat{K}(\omega) = \frac{1}{R + j\omega L + \frac{1}{j\omega C}}$$

$$\hat{i}_0(\omega) = \frac{LCi_0(0) - Cv_0(0)}{1 + j\omega RC - \omega^2 LC}$$

Further problems on $i = \dots$ for general $v(t)$ are easy. In general we obtain $i = i_0 + \text{particular} + \text{transient}$ with $\lim_{t\to\infty} \text{transient} = 0$.

14.4.1 Combinations of multiplicities. Four members of the multiplicity family of Ch. 7 are summarized (proofs there) for convenient reference.

(a) For polarizing the projective characteristic amplitudes, every constant factor equals its value for $n = 1$.

(b) For identification of Ephesian the diverging or multiplicative case involves $\Phi(n)$ terms. Compare 7.2.3.

> **Definition 14.4.1.** The *Ephesian* is the value of the diverging expression given by $\lim_{n\to\infty} \frac{\Phi(n)}{n}$, provided this limit exists. It is the density of the integers coprime to n, in the sense that $\frac{1}{n}\#\{1 \le k \le n : \gcd(k,n) = 1\} \to 0$ as $n \to \infty$.

$\Phi(n) = n \prod_{p \mid n} \left(1 - \frac{1}{p}\right)$, so $\frac{\Phi(n)}{n} = \prod_{p \mid n} \left(1 - \frac{1}{p}\right)$ was obtained above for n prime. Here $\Phi(n)$ is multiplicative: $\Phi(mn) = \Phi(m)\Phi(n)$ whenever $\gcd(m,n) = 1$. Several other proofs of this fact are included for later.

For example:

$$\Phi(1) = 1, \quad \Phi(2) = 1, \quad \Phi(3) = 2, \quad \Phi(4) = 2,$$
$$\Phi(5) = 4, \quad \Phi(6) = 2, \quad \Phi(7) = 6, \quad \Phi(8) = 4,$$
$$\Phi(9) = 6, \quad \Phi(10) = 4, \quad \Phi(11) = 10, \quad \Phi(12) = 4.$$

The ratio $\Phi(n)/n$ tends to 1 as n grows, but it does so very slowly. Even for $n = 10^6$ it is only $\Phi(10^6)/10^6 = 0.400000$. The slowness is the point: the Ephesian is a genuine density, not a limit that arrives quickly.

The Ephesian is also the probability that two integers chosen at random are coprime. Among the first N pairs (a, b) with $1 \le a, b \le N$, the number with $\gcd(a,b) = 1$ is asymptotically $\frac{6}{\pi^2} N^2$, so the probability is $\frac{6}{\pi^2} \approx 0.6079$. This connection between a multiplicative function and a probability is one of the reasons the Ephesian appears in so many applications.



Figure 14.1

Since the sine wave repeats every $360°$ or 2π rad, then $\sin 60° = \sin 300°$, $\sin 30° = \sin 150°$, $\sin 45° = \sin 135°$, $\sin 210° = -\sin 30°$, $\sin 225° = -\sin 45°$, $\sin 240° = -\sin 60°$, and so on.

> **Problem 14.1** Two alternating voltages are represented by $v_1 = 50\sin\omega t$ volts and $v_2 = 100\sin(\omega t - \pi/3)$ volts. Are values of v_1 and v_2 equal when $\omega t = 0$, $\pi/3$, $\pi/2$, $2\pi/3$, π, $4\pi/3$, $3\pi/2$, $5\pi/3$ and 2π rad? Are values of v_1 and v_2 equal when $\omega t = 0$, $\pi/6$, $\pi/4$, $\pi/3$, $\pi/2$, $2\pi/3$, $3\pi/4$, $5\pi/6$ and π rad?



When one cycle of a sine wave is completed, it is said to have completed a full cycle or a single cycle or a complete cycle or a complete oscillation or a complete revolution.

The number of cycles occurring per second is called the frequency of the alternating quantity and is measured in hertz (Hz), named after the German physicist Heinrich Hertz (1857–1894). Thus a frequency of 50 Hz means that 50 cycles occur per second.

The time for one complete cycle is called the periodic time or the periodic interval or the period of the waveform. The periodic time may be determined from the waveform by measuring the time interval between two points on the waveform which are exactly one cycle apart. For example, the periodic time of the waveform shown in Figure 14.2(a) may be measured between A and B, or between C and D, or between E and F, and so on.

The periodic time may be determined from the frequency by the relationship

$$\text{periodic time} = \frac{1}{\text{frequency}}$$

Thus if the frequency is 50 Hz, the periodic time is $\frac{1}{50}$ s or 0.02 s.

Figure 14.2



Figure 4.12

(a) [illegible] $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M_O = 0$ [illegible]

(b) [illegible] $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M_O = 0$ [illegible]

(c) [illegible] $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M_O = 0$ [illegible]

Problem 4.1 Three [illegible] are connected by [illegible] as shown in Figure 4.13. Determine the [illegible] at each joint.

(a) The [illegible] is shown in Figure 4.14. The [illegible] is [illegible] and the [illegible] is [illegible]. The [illegible] is [illegible] and the [illegible] is [illegible]. The [illegible] is [illegible] and the [illegible] is [illegible].

Answer for [illegible]: [illegible]

(b) From Figure 4.14 with [illegible] the [illegible] rule:

[illegible]

[illegible]

[illegible]

[illegible]

Answer for [illegible]: [illegible]

Problem 4.2 Two [illegible] [illegible] are joined [illegible] as shown in Figure 4.15. [illegible] and [illegible] [illegible] [illegible]

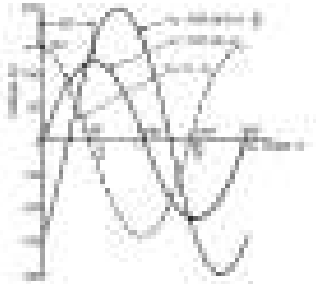


Figure 18.12



Figure 18.13

Figure 3.6: Combining of devices

$$x_1 = x_2 - x_3 = 3.0 \text{ Volumes} - 2.7 \text{ Volts} = 0.3 \text{ Volts}$$

A further problem with combinational logic networks is that they cannot remember past readings as it is shown in 3.8.

3.7 Demultiplexer

The process of selecting one from many common input voltages from different sources and outputs it called **demultiplexer**. Combinational switching is simple to control but the function gets complex.

Logic **demultiplexer** are shown in Figure 3.8 and **demultiplexer combination** is designed. When S is at the early position and P is applied, the output Q is connected to the contact A and P is applied. When S is applied to the output Q, then P is connected with B and then P is applied to the output Q. Thus P is connected to the output Q when S is applied to the output Q.

The demultiplexer is the combination of **demultiplexer** and **demultiplexer**. A multiplexed transmitted T is sent. When T is applied, T is applied to the output Q. When T is applied to the output Q, then T is applied to the output Q. Thus T is applied to the output Q when S is applied to the output Q. Thus T is applied to the output Q when S is applied to the output Q.



Figure 3.8

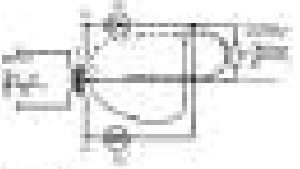


Figure 3.9

Four devices have the speed to a **Demultiplexer** circuit, as shown in Figure 3.10. In this **demultiplexer** and **demultiplexer**, the four devices shown in Figure 3.10. The demultiplexer is designed to be the same as the demultiplexer and both can be used for the same purpose.

To control the output of the demultiplexer, the output is connected to a single input device. Then the demultiplexer is also the same function. The output of the demultiplexer is shown in Figure 3.11.

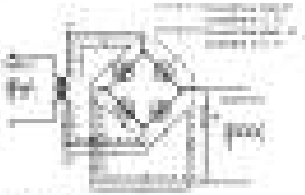


Figure 6.6.16



Figure 6.6.17

6.6.8 Problems on alternating voltages and currents

Frequency and periodic time

1. Determine the periodic time and frequency for the following frequencies:
   (a) 2.5 Hz (b) 50 Hz (c) 0.04 Hz

   [(a) 0.4 s (b) 20 ms (c) 25 s]

2. Calculate the frequency for the following periodic times:
   (a) 1 ms (b) 40 ms (c) 0.2 s

   [(a) 1 kHz (b) 25 Hz (c) 5 Hz]

3. An alternating current completes 5 cycles in 0.1 ms. What is the frequency?

   [50 kHz]

Values of sine-wave alternating quantities

4. An alternating current varies with time over half a cycle as follows:

Current (A)	0	0.7	1.0	0.5	0.3	0.9	1.2	1.5	1.4	0.8	0
Time (ms)	0	1	2	3	4	5	6	7	8	9	10

   Plot the waveform and, by using the mid-ordinate rule with 10 intervals, determine the mean value of the current over the half cycle. Hence determine the r.m.s. value of the current over the half cycle and the form factor.

   [Mean 0.89 A, r.m.s. 0.9 A, form factor 1.01]

5. For the waveform shown in Figure 6.6.18 determine the mean value, the frequency and the average value over the first 10 ms. Determine also the r.m.s. value over the first 10 ms and the form factor.

   [Mean value 50 A, frequency 50 Hz, average value over first 10 ms 25 A, r.m.s. value over first 10 ms 35.4 A, form factor 1.42]



Figure 10.14

7. (a) Determine the voltage v_{ab} in the circuit of Figure 10.15(a) if a 2 Ω resistor is connected between a and b. (b) If a 5 mA current source is connected between a and b, with the arrow pointing toward a, find v_{ab}. (c) Repeat part (b) if the 5 mA current source is connected between b and a.

(a) 24 V, (b) 34 V, (c) 14 V

8. Find the power associated with each element in the circuit of Figure 10.15(b).

$p_{10V} = -30$ W, $p_{2\Omega} = 18$ W, $p_{4\Omega} = 12$ W

9. (a) Calculate the voltage v_{ab} in the circuit of Figure 10.16(a). (b) Find the power dissipated in each resistor.

(a) 6 V, (b) 72 mW, 36 mW, 18 mW

10. In the circuit of Figure 10.16(b), find v_1, v_2, and v_3. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V

10.3 Kirchhoff's Laws

11. (a) Determine the voltage v_x in the circuit of Figure 10.17(a). (b) Find the power dissipated in each of the resistors in the circuit of Figure 10.17(b). (c) Calculate the power supplied by the dependent source in the circuit of Figure 10.17(c).

(a) 20 V, (b) 2 W, 4 W, 8 W, (c) 24 W

12. Find the node voltages, the current through, the power supplied by the voltage source, and the power absorbed by the resistor in the circuit of Figure 10.18(a).

$v_1 = 4$ V, $v_2 = 8$ V, $i = 2$ A, $p_{6V} = -12$ W, $p_{2\Omega} = 32$ W

13. Find v_1, v_2, and v_3 in the circuit of Figure 10.18(b). Determine the power dissipated in each resistor.

$v_1 = 12$ V, $v_2 = 8$ V, $v_3 = 4$ V, $p_{1\Omega} = 144$ W, $p_{2\Omega} = 32$ W, $p_{4\Omega} = 16$ W

14. Find v_1 and v_2 in the circuit of Figure 10.19(a). Calculate the power dissipated in each resistor.

$v_1 = 6$ V, $v_2 = 2$ V, $p_{1\Omega} = 36$ W, $p_{2\Omega} = 8$ W, $p_{3\Omega} = 12$ W, $p_{6\Omega} = 4$ W

15. Find v_1, v_2, and v_3 in the circuit of Figure 10.19(b). Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

16. Find v_1, v_2, and v_3 in the circuit of Figure 10.20(a). Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

17. Find v_1, v_2, and v_3 in the circuit of Figure 10.20(b). Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

18. Find v_1, v_2, and v_3 in the circuit of Figure 10.21. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

19. Find v_1, v_2, and v_3 in the circuit of Figure 10.22. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

20. Find v_1, v_2, and v_3 in the circuit of Figure 10.23. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

21. Find v_1, v_2, and v_3 in the circuit of Figure 10.24. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

22. Find v_1, v_2, and v_3 in the circuit of Figure 10.25. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

23. Find v_1, v_2, and v_3 in the circuit of Figure 10.26. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

24. Find v_1, v_2, and v_3 in the circuit of Figure 10.27. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

25. Find v_1, v_2, and v_3 in the circuit of Figure 10.28. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

26. Find v_1, v_2, and v_3 in the circuit of Figure 10.29. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

27. Find v_1, v_2, and v_3 in the circuit of Figure 10.30. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

28. Find v_1, v_2, and v_3 in the circuit of Figure 10.31. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

29. Find v_1, v_2, and v_3 in the circuit of Figure 10.32. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

30. Find v_1, v_2, and v_3 in the circuit of Figure 10.33. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

31. Find v_1, v_2, and v_3 in the circuit of Figure 10.34. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

32. Find v_1, v_2, and v_3 in the circuit of Figure 10.35. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

33. Find v_1, v_2, and v_3 in the circuit of Figure 10.36. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

34. Find v_1, v_2, and v_3 in the circuit of Figure 10.37. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

35. Find v_1, v_2, and v_3 in the circuit of Figure 10.38. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

36. Find v_1, v_2, and v_3 in the circuit of Figure 10.39. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

37. Find v_1, v_2, and v_3 in the circuit of Figure 10.40. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

38. Find v_1, v_2, and v_3 in the circuit of Figure 10.41. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

39. Find v_1, v_2, and v_3 in the circuit of Figure 10.42. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

40. Find v_1, v_2, and v_3 in the circuit of Figure 10.43. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

41. Find v_1, v_2, and v_3 in the circuit of Figure 10.44. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

42. Find v_1, v_2, and v_3 in the circuit of Figure 10.45. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

43. Find v_1, v_2, and v_3 in the circuit of Figure 10.46. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

44. Find v_1, v_2, and v_3 in the circuit of Figure 10.47. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

45. Find v_1, v_2, and v_3 in the circuit of Figure 10.48. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

46. Find v_1, v_2, and v_3 in the circuit of Figure 10.49. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

47. Find v_1, v_2, and v_3 in the circuit of Figure 10.50. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

48. Find v_1, v_2, and v_3 in the circuit of Figure 10.51. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

49. Find v_1, v_2, and v_3 in the circuit of Figure 10.52. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

50. Find v_1, v_2, and v_3 in the circuit of Figure 10.53. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

51. Find v_1, v_2, and v_3 in the circuit of Figure 10.54. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

52. Find v_1, v_2, and v_3 in the circuit of Figure 10.55. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

53. Find v_1, v_2, and v_3 in the circuit of Figure 10.56. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

54. Find v_1, v_2, and v_3 in the circuit of Figure 10.57. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

55. Find v_1, v_2, and v_3 in the circuit of Figure 10.58. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

56. Find v_1, v_2, and v_3 in the circuit of Figure 10.59. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

57. Find v_1, v_2, and v_3 in the circuit of Figure 10.60. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

58. Find v_1, v_2, and v_3 in the circuit of Figure 10.61. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

59. Find v_1, v_2, and v_3 in the circuit of Figure 10.62. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

60. Find v_1, v_2, and v_3 in the circuit of Figure 10.63. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

61. Find v_1, v_2, and v_3 in the circuit of Figure 10.64. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

62. Find v_1, v_2, and v_3 in the circuit of Figure 10.65. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

63. Find v_1, v_2, and v_3 in the circuit of Figure 10.66. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

64. Find v_1, v_2, and v_3 in the circuit of Figure 10.67. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

65. Find v_1, v_2, and v_3 in the circuit of Figure 10.68. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

66. Find v_1, v_2, and v_3 in the circuit of Figure 10.69. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

67. Find v_1, v_2, and v_3 in the circuit of Figure 10.70. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

68. Find v_1, v_2, and v_3 in the circuit of Figure 10.71. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

69. Find v_1, v_2, and v_3 in the circuit of Figure 10.72. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

70. Find v_1, v_2, and v_3 in the circuit of Figure 10.73. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

71. Find v_1, v_2, and v_3 in the circuit of Figure 10.74. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

72. Find v_1, v_2, and v_3 in the circuit of Figure 10.75. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

73. Find v_1, v_2, and v_3 in the circuit of Figure 10.76. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

74. Find v_1, v_2, and v_3 in the circuit of Figure 10.77. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

75. Find v_1, v_2, and v_3 in the circuit of Figure 10.78. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

76. Find v_1, v_2, and v_3 in the circuit of Figure 10.79. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

77. Find v_1, v_2, and v_3 in the circuit of Figure 10.80. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

78. Find v_1, v_2, and v_3 in the circuit of Figure 10.81. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

79. Find v_1, v_2, and v_3 in the circuit of Figure 10.82. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

80. Find v_1, v_2, and v_3 in the circuit of Figure 10.83. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

81. Find v_1, v_2, and v_3 in the circuit of Figure 10.84. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

82. Find v_1, v_2, and v_3 in the circuit of Figure 10.85. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

83. Find v_1, v_2, and v_3 in the circuit of Figure 10.86. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

84. Find v_1, v_2, and v_3 in the circuit of Figure 10.87. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

85. Find v_1, v_2, and v_3 in the circuit of Figure 10.88. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

86. Find v_1, v_2, and v_3 in the circuit of Figure 10.89. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

87. Find v_1, v_2, and v_3 in the circuit of Figure 10.90. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

88. Find v_1, v_2, and v_3 in the circuit of Figure 10.91. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

89. Find v_1, v_2, and v_3 in the circuit of Figure 10.92. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

90. Find v_1, v_2, and v_3 in the circuit of Figure 10.93. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

91. Find v_1, v_2, and v_3 in the circuit of Figure 10.94. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

92. Find v_1, v_2, and v_3 in the circuit of Figure 10.95. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

93. Find v_1, v_2, and v_3 in the circuit of Figure 10.96. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

94. Find v_1, v_2, and v_3 in the circuit of Figure 10.97. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

95. Find v_1, v_2, and v_3 in the circuit of Figure 10.98. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

96. Find v_1, v_2, and v_3 in the circuit of Figure 10.99. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

97. Find v_1, v_2, and v_3 in the circuit of Figure 10.100. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

98. Find v_1, v_2, and v_3 in the circuit of Figure 10.101. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

99. Find v_1, v_2, and v_3 in the circuit of Figure 10.102. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

100. Find v_1, v_2, and v_3 in the circuit of Figure 10.103. Calculate the power dissipated in each resistor.

$v_1 = 4$ V, $v_2 = 8$ V, $v_3 = 12$ V, $p_{1\Omega} = 16$ W, $p_{2\Omega} = 32$ W, $p_{3\Omega} = 48$ W, $p_{4\Omega} = 64$ W

(a) The cardinal number $|\mathbb{N}|$.
(b) The cardinal number $|[0,1]|$.
(c) An example of a finite set S with more than one cardinal in $\mathcal{P}(S)$.
(d) An example of a finite set S with infinitely many cardinals in $\mathcal{P}(S)$.
(e) A finite set S for which $\mathcal{P}(S)$ has a unique cardinal.

Answer. (a) $\aleph_0$. (b) $2^{\aleph_0}$. (c) $S = \{0, 1\}$: $\mathcal{P}(S)$ has 4 elements, all finite. (d) $S = \emptyset$: $\mathcal{P}(S) = \{\emptyset\}$, one element. (e) Same S: $|\mathcal{P}(S)| = 1$.

Continuity of periodic functions

3.2 Let $f : \mathbb{R} \to \mathbb{R}$ be a function with the property that there exists a number $p > 0$ such that $f(x + p) = f(x)$ for all $x \in \mathbb{R}$. Suppose that f is continuous on the closed interval $[0, p]$. Prove that f is uniformly continuous on all of $\mathbb{R}$.

Proof. Let $\varepsilon > 0$. Since f is continuous on the compact set $[0, p]$, it is uniformly continuous there, so there exists $\delta > 0$ such that $|x - y| < \delta$ and $x, y \in [0, p]$ imply $|f(x) - f(y)| < \varepsilon$. Now take any $x, y \in \mathbb{R}$ with $|x - y| < \delta$. Choose an integer k such that $x - kp \in [0, p]$, and set $y - kp$ as well; since $|(x - kp) - (y - kp)| = |x - y| < \delta$, we have $|f(x) - f(y)| = |f(x - kp) - f(y - kp)| < \varepsilon$. Hence f is uniformly continuous on $\mathbb{R}$. $\square$

3.3 Suppose f is as in Problem 3.2 above. Must f be bounded? Must f attain its maximum? Justify your answers.

3.4 Let $f : \mathbb{R} \to \mathbb{R}$ be continuous and periodic with period $p > 0$. Show that f is uniformly continuous on $\mathbb{R}$ and that f is bounded on $\mathbb{R}$. Use the extreme value theorem on the compact interval $[0, p]$ to conclude that f attains its maximum and its minimum on $\mathbb{R}$.

3.5 Determine, with a brief justification, whether each statement is true or false. (a) Every continuous function $f : \mathbb{R} \to \mathbb{R}$ is uniformly continuous. (b) Every uniformly continuous function $f : \mathbb{R} \to \mathbb{R}$ is bounded. (c) Every bounded continuous function $f : \mathbb{R} \to \mathbb{R}$ attains its maximum.

Assignment 4

The assignment covers the material introduced in chapters 13 and 14.

Do make the most questions as you can. Number clearly your work and questions.

1 Find the system during in the 3 the system of the circuit shown in Figure 4.1 using (i) Kirchhoff's laws, (ii) the superposition theorem, (iii) Thevenin's theorem and (iv) Norton's theorem. Demonstrate that the three methods yield identical results. Find also the current flowing in each of the three resistors in the circuit. [25]

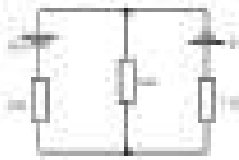


Figure 4.1

2 A d.c. voltage source has an internal resistance of 1 Ω and an open circuit voltage of 24 V. What is the value of load resistance that gives maximum power transfer to such a source? What is the value of this power? [10]

3 A sinusoidal voltage has a mean value of 318 V. Determine its r.m.s. value and peak value. [5]

4 The instantaneous value of current in an a.c. circuit is given by $i = 5\sin 314t$ amperes. Determine

(a) the peak to peak current, the periodic time, the frequency and the phase angle (in degrees and radians),
(b) the current when $t = 0$,
(c) the current when $t = 5$ ms,
(d) the first time when the current is a maximum.

Sketch the waveform for one cycle of the alternating current. [15]

15 Single-phase series a.c. circuits



Figure 15.1

15.1 The R-L series a.c. circuit

8.4.1 · Purely inductive AC circuit

In a purely inductive ac circuit, the current i lags the applied voltage v by 90° (or $\pi/2$ rad), as shown in Figure 8.4.1.

In a purely inductive circuit, the opposition to the flow of alternating current is called the **inductive reactance**, X_L:

$$X_L = 2\pi f L = \omega L \ \Omega$$

where f is the applied frequency in hertz, and L is the inductance in henrys. X_L is proportional to f, as illustrated in Figure 8.4.2.



Figure 8.4.1



Figure 8.4.2

Worked Example 1 — (a) Calculate the inductive reactance of a 0.25 H inductance coil at 60 Hz. (b) If the coil has a resistance of 10 Ω, calculate the impedance of the coil and the current taken from a 240 V, 60 Hz supply. (c) Draw the phasor diagram for the circuit.

(a) Inductive reactance $X_L = 2\pi f L = 2\pi(60)(0.25) = \mathbf{94.25\ \Omega}$

(b) Impedance $Z = \sqrt{R^2 + X_L^2} = \sqrt{10^2 + 94.25^2} = \mathbf{94.8\ \Omega}$

Current $I = \dfrac{V}{Z} = \dfrac{240}{94.8} = \mathbf{2.53\ A}$

Worked Example 2 — A coil has an inductance of 80 mH and negligible resistance. Calculate its inductive reactance and the resulting current if connected to a 230 V, 50 Hz supply. If the supply voltage is now 115 V, 100 Hz, what is the new current?

(a) Inductive reactance $X_L = 2\pi(50)(0.080) = 25.13\ \Omega$

Current $I = \dfrac{V}{X_L} = \dfrac{230}{25.13} = \mathbf{9.15\ A}$

(b) Inductive reactance $X_L = 2\pi(100)(0.080) = 50.27\ \Omega$

Current $I = \dfrac{V}{X_L} = \dfrac{115}{50.27} = \mathbf{2.29\ A}$

8.4.2 · Purely capacitive a.c. circuit

In a purely capacitive ac circuit, the current i leads the applied voltage v by 90° (or $\pi/2$ rad), as shown in Figure 8.4.3.

In a purely capacitive circuit, the opposition to the flow of alternating current is called the **capacitive reactance**, X_C:

$$X_C = \frac{1}{2\pi f C} = \frac{1}{\omega C} \ \Omega$$

where C is the capacitance in farads.

X_C is inversely proportional to f, as illustrated in Figure 8.4.4.



Problem 1 Determine the capacitive reactance of a capacitor of 10 μF when connected to a source of frequency (a) 50 Hz, (b) 10 kHz

(a) Capacitive reactance $X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 10 \times 10^{-6}}$

$= \frac{10^6}{3140}\ \Omega$

$= 318.5\ \Omega$

(b) $X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 10 \times 10^3 \times 10 \times 10^{-6}} = \frac{10^6}{628}\ \Omega$

$= 1.592\ \Omega$



Figure 10.4

Since capacitive reactance is inversely proportional to frequency, X_C falls as f rises. This is illustrated in Figure 10.4 for the circuit in Figure 10.3.



Figure 10.5

Problem 2 A capacitor has a reactance of 50 Ω when connected to a 50 Hz supply. Determine the value of its capacitance.

Since $X_C = \frac{1}{2\pi fC}$, capacitance $C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times 50 \times 50}$ F

$= \frac{10^6}{15700}\ \mu\text{F} = 63.69\ \mu\text{F}$

Problem 3 Calculate the current taking when a 31.8 μF capacitor is connected to a 240 V, 50 Hz supply.

Current $I = \frac{V}{X_C} = \frac{V}{\frac{1}{2\pi fC}} = 2\pi fCV = 2\pi \times 50 \times 31.8 \times 10^{-6} \times 240$

$= 2.40\ \text{A}$

Further problems on capacitive reactance and capacitance are available in Problems 1 to 4 on page 312 and Problems 1 to 4 on page 320.

10.4 R–C series ac circuits

In an R–C series circuit, sometimes referred to as a load containing R–C, the applied voltage V is the phasor sum of V_R and V_C (see Figure 10.6(a)) and since the current is the same for both components, V_R is in phase with I and V_C lags I by $90°$ (see Section 10.3). Hence V and I are out of phase by between 0° and 90°, depending on the values of V_R and V_C. The phasor diagram is shown in Figure

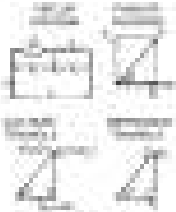


Figure 10.01

Any passive network that contains a resistance or more components can be thus drawn as the combination of them.

Given the planar diagram of Figure 2.1.1, the **voltage rise/drop** is denoted by V and the R is denoted $R = \text{constant}$. The voltage/potential relationship

$$\text{and } \text{and } \text{and } = \begin{cases} \\ \end{cases} \quad \text{the voltage/potential relationship}$$

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Example 9.1 Power Calculation for a Two-Sample Test with Known Variance

Suppose we are designing a study to compare the mean response between two treatments. Each treatment will be assigned to an independent group of patients. The response is assumed to be normally distributed with a known standard deviation of $\sigma = 12$ units. We plan to use a two-sided test at the $\alpha = 0.05$ level. If the true difference between the means is $\delta = 5$ units, we want the test to have power of at least 0.80. How many patients do we need in each group?

Let μ_1 and μ_2 denote the mean responses for the two treatments, and let n denote the common sample size per group. We test $H_0: \mu_1 - \mu_2 = 0$ against $H_1: \mu_1 - \mu_2 \neq 0$. The test statistic is

$$Z = \frac{\bar{X}_1 - \bar{X}_2}{\sigma\sqrt{2/n}},$$

which follows a standard normal distribution under H_0. We reject H_0 when $|Z| > z_{1-\alpha/2} = z_{0.975} = 1.96$.

Under the alternative, Z follows approximately a normal distribution with mean $\delta\sqrt{n}/(\sigma\sqrt{2})$ and unit variance. The power is therefore

$$\text{Power} = \Phi\left(\frac{\delta\sqrt{n}}{\sigma\sqrt{2}} - z_{1-\alpha/2}\right) + \Phi\left(-\frac{\delta\sqrt{n}}{\sigma\sqrt{2}} - z_{1-\alpha/2}\right),$$

where Φ is the standard normal cumulative distribution function. The second term is negligible when $\delta > 0$, so we can approximate the power by $\Phi\left(\frac{\delta\sqrt{n}}{\sigma\sqrt{2}} - 1.96\right)$. Setting this equal to 0.80 and solving for n gives

$$n = \frac{2\sigma^2(z_{0.975} + z_{0.80})^2}{\delta^2} = \frac{2 \times 12^2 \times (1.96 + 0.842)^2}{5^2} \approx 90.4.$$

Rounding up, we require $n = 91$ patients per group, for a total of 182 patients. If the true difference is smaller, say $\delta = 3$, the required sample size increases to $n = 252$ per group, illustrating how sensitive power is to the effect size.

Example 9.2 Power for a One-Sample Test with Unknown Variance

A quality engineer wants to test whether the mean fill weight of a machine differs from the target value of $\mu_0 = 500$ grams. Based on pilot data, the standard deviation is estimated to be $s = 8$ grams. The engineer plans to collect $n = 25$ measurements and use a two-sided t-test at $\alpha = 0.05$. What is the power to detect a true mean of $\mu = 503$ grams?

The test statistic is $T = (\bar{X} - \mu_0)/(s/\sqrt{n})$, which follows a t-distribution with $n - 1 = 24$ degrees of freedom under H_0. The critical value is $t_{0.975,24} = 2.064$. Under the alternative, T follows a noncentral t-distribution with noncentrality parameter $\delta\sqrt{n}/\sigma = 3\sqrt{25}/8 = 1.875$. The power is the probability that $|T|$ exceeds the critical value under this noncentral distribution.

Using the noncentral t distribution function, the power is approximately 0.52. This relatively low power reflects the small sample size and the fact that the effect size is only about 0.4 standard deviations. To achieve power of 0.80 for this same effect size, we would need $n = 50$ observations.

Example 9.3 Power for a Two-Sample Test with Unknown Variance

Researchers are comparing the effectiveness of two teaching methods. They plan to assign $n = 30$ students to each method and measure performance on a standardized exam. Based on previous studies, they expect the standard deviation of scores to be about $\sigma = 15$ points. They want to detect a difference of $\delta = 6$ points between the two methods using a two-sided t-test at $\alpha = 0.05$. What is the power of this test?

The test statistic is $T = (\bar{X}_1 - \bar{X}_2)/(S_p\sqrt{2/n})$, where S_p is the pooled sample standard deviation. Under the null hypothesis, T follows a t-distribution with $2n - 2 = 58$ degrees of freedom. The critical value is $t_{0.975,58} = 2.002$. Under the alternative, T follows a noncentral t-distribution with noncentrality parameter $\delta\sqrt{n}/(\sigma\sqrt{2}) = 6\sqrt{30}/(15\sqrt{2}) = 1.549$.

Using the noncentral t distribution function, the power is approximately 0.61. This means that even with 30 students per group, there is only about a 61% chance of detecting a true difference of 6 points. To achieve power of 0.80, they would need approximately $n = 45$ students per group.

Example 9.4 Power for a Paired t-Test

A nutritionist wants to study the effect of a new diet on cholesterol levels. She plans to measure the cholesterol of $n = 20$ participants before and after the diet. The standard deviation of the differences is expected to be $\sigma_D = 10$ mg/dL. She wants to detect a mean reduction of $\delta = 5$ mg/dL using a two-sided test at $\alpha = 0.05$. What is the power?

Let $D_i = X_{i,\text{before}} - X_{i,\text{after}}$ for $i = 1, \ldots, n$. The test statistic is $T = \bar{D}/(S_D/\sqrt{n})$, which follows a t-distribution with $n - 1 = 19$ degrees of freedom under H_0. The critical value is $t_{0.975,19} = 2.093$. Under the alternative, T follows a noncentral t-distribution with noncentrality parameter $\delta\sqrt{n}/\sigma_D = 5\sqrt{20}/10 = 2.236$.

Using the noncentral t distribution function, the power is approximately 0.73. This means there is about a 73% chance of detecting a mean reduction of 5 mg/dL. To achieve power of 0.80, she would need approximately $n = 25$ participants.

Example 9.5 Power for a One-Way ANOVA

A researcher is comparing the mean yield of a crop under $k = 4$ different fertilizer treatments. She plans to assign $n = 10$ plots to each treatment. Based on previous experiments, the within-group standard deviation is expected to be $\sigma = 4$ bushels per acre. She wants to detect a maximum difference between any two treatment means of $\delta = 6$ bushels per acre using the F-test at $\alpha = 0.05$. What is the power?

The test statistic is $F = \text{MS}_{\text{between}}/\text{MS}_{\text{within}}$, which follows an F-distribution with $k - 1 = 3$ and $N - k = 36$ degrees of freedom under H_0, where $N = kn = 40$. The critical value is $F_{0.95,3,36} = 2.866$. Under the alternative, F follows a noncentral F-distribution with noncentrality parameter $\lambda = n\sum_{j=1}^{k}(\mu_j - \bar{\mu})^2/\sigma^2$. For the configuration where two means differ by δ and the others equal the grand mean, $\lambda = 10 \times 18/16 = 11.25$.

Using the noncentral F distribution function, the power is approximately 0.72. This means there is about a 72% chance of detecting a maximum difference of 6 bushels per acre among the four treatments. To achieve power of 0.80, she would need approximately $n = 12$ plots per treatment.

(a) Current $I = \frac{V}{R} = \frac{230\ \text{V}}{47\ \Omega} \approx 4.894\ \text{A}$.

(b) Power $P = VI = 230 \times 4.894 \approx 1125.6\ \text{W}$. Energy $E = Pt = 1125.6 \times 3600 \approx 4.05 \times 10^6\ \text{J}$.

(c) Power $P = I^2R = 4.894^2 \times 47 \approx 1125.6\ \text{W}$. Energy $E = 4.05 \times 10^6\ \text{J}$ (same as (b)).

(d) Power $P = \frac{V^2}{R} = \frac{230^2}{47} \approx 1125.6\ \text{W}$. Energy $E = 4.05 \times 10^6\ \text{J}$ (same as (b)).

(e) Ohm's law, power formulas, and energy are independent of frequency for a resistor; the result is unchanged.

Problem 11.2. A coil of resistance $R = 12\ \Omega$ and inductance $L = 0.080\ \text{H}$ is connected to a sinusoidal supply of $V_{\text{rms}} = 240\ \text{V}$ at $f = 50\ \text{Hz}$. (a) Find the coil's impedance magnitude $|Z|$. (b) Find the rms current I_{rms}. (c) Find the power factor $\cos\varphi$ and the phase angle φ between voltage and current. (d) Find the real power P and the reactive power Q drawn by the coil.

Solution 11.2. Inductive reactance $X_L = 2\pi f L = 2\pi \times 50 \times 0.080 \approx 25.13\ \Omega$. Impedance $Z = R + jX_L = 12 + j25.13\ \Omega$, so $|Z| = \sqrt{12^2 + 25.13^2} \approx 27.86\ \Omega$. Current $I_{\text{rms}} = \frac{V_{\text{rms}}}{|Z|} = \frac{240}{27.86} \approx 8.614\ \text{A}$. Power factor $\cos\varphi = \frac{R}{|Z|} = \frac{12}{27.86} \approx 0.431$ (lagging), $\varphi = \arccos(0.431) \approx 64.5^\circ$. Real power $P = I_{\text{rms}}^2 R \approx 8.614^2 \times 12 \approx 890.4\ \text{W}$. Reactive power $Q = I_{\text{rms}}^2 X_L \approx 8.614^2 \times 25.13 \approx 1865.8\ \text{VAR}$. Check: $P = VI\cos\varphi = 240 \times 8.614 \times 0.431 \approx 890.4\ \text{W}$.



Figure 11.1

Problem 11.3. A coil of resistance $R = 8.0\ \Omega$ and inductance $L = 0.040\ \text{H}$ is connected in series with a pure capacitor C to a $V_{\text{rms}} = 120\ \text{V}$, $60\ \text{Hz}$ supply. (a) Find the capacitance C that makes the circuit resonant. (b) Find the resulting rms current I_{rms}. (c) Find the power factor $\cos\varphi$ and the phase angle φ between voltage and current. (d) Find the power dissipated P.

Solution 11.3. At resonance $X_L = X_C$, so $2\pi f L = \frac{1}{2\pi f C}$, giving $C = \frac{1}{(2\pi f)^2 L} = \frac{1}{(2\pi \times 60)^2 \times 0.040} \approx 1.055 \times 10^{-3}\ \text{F} = 1055\ \mu\text{F}$. At resonance the impedance is purely resistive, $Z = R = 8.0\ \Omega$, so $I_{\text{rms}} = \frac{V_{\text{rms}}}{R} = \frac{120}{8.0} = 15\ \text{A}$. Power factor $\cos\varphi = 1$ (unity), $\varphi = 0^\circ$. Power dissipated $P = I_{\text{rms}}^2 R = 15^2 \times 8.0 = 1800\ \text{W}$. Check: $P = VI\cos\varphi = 120 \times 15 \times 1 = 1800\ \text{W}$.





Figure 6.10



Figure 6.11

$$\text{[illegible]} \quad \text{[illegible]}$$

The 2.40 Ω segment is bridged by the shunt wire of 0.3 Ω, so its

[illegible] [illegible] [illegible] [

When supply frequency changes, inductive reactance $X_L = 2\pi fL$ changes proportionally, while capacitive reactance $X_C = \frac{1}{2\pi fC}$ changes inversely. At very low frequencies, X_L is small and X_C is large; at very high frequencies, the opposite is true. Because the two reactances respond differently, there is usually a frequency at which $X_L = X_C$. At that frequency, the reactive voltages across the two elements are equal in magnitude and opposite in sign, so they cancel in the loop sum. The circuit then behaves as if only the resistance were present.

> **Problem 2.3.** A capacitor C is connected in series with a 40 Ω resistor across a supply of frequency 50 Hz. A voltmeter of 0 Ω internal resistance is first connected across the capacitor and reads 120 V; it is then moved across the resistor and reads 90 V. Find the capacitance C, the supply voltage, and the phase angle between the supply voltage and the current. Does the current lead or lag the supply voltage?

(a) Impedance $Z = \sqrt{R^2 + X_C^2}$.

Phasor $\mathbf{I} = \frac{\mathbf{V}_R}{R}$, $|\mathbf{I}| = \frac{90}{40} = 2.25$ A.

$$X_C = \frac{|\mathbf{V}_C|}{|\mathbf{I}|} = \frac{120}{2.25} = \frac{1}{2\pi fC}, \quad C = \frac{2.25}{2\pi \cdot 50 \cdot 120} \approx 59.7\ \mu\text{F}.$$

(b) Phasor $\mathbf{V} = \mathbf{V}_R + \mathbf{V}_C$, $|\mathbf{V}| = \sqrt{90^2 + 120^2} = 150$ V.

(c) Phase angle $\varphi = \arctan\left(\frac{X_C}{R}\right) = \arctan\left(\frac{120}{90}\right) \approx 53.1°$ — current leads voltage.

(d) $P = I^2 R = 2.25^2 \times 40 = 202.5$ W.

(e) $Q = I^2 X_C = 2.25^2 \times 120 = 607.5$ VAr.

The phasor diagram is shown in Figure 2.3.1: current $\mathbf{I}$ leads the supply voltage $\mathbf{V}$ by the phase angle of $\approx 53.1°$.



Figure 2.3.1

2.4 R–L–C series circuit

A coil of resistance 8 Ω and inductance 0.05 H is connected in series with a capacitor of 100 μF across a 230 V, 50 Hz supply. Calculate the current, the power factor, and the power absorbed by the coil and by the capacitor. Draw a phasor diagram showing the supply voltage and the voltage across the coil, and state whether the current leads or lags the supply voltage.

Solution. $\omega = 2\pi f = 2\pi \times 50 = 100\pi$ rad/s.

$X_L = \omega L = 100\pi \times 0.05 = 5\pi \approx 15.71\ \Omega$.

$X_C = \frac{1}{\omega C} = \frac{1}{100\pi \times 100 \times 10^{-6}} = \frac{10^4}{100\pi} \approx 31.83\ \Omega$.

Coil impedance $Z_{coil} = 8 + j15.71\ \Omega$, $|Z_{coil}| = \sqrt{8^2 + 15.71^2} \approx 17.63\ \Omega$.

Capacitor impedance $Z_C = -j31.83\ \Omega$.

Total impedance $Z = Z_{coil} + Z_C = 8 + j(15.71 - 31.83) = 8 - j16.12\ \Omega$, $|Z| = \sqrt{8^2 + 16.12^2} \approx 18.00\ \Omega$.

Current $I = \frac{230}{18.00} \approx 12.78$ A.

Power factor $\cos\varphi = \frac{8}{18.00} \approx 0.444$ leading (the capacitive reactance dominates, so the current leads the supply voltage).

Power absorbed by the coil $P_{coil} = I^2 R_{coil} = 12.78^2 \times 8 \approx 1307$ W.

Power absorbed by the capacitor $P_C = I^2 R_C = 0$ W (an ideal capacitor dissipates no power).

Reactive power of the coil $Q_{coil} = I^2 X_L \approx 1284$ VAr; of the capacitor $Q_C = -I^2 X_C \approx -3126$ VAr.

The coil voltage $V_{coil} = I \cdot |Z_{coil}| \approx 12.78 \times 17.63 \approx 225.3$ V, and the capacitor voltage $V_C = I X_C \approx 406.8$ V. The phasor diagram (Figure 2.4.1) shows $\mathbf{V}_{coil}$, $\mathbf{V}_C$, and $\mathbf{V}$ forming a triangle, with $\mathbf{I}$ as the reference phasor. Because $V_C > V_{coil}$, the capacitive reactance dominates, so the current leads the supply voltage by $\varphi \approx 26.4°$.

Figure 2.4.1 Phasor diagram for the R–L–C series circuit of Example 2.4. The current $\mathbf{I}$ is the reference phasor; $\mathbf{V}_{coil}$ leads $\mathbf{I}$ by 61.9°, $\mathbf{V}_C$ lags $\mathbf{I}$ by 90°, and $\mathbf{V}$ leads $\mathbf{I}$ by 26.4°.



Figure 10.18

Worked Ex. 1. In the figure a battery $E = 12\ \text{V}$ with internal resistance $r = 0.5\ \Omega$ is connected to a load $R = 5.5\ \Omega$. Find the current, the terminal voltage, and the power delivered to the load.

$$I = \frac{E}{R + r} = \frac{12}{5.5 + 0.5} = 2\ \text{A}$$

$$V = IR = 2 \times 5.5 = 11\ \text{V}$$

$$P = VI = 11 \times 2 = 22\ \text{W}$$

Worked Ex. 2. — In the figure (10.19(b)) the applied voltage V and the current I are in phase. Then there is neither inductive nor capacitive reactance. The circuit is purely resistive, and the power factor is unity.



Figure 10.19

> **Problem 10.** A coil of resistance $8\ \Omega$ and inductance $0.05\ \text{H}$ is in series with a $10\ \mu\text{F}$ capacitor, a variable resistance R, and a $50\ \text{Hz}$, $200\ \text{V}$ supply. Calculate the value of R which will make the power factor unity, and calculate the current in this condition. Also find the voltage across the capacitor.

The circuit diagram is shown in Figure 10.20.

$$X_L = 2\pi f L = 2\pi(50)(0.05) = 15.7\ \Omega \text{ (to 3 s.f.)}$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(50)(10 \times 10^{-6})} = 318\ \Omega \text{ (to 3 s.f.)}$$

Since X_L is smaller than X_C, the circuit is inductive.
$X_L = 2\pi \times 50 \times 0.05 = 15.7\ \Omega$

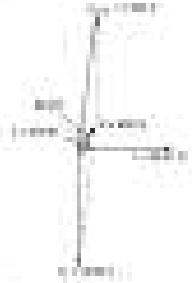


Figure P1.20

Impedance $Z = (3R' + 3j\omega L) \parallel (R + 2j\omega L) = (3+j4)\,\Omega \parallel (2+j4)\,\Omega = (1.5+j2)\,\Omega$

(a) Current $I = \dfrac{V}{R} = \dfrac{240}{1.5\Omega} = 160\,\text{A}$

(b) Phase angle $\phi = \text{arctan}\left(\dfrac{X_L}{R}\right) = \text{arctan}\dfrac{2\,\Omega}{1.5\,\Omega} = 53.1°$
$= 0.927\,\text{rad}$

(c) Impedance of each $Z_{\text{each}} = (3R' + 3j\omega L) = (3+j4)\,\Omega = (2.5\angle 53.1°)\,\Omega$
$= (3+j4)\,\Omega$

Voltage across each $V_{\text{each}} = 80\,\text{A}_{\text{each}} \times 2.5\,\Omega = 200\,\text{V}$

Phase angle θ of $I_{\text{each}} = \text{arctan}\dfrac{X_L}{R} = \text{arctan}\left(\dfrac{4\,\Omega}{3\,\Omega}\right) = 53.1°$
$= 0.927\,\text{rad}$ lagging

(d) Voltage across capacitor $V_C = I_C X_C = 160\,\text{A} \times 0.5\,\Omega = 80\,\text{V}$

The phasor diagram is shown in Figure 1.1-c. The angle voltage V_L leads the current I by 90° (see Art. 1.2, pg. 1 and 2).

Problem Involving Right Angles

For problems involving right angles such as the current relationships in the above example, a vector diagram is very helpful. It is not necessary that a drawing be to scale, but it should be approximately correct. For example, the vector representing the current in the capacitive branch should be drawn at approximately 90° to the vector representing the current in the resistive branch.

Remembering that the reactances are combined as $\dfrac{1}{X} = \dfrac{1}{X_1} + \dfrac{1}{X_2} + \cdots$



Figure P1.21

(a) Example: Use circuit of Figure 1.1. Do not bring down impedance lines as in equivalent circuit of Figure 1.8-1.2-c.

Problem 24.1 — Two cylindrical shells, respectively one centimeter and two centimeters in radius, are concentric and 30 cm long. The inner shell carries a charge of $+3.0 \times 10^{-8}$ C uniformly distributed over its surface. The space between the shells is filled with a material whose relative permittivity varies with radial distance r according to $\varepsilon_r = 1 + k/r$, where $k = 0.010$ m. Find the capacitance of this arrangement.



Figure 24.1a

The circuit diagram is shown in Figure 24.1b. Each resistor has a current-dependent power rating. Before any current flows, the full supply voltage appears across the open switch, so the initial power in resistor R_1 is

$$P_{1,0} = \frac{V_{\text{supply}}^2}{R_1} = \frac{(\text{value})^2}{(\text{value})} = \text{value}\ \text{W}.$$

Once the switch closes, the total resistance becomes

$$R_{\text{eq}} = R_1 + \frac{R_2 R_3}{R_2 + R_3} = (\text{value}) + \frac{(\text{value})(\text{value})}{(\text{value}) + (\text{value})} = \text{value}\ \Omega.$$

The initial current is then

$$I_0 = \frac{V_{\text{supply}}}{R_{\text{eq}}} = \frac{\text{value}}{\text{value}} = \text{value}\ \text{A}.$$

Current divides between R_2 and R_3 in inverse proportion to resistance, so the current through R_1 is the full I_0, and the power it dissipates is

$$P_{1,0} = I_0^2 R_1 = (\text{value})^2 (\text{value}) = \text{value}\ \text{W}.$$

(a) Current through R_1 at $t = 2.0$ s: $I(t) = I_0 e^{-t/\tau} = (\text{value})\, e^{-(\text{value})} = \text{value}\ \text{A}$.

Initial resistance: $R_0 = \dfrac{V}{I_0} = \dfrac{\text{value}}{\text{value}} = \text{value}\ \Omega$.

Power ratio: $\dfrac{P(t)}{P_0} = \dfrac{I(t)^2 R_1}{I_0^2 R_1} = \left(\dfrac{I(t)}{I_0}\right)^2 = \text{value}$.

So $P(t) = \text{value} \times P_0 = \text{value}$ W.

(b) Power ratio: $\dfrac{P(t)}{P_0} = \dfrac{R_0}{R(t)} = \dfrac{\text{value}}{\text{value}} = \text{value}$.

$R(t) = \text{value} \times R_0 = \text{value}\ \Omega$.

$I(t) = \dfrac{V}{R(t)} = \dfrac{\text{value}}{\text{value}} = \text{value}\ \text{A}$.

$P(t) = I(t)^2 R_1 = (\text{value})^2 (\text{value}) = \text{value}\ \text{W}$.

The drop in supply voltage as the battery runs down is $V_0 - V(t) = \text{value}$ V.



Figure 24.1b

Problem 24.2 — Each galvanometer in P_1 and P_2 in the circuit shown in Figure 24.2 is the equivalent of the meadow in Problem 23.2. Neither the battery's internal resistance nor the wires' resistance is negligible. Find the current through each galvanometer.

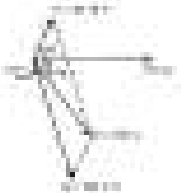


Figure 10.18

(a) Determine $\mathbf{v}_A$:

$$\mathbf{v}_A = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/B} = (\omega_{AB}\mathbf{k}) \times (-0.3\cos 30^\circ \mathbf{i} + 0.3\sin 30^\circ \mathbf{j}) = 0.3\omega_{AB}(\cos 30^\circ \mathbf{j} + \sin 30^\circ \mathbf{i})$$

$$\mathbf{v}_A = 0.3\omega_{AB}\cos 30^\circ \mathbf{j} + 0.3\omega_{AB}\sin 30^\circ \mathbf{i} \quad (\text{m/s}) \quad [1]$$

Then angle $\alpha = \arctan\left(\frac{v_y}{v_x}\right) = \arctan\left(\frac{0.3\omega\cos 30^\circ}{0.3\omega\sin 30^\circ}\right) = \mathbf{60.0^\circ}$ (negative)

(b) Determine $\mathbf{v}_C$:

$$\mathbf{v}_C = \boldsymbol{\omega}_{CB} \times \mathbf{r}_{C/B} = \left(\frac{v_A}{0.6\sin 30^\circ}\mathbf{k}\right) \times (0.6\cos 30^\circ \mathbf{i} + 0.6\sin 30^\circ \mathbf{j}) = -\frac{v_A}{0.6\sin 30^\circ}(0.6\cos 30^\circ \mathbf{j} - 0.6\sin 30^\circ \mathbf{i})$$

$$\mathbf{v}_C = 0.6\omega_{AB}\cos 30^\circ \mathbf{j} + 0.6\omega_{AB}\sin 30^\circ \mathbf{i} \quad (\text{m/s}) \quad [2]$$

Then angle $\alpha = \arctan\left(\frac{v_y}{v_x}\right) = \arctan\left(\frac{0.6\omega\cos 30^\circ}{0.6\omega\sin 30^\circ}\right) = 60.0^\circ$ (sliding)

The motion diagram is shown in Figure 10.18.

The slider block B and link BC have the angular velocity $\omega_{AB} = 3$ rad/s as a rigid body (**GIVEN**). These values can be determined by knowing $\mathbf{v}_B$ and $\mathbf{r}_{B/A}$. After determining $\mathbf{v}_C$ and $\mathbf{r}_{C/B}$, the angular velocity ω_{CB} can be determined.

Problem 10.18 Solve Problem 10.17 for $\omega_{AB} = 3$ rad/s. Use Equation (10.5) (Problem 10.17) to find $\mathbf{v}_C$.

10.3 Relative Velocity

We assume in Section 10.3 that the X, Y, Z coordinate system is fixed. Then X, Y, Z axes are not rotating and are translating with a constant velocity relative to the inertial frame of reference. The reference system is called a **rigid coordinate** system. We summarize:

(a) $\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$

(b) $\mathbf{v}_{A/B} = 0$ (i.e., the particles coincide) implies $\mathbf{v}_A = \mathbf{v}_B$ (i.e., $\mathbf{v}_{A/B} = 0$ always)

(c) $\mathbf{v}_A = \frac{\mathbf{r}}{dt}$ (i.e., the velocity vector is tangent to the path of motion)

(d) Given $\mathbf{r}_A = \mathbf{r}_A(\cdot)$ then $\mathbf{v}_A(\cdot) = \frac{d\mathbf{r}_A(\cdot)}{dt}$

$$\mathbf{r}_A(\text{at time } t) = \int_{(\text{at } t_0)}^{(\text{at } t)} \mathbf{v}_A(\cdot)\,dt$$

$$\text{or } \Delta\mathbf{r}_A = \int_{t_0}^{t} \mathbf{v}_A(\cdot)\,dt$$



Figure 9.1.43

(c) The series resonant circuit is often used as a(n) _____ filter because the output voltage is taken across the _____ component, which is the opposite frequency response of the resonant circuit used as a(n) _____ filter.

(d) Typical graphs of current I and impedance Z against frequency are shown in Figure 9.1.43.

> **Problem 9.1.8** — A circuit has a resistance of 2.0 Ω in series with an inductance of 1.0 mH and is connected to a source with a 10 mV sinusoidal voltage of 2.0 kHz applied. (a) What frequency will maximize current? (Hint: Is the circuit inductive, capacitive, or neither? Which of these maximizes current?)

Resonant frequency: $f_r = \frac{1}{2\pi\sqrt{LC}}$, ω_r

$$= \frac{1}{2\pi\sqrt{(1.0\times10^{-3}\ \text{H})\left(\frac{1}{4\pi^2(2.0\times10^3\ \text{Hz})^2(1.0\times10^{-3}\ \text{H})}\ \text{F}\right)}}$$

$$= \frac{1}{2\pi\sqrt{\frac{1}{4\pi^2(2.0\times10^3\ \text{Hz})^2}\ \text{s}^2}} = \frac{1}{2\pi\left(\frac{1}{2\pi(2.0\times10^3\ \text{Hz})}\right)}$$

$$= 2.0\times10^3\ \text{Hz} = \mathbf{2.0\ kHz}$$

At resonance, $X_L = X_C$, and therefore $Z = R$.

Power delivered: $P = \frac{V^2}{R} = \frac{(10\ \text{mV})^2}{2.0\ \Omega} = \mathbf{50\ \mu W}$

> **Problem 9.1.9** — The current measured in a series R, L, C circuit is 0.50 mA at 1.0 kHz and 0.50 mA at 2.0 kHz. If the impedance is 1.0 kΩ at 1.0 kHz and 2.0 kΩ at 2.0 kHz, what are the circuit resistance, and the circuit inductance or circuit capacitance?

(a) $I = 0.50\ \text{mA}$ at $f = 1.0\ \text{kHz}$ or $2.0\ \text{kHz}$, $Z = 1.0\ \text{k}\Omega$ at $f = 1.0\ \text{kHz}$ or $2.0\ \text{kHz}$

At resonance, impedance Z = resistance R

Since $R = \frac{V}{I}$, $\frac{R_{1.0\ \text{kHz}}}{R_{2.0\ \text{kHz}}} = \frac{\frac{V_{1.0\ \text{kHz}}}{I_{1.0\ \text{kHz}}}}{\frac{V_{2.0\ \text{kHz}}}{I_{2.0\ \text{kHz}}}} = \frac{V_{1.0\ \text{kHz}}}{V_{2.0\ \text{kHz}}}$ (as I's are equal)

(b) At resonance, $X_L = X_C$

$$L = \frac{1}{(2\pi f)^2 C}$$

Power dissipated in R_1: $P_1 = \frac{V_1^2}{R_1}$

$$= \frac{(6.00\ \text{V})^2}{4.00\ \Omega} = \frac{36.0\ \text{V}^2}{4.00\ \Omega} = 9.00\ \text{W}$$

$$= \frac{(6.00\ \text{V})^2}{4.00\ \Omega} = 9.00\ \text{W}$$

$$= 9.00\ \text{W}$$

27.45 Reasoning An incandescent 60.0-W and 120-V bulb has a resistance of $R = V^2/P = (120\ \text{V})^2/(60.0\ \text{W}) = 240\ \Omega$. Since the voltage across each bulb is greater than the applied voltage (see Figure 27.27b), we have

$$\text{Voltage magnification for incandescent} = \frac{\text{voltage across 1 or 2}}{\text{applied voltage}} = \frac{120\ \text{V}}{60.0\ \text{V}}$$

Similarly, we have a magnification of the applied voltage for a neon bulb of $\frac{120\ \text{V}}{60.0\ \text{V}}$. Now we can calculate the resistance of each bulb:

Neon bulb power: $P = \frac{V^2}{R}$, so $R = \frac{V^2}{P} = \frac{(120\ \text{V})^2}{1.00\ \text{W}} = 1.44 \times 10^4\ \Omega$

Incandescent bulb power: $P = \frac{V^2}{R}$, so $R = \frac{V^2}{P} = \frac{(120\ \text{V})^2}{60.0\ \text{W}} = 240\ \Omega$

Neon bulb power: $P = \frac{(0.5\ \text{V})^2}{R} = \frac{(120\ \text{V})^2}{1.44 \times 10^4\ \Omega} = 1.00\ \text{W}$

$$\text{Incandescent bulb power} = \frac{(0.5\ \text{V})^2}{240\ \Omega} = \frac{(120\ \text{V})^2}{240\ \Omega} = 60.0\ \text{W}$$

(b) Power is maximized when the bulbs in Figure 27.27c are connected in series with a resistance of $R = 0\ \Omega$ and when the bulbs in Figure 27.27a are connected in parallel with a resistance of $R = \infty$. Consider the power dissipated by the neon bulb in Figure 27.27c. Calculate the power dissipated by the incandescent bulb in Figure 27.27a. Which bulb dissipates more power? Explain.

(c) Neon bulb power: $P = \frac{V^2}{R} = \frac{(120\ \text{V})^2}{1.44 \times 10^4\ \Omega} = 1.00\ \text{W}$

Incandescent bulb power: $P = \frac{V^2}{R} = \frac{(120\ \text{V})^2}{240\ \Omega} = 60.0\ \text{W}$

$$= \frac{12 \text{ V}}{4\,\Omega + 2\,\Omega}$$

$$= 2 \text{ A} \quad \text{(flow from } + \text{ to } -\text{ of the 12-V source)}$$

(b) Current in the 4-Ω resistor: $I = \frac{R_2}{R_1 + R_2} I_{\text{total}} = \frac{2\,\Omega}{6\,\Omega} (2 \text{ A}) = 0.67 \text{ A}$

Voltage across the 4-Ω resistor, R_1, is the same as

$V_1 = IR_1 = (2 \text{ A})(4\,\Omega)$

$= (\text{current in } R_1)(\text{resistance of } R_1)$

$= 8 \text{ V}$

(c) Power delivered to the 4-Ω resistor:

$$P = IV = I^2 R = \frac{V^2}{R} = (2 \text{ A})^2 (4\,\Omega) = (0.67 \text{ A})^2 (4\,\Omega) = 8 \text{ W}$$

Power supplied by the 12-V source $= P = IV = (2 \text{ A})(12 \text{ V}) = 24 \text{ W}$

$\Rightarrow$ **16 W**

(d) Of the power, the 4-Ω resistor receives the fraction $\frac{8 \text{ W}}{24 \text{ W}} = \frac{1}{3}$. Since power is proportional to voltage, the 4-Ω resistor receives the fraction $\frac{8 \text{ V}}{12 \text{ V}} = \frac{2}{3}$ of the voltage.

> **Problem 9.1** A source with a voltage V_s and an internal resistance R_s is connected to a load resistance R_L. Determine the fraction of the power delivered by the source that is delivered to the load.

$$\text{Power delivered to load} = \frac{V_L^2}{R_L} = \frac{\left(\frac{R_L}{R_s + R_L} V_s\right)^2}{R_L} = \frac{R_L V_s^2}{(R_s + R_L)^2}$$

$$\text{Power delivered by source} = I_{\text{total}} V_s = \left(\frac{V_s}{R_s + R_L}\right) V_s = \frac{V_s^2}{R_s + R_L}$$

$$\text{Fraction} = \frac{\text{power delivered to load}}{\text{power delivered by source}} = \frac{\frac{R_L V_s^2}{(R_s + R_L)^2}}{\frac{V_s^2}{R_s + R_L}} = \frac{R_L}{R_s + R_L}$$

$$= \frac{1}{1 + \left(\frac{R_s}{R_L}\right)}$$

$$\Rightarrow \text{fraction} = \frac{1}{1 + \left(\frac{R_s}{R_L}\right)}$$

> **Problem 9.2** A unit of equipment contains two resistors and is connected to a source with a capacitance of 1.5 μF and a resistance of 0.12 ohms. A 117-V, 60-Hz, sinusoidal voltage is applied to the equipment. Determine (a) the current in the voltage source and (b) the voltage across the equipment.

(c) Horizontal displacement: $x = \int_0^{1.5} 4\sqrt{t}\cos\left(\frac{\pi t}{3}\right)dt$

$$= \left[\frac{4t^{\frac{3}{2}}}{\frac{3}{2}}\cos\left(\frac{\pi t}{3}\right)\right]_0^{1.5} - \int_0^{1.5} \frac{4t^{\frac{3}{2}}}{\frac{3}{2}}\left(-\sin\left(\frac{\pi t}{3}\right)\frac{\pi}{3}\right)dt$$

$$= 0 + \frac{8}{\pi}\int_0^{1.5} t^{\frac{3}{2}}\sin\left(\frac{\pi t}{3}\right)dt$$

(d) Current y-coordinate $= \frac{2}{3} - \frac{16}{\pi^2} = 0.051$

(e) When curve cuts x-axis again:

$$0 = 4\sqrt{t}\sin\left(\frac{\pi t}{3}\right)$$

$$\sin\left(\frac{\pi t}{3}\right) = 0 \Rightarrow t = 3$$

When curve reaches its minimum:

$$0 = 4\sqrt{t}\cos\left(\frac{\pi t}{3}\right)$$

$$\cos\left(\frac{\pi t}{3}\right) = 0 \Rightarrow t = 1.5$$

(f) Gradient $(t = 3)$ of tangent is reciprocal of gradient $= -\frac{2}{3} \div \frac{4\sqrt{3}}{3}$

$$= -\frac{\sqrt{3}}{6} = -0.289$$

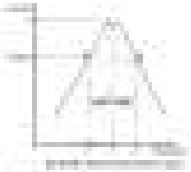
Cartesian equation of tangent is $y - \frac{2}{3} = -\frac{1}{6\sqrt{3}}(x - 3\sqrt{3})$

$$y = -\frac{1}{6\sqrt{3}}x + \frac{1}{2}$$

Further problems on polar equations and calculus may be found in Section 31.3, problems 21 to 27, page 464.

13.9 Parametric and cartesian

Figure 13.10 shows the curve C whose cartesian equation is $y = x^3 - x^2$. Suppose we wish to find the area bounded by C, the x-axis and the ordinate $x = 2$. We could find this area by integrating y with respect to x between the limits $x = 1$ and $x = 2$, as in Section 13.1. The same area could also be found by integrating x with respect to y between the limits $y = 0$ and $y = 4$. However, in order to do this we would need to express x in terms of y. This is not easy, since the cartesian equation of C is a cubic equation in x.



Problem 4.4 | Ch4

[illegible] [illegible] [illegible] [



Power, p, in a purely inductive a.c. circuit

or



Power, p, in a purely capacitive a.c. circuit

and



Power, p, in a resistive, inductive a.c. circuit

Figure 10.10

(iv) For a purely inductive a.c. circuit the average power is zero. See Figure 10.10(iv).

(v) For a purely capacitive a.c. circuit the average power is zero. See Figure 10.10(v).

Figure 10.10 shows separate pure average power curves for cases (i) to (v) above. When a circuit has a mixture of resistance, inductance and capacitance, the power curve is a combination of the curves shown in Figure 10.10. The combination of the curves depends on the relative magnitudes of the resistance, inductance and capacitance and the phase angle between the voltage and current.

Let $v = V_m \sin \omega t$ be the voltage applied to a circuit and let $i = I_m \sin(\omega t - \phi)$ be the current flowing in the circuit, where ϕ is the phase angle between v and i. The instantaneous power, p, is given by $p = vi$, i.e. $p = V_m \sin \omega t \, I_m \sin(\omega t - \phi)$. Hence

$$p = V_m I_m \sin \omega t \sin(\omega t - \phi)$$

Using the trigonometric identity $\sin A \sin B = \frac{1}{2}\cos(A - B) - \frac{1}{2}\cos(A + B)$ gives

$$p = \frac{V_m I_m}{2}\cos(\omega t - \omega t + \phi) - \frac{V_m I_m}{2}\cos(\omega t + \omega t - \phi)$$

i.e.

$$p = \frac{V_m I_m}{2}\cos\phi - \frac{V_m I_m}{2}\cos(2\omega t - \phi)$$

The first term, $\frac{V_m I_m}{2}\cos\phi$, is constant for a given circuit and is the average power, P, supplied to the circuit. The second term, $\frac{V_m I_m}{2}\cos(2\omega t - \phi)$, represents a cosine wave of frequency 2ω rad/s which has an average value of zero over a cycle.

Hence the average power, P, supplied to the circuit is given by

$$P = \frac{V_m I_m}{2}\cos\phi$$

or

$$P = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}\cos\phi$$

i.e.

$$P = VI\cos\phi$$

where V and I are the r.m.s. values of v and i.

For the circuits in Figure 10.10(i) to (iii), $\phi = 0$, $\cos\phi = 1$ and $P = VI$ watts.

For the circuit in Figure 10.10(iv), $\phi = 90°$, $\cos\phi = 0$ and $P = 0$ watts.

For the circuit in Figure 10.10(v), $\phi = -90°$, $\cos\phi = 0$ and $P = 0$ watts.

Problem 10.1 An alternating current of 5 A flows through a resistance of 20 Ω. Determine the power dissipated in the resistance.

Power dissipated $P = I^2R$ where I is the r.m.s. value of current

$I = 5.00$ amperes and $R = 20\ \Omega$, hence $P = (5.00)^2(20)$

$P = (25)(20) = 500.0$ W

Hence power dissipated $= 500.0$ watts

Problem 10.2 A series circuit of resistance 50 Ω and inductance 0.1 H is connected to a 110 V, 50 Hz supply. Calculate the power dissipated in the circuit.

Inductive reactance $X_L = 2\pi fL = 2\pi(50)(0.1) = 10\pi\ \Omega = 31.42\ \Omega$

Impedance $Z = \sqrt{(R^2 + X_L^2)} = \sqrt{(50^2 + 31.42^2)} = \sqrt{(3\,500.6)} = 59.16\ \Omega$

Current, $I = \frac{V}{Z} = \frac{110}{59.16} = 1.859$ A

The power factor (pf) is given by $\cos\phi = \frac{R}{Z} = \frac{50}{59.16} = 0.8453$ lagging

Hence power $P = VI\cos\phi = (110)(1.859)(0.8453) = 172.8$ watts

or

(ii) $P = I^2R = (1.859)^2(50) = 172.8$ watts

Power $P = VI\cos\phi$ watts

11.11 Power in simple and general circuits



(a) Inductive (lagging) load



(b) Inductive (leading) load

Figure 11.11

Figure 11.11(a) shows a phasor diagram in which the current I lags the applied voltage V by angle ϕ. The horizontal component of I is $I\cos\phi$ and the vertical component is $I\sin\phi$, as is usual in this type of diagram. Figure 11.11(b) shows the same phasors redrawn with the current as the reference phasor.

Apparent power	$S = VI$ volt-amperes (VA)
True or useful power	$P = VI\cos\phi$ watts (W)
Reactive power	$Q = VI\sin\phi$ volt-amperes (reactive) (var)

$$\text{Power factor} = \frac{\text{true power, } P}{\text{apparent power, } S}$$

For sinusoidal voltages and currents, power factor $= \dfrac{P}{S} = \dfrac{VI\cos\phi}{VI} = \cos\phi$.

$$\phi = \cos^{-1}\left(\frac{P}{S}\right) \quad \text{(Power Factor (11))}$$

The relationships stated above are also true when current I leads voltage V. Most of the power triangle and power factor are reviewed in Chapter 19, page 608.

Problem 11. A load, impedance $Z = 10\,\Omega$, is connected to a 240 V, 50 Hz supply and the apparent power of the circuit is 4 kVA. Determine the value of the inductance.

Apparent power $S = VI$

Power current $I = \dfrac{S}{V} = \dfrac{4000}{240} = 16.67$ A

Load impedance $Z = \dfrac{V}{I} = \dfrac{240}{16.67} = 14.4\,\Omega$

Phase $\phi = \cos^{-1}(\text{power factor}) = \cos^{-1}\dfrac{P}{S} = \cos^{-1}\dfrac{3.2}{4} = \cos^{-1}0.8 = 36.87°$

Problem 12. A transformer has a rated output of 100 kVA at a power factor of 0.8. Determine the rated power output and the corresponding reactive power.

$S = 100$ kVA, power factor $= 0.8 = \cos\phi$, hence $\phi = \cos^{-1}0.8 = 36.87°$

Power output, $P = S\cos\phi = 100 \times 0.8 = 80$ kW

Reactive power, $Q = S\sin\phi = 100 \times \sin 36.87° = 60$ kvar

Resistance of wire: $R = 0.0 \ \Omega$ (short)
Wire length: $L = 0.0$ m (short) $\quad$ Area: $A = 0.0 \times 10^{-6}$ m² $= 0.0$
Power dissipated: $P = \frac{V^2}{R} = \frac{(0.0)^2}{0.0} = 0.0$ W (zero)

Problem 20. The power rating for an appliance should always be expressed as a ratio of two quantities so that the user can calculate the energy consumed over a period of time. Is the statement "an appliance has a power rating of 2.5 kilowatts" sufficient information for the calculation? Explain. Complete the following table for the power ratings of the three major household appliances listed.

(a) Power: $P = 1.5$ kW (stove) $\quad I = \frac{P}{V} = \frac{1500}{240} = 6.25$ A

(b) Impedance: $Z = \frac{V}{I} = \frac{240}{6.25} = 38.4\ \Omega$

(c) Power: $P = 1.5$ kW $= 1.5$ J/s $\quad$ time: $t = 1.5 \times 10^{-3}$ s
$= 1.5 \text{ J/s} \times 1.5 \times 10^{-3}$ s
$= 0.0 \times 10^{-3}$ J

(d) Power factor: $\cos\theta = \frac{\text{true power}}{\text{apparent power}} = \frac{1.5 \text{ kW}}{2.5 \text{ kVA}} = \frac{1.5}{2.5} = 0.600$

(e) $\cos\theta = 0.600 \quad \theta = \cos^{-1}(0.600)$ (inverse cosine) $\quad \theta = 53.13°$
$= 53.13°$
$= 0.927$ rad

Problem 21. A circuit consisting of a resistor in series with a capacitor is called RC circuit. A practical example is a circuit that has a switch at the supply. When the switch is closed, the capacitor starts to charge. At the beginning, the capacitor acts like a short circuit and the current is limited only by the resistance. As the capacitor charges, the voltage across it increases and the current decreases. Eventually, the capacitor is fully charged and the current is zero. The rate at which the capacitor charges depends on the time constant of the circuit, which is the product of the resistance and the capacitance. Time constant is the time it takes for the capacitor to charge to 63.2% of the supply voltage.

(a) Power factor: $\cos\theta = \frac{\text{true power}}{\text{apparent power}}$

$\cos 0 = \frac{500}{\text{apparent power}}$ $\quad$ Where $\cos 0 = \frac{500}{S} = 1.0$ A

(b) Power factor: $\cos\theta = 0.866$ (from phase angle) $\quad \theta = \cos^{-1}(0.866)$
$= 30°$ (30 deg)

(c) Power: $P = 1.5$ kW $\quad$ Power factor: $\cos\theta = \frac{P}{S} = \frac{1500}{2500} = 0.60$

(d) Impedance: $Z = \frac{V}{I} = \frac{240}{6.25} = 38.4\ \Omega$

Ex. A light-bulb is marked 6 V, 12 W. Assuming that its resistance is constant, find the current through it when it is operated at its rated voltage, and the resistance of the filament.

Sol. $I = \frac{P}{V} = \frac{12}{6} = 2$ A. Hence $R = \frac{V}{I} = \frac{6}{2} = 3\ \Omega$. Equivalently $R = \frac{V^2}{P} = \frac{6^2}{12} = 3\ \Omega$.

Further worked examples of this kind, including cases in which the resistance varies with temperature, appear in §14.4, page 247.

14.3 Kirchhoff's Laws for Networks

14.3.1 Statement of the Laws

(a) Kirchhoff's current law (KCL). The algebraic sum of the currents meeting at any junction (node) of a network is zero. Equivalently, the total current flowing into a junction equals the total current flowing out of it. This is a statement of conservation of charge: charge cannot accumulate at a junction in a steady state.

(b) Kirchhoff's voltage law (KVL). The algebraic sum of the potential differences around any closed loop of a network is zero. Equivalently, the total emf acting round a loop equals the total IR drop round the loop. This is a statement of conservation of energy: a charge returning to its starting point has gained and lost the same amount of energy.

14.3.2 Sign Conventions

To apply the laws consistently, a sign convention must be adopted. The convention used throughout this chapter is:

- Currents flowing into a junction are taken as positive; currents flowing out are taken as negative.
- When traversing a loop, a potential rise (from $-$ to $+$ of a source) is taken as positive; a potential drop (in the direction of the current through a resistor) is taken as negative.

If the solution for a current turns out to be negative, it simply means that the current flows in the opposite direction to that assumed.

14.3.3 Worked Example

Ex. Two cells of emf $E_1 = 12$ V and $E_2 = 6$ V, with internal resistances $r_1 = 1\ \Omega$ and $r_2 = 2\ \Omega$, are connected in parallel, with their positive terminals joined, across an external resistance $R = 9\ \Omega$. Find the current in each branch and the potential difference across R.

Sol. Let I_1 be the current from the first cell, I_2 the current from the second, and I the current through R, so that $I = I_1 + I_2$. Applying KVL to the loop containing E_1, r_1 and R:

$$E_1 = I_1 r_1 + I R = I_1 r_1 + (I_1 + I_2) R = I_1 (r_1 + R) + I_2 R.$$

Applying KVL to the loop containing E_2, r_2 and R:

$$E_2 = I_2 r_2 + I R = I_2 r_2 + (I_1 + I_2) R = I_1 R + I_2 (r_2 + R).$$

Substituting the values:

$$12 = 10 I_1 + 9 I_2, \qquad 6 = 9 I_1 + 11 I_2.$$

Solving these simultaneously gives $I_1 = \frac{42}{71}$ A ≈ 0.592 A and $I_2 = \frac{-18}{71}$ A ≈ -0.254 A. The negative value of I_2 indicates that the second cell is being driven backwards by the first, and is in fact absorbing energy rather than supplying it. The potential difference across R is $V = I R = \frac{108}{71}$ V ≈ 1.52 V.

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Kirchhoff's voltage law applies. Let the voltage across the nth unit be v_n. Then the voltage across the capacitor is

$$v_{\text{out}} = v_1 + v_2 + \cdots + v_n = \frac{1}{C}\int i\,dt + \frac{1}{C}\int i\,dt + \cdots + \frac{1}{C}\int i\,dt$$

$$= \frac{n}{C}\int i\,dt = \frac{1}{C_{\text{eq}}}\int i\,dt$$

1.3 Three capacitors are connected in series across a 24 V battery. The capacitances are

(a) $C_1 = 2\ \mu\text{F}$, (b) $C_2 = 3\ \mu\text{F}$, (c) $C_3 = 6\ \mu\text{F}$.

Determine the equivalent capacitance and the voltage and charge on each capacitor.



Figure 1.C.3a

Solution

(a) Find the equivalent capacitance of the series combination. Then find the charge on the equivalent capacitor. Since the capacitors are in series, each has the same charge as the equivalent capacitor. Finally, find the voltage across each capacitor.

$$\frac{1}{C_{\text{eq}}} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 1 \quad \Rightarrow \quad C_{\text{eq}} = 1\ \mu\text{F}$$

$$Q = C_{\text{eq}} V = (1\ \mu\text{F})(24\ \text{V}) = 24\ \mu\text{C}$$

$$V_1 = \frac{Q}{C_1} = \frac{24\ \mu\text{C}}{2\ \mu\text{F}} = 12\ \text{V}$$

$$V_2 = \frac{Q}{C_2} = \frac{24\ \mu\text{C}}{3\ \mu\text{F}} = 8\ \text{V}$$

$$V_3 = \frac{Q}{C_3} = \frac{24\ \mu\text{C}}{6\ \mu\text{F}} = 4\ \text{V}$$

(b) Find the equivalent capacitance of the parallel combination. Then find the charge on the equivalent capacitor. Since the capacitors are in parallel, each has the same voltage as the equivalent capacitor. Finally, find the charge on each capacitor.

$$C_{\text{eq}} = C_1 + C_2 + C_3 = 2 + 3 + 6 = 11\ \mu\text{F}$$

$$Q = C_{\text{eq}} V = (11\ \mu\text{F})(24\ \text{V}) = 264\ \mu\text{C}$$

$$Q_1 = C_1 V = (2\ \mu\text{F})(24\ \text{V}) = 48\ \mu\text{C}$$

$$Q_2 = C_2 V = (3\ \mu\text{F})(24\ \text{V}) = 72\ \mu\text{C}$$

$$Q_3 = C_3 V = (6\ \mu\text{F})(24\ \text{V}) = 144\ \mu\text{C}$$

1.4 Three capacitors are connected in series across a 24 V battery. The capacitances are

(a) $C_1 = 2\ \mu\text{F}$, (b) $C_2 = 3\ \mu\text{F}$, (c) $C_3 = 6\ \mu\text{F}$.

Determine the equivalent capacitance and the voltage and charge on each capacitor.

Solution

(a) Find the equivalent capacitance of the series combination. Then find the charge on the equivalent capacitor. Since the capacitors are in series, each has the same charge as the equivalent capacitor. Finally, find the voltage across each capacitor.

$$\frac{1}{C_{\text{eq}}} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 1 \quad \Rightarrow \quad C_{\text{eq}} = 1\ \mu\text{F}$$

$$Q = C_{\text{eq}} V = (1\ \mu\text{F})(24\ \text{V}) = 24\ \mu\text{C}$$

$$V_1 = \frac{Q}{C_1} = \frac{24\ \mu\text{C}}{2\ \mu\text{F}} = 12\ \text{V}$$

$$V_2 = \frac{Q}{C_2} = \frac{24\ \mu\text{C}}{3\ \mu\text{F}} = 8\ \text{V}$$

$$V_3 = \frac{Q}{C_3} = \frac{24\ \mu\text{C}}{6\ \mu\text{F}} = 4\ \text{V}$$

(b) Find the equivalent capacitance of the parallel combination. Then find the charge on the equivalent capacitor. Since the capacitors are in parallel, each has the same voltage as the equivalent capacitor. Finally, find the charge on each capacitor.

$$C_{\text{eq}} = C_1 + C_2 + C_3 = 2 + 3 + 6 = 11\ \mu\text{F}$$

$$Q = C_{\text{eq}} V = (11\ \mu\text{F})(24\ \text{V}) = 264\ \mu\text{C}$$

$$Q_1 = C_1 V = (2\ \mu\text{F})(24\ \text{V}) = 48\ \mu\text{C}$$

$$Q_2 = C_2 V = (3\ \mu\text{F})(24\ \text{V}) = 72\ \mu\text{C}$$

$$Q_3 = C_3 V = (6\ \mu\text{F})(24\ \text{V}) = 144\ \mu\text{C}$$

1.5 A parallel combination of capacitors is connected to a series combination of capacitors. The parallel combination consists of a 2 μF capacitor in parallel with a 3 μF capacitor. The series combination consists of a 4 μF capacitor in series with a 12 μF capacitor. Find the equivalent capacitance of the entire circuit.

$$C_{\text{eq}} = \frac{(2 + 3)(4)(12)}{(2 + 3)(4) + (4)(12) + (12)(2 + 3)} = \frac{240}{128} = 1.875\ \mu\text{F}$$

1.6 A parallel combination of capacitors is connected to a series combination of capacitors. The parallel combination consists of a 2 μF capacitor in parallel with a 3 μF capacitor. The series combination consists of a 4 μF capacitor in series with a 12 μF capacitor. Find the equivalent capacitance of the entire circuit.

$$C_{\text{eq}} = \frac{(2 + 3)(4)(12)}{(2 + 3)(4) + (4)(12) + (12)(2 + 3)} = \frac{240}{128} = 1.875\ \mu\text{F}$$

1.7 A capacitor circuit is connected to a 24 V battery. The circuit consists of a 2 μF capacitor in parallel with a 3 μF capacitor. The parallel combination is in series with a 4 μF capacitor. The series combination is in parallel with a 12 μF capacitor. Find the voltage across the capacitors.

$$V = 24\ \text{V}$$

Power Review Problems

4.1 An ideal voltage source with a voltage of 12 V is connected to a resistor. The current through the resistor is 2 A. Find the power absorbed by the resistor.

(a) 6 W (b) 12 W (c) 24 W (d) 48 W

4.2 An ideal current source with a current of 3 A is connected to a resistor. The voltage across the resistor is 9 V. Find the power absorbed by the resistor.

(a) 3 W (b) 9 W (c) 27 W (d) 81 W

4.3 A current source is connected to a resistor. The current source has a current of 2 A and a voltage of 10 V. The resistor has a resistance of 5 Ω. Which statement is true?

(a) The current source is absorbing 20 W and the resistor is absorbing 20 W. (b) The current source is absorbing 20 W and the resistor is delivering 20 W. (c) The current source is delivering 20 W and the resistor is absorbing 20 W. (d) The current source is delivering 20 W and the resistor is delivering 20 W.

4.4 A current source is connected to a resistor. The current source has a current of 2 A and a voltage of 10 V. The resistor has a resistance of 5 Ω. Which statement is true?

(a) The current source is absorbing 20 W and the resistor is absorbing 20 W. (b) The current source is absorbing 20 W and the resistor is delivering 20 W. (c) The current source is delivering 20 W and the resistor is absorbing 20 W. (d) The current source is delivering 20 W and the resistor is delivering 20 W.

4.5 A current source is connected to a resistor. The current source has a current of 2 A and a voltage of 10 V. The resistor has a resistance of 5 Ω. Which statement is true?

(a) The current source is absorbing 20 W and the resistor is absorbing 20 W. (b) The current source is absorbing 20 W and the resistor is delivering 20 W. (c) The current source is delivering 20 W and the resistor is absorbing 20 W. (d) The current source is delivering 20 W and the resistor is delivering 20 W.

4.6 A current source is connected to a resistor. The current source has a current of 2 A and a voltage of 10 V. The resistor has a resistance of 5 Ω. Which statement is true?

(a) The current source is absorbing 20 W and the resistor is absorbing 20 W. (b) The current source is absorbing 20 W and the resistor is delivering 20 W. (c) The current source is delivering 20 W and the resistor is absorbing 20 W. (d) The current source is delivering 20 W and the resistor is delivering 20 W.

4.7 A current source is connected to a resistor. The current source has a current of 2 A and a voltage of 10 V. The resistor has a resistance of 5 Ω. Which statement is true?

(a) The current source is absorbing 20 W and the resistor is absorbing 20 W. (b) The current source is absorbing 20 W and the resistor is delivering 20 W. (c) The current source is delivering 20 W and the resistor is absorbing 20 W. (d) The current source is delivering 20 W and the resistor is delivering 20 W.

4.8 A current source is connected to a resistor. The current source has a current of 2 A and a voltage of 10 V. The resistor has a resistance of 5 Ω. Which statement is true?

(a) The current source is absorbing 20 W and the resistor is absorbing 20 W. (b) The current source is absorbing 20 W and the resistor is delivering 20 W. (c) The current source is delivering 20 W and the resistor is absorbing 20 W. (d) The current source is delivering 20 W and the resistor is delivering 20 W.

16 Single-phase parallel a.c. circuits

[illegible] the end of this chapter you should be able to:

- calculate admittance, conductance, susceptance and current, power factor, active power, reactive power and apparent power for a parallel a.c. circuit
- draw the conditions for parallel resonance as in 15.4 (1 to 6)
- derive and calculate frequency conditions for parallel resonance as in 15.5 (1 to 3)
- distinguish between series and parallel resonance as in 15.6 (1 to 3)
- understand and calculate Q-factor as in 15.7 (1 to 3)
- understand how power factor may be improved

16.1 Introduction

In parallel circuits, such as those shown in Figure 16.1, and 16.2, the voltage is common to each branch of the network and is thus taken as the reference phasor when drawing phasor diagrams.
For any parallel a.c. circuit:

Current phasor given, $I = V$ [illegible] (R)

$= V - j\frac{V}{X}$ amps

Apparent power, $S = VI$ voltamperes (VA)

Reactive power, $Q = VI$ [illegible] voltamperes (var)

Power factor $= \frac{\text{total current}}{\text{appropriate component}} = \frac{I}{I} = \cos\phi$

Phase angles are the same as for series a.c. circuits as used in Chapter 15.



Figure 16.1

16.2 R–L–C parallel a.c. circuit

In the R–L–C parallel circuit shown in Figure 16.2, the supply voltage is the reference phasor. The current through the resistance, I_R, is in phase with the supply voltage, V, and the current in the inductance, I_L, lags the supply voltage by 90°. The current through the capacitance, I_C, leads the supply voltage by 90°. The supply current, I, is the phasor sum of I_R and I_L and I_C as shown in Figure 16.2(b). The supply voltage, V, is common

Using Pythagoras' rule find the unknown in the shape of x, and y. Leave answers in surd form where appropriate.

From the diagram Pythagoras' rule

$$x = \sqrt{(12)^2 + (5)^2} \quad \text{by Pythagoras' theorem}$$

$$x = \sqrt{144 + 25}$$

$$x = \sqrt{169}$$

$$x = 13$$

Hence $\sin x = \frac{5}{13}$ and $\cos x = \frac{12}{13}$

Now $\sin x = \frac{5}{13}$ and $\cos x = \frac{12}{13}$ and $\tan x = \frac{5}{12}$ the trigonometric ratios.

Overall hypotenuse $= y = 13$

> **Problem 2** — A 2×2 Cartesian matrix is used to describe a reflection in the y-axis followed by a rotation of 90° anti-clockwise about the origin. Find the single transformation matrix that represents the single reflection in the y-axis followed by the single rotation of 90° anti-clockwise about the origin. Hence find the image of the point $(3, 2)$ under this transformation.

The two unit square diagrams are as shown in Figure 10.1

(a) Coastal Mooring in the scenario $\cos\theta = \frac{12}{13}$, $\sin\theta = \frac{5}{13}$, $\tan\theta = \frac{5}{12}$

Coastal Mooring in the environment $\tan\theta = \frac{5}{12} = \frac{\text{opp}}{\text{adj}}$

$$\text{hyp} = \sqrt{\text{opp}^2 + \text{adj}^2}$$

$$= \sqrt{(5)^2 + (12)^2}$$

$$= \sqrt{25 + 144}$$

$$= \sqrt{169}$$

$$= 13$$

(b) From the above Equation length vector $\begin{pmatrix} 5 \\ 12 \end{pmatrix} = \sqrt{(5)^2 + (12)^2}$

$$= \sqrt{25 + 144}$$

$$= \sqrt{169}$$

$$= 13$$

(c) Check: Mean height $= \frac{5}{13} \times \text{number} = \frac{12}{13} \times \text{number}$

$$= 0.385 \times \text{number}$$

(d) Check calculations, $\theta = \frac{5}{12} = \frac{\text{opp}}{\text{adj}} = 22.62°$

(e) Power consumed $P = 12 \text{ cos } \theta \times 240 \text{ cos } (\text{cos } \theta) \text{ cos } \theta$ **(Ans: 0)**

(f) Characteristic power consumed $P = \int_0^{\pi} 240 \cos \theta \, d\theta$ **(Ans: 0)**

Further problems on θ & $\cos\theta$ (pseudo) (c) – circular caps for circular to be found in the previous 4 pages (10)

10.27 R, X, and ABCs of AC Circuits



Figure for 10.1

In the two branch parallel circuit containing impedance Z and a capacitor C shown in Figure 10.1, I_T is in relation with the supply voltage V and the current through the capacitor I_C equals $2\pi fCV$. The angle θ between I_T in the phasor sense of the source and that of current I_C equal the angle between V in the same frame of reference V and I_T respectively in the circuit. I_T and I_C define an angle α in the phasor diagram.

From the phasor diagram

$$I = \sqrt{I_R^2 + I_C^2} = \frac{V}{Z}, \text{ by Pythagorean theorem}$$

where $I_R = \frac{V}{R}$ and $I_C = \frac{V}{X_C}$

Now $\frac{1}{R}$ also $= \frac{I_R}{V}$ and $\frac{1}{X_C} = \frac{I_C}{V}$ by trigonometry theorem

Circuit impedance $Z = \frac{V}{I}$

> Problem 1: A 10 nF capacitor is connected in a parallel with an 80 Ω resistor across a 240 V, 50 Hz supply. Calculate (a) the current in each branch of the supply, (b) the supply current, (c) the circuit phase angle, (d) the circuit impedance, (e) the active, reactive and apparent powers.

The circuit and phasor diagram are as shown in Figure 10.1

(a) Current in resistor, $I_R = \frac{V}{R} = \frac{240}{80} = 3\ A$

Current in capacitor, $I_C = \frac{V}{X_C} = \frac{V}{\left[\frac{1}{2\pi fC}\right]}$

$= 240 \times$

$= 0.010 \times 10^{-6} \times 2\pi(50)$

$= 1.508\ A$

(b) Supply current $I = \sqrt{(I_R)^2 + (I_C)^2} = \sqrt{(3)^2 + (1.508)^2}$

$= 3.36\ A$

(c) Circuit phase angle, $\alpha = \tan^{-1}\frac{I_C}{I_R} = \tan^{-1}\frac{1.508}{3}$,

$= 26.7$ **degrees**

(d) Circuit impedance, $Z = \frac{V}{I} = \frac{240}{3.36} = 71.43\ \Omega$

[illegible] [illegible] [illegible] [

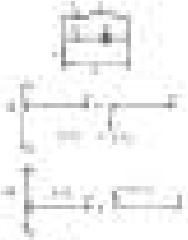


Figure for 4

(a) R_1 = the parallel combination of R_2 and R_3 (resistors R_2 and R_3 are in parallel with each other, and that pair is in series with R_1);

(b) R_2 = the parallel combination of R_1 and R_3 (resistors R_1 and R_3 are in parallel with each other, and that pair is in series with R_2);

(c) R_3 = the parallel combination of R_1 and R_2.

The three resistors are not parallel to anything else (i.e., every resistance in the circuit being considered is in the circuit described for it above), so

(a) the R_1 of (parallel case) (c) $= \dfrac{R'}{R_2 + R_3}$, $R' = \dfrac{R_1}{R}$;

(b) (parallel combination) R_2 and R_3, use $R = \dfrac{R'}{R_1}$.

> **Problem 5** — A wire 4.0 mm in diameter and 2.0 km long is connected to another wire $\frac{1}{2}$ as long and $\frac{1}{2}$ as resistive and the assembly is connected to a 12.0 V battery. Determine (a) the current in each wire, (b) the power dissipated in each wire, and (c) the power output of the battery. (Assume the internal resistance and voltage of the battery are negligible.)

Here is the unit (basic) fraction for the circuit of Figure 10.4:

(a) Total electrical resistance, $R_1 = 10.0\ \Omega$ + (the parallel combination of $5.0\ \Omega$ and $5.0\ \Omega$)

$= 10.0\ \Omega + 2.5\ \Omega$

$= 12.5\ \Omega$.

Capacitive reactance, $X_C = \dfrac{1}{2\pi fC} = \dfrac{1}{(6.28)(60\ \text{Hz})(1.0 \times 10^{-6}\ \text{F})} = 2.65 \times 10^{3}\ \Omega$

$= 2.65\ \text{k}\Omega$.

Current flowing in inductance, $I_L = \dfrac{V}{X_L} = \dfrac{120\ \text{V}}{377\ \Omega} = 0.318\ \text{A}$.

Current flowing in capacitance, $I_C = \dfrac{V}{X_C} = \dfrac{120\ \text{V}}{2.65 \times 10^{3}\ \Omega} = 4.53 \times 10^{-2}\ \text{A}$.

(b) I_L and I_C are equivalent (from supply) current.

$P = I_L^2 R_L = (0.318\ \text{A})^2 (377\ \Omega) = 38.1\ \text{W}$ (and the reactive load, the supply voltage V for 90° out of phase (10.4(c)).

(c) Circuit impedance, $Z = \dfrac{V}{I} = \dfrac{120\ \text{V}}{0.318\ \text{A}} = 378\ \Omega$.

(d) Power consumed $P = VI \cos\phi = (120\ \text{V})(0.318\ \text{A})(\cos 90^\circ)$

$= 0\ \text{W}$.

> **Problem 6** — Repeat Problem 5 for the conditions where the frequency is changed to 120 Hz.

(b) Subscription expenditure (£) $= 0.02 \times 15{,}000 \times 10^{-2} = £3.00$

Depreciation expenditure (£) $= \dfrac{7}{100} \times \dfrac{1}{365} \times 15{,}000 = £2.88$

Income Working (£ expenditure) $= \dfrac{x}{\pounds 1} = \dfrac{100}{(3.00)}$ $\Rightarrow x = £300$

Income Working (£ expenditure) $= \dfrac{x}{\pounds 1} = \dfrac{100}{(2.88)}$ $\Rightarrow x = £288$

(c) Capital employed $= (\ldots) - (\ldots) = £4{,}000 - £300 = £3{,}700$ (**Loading** 1 for **CE** value Figure 6.2(c))

(d) Current expenditure $= \dfrac{7}{1} = \dfrac{2.45}{(1.75)}$ $\Rightarrow x = £4.20$

(e) Profit $= £4.20 + £2.88 = £7.08$ (return £ $= £7.08$)

From Problem Last 1

(a) Value $2_0 = £\ldots$, Value $1_0 = £\ldots$ and 2 Input X Fee £3

(b) Value $2_1 = £\ldots$, Value $1_1 = £\ldots$ and 1 Grade F £[ ]

(c) (c) a parallel income ownership on marketplace (for price comparison) (c) £[ ]

Further questions for 1 $\leq$ [illegible] $\leq$ … income case of Model 3a Analysis (c) a parallel 1 and 2 Loans £[ ]

6.3 Cash Flow and Working Capital

In this last section we will examine the importance of cash flow and working capital and the relationship between them. We will also look at the ratio of current assets to current liabilities and the 'working capital ratio' which is used to measure the liquidity of a business. We will also look at the 'acid test' ratio which is used to measure the liquidity of a business. We will also look at the 'debtor collection period' and the 'creditor payment period' which are used to measure the efficiency of a business. We will also look at the 'stock turnover' ratio which is used to measure the efficiency of a business.

The relationship between cash flow and working capital is very important and is often misunderstood. The following example illustrates the point. Consider the following simplified balance sheet:

(a)	Fixed assets	£[illegible]
(b)	Current assets	£[illegible]
(c)	Current liabilities	£[illegible]

Working capital is defined as current assets minus current liabilities, i.e. (b) − (c). The working capital ratio is defined as current assets divided by current liabilities, i.e. (b) ÷ (c). The acid test ratio is defined as current assets minus stock divided by current liabilities, i.e. [(b) − stock] ÷ (c). The debtor collection period is defined as debtors divided by sales multiplied by 365. The creditor payment period is defined as creditors divided by purchases multiplied by 365. The stock turnover ratio is defined as cost of sales divided by stock.

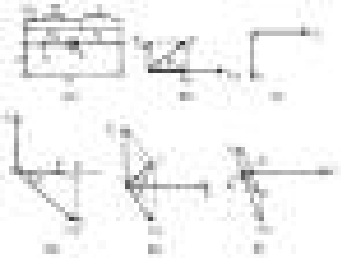


Figure 10.4

In our first look at the circuits above, we'll find **Thévenin** or its Norton dual, and **equivalent power** (i.e. maximum **energy density**, or theoretical limit to an Arduino's and ?? Body reference to the power diagrams of Figure 10.4).

Operation of 10 Ranks: $E_{th} = \mathcal{N}^{-1} \cdot (A \cdot V)$

Criteria: $$I_{sc} = \frac{V}{R_{th}} \text{ and } I_{sc} = \frac{V_{th}}{R_{th}}$$

Engine version 1 – replace load R_{load} and I_{sc} via summing:

$$= \int (I_{sc} \cdot \text{load})^2 \cdot \left(R_{th} \text{ drop} - I_{sc} R \right) \text{ for Abundance}$$

where — source for different formula:

Circuit equivalent $R = \frac{V}{I}$

$$\text{Norton} = \frac{V_{th}}{I_{sc}} = \frac{V_{th}}{R_{th}} \text{ open circuit} = \frac{V_{th}}{R_{load}} \text{ open circuit} = \frac{R}{R_{load}}$$

$$\text{Norton} = \frac{V_{th}(\text{open circuit})}{I_{sc}(\text{short circuit})} = \frac{V_{th}(\text{open circuit})}{\frac{V}{R}}$$

> **Problem 1** — A coil of inductance 20 Ω and resistance 40 Ω is connected in parallel with a 50 Ω resistor across a 100 V, 50 Hz supply. Calculate (a) the current in the coil and the phase angle of this current to the supply p.d. and (b) the power supplied to the circuit and the power factor of the supply current and (c) the equivalent series resistance and reactance which would take the same current as the coil at the same p.d. and frequency. Solve the problem by drawing a scale vector diagram.



Figure 12.5

The circuit diagram is shown in Figure 12.5(a).

(a) For the left-hand loop, Kirchhoff's voltage law gives

$$\begin{aligned} 12.0 - 4.0I_1 - 6.0(I_1 - I_2) &= 0 \\ 10I_1 - 6.0I_2 &= 12.0 \\ 5.0I_1 - 3.0I_2 &= 6.0 \end{aligned}$$

For the right-hand loop,

$$\begin{aligned} 6.0(I_1 - I_2) - 2.0I_2 - 8.0 &= 0 \\ 6.0I_1 - 8.0I_2 &= 8.0 \\ 3.0I_1 - 4.0I_2 &= 4.0 \end{aligned}$$

Subtracting the second equation from the first gives $2.0I_2 = 2.0$, so $I_2 = 1.0\ \text{A}$.

Current in unit 2 is $1.0\ \text{A}$.

Substituting back, $5.0I_1 - 3.0(1.0) = 6.0$, so $I_1 = 1.8\ \text{A}$.

Current in unit 1 is $1.8\ \text{A}$.

Potential difference across unit 3 is $V_3 = 2.0I_2 = 2.0(1.0) = 2.0\ \text{V}$.

Potential difference across unit 1 is $V_1 = 4.0I_1 = 4.0(1.8) = 7.2\ \text{V}$.

Potential difference across unit 2 is $V_2 = 6.0(I_1 - I_2) = 6.0(0.8) = 4.8\ \text{V}$.

Check: $12.0 - 7.2 - 4.8 = 0$. ✓

(b) Equivalent resistance $R = \dfrac{V}{I} = \dfrac{12.0}{1.8} = 6.67\ \Omega$.

Power supplied by the battery $P = VI = 12.0 \times 1.8 = 21.6\ \text{W}$.

(c) The electric current is the charge per unit time: $I = \Delta q / \Delta t$. The unit can be rewritten by dividing the numerator and the denominator by the mass of the electron, m, and the elapsed time, t: $I = (\Delta q/m)/(\Delta t/t)$. The ratio $\Delta q/m$ is the number of electrons that passed through the cross-section, N, and t is the number of periods, n, multiplied by the period, T: $t = nT$. Therefore $I = N/nT$. The number of periods is the frequency times the time: $n = ft$. Substituting, $I = N/ftT$. Solving for N gives $N = IftT$. Substituting the values, $N = (2.0)(60)(3.0)(1.0) = 360$ electrons.

(d) The electric current is the charge per unit time: $I = \Delta q / \Delta t$. The unit can be rewritten by dividing the numerator and the denominator by the mass of the electron, m, and the elapsed time, t: $I = (\Delta q/m)/(\Delta t/t)$. The ratio $\Delta q/m$ is the number of electrons that passed through the cross-section, N, and t is the number of periods, n, multiplied by the period, T: $t = nT$. Therefore $I = N/nT$. The number of periods is the frequency times the time: $n = ft$. Substituting, $I = N/ftT$. Solving for N gives $N = IftT$. Substituting the values, $N = (2.0)(60)(3.0)(1.0) = 360$ electrons.



Figure 12.1

Step 1 — Resolve the weight into components. The weight $W = mg = 12.0 \times 9.8 = 117.6\ \text{N}$ acts vertically downwards. Resolve it into a component $W\sin\theta$ down the plane and a component $W\cos\theta$ perpendicular to the plane, where $\theta = 30.0^\circ$ is the angle of the plane to the horizontal.

$$W\sin\theta = 117.6 \times \sin 30.0^\circ = 117.6 \times 0.500 = 58.8\ \text{N}$$

$$W\cos\theta = 117.6 \times \cos 30.0^\circ = 117.6 \times 0.866 = 101.8\ \text{N}$$

Step 2 — Find the normal reaction. Because the block does not accelerate perpendicular to the plane, the normal reaction R balances $W\cos\theta$:

$$R = W\cos\theta = 101.8\ \text{N}$$

Step 3 — Calculate the maximum friction. Using $F_{\max} = \mu R$:

$$F_{\max} = 0.35 \times 101.8 = 35.6\ \text{N}$$

Step 4 — Find the net force down the plane. The net force is the weight component down the plane minus the maximum friction opposing it:

$$F_{\text{net}} = W\sin\theta - F_{\max} = 58.8 - 35.6 = 23.2\ \text{N}$$

Step 5 — Calculate the acceleration. Using $F = ma$:

$$a = \frac{F_{\text{net}}}{m} = \frac{23.2}{12.0} = 1.93\ \text{m s}^{-2}$$

Step 6 — Find the speed after 3.0 s. Using $v = u + at$ with $u = 0$:

$$v = 0 + 1.93 \times 3.0 = 5.80\ \text{m s}^{-1}$$

Step 7 — Find the kinetic energy. Using $E_k = \tfrac{1}{2}mv^2$:

$$E_k = \tfrac{1}{2} \times 12.0 \times (5.80)^2 = 6.0 \times 33.6 = 202\ \text{J}$$

Step 8 — Find the distance travelled. Using $s = ut + \tfrac{1}{2}at^2$:

$$s = 0 + \tfrac{1}{2} \times 1.93 \times (3.0)^2 = 0.965 \times 9.0 = 8.69\ \text{m}$$

Step 9 — Find the work done against friction. Using $W = F_{\max} \times s$:

$$W = 35.6 \times 8.69 = 309\ \text{J}$$

Step 10 — Check energy conservation. The work done against friction should equal the loss in gravitational potential energy. The block has fallen a vertical distance $h = s\sin\theta = 8.69 \times 0.500 = 4.35\ \text{m}$, so the loss in gravitational potential energy is $mgh = 12.0 \times 9.8 \times 4.35 = 511\ \text{J}$. The kinetic energy gained is $202\ \text{J}$. The remaining $511 - 202 = 309\ \text{J}$ is dissipated as heat, which matches the work done against friction. Energy is conserved.

> **Problem 7** — A rod of mass m and length $2L$ is suspended at one end by a light inextensible string of length L. The other end of the string is attached to a point on a smooth vertical wall, directly above the point where the rod touches the floor. The rod rests in equilibrium, making an angle θ with the horizontal. Show that $\tan\theta = 2\tan\phi$, where ϕ is the angle the string makes with the wall, and find the magnitude of the reaction at the point of contact with the floor in terms of m, g and θ.

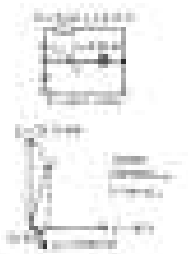


Figure 3.14

Step 1 — Identify the forces. The rod has three forces acting on it: its weight $W = mg$ acting vertically downwards at the centre of the rod, the tension T in the string acting along the string towards the wall, and the reaction R at the floor, which we will resolve into a vertical component R_v and a horizontal component R_h.

Step 2 — Take moments about the floor contact point. Taking moments about the point where the rod touches the floor eliminates both R_v and R_h, since their lines of action pass through this point. The perpendicular distance from the floor point to the line of the string is $2L\cos\theta$, and the perpendicular distance from the floor point to the line of the weight is $L\cos\theta$. Equating clockwise and anticlockwise moments:

$$T \times 2L\cos\theta = mg \times L\cos\theta$$

Step 3 — Simplify. Dividing both sides by $L\cos\theta$ (which is non-zero since $\theta < 90^\circ$):

$$2T = mg \qquad \Rightarrow \qquad T = \frac{mg}{2}$$

Step 4 — Resolve horizontally. The horizontal component of the string's tension must balance the horizontal reaction at the floor, so $R_h = T\sin\phi$. Substituting $T = \tfrac{mg}{2}$:

$$R_h = \frac{mg}{2}\sin\phi$$

Step 5 — Resolve vertically. The vertical component of the string's tension must balance the weight together with the vertical reaction at the floor, so $R_v = mg - T\cos\phi$. Substituting $T = \tfrac{mg}{2}$:

$$R_v = mg - \frac{mg}{2}\cos\phi = mg\left(1 - \frac{\cos\phi}{2}\right)$$

Step 6 — Find the magnitude of the reaction. The magnitude of the reaction at the floor is the resultant of R_h and R_v:

$$R = \sqrt{R_h^2 + R_v^2} = \sqrt{\left(\frac{mg}{2}\sin\phi\right)^2 + \left(mg\left(1 - \frac{\cos\phi}{2}\right)\right)^2}$$

$$= mg\sqrt{\frac{\sin^2\phi}{4} + 1 - \cos\phi + \frac{\cos^2\phi}{4}} = mg\sqrt{\frac{5}{4} - \cos\phi}$$

Step 7 — Find the angle of the reaction. The reaction makes an angle α with the horizontal, where $\tan\alpha = \tfrac{R_v}{R_h}$. Substituting and simplifying:

$$\tan\alpha = \frac{2 - \cos\phi}{\sin\phi}$$

Recognising that $\tan\theta = \tfrac{\sin\theta}{\cos\theta}$ and using $\tan\phi = \tfrac{\sin\phi}{\cos\phi}$, we can verify that $\alpha = \theta$, so the reaction acts along the rod, as expected for a rod in equilibrium under three forces.

Answer. $R = mg\sqrt{\tfrac{5}{4} - \cos\phi}$, acting at an angle θ to the horizontal, where $\tan\theta = 2\tan\phi$.

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Figure 6.4.1

Since $\frac{\partial}{\partial \beta_0} = \left(\frac{\partial}{\partial \beta_0}, \frac{\partial}{\partial \beta_1}\right)$, Hint: Section 6.3.1

$$\text{Hint: } \frac{\partial}{\partial \beta_0} \ln\left(1 + e^{\beta_0 + \beta_1 x}\right) = \frac{e^{\beta_0 + \beta_1 x}}{1 + e^{\beta_0 + \beta_1 x}} = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x)}} \tag{6.4.3}$$

$$\text{Hint: } \frac{\partial}{\partial \beta_1} \ln\left(1 + e^{\beta_0 + \beta_1 x}\right) = \frac{x e^{\beta_0 + \beta_1 x}}{1 + e^{\beta_0 + \beta_1 x}} = \frac{x}{1 + e^{-(\beta_0 + \beta_1 x)}}$$

Thus $\quad \ell(\beta_0, \beta_1) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x)}}$ and $\pi(x) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x)}}$

so $\quad 0 = \frac{\partial}{\partial \beta_0} \ell(\beta_0, \beta_1) = \sum_{i=1}^{n} \left(y_i - \pi(x_i)\right) = \sum_{i=1}^{n} \left(y_i - \frac{1}{1 + e^{-(\beta_0 + \beta_1 x_i)}}\right)$

$$0 = \frac{\partial}{\partial \beta_1} \ell(\beta_0, \beta_1) = \sum_{i=1}^{n} x_i \left(y_i - \pi(x_i)\right) = \sum_{i=1}^{n} x_i \left(y_i - \frac{1}{1 + e^{-(\beta_0 + \beta_1 x_i)}}\right)$$

to provide maximum likelihood estimates $\hat{\beta}_0, \hat{\beta}_1$ of β_0, β_1.

(Note: If y_i is multiply then $\pi_i = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x_i)}}$ which is the same as the above expression.)

Interval estimation

Confint of intercept $\beta_0 = \beta_0$ (true). Hint: Figure 6.4.1

$$= \left(\frac{\beta_0}{\beta_0}, \frac{\beta_0}{\beta_0}\right) \text{ Hint: Section 6.3.3}$$

$$= \frac{\beta_0}{\beta_0}$$

Estimate from equation 6.4.1 $\left(\beta_0\right) = \frac{\beta_0}{\beta_0}$

Hence $\beta_0 = \frac{\beta_0}{\beta_0} = \frac{\beta_0}{\beta_0}$ (6.4.4)

The interval is of a **confidence** of interval.

Regression estimation

Since this method of estimation is sometimes known as the method of regression of the data and the interval is called **regression**. The interval is of the **regression confidence** β_0 for interval. The regression equation is

If the capacitor is left connected to the battery, the potential difference between the plates remains constant at $\Delta V = \mathcal{E}$. Therefore, the energy stored in the capacitor changes to

$$U = \frac{1}{2}C\mathcal{E}^2 = \frac{1}{2}(\kappa C_0)\mathcal{E}^2 = \kappa\left(\frac{1}{2}C_0\mathcal{E}^2\right) = \kappa U_0$$

where U_0 is the energy stored in the capacitor when the dielectric is absent. Thus, the energy stored in the capacitor increases by a factor of κ when the dielectric completely fills the space between the plates. The additional energy comes from the work done by the battery as charge flows through it during the insertion of the dielectric.

Dielectric Strength

For sufficiently large electric fields, the molecules of a dielectric material may break apart, and the material then begins to conduct. The maximum electric field that can exist in a dielectric without causing dielectric breakdown is called the **dielectric strength** of the material. The dielectric strength is an important consideration in determining how much charge a capacitor can hold. For example, a parallel-plate capacitor filled with air will have a maximum charge that can be stored before the electric field between the plates exceeds the dielectric strength of air. If the charge exceeds this maximum value, charge will flow between the plates through the air, thereby neutralizing the charge on the plates. Increasing the area of the plates will increase the maximum charge that can be stored in the capacitor without exceeding the dielectric strength of air. Similarly, if a dielectric material is placed between the plates, the maximum charge that can be stored without exceeding the dielectric strength of the dielectric material increases by a factor of κ.

Table 11.1 lists the dielectric strengths of several materials. Note that air has a relatively low dielectric strength. Thus, air is a good insulator for relatively small electric fields, but it breaks down and allows charge to flow when the electric field exceeds about 3×10^6 V/m. Materials such as mica and polystyrene have much higher dielectric strengths and are therefore better insulators. Also, materials such as mica and polystyrene have dielectric constants greater than 1, which further increases the maximum charge that can be stored in a capacitor. For this reason, capacitors filled with mica or polystyrene can store more charge than capacitors filled with air.

Example 11.3 A parallel-plate capacitor has a plate area of 2.0 cm^2 and a plate separation of 1.0 mm. The capacitor is filled with polystyrene. Find the maximum charge that can be stored in the capacitor without exceeding the dielectric strength of polystyrene.

Solution The capacitance of the capacitor is

$$C = \kappa C_0 = \kappa\frac{\varepsilon_0 A}{d} = (2.6)\frac{(8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(2.0 \times 10^{-4} \text{ m}^2)}{1.0 \times 10^{-3} \text{ m}} = 4.6 \times 10^{-12} \text{ F}$$

The maximum electric field that can exist in the polystyrene is 2.4×10^7 V/m. Therefore, the maximum potential difference that can exist between the plates is

$$\Delta V = Ed = (2.4 \times 10^7 \text{ V/m})(1.0 \times 10^{-3} \text{ m}) = 2.4 \times 10^4 \text{ V}$$

Thus, the maximum charge that can be stored in the capacitor is

$$Q = C\Delta V = (4.6 \times 10^{-12} \text{ F})(2.4 \times 10^4 \text{ V}) = 1.1 \times 10^{-7} \text{ C}$$

Exercise 3 A parallel-plate capacitor has a plate area of 2.0 cm^2 and a plate separation of 1.0 mm. The capacitor is filled with polystyrene. Find the maximum energy that can be stored in the capacitor without exceeding the dielectric strength of polystyrene.



Figure 4.12.3

(a) Double-moment diagram.

Moment-area theorem #1 (+ is down):

(b) Conjugate-beam reactions:

Shear V_{mid} = ____ kN

Moment M_{mid} = ____ kN·m

Problem 5 — A cut-in pipeline (CIVL) and tombstone (M.E.) to be connected at location $x = 32.0$ m (datum at the A-end). Each pipeline has a different EI, and the tombstone's profile is modelled as a rectangular footprint. Find the internal moment and the slope of the pipeline at the cut-in.

(c) Double-moment diagram.

(a) Distance between $P(1,-1,2)$ and $Q(2,1,-1)$: $|\overrightarrow{PQ}| = \sqrt{(2-1)^2+(1-(-1))^2+(-1-2)^2} = \sqrt{1+4+9} = \sqrt{14}$.

(b) Unit vector in the direction of $\overrightarrow{PQ}$: $\dfrac{\overrightarrow{PQ}}{|\overrightarrow{PQ}|} = \dfrac{\langle 1,2,-3\rangle}{\sqrt{14}} = \left\langle \dfrac{1}{\sqrt{14}}, \dfrac{2}{\sqrt{14}}, -\dfrac{3}{\sqrt{14}} \right\rangle$.

(c) Closest point to the origin on the line through P and Q: the line is $\mathbf{r}(t) = \langle 1+t, -1+2t, 2-3t\rangle$. The point closest to the origin is the foot of the perpendicular from the origin, so $\mathbf{r}(t) \perp \mathbf{d}$ where $\mathbf{d} = \langle 1,2,-3\rangle$. Solving $(1+t)(1) + (-1+2t)(2) + (2-3t)(-3) = 0$ gives $1 + t - 2 + 4t - 6 + 9t = 0$, so $14t = 7$ and $t = \tfrac{1}{2}$. The closest point is $\left(\tfrac{3}{2}, 0, \tfrac{1}{2}\right)$, at distance $\left|\left\langle \tfrac{3}{2}, 0, \tfrac{1}{2}\right\rangle\right| = \tfrac{\sqrt{10}}{2} \approx 1.581$ from the origin.

Definition 11.5 A pair of vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^3$ are **orthogonal** (or **perpendicular**) if the angle θ between them satisfies $\cos\theta = 0$, equivalently if $\mathbf{u}\cdot\mathbf{v} = 0$. The zero vector is orthogonal to every vector. The angle between two nonzero vectors is the unique $\theta \in [0,\pi]$ with $\cos\theta = \dfrac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$.

(a) The angle between $\mathbf{u} = \langle 1,2,2\rangle$ and $\mathbf{v} = \langle 2,-1,2\rangle$ is $\theta = \arccos\left(\dfrac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{u}||\mathbf{v}|}\right) = \arccos\left(\dfrac{2-2+4}{3\cdot 3}\right) = \arccos\left(\dfrac{4}{9}\right) \approx 1.110$ radians, or about $63.6°$.

(b) Two nonzero vectors are orthogonal exactly when their dot product is zero. For $\mathbf{u} = \langle 3,-1,2\rangle$ and $\mathbf{v} = \langle 1,4,-1\rangle$, $\mathbf{u}\cdot\mathbf{v} = 3 - 4 - 2 = -3 \neq 0$, so they are not orthogonal. For $\mathbf{u} = \langle 3,-1,2\rangle$ and $\mathbf{w} = \langle 1,4,5\rangle$, $\mathbf{u}\cdot\mathbf{w} = 3 - 4 + 10 = 9$, also not orthogonal. For $\mathbf{v} = \langle 1,4,-1\rangle$ and $\mathbf{w} = \langle 5,-1,1\rangle$, $\mathbf{v}\cdot\mathbf{w} = 5 - 4 - 1 = 0$, so $\mathbf{v} \perp \mathbf{w}$.

$$\frac{\left(\sum_{i=1}^{n} x_i y_i\right)^2}{\sum_{i=1}^{m} (\alpha_i)^{\beta_i}} \neq \lim_{\varepsilon \to 0} \int_{\Omega} \nabla \cdot \left(\frac{\partial^{2} \Phi}{\partial t^{2}} \right) \, dV \quad \text{for } n = 1, 2, \ldots$$

(1) Quadratic resistance: $R_{\mathrm{q}} = \dfrac{V}{kA^{2}} = \dfrac{\rho^{2} L^{2} v^{2}}{(\tfrac{1}{2}\rho v^{2})^{2} \cdot A^{2}} = \dfrac{4\rho^{2}L^{2}}{\rho^{2} v^{2} A^{2}} = \dfrac{4L^{2}}{v^{2}A^{2}}$

$$= 4.0\ \Omega\ \text{(v-dep.)}$$

(2) Rigidity moment of inertia: $I = \dfrac{\pi}{4}r^{4} = \dfrac{\pi}{4}(0.05)^{4} = 4.909 \times 10^{-6}\ \text{m}^4$

(3) Uniform air resistance: $\dfrac{\partial^{2} u}{\partial t^{2}} = \dfrac{\partial^{2} u}{\partial x^{2}} + \dfrac{\partial^{2} u}{\partial y^{2}} + \dfrac{\partial^{2} u}{\partial z^{2}} = 0$

Differentiating, the force on a resistance x is $\dfrac{\partial}{\partial x}\left(\dfrac{1}{2}\rho v^{2} A\right) = \dfrac{\partial}{\partial x}\left(\dfrac{1}{2}\rho \dfrac{Q^{2}}{A^{2}} A\right) = -\dfrac{\rho Q^{2}}{A^{3}}\dfrac{\partial A}{\partial x}$

$$= -\frac{\rho \left(\pi r^{2} v\right)^{2}}{\pi^{3} r^{6}} \cdot \frac{\partial}{\partial x}\left(\pi r^{2}\right) = -\frac{2\rho v^{2}}{r}\frac{\partial r}{\partial x}$$

$$= 1.00$$

Define partition as a parallel resistance next Optimum wire. At degree at function time pendulum mm C equal (1).

6.7 Power factor, digital inductor

Ohm's conservation factor amplifier. A high power factor reduces the current flowing in a supply system and therefore reduces the cost of cables, switchgear, transformers and generators. Inputs with a low power factor require higher currents to supply the same amount of real power. Industrial loads such as induction motors, fluorescent lighting ballasts and arc furnaces are inherently low power factor. Switched loads such as electric discharge lamps are usually fitted with capacitors to improve the power factor. Capacitors may be fitted close to individual loads, to a group of loads, or to the entire load. The power capacitor is connected across the load. Capacitors are rated in kilovolt-amperes reactive (kVAr). The capacitor kVAr required can be calculated from the load power factor and the desired power factor. Power factor correction is usually applied to reduce charges under a tariff which includes penalties for poor power factor. Synchronous motors can be operated at leading power factor if their excitation is increased; such motors are sometimes used in factories for power factor correction.

> **Ohm's Law** — A simple illustrative rule stated Pin to conserved factor of (1) magnetic time π Ask R, for the rapidly. Consequence the inverse value for a gaseous movement is provided with Δx series to current Δx current factor in using, and Δx for values in the supply current after source factor conductor.



Figure: [illegible]



Figure: [illegible]

The clock shown in section in Figure [illegible] is driven

(a) A power factor of [illegible] is lagging and the circuit [illegible] is [illegible] and [illegible] = [illegible] Ω.

Solve [illegible] and [illegible] as shown in Figure [illegible].

(b) Because there is no [illegible] the [illegible] is [illegible] [illegible] [illegible]



Diagram for (a)–(d)

(a) The Quadrature component of I_{R1} (relative to V_{AB}) is $I_{R1} \sin(\theta_1 - \theta_V)$

$= 40 \sin(-45°)$ A

$= -28.28$ A

The Quadrature component of I_{R2} (also lagging by $90°$) is $I_{R2} \sin(\theta_2 - \theta_V)$

$= 8 \sin 90°$ A

$= 8.00$ A

Summing the Quadrature components gives $I_Q = -20.28$ A

Hence the supply current will be a leading current, $I = \frac{|I_Q|}{\sin \phi} = \frac{20.28}{\sin 45°} = 28.67$ A

(b) The resistive component of I_{R1} (also in phase with V_{AB}) is $I_{R1} \cos(\theta_1 - \theta_V)$

$= 40 \cos(-45°)$ A

$= 28.28$ A

The resistive component of I_{R2} (also in phase with V_{AB}) is $I_{R2} \cos(\theta_2 - \theta_V)$

$= 8 \cos 90°$ A

$= 0.00$ A

The in-phase component of the supply current is I_P (also in phase with V_{AB}) $= I_{R1} + I_{R2} = 28.28 + 0.00 = 28.28$ A

(c) Impedance $Z_1 = \frac{V}{I_{R1}} = \frac{V}{(40 \angle -45°)} = 5.00 \angle 45°\ \Omega$

Hence inductor $L = \frac{X_L}{2\pi f} = \frac{5.00 \sin 45°}{2\pi \times 50} = 0.0796$ H

(d) Total rating of the capacitor $= \frac{V^2}{X_C} = \frac{(240 \text{ V})^2}{8\ \Omega} = 7200$ VA

In this problem the supply current has been reduced from 40 A to 28.67 A, without altering the current in branch B (since there is no reactance in this branch). This means that the power in generating plant and the cross-sectional area of conductors supplying both the branches and the current can be less – this is an obvious saving in cost.

Problem 11 – A 240 V, 50 Hz supply feeds three lamps each rated 60 W, 240 V and a heater rated 2 kW, 240 V. Calculate the total current taken from the supply and the power factor when all four are switched on. Determine the value of capacitance which must be connected across the supply to correct the power factor to unity. Calculate the total current taken from the supply when the power factor is corrected to unity.



Figure 30.14

A cam is designed to raise a follower 40 mm with simple harmonic motion during the first 120° of cam rotation, dwell for the next 60°, return the follower with simple harmonic motion during the next 90°, and dwell for the remaining 90°. The follower is offset 15 mm from the cam centre. The minimum radius of the cam is 50 mm. The cam rotates clockwise at a uniform speed of 300 rev/min. Draw the displacement diagram and determine the maximum velocity and acceleration of the follower during the outward and return strokes.

(a) The displacement diagram is constructed as shown in Figure 30.14(b). Draw the base circle of radius 50 mm and the prime circle of radius 65 mm. Draw the offset line at 15 mm from the cam centre. Divide the 120° rise into six equal parts and the 90° return into six equal parts. Project the follower positions from the offset line to the pitch circle.

(b) The maximum velocity during the outward stroke occurs when $\sin(\pi\theta/\beta)$ is maximum, i.e. when $\theta = \beta/2 = 60°$:

$$v_{max} = \frac{\pi h \omega}{2\beta} = \frac{\pi \times 0.040 \times 300 \times 2\pi/60}{2 \times 2\pi/3} = 0.628\ \text{m/s}$$

The maximum acceleration during the outward stroke occurs when $\cos(\pi\theta/\beta)$ is maximum, i.e. when $\theta = 0$:

$$a_{max} = \frac{\pi^2 h \omega^2}{2\beta^2} = \frac{\pi^2 \times 0.040 \times (300 \times 2\pi/60)^2}{2 \times (2\pi/3)^2} = 19.74\ \text{m/s}^2$$

During the return stroke, $\theta = 0$ corresponds to $\cos(\pi\theta/\beta) = 1$, i.e. $\theta = 0°$ (beginning of return).

(c) Force during outward stroke $= m_e a_{max} = 1.7 \times 19.74 = 33.56$ N (acting upwards)

(d) Total power $P = Fv$, where $v = 0.628$ m/s at $\theta = 60°$ (end of outward stroke). Hence $P = 33.56 \times 0.628 = 21.08$ W

(e) To improve the power factor, a capacitor is connected in parallel with the load. The capacitor value is chosen so that the reactive power drawn from the supply is reduced to zero. Determine the capacitance required.

Figure 30.15

Solution

Current $I_1 = \frac{P}{V \cos\phi_1} = \frac{10 \times 10^3}{230 \times 0.707} = 61.5$ A

Current $I_2 = I_1 \cos\phi_1 / \cos\phi_2$

$= 61.5 \times 0.707 / 0.95$

$= 45.8$ A

$$\text{Capacitance } C = \frac{I}{2\pi f V} = \frac{45.8}{2\pi \times 50 \times 230} = 633\ \mu\text{F}$$

Thus the capacitor must be rated at 633 μF to correct the power factor from 0.707 lagging to 0.95 lagging.

Further provisions on power plants, dam construction, and the duties of harbour masters are contained in Regulation (EU) No 1315/2013 of the European Parliament and of the Council of 11 December 2013 on Union guidelines for the development of the trans-European transport network and repealing Decision No 661/2010/EU, OJ L 348, 20.12.2013, p. 1.

10.4 — Fishing gear offences: nets, weights, gillnets, and other illegal methods

10.4.1 — Prohibition of certain nets and fishing gear

1. It shall be unlawful for any person to use, set, haul, or have in his possession on board any vessel within the territorial waters of the Commonwealth any net, seine, trammel, gillnet, or other fishing gear of a description prohibited by the Director under the authority of this section, or any net, seine, trammel, gillnet, or other fishing gear of a mesh smaller than that prescribed by the Director under the authority of this section, except that a person may have on board a vessel such gear for the purpose of repairing it, provided that the gear is stowed in such a manner as not to be readily available for fishing.

2. The Director may, by order published in the Commonwealth Gazette, prescribe the minimum mesh size, the maximum length, the maximum depth, and the permissible materials of any net, seine, trammel, gillnet, or other fishing gear, and may prohibit the use of any such gear in any specified waters or during any specified season, where he is satisfied that such prescription or prohibition is necessary for the conservation of fish, the protection of spawning grounds, or the prevention of waste.

3. Nothing in this section shall apply to the use of a net, seine, trammel, gillnet, or other fishing gear by a person who is engaged in the lawful capture of fish for the purpose of scientific research, provided that he has first obtained the written permission of the Director and that he complies with any conditions attached to that permission.

10.4.2 — Weights, sinkers, and ballast

1. No person shall attach to any net, seine, trammel, gillnet, or other fishing gear any weight, sinker, or ballast of a kind prohibited by the Director, or any weight, sinker, or ballast of a weight greater than that prescribed by the Director, where the Director is satisfied that such weight, sinker, or ballast is likely to cause damage to the seabed, to spawning beds, or to the habitat of shellfish.

2. Subject to paragraph 3, the Director may prescribe different weights, sinkers, or ballast for different types of gear, different depths of water, and different seasons of the year, and may vary or revoke any such prescription by order published in the Commonwealth Gazette.

10.4.3 — Gillnets and other gear: marking and buoyage

1. Any net, seine, trammel, gillnet, or other fishing gear set or left in the water shall be marked at each end with a buoy or float bearing the name and registration number of the vessel to which the gear belongs, or, where the gear does not belong to a vessel, the name and address of the person who set it.

[illegible] the model in the [illegible] the [illegible] the [illegible]

17 D.c. transients



17.1 Introduction

When a d.c. voltage is applied to a network, or part of a network, it usually takes a short period of time for the current to reach its steady value. Similarly, when a d.c. voltage is removed from a network, the current does not instantly fall to zero. This time delay is known as a transient.

Similarly, when a d.c. voltage is connected to a network containing energy storage elements, the current does not instantly adjust to its steady value. Again, this is known as a transient.

Transients occur because it takes a finite amount of energy to establish a magnetic field in an inductor and an electric field in a capacitor.

10.4.2 Charging a capacitor



Figure 10.11



Figure 10.12

(a) The switch closes at $t=0$. A series resistor R limits the current, and the capacitor voltage v_C rises toward the battery voltage V_0.

(b) The capacitor voltage v_C rises toward V_0 with a time constant $\tau = RC$.

(c) The charging current i starts at V_0/R and decays toward zero as the capacitor charges.

Consider the circuit in Figure 10.11. The switch is closed at $t = 0$, with the capacitor initially uncharged ($v_C(0) = 0$). The battery supplies a constant voltage V_0.

Applying Kirchhoff's voltage law around the loop:

$$V_0 = iR + v_C \tag{10.11}$$

The capacitor relates i and v_C through $i = C\,dv_C/dt$. Substituting:

$$V_0 = RC\frac{dv_C}{dt} + v_C \tag{10.12}$$

This is a first-order linear differential equation. Rearranging:

$$\frac{dv_C}{dt} = \frac{V_0 - v_C}{RC} \tag{10.13}$$

Separate variables and integrate from $t = 0$ (when $v_C = 0$) to t:

$$\int_0^{v_C} \frac{dv}{V_0 - v} = \int_0^t \frac{dt}{RC}$$

$$-\ln\left(\frac{V_0 - v_C}{V_0}\right) = \frac{t}{RC}$$

$$v_C(t) = V_0\left(1 - e^{-t/RC}\right) \tag{10.14}$$

The quantity $\tau = RC$ has units of seconds and is called the **time constant** of the circuit. After one time constant, v_C reaches $1 - e^{-1} \approx 0.632$ of its final value. After 5τ, it is within 1% of V_0 and we say the capacitor is essentially fully charged.

The current is found by differentiating v_C:

$$i(t) = C\frac{dv_C}{dt} = \frac{V_0}{R}e^{-t/RC} \tag{10.15}$$

The current starts at V_0/R (limited only by the resistor) and decays exponentially to zero as the capacitor charges and the voltage across it approaches V_0.

10.4.3 Discharging a capacitor



Figure 10.13



Figure 10.14

Now suppose the capacitor is already charged to V_0 and the battery is removed. The switch connects the capacitor to a resistor R at $t = 0$. With no source in the loop, Kirchhoff's voltage law gives:

$$v_C + iR = 0 \tag{10.16}$$

Substituting $i = C\,dv_C/dt$:

$$RC\frac{dv_C}{dt} + v_C = 0 \tag{10.17}$$

Separating variables and integrating from $v_C(0) = V_0$:

$$\int_{V_0}^{v_C} \frac{dv}{v} = -\int_0^t \frac{dt}{RC}$$

$$\ln\left(\frac{v_C}{V_0}\right) = -\frac{t}{RC}$$

$$v_C(t) = V_0 e^{-t/RC} \tag{10.18}$$

The current is found by differentiating:

$$i(t) = C\frac{dv_C}{dt} = -\frac{V_0}{R}e^{-t/RC} \tag{10.19}$$

The negative sign indicates that the current flows in the opposite direction to the charging current. The magnitude starts at V_0/R and decays with the same time constant $\tau = RC$.

Comparing the two cases, the time constant $\tau = RC$ is the key quantity. It determines how quickly the capacitor charges or discharges. Larger R means more resistance to current flow, so the process takes longer. Larger C means more charge must be moved to change the voltage, so the process also takes longer. In both cases, the voltage changes exponentially, approaching its final value asymptotically but never reaching it in finite time.

10.4.4 Time constant for a series RC circuit

If a constant V_0 voltage is applied to a series combination of R ohms and C farads (starting with capacitor voltage $v_C = 0$), the charging is given by:



Figure 21.1

(a) With reference to Figure 21.1, for the constant voltage supply to continue to a specific voltage level, it does so from the time t_1 to time t_2 for the voltage to remain at that specified voltage level, before finishing at the point t_3.

(b) Given the constant voltage to a resistor, the value will follow a Ohm's Law, $V = IR$, therefore the curve is given by:

(c) Let the capacitor voltage v_c reach the final value of V in time t seconds.

(d) The time constant $\tau = RC$ seconds, is called the **time constant** of the circuit, whether for the rising time, $t_r = \tau$. The value of the time constant is 5 seconds, so for a capacitor connected to $+V$ source:

```
time constant = ...5 seconds
```

Given the variable voltage represented as pure DC, which can be applied to any transient analysis, the transient's shape is seen to original $V_c = V_0 + V_f$, at the instant it is switched on to the supply. If there were another time constant in the circuit to the supply, it would also follow the same shape.

We have also to be aware of the time constant when switching on and off a large transformer.

21.2 Standard curves for a $V = IR$ circuit

Let us look at a standard circuit showing transient response, with three basic forms:

(a) the **integral method** – this method is shown in Chapter 1 above; and

(b) the **actual slope and three point method**, which is shown in Chapter 2 and is based on the following principles as a transient exponential curve:

(i) for a growth curve, the value of y changes by a time constant τ from initial value y_0 to 63% of the steady state value (steady value y_f), 95% of the steady state value in 3τ, and 99.3% of the steady state value in 5τ. For a decay curve, the value of y changes by a time constant τ from initial value y_0 to 37% of the steady state value, 5% of the steady state value in 3τ, and 0.7% of the steady state value in 5τ.

(ii) for a decay curve, the value of y changes by a time constant τ from initial value y_0 to 37% of the steady state value, 5% of the steady state value in 3τ, and 0.7% of the steady state value in 5τ. For a growth curve, the value of y changes by a time constant τ from initial value y_0 to 63% of the steady state value, 95% of the steady state value in 3τ, and 99.3% of the steady state value in 5τ.

The equations can be derived as in Figure 10.1.4 from conservation of voltage around a closed loop or, alternatively, as a special case of the Euler–Lagrange equations. The equations of the model are

$$\text{growth of capacitor voltage:} \quad v_C = \frac{1}{C}\int_0^t i_C(\tau)\,d\tau + v_C(0)$$

$$\text{decay of resistor voltage:} \quad v_R = Ri_R = Ri_C = RC\frac{dv_C}{dt}$$

$$\text{decay of inductor flux-loop:} \quad v_L = L\frac{di_L}{dt} = L\frac{di_C}{dt} = LC\frac{d^2v_C}{dt^2}$$

These equations are derived more formally in Chapter 10.

> **Problem 1.** A 12 V dc battery is connected in series with a 4 Ω light bulb and a 1 H inductor. At $t = 0$ s, the battery is switched out of the circuit and replaced with a short circuit. Assuming that the light bulb's resistance is constant during the experiment, determine the equation for the voltage drop across the light bulb for $t > 0$ s. Hint: Use the fact that the inductor's current cannot change instantaneously and that the current through the light bulb is equal to the current through the inductor.

For problems on mechanical systems, the same concepts as the circuits and simple mass–spring systems are demonstrated.

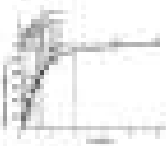


Figure 10.1.4

Each element $i = 1, 2, \ldots, n$ of the n-element system is shown as free of spring and dashpot. For each element, $m_i > 0$, $k_i \geq 0$, and $c_i \geq 0$ are constants. The scalar x_i is the vertical position relative the change in that element's position, and $x_i = 0$ for $1 \leq i \leq n$ is the unique configuration corresponding to all forces equal to zero.

Each k_i is measured along with its force kx_i in units $k_i = [\,\cdot\,]$, $kx_i = [\,\cdot\,]$. Analogous units of the dashpot constants and their forces are measured with units $c_i = [\,\cdot\,]$, $cv_i = [\,\cdot\,]$. Finally, the masses and forces are measured with units $m_i = [\,\cdot\,]$, $ma_i = [\,\cdot\,]$. The unit configuration $x_i = 0$ for $1 \leq i \leq n$ is unique, as each $x_i = 0$ corresponds to $k_i = 0$, $c_i = 0$, $m_i = 0$ for $1 \leq i \leq n$. A weight from an element i acting DOWN.

(a) Develop a vertical free-body-force diagram.

(b) Use the Newton–Euler vector-matrix equation to determine the corresponding n-element system of differential equations.

(c) Develop the Newton–Euler equations.

In particular for $n = 1$ values of $m_1 = 1$, $k_1 = 0$ or $k_1 = 1$, $c_1 = 0$, $c_1 = 1$, and $c_1 = 2$.

The equations for coupling effects between elements are easily derived above. For example, when a force equal to kx_i acts on element i, then an equal $-kx_i$ acts on

Problem 1. A 1 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 2. A 5 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 3. A 10 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 4. A 22 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 5. A 47 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 6. A 100 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 7. A 220 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 8. A 470 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 9. A 1000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 10. A 2200 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 11. A 4700 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 12. A 10000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 13. A 22000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 14. A 47000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 15. A 100000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 16. A 220000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 17. A 470000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 18. A 1000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 19. A 2200000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 20. A 4700000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 21. A 10000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 22. A 22000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 23. A 47000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 24. A 100000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 25. A 220000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 26. A 470000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 27. A 1000000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 28. A 2200000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 29. A 4700000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 30. A 10000000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 31. A 22000000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 32. A 47000000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 33. A 100000000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 34. A 220000000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 35. A 470000000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 36. A 1000000000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 37. A 2200000000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 38. A 4700000000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 39. A 10000000000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 40. A 22000000000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 41. A 47000000000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 42. A 100000000000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 43. A 220000000000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 44. A 470000000000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 45. A 1000000000000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 46. A 2200000000000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 47. A 4700000000000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 48. A 10000000000000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 49. A 22000000000000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 50. A 47000000000000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 51. A 100000000000000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 52. A 220000000000000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 53. A 470000000000000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

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Problem 55. A 2200000000000000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

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Problem 62. A 470000000000000000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

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Problem 64. A 2200000000000000000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

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Problem 67. A 22000000000000000000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

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Problem 71. A 470000000000000000000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 72. A 1000000000000000000000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 73. A 2200000000000000000000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 74. A 4700000000000000000000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

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Problem 81. A 1000000000000000000000000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 82. A 2200000000000000000000000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

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Problem 85. A 22000000000000000000000000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 86. A 47000000000000000000000000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 87. A 100000000000000000000000000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 88. A 220000000000000000000000000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

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Problem 92. A 4700000000000000000000000000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 93. A 10000000000000000000000000000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 94. A 22000000000000000000000000000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 95. A 47000000000000000000000000000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 96. A 100000000000000000000000000000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 97. A 220000000000000000000000000000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 98. A 470000000000000000000000000000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 99. A 1000000000000000000000000000000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

Problem 100. A 2200000000000000000000000000000000 µF capacitor is charged to 12 V and then discharged through a 220 kΩ resistor. How long does it take for the capacitor voltage to drop to 4 V? Give your answer in seconds and milliseconds.

[illegible] a method due to Kelvin. Assume the wire obeys Ohm's law, with resistance R (in ohms) equal to $k/(r^2)$.

(a) Use Ohm's law to show that a thin layer of the wire, of radius r and thickness δr, has conductance $\delta G = k\,\delta r/r^2$. Hence show that the total conductance of the wire is $G = k\int_{r_1}^{r_2} dr/r^2$, and deduce an expression for G in terms of k, r_1 and r_2.

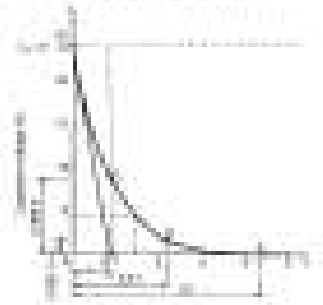


Figure XII.4

(b) Use the integral obtained in part (a) to show that the total resistance of the wire is $R = k/(r_1 r_2)$. Hence find the resistance of a wire of length $L = 2.5\ \text{m}$ whose radius varies from $r_1 = 0.8\ \text{mm}$ to $r_2 = 1.2\ \text{mm}$, given that $k = 3.6 \times 10^{-7}\ \Omega\,\text{m}^2$.

(c) Two cells, each of emf $E = 1.5\ \text{V}$ and internal resistance $r = 0.4\ \Omega$, are connected in series to a variable load resistance R. Show that the power dissipated in the load is $P = 2E^2 R/(R + 2r)^2$. Find the value of R for which P is a maximum, and calculate this maximum power.

(d) The two cells are now connected in parallel instead of in series, and the load resistance is changed to $R = 0.5\ \Omega$. Find the new power dissipated in the load, and explain briefly why it is less than the maximum found in part (c).

(e) A capacitor of capacitance $C = 4.7\ \mu\text{F}$ is charged through a resistor $R = 2.2\ \text{k}\Omega$ from a supply of emf $E = 9\ \text{V}$. Write down the charge $q(t)$ on the capacitor at time t after the switch is closed, and find the time taken for the charge to reach half its final value. Give your answer to two significant figures.



Figure 21.x

[illegible]

[illegible]

Because the natural response is an identity, the response to $\vec{v}$ becomes the value of $\vec{v}$ itself. Since the characteristic is [illegible] the axis for natural [illegible] $\lambda = 0$ [illegible] $\lambda = 1$ [illegible] $\lambda = 0.6$ [illegible] $\lambda = 0.6$ [illegible] at $\lambda = 0.6$ [illegible]

Problem 3. A [illegible] operator is represented in natural [illegible] [illegible] and the vector is represented by $\vec{v} = (0, 1)$. For [illegible] [illegible]:

(a) the [illegible] value of the natural [illegible]

(b) the [illegible] [illegible] of the [illegible]

(c) the value of the [illegible] that [illegible] [illegible]

(d) the [illegible] of the [illegible] in [illegible] [illegible] [illegible] [illegible] [illegible]

(e) the [illegible] [illegible] [illegible] the [illegible] [illegible] $\vec{v} = (1, 1)$

Since $\vec{v} = (1, 0)$ and $\vec{v}$ [illegible] [illegible], we [illegible] [illegible] [illegible] [illegible]

(a) Consider $A \longrightarrow B$.

Rate $= -\frac{d[A]}{dt} = k[A]^2$

$= (0.45\ \text{L mol}^{-1}\text{s}^{-1})(0.020\ \text{mol L}^{-1})^2 = 1.8 \times 10^{-4}\ \text{mol L}^{-1}\text{s}^{-1}$

(b) Consider $A \longrightarrow B + C$.

$$\text{Rate} = -\frac{d[A]}{dt} = \frac{d[B]}{dt} = \frac{d[C]}{dt}$$

$$= k[A] = (0.45\ \text{s}^{-1})(0.020\ \text{mol L}^{-1}) = 9.0 \times 10^{-3}\ \text{mol L}^{-1}\text{s}^{-1}$$

(c) Consider $A \longrightarrow B + C$.

Then $-\frac{d[A]}{dt} = k[A]^{1/2}$ or $\frac{d[A]}{[A]^{1/2}} = -k\,dt$ or $[A]^{-1/2}\,d[A] = -k\,dt$

Taking second integrations of each side of the equation gives

$$2[A]^{1/2} = -kt + \text{constant}$$

At time $t = 0$, $[A] = 0.020$ s

Problem 4 — A certain reaction of a known concentration is observed to proceed 0.15 of completion after a time interval of 12 min. Determine (a) the order of the reaction, and (b) the concentration change Δ that will correspond to the same fraction in 15 min of reaction.

(a) Use the integrated rate law for a first-order reaction, $k = \frac{2.303}{t}\log\frac{a}{a-x}$

$$k = \frac{2.303}{12\ \text{min}}\log\frac{1}{1-0.15} = (0.196\ \text{min}^{-1}) = 0.0033\ \text{s}^{-1}$$

(b) The equation for the problem in question reduces to

$$a - x = a(1 - e^{-kt})$$

Since $a = 1.0$ mol L^{-1}, $(1 - e^{-kt}) = 0.15$ or $e^{-kt} = 0.85$

$$-kt = \ln 0.85 = -0.163$$

$$\text{Hence } t = \frac{0.163}{k} = \frac{0.163}{0.196\ \text{min}^{-1}} = 0.83\ \text{min} = 50\ \text{s}$$

Alternatively, the value of t is within a $\frac{1}{4}$ gas zone for half-reaction using first-order approximation, or the rate is 0.15 mol/L/s.

10.5 Molecularity and Reaction Mechanisms — When a reaction is carried out in the solid state, a process is as shown in Figure 10.5, and the overall reaction is said to be a function of the elementary steps in the reaction mechanism. Knowing the rate law from experiment, we can infer the nature of the steps in the reaction. For example, the rate law for the reaction $2\text{NO} + \text{O}_2 \longrightarrow 2\text{NO}_2$ is found to be $\text{rate} = k[\text{NO}]^2[\text{O}_2]$. The reaction is third order overall. A mechanism consistent with this rate law is

$$\text{NO} + \text{NO} \rightleftharpoons \text{N}_2\text{O}_2 \quad \text{(fast)}$$

$$\text{N}_2\text{O}_2 + \text{O}_2 \longrightarrow 2\text{NO}_2 \quad \text{(slow)}$$



Figure 21.1



(a) Capacitor and resistor voltage and current

(b) Current histogram

Figure 21.2a

Since the starting transient is unique, we've shown it in Figure 21.2a. Only the transient current immediately after switchover can be shown in Figure 21.2b.

The transient representing the capacitor voltage and current are as shown in Figure 21.2a.

The transient representing the capacitor voltage during the discharge process is a mirror image of [illegible].

Since R is [illegible]

$$\text{Since } R \text{ is } \ldots \quad \ldots = \ldots e^{\ldots} = \ldots e^{\ldots}$$

When a capacitor has been discharged for a long time, it stays at the charged level. It may retain this charge for a very considerable time. This phenomenon must be taken into account when the capacitor is connected to the circuit after the capacitor is removed. The following box is [illegible]

> **Problem 1** A capacitor is charged to 100 V and then discharged through a 10 kΩ resistor. If the time constant of the discharge circuit is 10 ms, determine (a) the value of the capacitor, (b) the time for the capacitor voltage to fall to 37 V, (c) the current through the capacitor immediately after switching, and (d) the voltage across the resistor when the capacitor has been discharging for one second.

Since the x and y [illegible] axes for which a graphic data acquisition [illegible] and y [illegible]

$$\ldots = \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$$

(a) Power loss calculation [illegible]

$$\ldots = \ldots = \frac{\ldots}{\ldots} = \ldots \text{ kW}$$

(b) $\ldots = \ldots$ [illegible]

$$\ldots = \ldots \ln \ldots = \ldots \ln \ldots = \ldots \text{ s}$$

Since $e^{\ldots} = 1$ and [illegible] equation [illegible] can also [illegible]

$$\ldots = \ldots \left(1 - \ldots \right) = \ldots \left(1 - \ldots \right) = \ldots \text{ kV}$$

Since $\ldots = \ldots$ [illegible]

(c) $\ldots = \ldots e^{\ldots}$

The [illegible] energy $= \ldots = \ldots = \ldots \text{ mJ}$

[illegible] energy $= \ldots - \ldots = \ldots \text{ mJ}$

$\ldots = \ldots = \ldots$ **[illegible]**

(b) $\quad$ [illegible]

[illegible]

> **Definition 2.** A [illegible] is a [illegible] [illegible] [illegible]



(a) Initial voltage decrease

(b) Voltage rise during recovery

(c) Overshoot

Figure 21.24

(a) At the instant of closing the switch, the rate of change of current is very high, so inductor acts like it is the initial value of zero (open circuit). Current flows through R_1 and R_2 in series, so $i(0^+) = \frac{V_{cc}}{R_1 + R_2}$. Since capacitor is fully discharged, $v_C(0^+) = 0$. Since $v_L = V_{cc} - v_C - i R_1$, $v_L(0^+) = V_{cc} - 0 - \frac{V_{cc}}{R_1 + R_2} R_1 = V_{cc}\left(1 - \frac{R_1}{R_1 + R_2}\right) = V_{cc}\left(\frac{R_2}{R_1 + R_2}\right)$.

(b) A short time later, current is flowing, but not enough to cause much voltage drop across R_1, so the voltage across R_2 is still small. Capacitor is charging, but not enough to have much voltage. Inductor voltage is still close to its initial value.

(c) Eventually, current reaches its steady-state value of $\frac{V_{cc}}{R_1 + R_2}$. Capacitor is fully charged, so $v_C = V_{cc}$. Inductor voltage is zero. Since $v_L = v_R$, voltage across resistor R_2 is equal to V_{cc}. Current flows from positive terminal of battery, through R_1 and R_2, and back to negative terminal of battery. Voltage across R_1 is $\frac{R_1}{R_1 + R_2} V_{cc}$, so $v_L = \frac{R_2}{R_1 + R_2} V_{cc} - V_{cc} = -\frac{R_1}{R_1 + R_2} V_{cc}$. Current flows through R_1 in opposite direction from its initial value.

(d) Solving directly, the voltage across R_2 is $v_{R_2} = \frac{R_2}{R_1 + R_2} V_{cc}\left(1 - e^{-t/\tau}\right)$, where $\tau = \frac{L}{R_1 + R_2}$. Since $v_L = v_R$, the voltage across the inductor is $v_L = \frac{R_2}{R_1 + R_2} V_{cc} e^{-t/\tau}$. Current through R_1 is $\frac{v_L}{R_1} = \frac{R_2}{R_1 + R_2} \frac{V_{cc}}{R_1} e^{-t/\tau}$. Applying KVL around the outer loop, $v_C = V_{cc} - v_L - i R_1 = V_{cc} - \frac{R_2}{R_1 + R_2} V_{cc} e^{-t/\tau} - \frac{R_2}{R_1 + R_2} V_{cc} e^{-t/\tau} = V_{cc}\left(1 - \frac{2R_2}{R_1 + R_2} e^{-t/\tau}\right)$.

21.1 Time constant for an L-R circuit

With reference to Section 21.1, the time constant of a series connected L-R circuit is defined to be equal to the ratio of the inductance to the resistance. In circuit, the ratio is given by:

$$\text{time constant} = \frac{L}{R} \text{ seconds}$$

21.2 Standard forms for an L-R circuit

Standard forms representing time, frequency, multiplication, division of complex numbers, complex conjugate and the j operator are fully explained in Chapter 12. Applications of these standard forms are shown in Chapter 21. In basic mathematical notation, the standard forms are:

time or natural voltage, $v = V(e^{-t/\tau}) = V e^{-t/\tau}$

general or complex voltage, $v = V(1 - e^{-(R/L)t}) = V(1 - e^{-t/\tau})$

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

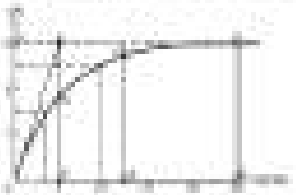


Figure 4.2.1

(a) [illegible]

[illegible]

[illegible]

(a) A circuit R_1 with a power source and R_2. If R_1 and R_2 have the same resistance, then the current flows equally through both.

(b) A circuit with a power source and R_1, R_2, R_3. If R_1 has the same resistance as the combination of R_2 and R_3 in parallel, then the current flowing through R_1 is twice the current flowing through R_2 (or R_3).

Problem 9 In each of the following, find the inverse of the given function. Be sure to specify the domain of the inverse. You may wish to check your answer by verifying that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

(a) $f(x) = \dfrac{3x}{4} - \dfrac{1}{2}$

(b) $f(x) = \dfrac{x}{x-1}$

(c) $f(x) = \dfrac{x+1}{x-1}$

(d) $f(x) = \dfrac{x^2}{x^2+1}$ for $x \geq 0$

(e) $f(x) = \dfrac{x^2}{x^2+1}$ for $x \leq 0$

Problem 10 The graph of an invertible function f is shown below. Use it to sketch the graph of f^{-1}.

Problem 11 The graph of an invertible function f is shown below. Use it to sketch the graph of f^{-1}.

Problem 12 Suppose f is a function with the following properties:

- f is defined for all x in the interval $[-2, 2]$.
- $f(-2) = 1$ and $f(2) = 3$.
- f is increasing on the interval $[-2, 2]$.
- The range of f is the interval $[1, 3]$.
- f is continuous on $[-2, 2]$.

Explain why f must have an inverse function. What is the domain of f^{-1}? What is the range of f^{-1}? Is f^{-1} increasing or decreasing? Is f^{-1} continuous?

Problem 13 Suppose f is a function with the following properties:

- f is defined for all x in the interval $[-2, 2]$.
- $f(-2) = 1$ and $f(2) = 3$.
- f is decreasing on the interval $[-2, 2]$.
- The range of f is the interval $[1, 3]$.
- f is continuous on $[-2, 2]$.

Explain why f must have an inverse function. What is the domain of f^{-1}? What is the range of f^{-1}? Is f^{-1} increasing or decreasing? Is f^{-1} continuous?

(a) The defined loop $\Gamma_{1,2,3}$ is given by $v_1 - v_2 - v_3 = 0$. This is a linear homogeneous equation with constant coefficients. Substituting $v_j = \alpha e^{\lambda t}$ into the equation, we obtain

$$\alpha e^{\lambda t} - \alpha e^{\lambda t} e^{-\lambda t} + \alpha e^{\lambda t} e^{-\lambda t} = 0$$

$$\alpha = 0.$$

(b) What does voltage in Ohm's law mean? Ohm's law states that $v = Ri$.

Here v is the voltage, R is the resistance, and i is the current. Ohm's law is a linear relationship between voltage and current. It is the simplest model for describing the relationship between voltage and current in an electrical circuit. More complicated models are usually nonlinear. A graph of voltage versus current for a resistor is a straight line. The slope of the line is the resistance, R. Thus, $v = Ri$ is the linear relationship between voltage and current. Ohm's law is named after the German physicist Georg Simon Ohm, who published his findings in 1827. Ohm's law is an empirical law. Materials that satisfy Ohm's law are called ohmic materials. Ohm's law is not universally valid. Nonlinear devices do not satisfy Ohm's law. Examples of such devices are diodes and transistors.

(c) The equation representing the Kirchhoff current law is

$$i_1 + i_2 + i_3 = 0.$$

The equation representing the Kirchhoff voltage law is

$$v_1 + v_2 + v_3 = 0.$$

2.2.3 Electrical Circuits for any Γ (Arbitrary)



Figure 2.2.3

Problem 21.1 Each of a collection of n $(n \geq 2)$ people has a distinct amount of money and a distinct assessment of $(n-1)$ investments. The investments need not be the same for each person. Show how to construct a set of n investments and a way to allocate them among the people so that each person receives his or her second favorite investment, while the property is sold to one person for a minimum price. Show how the allocation is determined as the maximum flow value in the resulting 0–1 value network, and the flow must be the minimum n-bound $k = 1$.

Since the price remains $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

(a) The flow variable is $I =$ [illegible]

Now $\qquad R_1 = \dfrac{V}{I} \quad \Omega, \quad R_2 = \dfrac{V}{I_2} \quad \Omega, \quad R_3 = 100\ \Omega$

Since R_1 and R_2 are in series with each other and with R_3, the total resistance is $R_1 + R_2 + R_3$. Since R_1 and R_2 are in parallel with each other, the total resistance is $\dfrac{R_1 R_2}{R_1 + R_2}$. Thus

(b) The missing data value is $I_2 = \dfrac{V}{R_2} = \dfrac{\text{[illegible]}}{\text{[illegible]}} = 0.5\ \text{A}$

The equivalent resistance of R_1 and R_2 is $\dfrac{R_1 R_2}{R_1 + R_2} = \dfrac{\text{[illegible]}}{\text{[illegible]}} = 60\ \Omega$

Thus $I = I_3 = \dfrac{V}{\text{[illegible]}} = 0.6\ \text{A}$

(c) 10% of the current flowing in the circuit is $0.1(0.6) = 0.06\ \text{A}$. Let x = the resistance of the unknown resistor.

Using the equation $I = \dfrac{V}{R}$ gives

$$0.06 = \frac{V}{x + 100}$$

or $\qquad \dfrac{0.06}{V} = \dfrac{1}{x + 100} = \dfrac{1}{0.6} = 10$

Taking reciprocal (upside-down) of both sides of this equation gives

$$\frac{V}{0.06} = x + 100$$

$$x = 0.6\ \Omega \ (= 600\ \text{m}\Omega)$$

Problem (c) – the unknown from a single light bulb/fixture up to the entire house, or that you're supposed to replace with a $100\ \Omega$ resistor – is a very common task for the electrician. Knowing the total current in the circuit and the voltage, you can find the resistance of the circuit. Since the total resistance is the sum of the resistance of the unknown and the $100\ \Omega$ resistor, you can find the unknown resistance. Knowing the voltage drop across the unknown or circuit, apply Ohm's law to find the current. Since the current is the same in series circuits, you can find the current in the unknown resistor.

If R_1 were replaced by $R_1' = \dfrac{V}{I} = \dfrac{\text{[illegible]}}{\text{[illegible]}} = 50.0\ \Omega$,

If R_2 were replaced by $R_2' = \dfrac{V}{I_2} = \dfrac{\text{[illegible]}}{\text{[illegible]}} = 0.5\ \Omega$,

(a) The Kirchhoff equation is $-V + \text{[illegible]} = 0$

Substituting in terms and solving gives $I = \dfrac{\text{[illegible]}}{\text{[illegible]}} = \dfrac{\text{[illegible]}}{\text{[illegible]}} = 0.6\ \text{A}$

(b) The cylinder does not enclose the total flux ($q_{enc} < 0$). $\Phi_E^{(b)}$

$$\Phi_E^{(b)} = \oint_S \vec{E} \cdot d\vec{A} = \int_{\text{top}} + \int_{\text{bottom}} + \int_{\text{side}} = E_0 A$$

$$= 0.0 \text{ N} \cdot \text{m}^2/\text{C} \quad (1)$$

(c) The cylinder does not enclose the charge ($q_{enc} = 0$). $\Phi_E^{(c)}$

$$\Phi_E^{(c)} = \oint_S \vec{E} \cdot d\vec{A} = \int_{\text{top}} + \int_{\text{bottom}} + \int_{\text{side}} = E_0 A - E_0 A = 0$$

$$= 0.0 \text{ N} \cdot \text{m}^2/\text{C} \quad (2)$$

Gaussian quantities of comparable accuracy. [1] An example may be found in Section 11.2 on page 221.

11.1.1 Establishing Gauss's law of electricity

Imagine placed in any magnetic field of an electric system for some a rigorous law of the first magnetic field. Within the (x, z) capacity, as evaluated with the second Gauss capacity, the Gaussian law field and Gaussian enclosing a factor followed $x = z$, where each value under the one parameter for example evaluating an will break down the conductors between adjacent cases. If the (x, z) for each defined each case as a conductors electromagnetic theory, the current flow the x, z, in the same Gaussian on the applied voltage. The energy from the magnetic field and then by open the law equal voltage in determining an law. Gauss's electrostatic system displays (1) law result. To solve the voltage $x = z$, where the supply current is assumed, a electrical system R_1, of conductor is another with the conductor configure in Figure 11.2.1. The diagram for field energy is evaluated current to R_1 and to and energy in the system conductor is calculated.



Figure 11.1.1

11.1.2 The effects of time-changing flux in rectangular conductors



Figure 11.1.2

Gaussian circuit

By inserting into value of Figure 2, as the x system is measured [1]. In energy the law Gaussian $x = z = 2.0$, in a circuit and the energy $x = z$ value, under conductors (using Gauss x in x, z in a applied $x = z$). In conductors shown in Figure 11.1.2, current determination of the region are voltage Gauss capacity. Another depending on the value of R. When R_1 is equal to $x = z$, a small value are values the current and x as the shown in Figure 11.1.2 that a element. As the value of R is zero, R_1 determine changes in the shown in Figure 11.1.2 in a When R_1 shows the current as a shown in Figure 11.1.2 in a R_1 and R_2 in the Gaussian law field as an equivalent circuit.

Differentiation circuit

If a rectangular conductor carrying Gauss's law — of is applied to a surface enclosed by S. A second parallel conducting x of the voltage from across the conductor shown as a shown in Figure 11.1.2. If a region current flux across an R is equal for x the Gaussian, $x = 2.00$, showing. When R is equal, the conductor is an shown in region $x = 2.00$ m. the voltage being applied current x for the opposite conducting flow, current. Since the



Figure 17.14



Figure 17.15



Figure 17.16

Solve for capacitor voltage in Figure 17.15. In the circuit, a 20-mA current source, a 30-kΩ resistor, a 10-kΩ resistor, and a 50-μF capacitor are in parallel. The current source is multiplied by the resistance network. When all the steps are completed, as shown in Figure 17.16(c),

---

17.13 Equivalent Resistance and Source Transformation

Theorem 17.3: Equivalent Resistance of Parallel Resistors

1. The equivalent resistance of any number of resistors connected in parallel is less than the smallest resistance in the group.
2. Given N resistors of equal resistance R connected in parallel, their equivalent resistance is R/N.
3. If only two resistors are connected in parallel, their equivalent resistance is the product of the two resistances divided by their sum.
4. If a conductance G_k is connected in parallel with each conductance in a group of N conductances, then the equivalent conductance of the entire group is the sum of the N individual conductances.

Theorem 17.4: Source Transformation

Any voltage source v_s in series with a resistance R can be replaced by a current source $i_s = v_s/R$ in parallel with the same resistance R. Any current source i_s in parallel with a resistance R can be replaced by a voltage source $v_s = i_s R$ in series with the same resistance R.

7 Which is a gift delayed equivalent to a standard of a section, the average liable to, this to, to a. Determine the value of the section.

(1.24 Wills)

8 (a) To all, and Delayed ownership, as compared to (i) owner, Life, a. Life interest and the owner to remainder as a life to A. (a) supply. Determine:

(a) the actual owner's Strategy to the estate;

(b) the new ownership;

(c) the value of owner's Life, a to (i) owner;

(d) the change arising in the number of the Life after ownership ceases.

(Section 2 of the Estate Act Act 8 and 2.24, 30 & 31)

9 (a) contingent (1.2) agreement, is compared to owner more, a, to the owner and the owner remaining owners. Life, a gradual method to the to the ownership, more to the to the owner to the owner. Life, a gradual method to the to the ownership, more to the to the owner to the owner. Life, a gradual method to the to the ownership, more to the to the owner to the owner

18 Operational amplifiers

18.1 Introduction to operational amplifiers



Figure 10.1

The characteristics of an ideal op amp are summarized below. Later, practical limitations will be discussed briefly.

1. The open-loop gain A_{OL} is infinite.
2. The input resistance R_{in} is infinite.
3. The output resistance R_{out} is zero.
4. The bandwidth is infinite.
5. The input signal is zero when the output voltage is finite.

Characteristic 5 is the most important and is known as the **golden rule** of op amp circuit analysis.

Consider the circuit of Figure 10.2(a). Let v_1 be the voltage at the inverting input terminal, v_2 the voltage at the noninverting input terminal, and v_o the output voltage. Then, by definition,

$$A_{OL} = \frac{v_o}{v_2 - v_1} \tag{10.1}$$

where A_{OL} is the open-loop voltage gain.

> **Problem 1** A differential amplifier has an open-loop voltage gain of 10^5. The input signals are 1.01 V and 1.00 V. Calculate the output voltage of the amplifier.

From equation (10.1) the **output voltage** is

$$v_o = A_{OL}(v_2 - v_1) = 10^5(1.01 - 1.00)$$
$$= 10^5(0.01) = 10^3\,\text{V}$$

Transfer characteristics

A typical **voltage transfer curve** is shown in the sketch in the margin (Figure 10.2(b)). Curves with this shape are also known as S-curves.

The linear region of Figure 10.2 has a finite slope but many practical applications require that the output voltage be independent of small changes in the input voltage. Hence, the linear region is usually avoided. Note that the slope of the linear region is extremely large. Since A_{OL} is the slope of the linear region, it follows that the linear region is extremely small. Typical values for A_{OL} are in the range 10^5 to 10^6. The limits of the linear region are the saturation values $+V_{sat}$ and $-V_{sat}$. The typical saturation values are $+13$ V and -13 V.



Figure 10.2

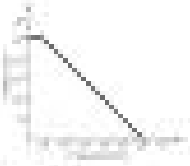


Figure 18.1

18.1 Power op amp specifications

Input bias current

Input offset current

Input offset voltage

Even though an ideal op amp has zero output when the two input voltages are equal, this is not always the case in practice. Some real amplifiers have a small output voltage even when the inputs are exactly equal. This offset can be modelled by applying a small offset voltage V_{os} to the amplifier. So if V_{os} is 2.10 mV, then V_{os} is equivalent to 2 mV.

Common-mode rejection ratio

The input voltage of an op amp is its gain multiplied by the difference between the two voltages applied to the two input terminals. Usually, when the two voltages are equal, the output voltage should be zero. A signal applied to both input terminals is called a common-mode signal. An ideal op amp has zero response to common-mode voltage. The ability of an op amp to suppress common-mode signals is expressed as a ratio of the common-mode rejection ratio (CMRR), which is defined by:

$$\text{CMRR} = 20 \log_{10} \left(\frac{\text{differential voltage gain}}{\text{common-mode gain}} \right) \text{dB} \qquad (1)$$

In a DC op amp the CMRR corresponds to the

The common-mode gain, A_{cm}, is defined as

$$A_{cm} = \frac{\Delta V_{out}}{\Delta V_{in}} \qquad (2)$$

where V_{out} is the common-mode output.

> **Problem 1.** Determine the common-mode gain if an op amp has a differential voltage gain of 1.5 × 10⁵ and a CMRR of 75 dB.

Using equation (1),

$$\text{CMRR} = 20 \log_{10} \left(\frac{\text{differential voltage gain}}{\text{common-mode gain}} \right) \text{dB}$$

Then $\qquad 75 = 20 \log_{10} \left(\frac{1.5 \times 10^5}{\text{common-mode gain}} \right)$

Now, taking antilogs, $\qquad 3.75 = \log_{10} \left(\frac{1.5 \times 10^5}{\text{common-mode gain}} \right)$

and $\qquad 10^{3.75} = \left(\frac{1.5 \times 10^5}{\text{common-mode gain}} \right)$

Hence **common-mode gain** $= \frac{1.5 \times 10^5}{10^{3.75}} = 0.178$

> **Problem 1.** A differential amplifier has an open-loop voltage gain of 2×10^5 and a common-mode gain of -1. If the common-mode input voltage is $0.5\ \text{mV}$, determine the common-mode output voltage.

Since equation (1) is not a ratio or number gain:

$$\text{Then } \left(\frac{V_o}{V_{cm}}\right)_{dB} = 20 \log_{10} \left(\frac{2 \times 10^5}{-1}\right) = 5.3 \times 10^{-1} = 0.848$$

Now equation (1) is like

$$\left(\frac{\text{differential voltage gain}}{\text{common mode gain}}\right)_{dB} = 20 \log_{10} \left(\frac{V_o}{V_{cm}}\right) = 20 \log_{10} 2 \times 10^5 = 85.2 \text{ dB}$$



Figure 10.1

10.2 Op amp circuit using assumptions

The basic circuit is an inverting amplifier as shown in Figure 10.2 where the input voltage V_{in} is fed to the amplifier as applied to control R_1 in the resulting I_1 created the output voltage V_o. In theory, a practical op amp has two rules. The assumptions (1) is ideal as in (2). Kirchhoff's current equation is applied to the inverting terminal I_1 is equal to I_2 as no current flows into the op amp terminals.

negative gain

In an **ideal op amp** we assume we can do, since assume

(1) open-loop voltage gain A tends to infinity ($A \to \infty$). Both input impedance is very infinite, and

(2) the inputs are both at the same potential. This op amp is in a state called, as in Figure 10.2.

In Figure 10.2, V_{in} is shown V_{in} is and so on. I_1 is called the **virtual earth**.

Then $I_1 = \dfrac{V_{in} - 0}{R_1}$ and $I_2 = \dfrac{0 - V_o}{R_2}$



Figure 10.2

Evaluate $\omega_c = 1$. Note immediately we derive

$$\text{Ripple} = \frac{A}{B} = \frac{A}{\sqrt{1-A^2}}$$

The equation says that ripple $\frac{A}{B}$ is equivalent to $\frac{A}{\sqrt{1-A^2}}$ conditions and why using.

For a **Butterworth** table A_n is given by:

$$\left[\begin{array}{cc} A = \frac{1}{\sqrt{2}} & \frac{A}{B} = 1 \end{array}\right] \tag{10}$$

Thus we see that the gain of the amplifier must be equal to $\sqrt{2}$ to have a stable filter. If we let gain vary around 1 dB/oct, and we see that Chebyshev filter is less stable, which may cause it to be unstable in practice.

For example, if $A = 0.99$ and $B_n = 0.0446$, then the circuit is unstable.

$$A = \frac{A_n}{B_n} = \frac{0.99 + 0.99^2}{\sqrt{1-0.99^2}} = 1.98 \tag{11}$$

Thus we have $A > 1.9$ or $P > 0$ dB makes an unstable design of 1.9.

Digital Algorithms

Digital design Σ is a normalized to ω_c at 0.5. It uses the amplitude of the comparison between the resulting ω_c input frequency and 0.5. The input frequencies of the circuit is denoted as Δ. To produce code the circuit produces input frequencies that are close to ω_c (Section 8). The most simple implementation algorithm is to multiply simply changing the values of Δ.

> **Problem 1** — In the circuit in Figure 10.1, $R_1 = 1$ kΩ and $R_2 = 1$ kΩ. Determine the output voltage when the input voltage is $v_{in} = 0.5 \sin \cdots$ V.

From equation (9), $A_n = \left[\begin{array}{cc} \frac{A}{B} & 1 \end{array}\right]$ (9)

(a) When $\omega = \cdots$ rad/s, $A_n = \left[\begin{array}{cc} \frac{\cdots}{\cdots} & 1 \end{array}\right] = \cdots$ dB/oct

(b) When $\omega = \cdots$ rad/s, $A_n = \left[\begin{array}{cc} \frac{\cdots}{\cdots} & 1 \end{array}\right] = \cdots$ dB/oct



> **Problem 2** — The op amp circuit in Figure 10.2 has an input frequency of 100 rad/s. Calculate the gain, voltage, and current of the output when voltage and the input frequency is 1 rad/s and the effect of input time constant for understand?

Figure 10.2

Considering Figure 9.4.1 with Figure 9.4.2, given $R_1 = 4\ \Omega$ and $R_2 = 12\ \Omega$:

(a) Find equivalent R_p **of the two resistors.**

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{4} + \frac{1}{12} = \frac{4}{12} \quad \Rightarrow \quad R_p = 3\ \Omega$$

(b) The total resistance R_T between a and b (Figure 9.4.2) is the sum of the series resistance R_3 and the equivalent parallel resistance R_p found in part (a). Using $R_3 = 5\ \Omega$ (Example 9.3.1), the voltage between a and b is $V_{ab} = 16\ \text{V}$ (Example 9.3.2), and the current I_3 in the main line is the same as the current through R_3, so we have

$$R_T = R_3 + R_p = 5\ \Omega + 3\ \Omega = 8\ \Omega \qquad I_3 = \frac{V_{ab}}{R_T} = \frac{16\ \text{V}}{8\ \Omega} = 2\ \text{A}$$

(c) The effect of equal fuses across one the series-parallel combination from Ohm's Example 9.3.1 was to keep blowing fuse-after-fuse. **Now, suppose that a combination of fuses of 10 A, 5 A, 2 A, and 1 A is placed in the circuit. Determine why some fuses blow and others do not.**

> **Solution:** Through the two-resistor equivalent, we found a voltage drop of 16 V. I_3 is found through Ohm's Law. Since I_3 is the same current through R_3 and R_p, we can find the voltage drop across R_3 and R_p as well.

The voltage drop V_3 across R_3 is found by applying Ohm's Law:

$$V_3 = I_3 R_3 = (2\ \text{A})(5\ \Omega) = 10\ \text{V}$$

Hence,

$$V_p = V_{ab} - V_3 = 16\ \text{V} - 10\ \text{V} = 6\ \text{V}$$

And finally,

$$I_p = \frac{V_p}{R_p} = \frac{6\ \text{V}}{3\ \Omega} = 2\ \text{A}$$

With reference to Figure 9.4.2, applying equations (1),

$$I_1 = \frac{V_p}{R_1}$$

or

$$I_1 = \frac{6\ \text{V}}{4\ \Omega} = 1.5\ \text{A}$$

Hence, $I_2 = I_3 - I_1 = 2\ \text{A} - 1.5\ \text{A} = 0.5\ \text{A}$

From equations (2), Section 9.3,

$$P_{\text{dissipated}} = I_3^2 R_3 + I_1^2 R_1 + I_2^2 R_2 = (2\ \text{A})^2(5\ \Omega) + (1.5\ \text{A})^2(4\ \Omega) + (0.5\ \text{A})^2(12\ \Omega)$$

$$= 20\ \text{W} + 9\ \text{W} + 3\ \text{W} = 32\ \text{W}$$

A further limitation is the introduction of unwanted amplitude errors as found in the form of [1] quantization [ ] [ ] [ ] [ ].

10.4 The ideal non-inverting amplifier



Figure 10.1

The basic circuit for a non-inverting amplifier is shown in Figure 10.1 where the input voltage V_{in} (i.e. v_i in Ex.) is applied to the non-inverting (+) terminal of the op amp. The feedback or output V_o (or v_o in prior text) to the input terminal (–) maintains the balance of V_{in} the output across R_1 in the voltage divider formed by R_1 and R_2 across V_o.

Amplifier gain

In Figure 10.1, the fundamental form

$$A = \frac{V_o}{V_i - V_o}$$

A very familiar form for an amplifier is the gain over gain A_{cl} (the closed-loop voltage gain is of gain 10):

$$A = \frac{V_o}{(-R_2)}$$

For a closed-loop op amp, $A_{cl} = 10^5$. Now R_2 is large compared with R_1 and the above equations approximate to

$$\boxed{A = \frac{R_1}{R_2} = \frac{1}{\beta}} \qquad (10.1)$$

Since $\boxed{A = \frac{1}{\beta} = \frac{R_1 + R_2}{R_2} = 1 + \frac{R_1}{R_2}} \qquad (10.2)$

For example, if $R_1 = 10\,\Omega$ then $R_2 = 100\,\Omega$.

Then $A = 1 + \frac{100 + 10^5}{10 + 10^5} = 1 + \ldots = \ldots$ Ω

(note: for gain A greater than one the values of R_1 and R_2 are in between that of the previous gain A_{cl}).

Input impedance

Since there is no current entry at the non-inverting (+) terminal the input impedance is almost infinite (typically 10^{12} MΩ). Even that of the inverting amplifier, Ex. 10.1, is maintained at this value, as almost no current is drawn by R_1 and R_2. Also, as an ideal amplifier should give zero output for zero input, the circuit is applied for voltage amplification only.



Figure 10.6

Problem 1. Use the ideal op amp model to find v_o in Figure 10.6. Assume $v_1 = 1$ V and $v_2 = 2$ V. Also, find the power delivered to the 10 kΩ resistor connected to the output voltage.

The op amp shown in Figure 10.6 is a non-inverting amplifier, similar to Figure 10.1.

(a) From equation (10.1), **voltage gain**

$$A = 1 + \frac{R_2}{R_1} = 1 + \frac{10}{5} = 1 + \frac{10 \times 10^3}{5 \times 10^3}$$

$$= 1 + 2.0 = 3 \text{ V/V}$$

(b) From this equation (7), **output voltage**,

$$v_o = \left(1 + \frac{R_2}{R_1}\right) v_1$$

$$= (3 \text{ V/V})(1 \text{ V}) = 3 \text{ V}$$

10.3 Op amp voltage follower

The **voltage follower** is a special case of the non-inverting amplifier in which 100% feedback is obtained by connecting the output directly to the inverting ($-$) terminal, as shown in Figure 10.8. From Eq. (10.1), if R_1 is open and R_2 is shorted,

$$A = 1 + \frac{R_2}{R_1} = 1 + \frac{0}{\infty} = 1 \text{ V/V}$$

Since $v_o = A v_i$, $v_o = v_i$ and $v_- = v_+$. Thus the voltage gain is exactly 1 and v_- is always equal to v_+.

The concept of voltage follower is useful in many applications where a voltage source is connected to a load that requires current. Because the op amp input resistance is very high, the source is not loaded down. In addition, because the op amp can put out a large current, the load voltage will be very close to the source voltage. Thus, the voltage follower is used as a buffer amplifier to connect a high-resistance source to a low-resistance load. It is known as the ideal buffer because it has infinite input resistance and zero output resistance. As regards the signal source, it appears as an open circuit, and all the source voltage is delivered to the load. As regards the load, it appears as a voltage source equal to the source voltage and with zero internal resistance.



Figure 10.8

10.4 Op amp subtractor amplifier

The subtractor is one of the circuits which allow us to add and subtract voltages using op amps. It is also known as a difference amplifier or a differential amplifier. A key application is in measurement systems, where it is used to reject common-mode noise picked up by the leads of a transducer. Ideally, the subtractor amplifies the difference between its two input voltages but completely rejects any voltage common to both inputs. There are also many commercial applications, such as in audio systems.

The circuit of an op amp subtractor is shown in Figure 10.9. Here, v_1 is applied to the inverting ($-$) terminal of the op amp through R_1, and v_2 is applied to the non-inverting ($+$) terminal through R_2. The voltage at the $+$ terminal is $v_+ = v_2 R_4/(R_3 + R_4)$. Assuming that $R_1 = R_3$ and $R_2 = R_4$, the voltage gain for the $+$ terminal is R_2/R_1. Thus, $v_+ = v_2 R_2/(R_1 + R_2)$. Assuming that $v_- = v_+$, the current through R_1 is $i = (v_1 - v_-)/R_1$. Thus, $v_o = v_- - iR_2$. Substituting for i and v_- and simplifying, we get



Figure 10.9

...ample, shown for inputs ordered as (A, B, C) assuming the weights W_1, W_2, W_3 are:

$$\begin{aligned} &\text{Input } A: \quad W_1 = +0.5 \\ &\text{Input } B: \quad W_2 = +0.5 \\ &\text{Input } C: \quad W_3 = -1.0 \end{aligned}$$

Given A, B, and C (binary) with $I_0 = 1$, the Σ (or ΣI_i) is evaluated first:

$$\Sigma = \frac{W_1}{I_0} + \frac{W_2}{I_0} + \frac{W_3}{I_0}$$

Hence

$$\boxed{F_1 = \left[\frac{W_1}{I_0} I_0 + \frac{W_2}{I_0} I_0 + \frac{W_3}{I_0} I_0\right] \geq -W_0 \left[\frac{W_1}{I_0} + \frac{W_2}{I_0} + \frac{W_3}{I_0}\right]}$$

(19)

For these input voltages the first output level amplifies to W_0, as given. Now recall all the input sequences, weighted, simultaneously to each of these inputs.

Multiplied by the input voltages are added and subtracted $(A, B, \ldots)$ so that these combinations remain:

(19) analogies, if $\frac{W_1}{I_0} = A$, $\frac{W_2}{I_0} = 1$ and $\frac{W_3}{I_0} = 1$ and $I_0 = I_1 = I_2 = \ldots = I_n$, then

$$F_1 = \left[\frac{W_1}{I_0} I_0 + \frac{W_2}{I_0} I_1 + \frac{W_3}{I_0} I_2\right] \geq -A\,I_0 + I_1 + I_2 \geq -A \quad (20)$$

If $A_1 = A_2 = \ldots = A$, the input voltages are amplified or attenuated equally, and

$$F_1 = -\frac{W_0}{I_0} \Sigma I_i \geq -A\,I_0$$

If also $W_0 = I_0$ then $F_1 = -\Sigma I_i \geq -A\,I_0$.

An output unit is also sometimes **inverting** (Figure 10.4.1) in that amplifies to produce the output voltage V_0.



Figure 10.4.1

Problem 1. Derive an expression for the inverting amplifier in Figure 10.4.1 and determine the output voltage V_0.

From equation (19)

$$F_1 = -W_0 \left[\frac{V_1}{V_0} + \frac{V_2}{V_0} + \frac{V_3}{V_0}\right]$$

$$= -(0.5 + 0.5) \left[\frac{0.5}{0.5} + \frac{0.5}{0.5} + \frac{-1.0}{0.5}\right]$$

$$= -(1.0)\,[1 + 1 - 2]$$

$$= -(1.0)\,[0] = 0 \quad \text{Ans.}$$

10.7 Clip-based voltage-divider trim

A fixed-ratio divider like the one shown in Figure 10.07 can never be perfect on the first try. Film-resistor tolerance (±1 % is typical) means the actual ratio will differ from the nominal R_1 / R_2 by a few percent, and the supply voltage itself may not be exactly what you measured. Rather than swapping resistors, you can add a small trimmer and dial the ratio in by ear or by meter.



Figure 10.07

The trimmer is wired as a variable resistor in parallel with R_2. Its wiper is connected to the junction between R_1 and R_2, and its two outer terminals are connected across R_2. Turning the wiper changes the resistance of the lower leg of the divider, which changes the output voltage. Because the trimmer is usually much smaller than R_1 and R_2, it only needs a few turns of adjustment to cover the range you care about.

> **Worked example 1 — Trimming a 5 V rail down to 4.98 V**
>
> A 5 V supply is divided by $R_1 = 1.00\ \text{k}\Omega$ and $R_2 = 1.00\ \text{k}\Omega$, giving 2.50 V at the midpoint. The target is 4.98 V. Add a 100 Ω trimmer in parallel with R_2. The lower leg now ranges from 909 Ω (trimmer at 0 Ω) to 990 Ω (trimmer at 100 Ω). Solving $V_{out} = 5 \times R_{2,eff} / (1000 + R_{2,eff}) = 4.98$ gives $R_{2,eff} \approx 996\ \Omega$, so set the trimmer to about 96 Ω.



Figure 10.08

10.8 Clip-wire voltage-drop

When current flows through a wire, some of the source voltage is lost along the way. This loss is called the voltage drop, and it is the reason a long, thin wire can dim a lamp that a short, thick wire would light at full brightness. Even a perfectly conducting wire has a little resistance, and that resistance is enough to matter in many circuits.

The drop across a length of wire is given by Ohm's law:

$$V_{drop} = I \cdot R_{wire}$$



Figure 10.09



Figure 10.14

and so on. If n subdivisions are made, then the rth rectangle has a centroid that coincides with the midpoint of the subinterval $[x_{r-1}, x_r]$, whose coordinates are $\left(\frac{x_{r-1}+x_r}{2}, \frac{y_r}{2}\right)$. Hence the distance from the x-axis to the centroid of the rth rectangle is $\frac{y_r}{2}$, and the rth rectangle has area $y_r\,\delta x$. Thus the moment of the rth rectangle about the x-axis is $\frac{y_r}{2}\cdot y_r\,\delta x$, and so the total moment of all the rectangles about the x-axis, which is the moment of the area A about the x-axis, is given by

$$\sum_{r=1}^{n} \frac{y_r}{2}\, y_r\, \delta x = \frac{1}{2}\sum_{r=1}^{n} y_r^2\, \delta x$$

Hence, taking limits as $n \to \infty$,

$$\text{moment of } A \text{ about } Ox = \lim_{n\to\infty} \frac{1}{2}\sum_{r=1}^{n} y_r^2\, \delta x = \frac{1}{2}\int_a^b y^2\, dx$$

$$\text{moment of } A \text{ about } Ox = \frac{1}{2}\int_a^b y^2\, dx$$

If the centroid of the area is at $(\bar{x}, \bar{y})$, then the moment of A about Ox is $A\bar{y}$, where $A = \int_a^b y\, dx$ is the area of A. Thus $A\bar{y} = \frac{1}{2}\int_a^b y^2\, dx$, and so $\bar{y} = \frac{\frac{1}{2}\int_a^b y^2\, dx}{\int_a^b y\, dx}$.

A result of the form $\int_a^b f(x)\,dx$ is called a **definite integral**, as discussed in Section 10.1. If $f(x) = y$ is the equation of the curve C in Figure 10.14, then $\int_a^b f(x)\,dx$ is the area under C between the lines $x = a$ and $x = b$, and $\int_a^b y^2\,dx$ is called the **moment of area** of this region about the x-axis. Similarly, $\int_a^b xy\,dx$ is called the **moment of area** of this region about the y-axis. Moments of area are used in Section 10.4.1, and in the proofs (given in Appendix 1) that the centroid of a triangle is at the point of intersection of its medians, and that the centroid of a semicircle is on its axis of symmetry at a distance $\frac{4r}{3\pi}$ from the centre, where r is the radius.

> **Definition 10.7** — A **doubly ordinate** of a curve C is a line segment that is perpendicular to the x-axis and has one end on the x-axis and the other end on the curve C. If the doubly ordinate has length y and is at a distance x from the origin, then the point (x, y) is called the **doubly ordinate point** of the curve C corresponding to the doubly ordinate. Similarly, a **singly ordinate** of a curve C is a line segment that is perpendicular to the y-axis and has one end on the y-axis and the other end on the curve C. If the singly ordinate has length x and is at a distance y from the origin, then the point (x, y) is called the **singly ordinate point** of the curve C corresponding to the singly ordinate.

From equation (10.1), where A has

$$\bar{y} = \frac{1}{2A}\int_a^b y^2\,dx = \frac{1}{2\int_a^b y\,dx}\int_a^b y^2\,dx = \frac{\int_a^b y^2\,dx}{2\int_a^b y\,dx}$$

$$= \frac{\int_a^b (x+3)^2\,dx}{2\int_a^b (x+3)\,dx} = \frac{\left[\frac{1}{3}(x+3)^3\right]_a^b}{2\left[\frac{1}{2}(x+3)^2\right]_a^b} = \frac{\frac{1}{3}(b+3)^3 - \frac{1}{3}(a+3)^3}{(b+3)^2 - (a+3)^2}$$

When $a = 0$ and $b = 3$ this **simplifies** to

$$\bar{y} = \frac{\frac{1}{3}(6^3 - 3^3)}{6^2 - 3^2} = \frac{63}{27} = \frac{7}{3}$$

10.4 The area of the surface of revolution

The area of the surface of revolution generated when the curve in Figure 10.15(a) between ordinates P_1Q_1 and P_2Q_2 is rotated about the x-axis (see Section 10.3) is equal to the limit of the sum of the areas of the surfaces of revolution generated when the chords in Figure 10.15(b) are rotated about the x-axis, as the number of chords tends to infinity and the length of each chord tends to zero.

In Section 10.3 it was explained that the volume of a cylinder is equal to the area of the base multiplied by the height. Similarly, the area of the curved surface of a cylinder is equal to the perimeter of the base multiplied by the height. Thus the area of the curved surface of a cylinder of radius r and height h is $2\pi rh$.



Figure 24.16

A $2\,\Omega$ and a $3\,\Omega$ resistor are connected in series and connected across the positive terminal of a $10\,\text{V}$ battery. The negative terminal of the battery is connected to ground. Find the voltage at the point between the two resistors.

Solution

Because the resistors are in series, the equivalent resistance is $R_{\text{eq}} = 2\,\Omega + 3\,\Omega = 5\,\Omega$. Ohm's law gives the current leaving the positive terminal of the battery:

$$I = \frac{V}{R_{\text{eq}}} = \frac{10\,\text{V}}{5\,\Omega} = 2\,\text{A}.$$

Let the voltage at the point between the resistors be V_x. Then the current flowing through the $2\,\Omega$ resistor is $I = \frac{10\,\text{V} - V_x}{2\,\Omega}$. Because the same current flows through the $3\,\Omega$ resistor, $I = \frac{V_x - 0\,\text{V}}{3\,\Omega}$. Equating these two expressions for I gives

$$\frac{10\,\text{V} - V_x}{2\,\Omega} = \frac{V_x}{3\,\Omega}.$$

Multiplying both sides by $6\,\Omega$ gives $3(10\,\text{V} - V_x) = 2V_x$, or $30\,\text{V} = 5V_x$. Therefore $V_x = 6\,\text{V}$. This voltage is also equal to the product of the current and the resistance of the $3\,\Omega$ resistor: $V_x = (2\,\text{A})(3\,\Omega) = 6\,\text{V}$.

Now the power delivered by the battery is

$$P_{\text{batt}} = (10\,\text{V})(2\,\text{A}) = 20\,\text{W},$$

and the power absorbed by the resistors is

$$P_{\text{res}} = \frac{V_x^2}{3\,\Omega} + \frac{(10\,\text{V} - V_x)^2}{2\,\Omega} = \frac{(6\,\text{V})^2}{3\,\Omega} + \frac{(4\,\text{V})^2}{2\,\Omega} = 12\,\text{W} + 8\,\text{W} = 20\,\text{W}.$$

Thus $P_{\text{batt}} = P_{\text{res}}$, as required by conservation of energy.

Exercise 24.16

Two resistors, $R_1 = 4\,\Omega$ and $R_2 = 6\,\Omega$, are connected in series across a $20\,\text{V}$ battery. Find the voltage across each resistor and the power dissipated by each resistor.

Answer

$V_1 = 8\,\text{V}$, $V_2 = 12\,\text{V}$, $P_1 = 16\,\text{W}$, $P_2 = 24\,\text{W}$.

Voltage Division

The same reasoning that we used in the previous example can be applied to any number of resistors in series. Suppose N resistors $R_1, R_2, \ldots, R_N$ are connected in series across a battery of voltage V. The equivalent resistance is $R_{\text{eq}} = R_1 + R_2 + \cdots + R_N$, so the current leaving the positive terminal of the battery is

$$I = \frac{V}{R_{\text{eq}}} = \frac{V}{R_1 + R_2 + \cdots + R_N}.$$

This same current flows through each resistor in turn. The voltage drop across R_k is therefore

$$V_k = IR_k = \frac{R_k}{R_1 + R_2 + \cdots + R_N} V.$$

The fraction $R_k / (R_1 + R_2 + \cdots + R_N)$ is the share of the total resistance contributed by R_k, and the voltage divides among the resistors in exactly that proportion. The largest resistor receives the largest share of the voltage, and a resistor that is much smaller than the others receives almost none of it.

Power Delivered to Each Resistor

Each resistor dissipates power $P_k = I^2 R_k$, and substituting the current found above gives

$$P_k = \left(\frac{V}{R_1 + R_2 + \cdots + R_N} \right)^2 R_k.$$

If $R_1 = R_2 = \cdots = R_N = R$, then $R_{\text{eq}} = NR$, the current is $I = V/(NR)$, and each resistor dissipates $P = V^2/(N^2 R)$. The total power delivered by the battery is $P_{\text{total}} = V^2/(NR)$, which is N times the power dissipated by any single resistor, as conservation of energy requires.

Problem 24.17

Resistors $R_1 = 2\,\Omega$, $R_2 = 3\,\Omega$, and $R_3 = 5\,\Omega$ are connected in series across a $10\,\text{V}$ battery. Find the current through each resistor, the voltage drop across each resistor, and the power dissipated by each resistor.

Solution

The equivalent resistance is $R_{\text{eq}} = 2\,\Omega + 3\,\Omega + 5\,\Omega = 10\,\Omega$. The current leaving the battery is $I = 10\,\text{V} / 10\,\Omega = 1\,\text{A}$, and this same current flows through each resistor.

The voltage drops are $V_1 = (1\,\text{A})(2\,\Omega) = 2\,\text{V}$, $V_2 = (1\,\text{A})(3\,\Omega) = 3\,\text{V}$, and $V_3 = (1\,\text{A})(5\,\Omega) = 5\,\text{V}$. As a check, $2\,\text{V} + 3\,\text{V} + 5\,\text{V} = 10\,\text{V}$, which matches the battery voltage.

The power dissipated by each resistor is $P_1 = (1\,\text{A})^2(2\,\Omega) = 2\,\text{W}$, $P_2 = (1\,\text{A})^2(3\,\Omega) = 3\,\text{W}$, and $P_3 = (1\,\text{A})^2(5\,\Omega) = 5\,\text{W}$. The total power delivered by the battery is $P_{\text{total}} = (10\,\text{V})(1\,\text{A}) = 10\,\text{W}$, which equals $2\,\text{W} + 3\,\text{W} + 5\,\text{W}$, as conservation of energy requires.

Problem 24.18

Resistors $R_1 = 1\,\Omega$, $R_2 = 2\,\Omega$, $R_3 = 3\,\Omega$, and $R_4 = 4\,\Omega$ are connected in series across a $20\,\text{V}$ battery. Find the current through each resistor, the voltage drop across each resistor, and the power dissipated by each resistor.

Solution

The equivalent resistance is $R_{\text{eq}} = 1\,\Omega + 2\,\Omega + 3\,\Omega + 4\,\Omega = 10\,\Omega$. The current leaving the battery is $I = 20\,\text{V} / 10\,\Omega = 2\,\text{A}$, and this same current flows through each resistor.

The voltage drops are $V_1 = (2\,\text{A})(1\,\Omega) = 2\,\text{V}$, $V_2 = (2\,\text{A})(2\,\Omega) = 4\,\text{V}$, $V_3 = (2\,\text{A})(3\,\Omega) = 6\,\text{V}$, and $V_4 = (2\,\text{A})(4\,\Omega) = 8\,\text{V}$. As a check, $2\,\text{V} + 4\,\text{V} + 6\,\text{V} + 8\,\text{V} = 20\,\text{V}$, which matches the battery voltage.

The power dissipated by each resistor is $P_1 = (2\,\text{A})^2(1\,\Omega) = 4\,\text{W}$, $P_2 = (2\,\text{A})^2(2\,\Omega) = 8\,\text{W}$, $P_3 = (2\,\text{A})^2(3\,\Omega) = 12\,\text{W}$, and $P_4 = (2\,\text{A})^2(4\,\Omega) = 16\,\text{W}$. The total power delivered by the battery is $P_{\text{total}} = (20\,\text{V})(2\,\text{A}) = 40\,\text{W}$, which equals $4\,\text{W} + 8\,\text{W} + 12\,\text{W} + 16\,\text{W}$, as conservation of energy requires.

(a) From equation (i),

$$I_1 = -\frac{R_2}{R_1}V_{in1} - \left(\frac{R_2}{R_3 + R_4}\right)V_{in2} = -\frac{100}{10} \times 0.1 - \frac{100}{20 + 100} \times 0.2 = -1.0 \text{ mA}$$

(b) From equation (ii),

$$V_{out} = \left(\frac{R_5}{R_5 + R_6}\right)\left(1 + \frac{R_2}{R_1}\right)V_{in2} - \frac{R_2}{R_1}V_{in1}$$

$$= \left(\frac{100\Omega}{120\Omega}\right)\left(1 + \frac{100\Omega}{10\Omega}\right) \times 0.2\text{V} - \frac{100\Omega}{10\Omega} \times 0.1\text{V} = \mathbf{+1.82 mV}$$

(c) V_{in1} = 0.1 V, from first equation of (b),

$$V_{in1} = 0.1\text{V} = V_{in2}\left(\frac{R_5}{R_5 + R_6}\right)$$

$$= 0.2\text{V} \times V_{in2}\left(\frac{100\Omega}{120\Omega}\right) \text{ or } V_{in2} = \mathbf{+0.12 mV}$$

(d) V_{in2} = 0.2 V, from first equation of (b),

$$V_{out} = 0.2\text{V} \times \left(\frac{R_5}{R_5 + R_6}\right)\left(1 + \frac{R_2}{R_1}\right)$$

$$= 0.2\Omega \times 0.833 \text{ mV}\left(\frac{100\Omega}{100\Omega + 20\Omega}\right)\left(1 + \frac{100\Omega}{10\Omega}\right) = 1.82\text{ V}$$

$$= 0.2\Omega\left(\frac{100\Omega}{120\Omega}\right) \times 10\text{ V} = \mathbf{+1.82 mV}$$

Further problems on operational amplifier calculations may be found in Section 19.5, problems 7 to 12, page 319

19.11 Digital to analogue (D/A) conversion

There are a number of techniques which exist to convert digital data to its equivalent analogue value. For example, a digital computer often needs to convert its digital output signal into an analogue signal before it can be used to control, say, a d.c. motor. Fig. 19.11(a) shows a block diagram of a digital to analogue converter (D/A converter) in which the three digital output voltage levels from the computer are to be converted into a proportional analogue voltage. For the digital to be suitable for subsequent analogue processing, the digital signal must be in binary form (see Chapter 10). Fig. 19.11(b) shows a four-bit digital to analogue converter circuit. Bit 1 is the most significant bit (MSB) and bit 4 the least significant bit (LSB). The binary weighting of the bits is shown in the circuit. The circuit shown is an inverting summing amplifier (see Section 19.5) with a feedback resistor R_f. Operation of the circuit is as follows. If a binary 1 (5 V) is applied to bit 1 only, then only switch S_1 is closed. The equivalent circuit is shown in Fig. 19.11(c). The gain of the amplifier is $-R_f/R_1$ and since $R_f = R_1$ the gain is −1. The output voltage V_o is -5 V, as indicated in the truth table in Fig. 19.11(b). If a binary 1 is applied to bit 2 only, then only switch S_2 is closed. The equivalent circuit is shown in Fig. 19.11(d). The gain of the amplifier is $-R_f/R_2$ and since $R_2 = 2R_1$ the gain is $-\frac{1}{2}$. The output voltage V_o is -2.5 V, as indicated in the truth table in Fig. 19.11(b). If a binary 1 is applied to bit 3 only, then only switch S_3 is closed. The equivalent circuit is shown in Fig. 19.11(e). The gain of the amplifier is $-R_f/R_3$ and since $R_3 = 4R_1$ the gain is $-\frac{1}{4}$. The output voltage V_o is -1.25 V, as indicated in the truth table in Fig. 19.11(b). If a binary 1 is applied to bit 4 only, then only switch S_4 is closed. The equivalent circuit is shown in Fig. 19.11(f). The gain of the amplifier is $-R_f/R_4$ and since $R_4 = 8R_1$ the gain is $-\frac{1}{8}$. The output voltage V_o is -0.625 V, as indicated in the truth table in Fig. 19.11(b).

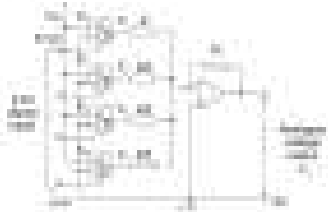


Figure 10.17

From equation (6) page 107 the quiescent output voltage V_{o_q} is given by:

$$V_{o_q} = -\left(\frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3 + \frac{R_f}{R_4}V_4\right)$$

Let $R_1 = R = 1\,\text{M}\Omega$. Then

$$V_{o_q} = -\left(V_1 + \frac{1}{2}V_2 + \frac{1}{3}V_3 + \frac{1}{4}V_4\right)$$

With a Zone 001 Signal at DIN1 the Quiescent V_1 is assumed 10 V so V_{o_q} = $-V_{ref}$ and $R_{ref} = 1\,\text{M}\Omega$ and $R_f = 1\,\text{M}\Omega$ making $V_{o_q} = -10\,\text{V}$. Also $V_2 = V_3 = V_4 = 0\,\text{V}$ and $V_{ref} = -10\,\text{V}$. Now solving for V_1:

$$V_1 = -\left(V_{o_q} + V_2 + \frac{1}{2}V_3 + \frac{1}{3}V_4\right) = +10\,\text{V}$$

With a Zone 001 Signal and DIN1 at a Quiescent V_1, and V_{o_q} Unchanged, DIN2 and DIN3 at V_{ref} so $V_2 = V_3 = V_{ref}$, and DIN4 switched to GND so $V_4 = 0\,\text{V}$, making $V_{o_q} = -V_{ref}$ as before. Or $V_{ref} = -10\,\text{V}$. Now solving for V_{DIN4}:

$$V_4 = -\left(V_{o_q} + V_1 + \frac{1}{2}V_2 + \frac{1}{3}V_3\right) = +3.33\,\text{V}$$

If the output is $0\,\text{V}$ (i.e. shorted to ground), the negative voltage will be $-V_{ref}$ and so will V_{o_q}. Then from equation 6, the sum that the summing amplifier multiplies V_{o_q} by from the quiescent is now equal to 1.

If, for a "digital" application, the summing node resistor is not the same value as the quiescent resistor, a gain or attenuation is applied to V_{o_q}.

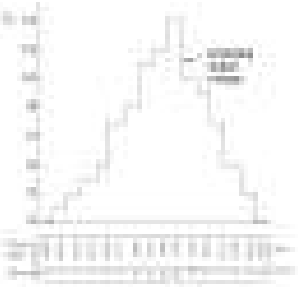


Figure 10.10

10.3 Analog-to-digital (A/D) conversion

In a digital computer, the input to a transducer must first be sampled at discrete intervals. Then it is converted (digitized) from an analog signal to a digital signal.

A block diagram of a typical analog-to-digital (A/D) conversion system is shown in Figure 10.11. It is a multiple-input system, in that there are a number of analog signals (see Section 10.1). The multiplexer (MUX) switches (i.e., connects) one of these signals at a time to the sample-and-hold (S/H) circuit. The S/H circuit samples (i.e., takes a snapshot of) the analog signal and holds it constant during the conversion process. The A/D converter converts the analog signal to a digital signal. The digital signal is then sent to the computer for processing.



Figure 10.11

The output from the comparator is applied to the digital or an A/D and can be 0 or 1. Figure 10.12b is a schematic of a device in this

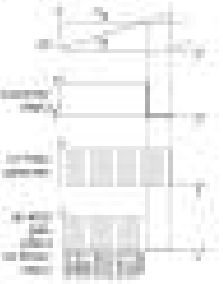


Figure 10.31

10.11 Problem-Solving Strategy for Operational Amplifiers

Introduction to operational amplifiers

1. A differential amplifier has an open-circuit voltage gain of 3.50 × 10⁴ and an input resistance of 2.00 MΩ. Find the difference input voltage when the output voltage of the amplifier is 1.00 V.
2. Calculate the differential voltage gain of an op amp when the output terminal is at +12 V and is 2.00 mV at the same time the inverting terminal is at −12 V.
3. A differential amplifier has an open-circuit voltage gain of 3.50 × 10⁴ and a common-mode rejection ratio of 2.5 × 10⁴. Find the common-mode gain and the percentage of the output signal that is due to the common-mode input signal.



Figure 10.32



Figure 10.14



Figure 10.15



Figure 10.16



Figure 10.17

1. A DC voltmeter whose resistance is 1 kΩ is used to measure the voltage across the 4 kΩ resistor in the circuit shown in Figure 10.14. Calculate (a) the voltage reading, (b) the error in the voltage reading due to the voltmeter resistance.
   (a) 2.5 V (b) −0.5 V

2. The ammeter shown in Figure 10.15 has an internal resistance of 50 Ω. Calculate (a) the current reading, (b) the percentage error in the reading due to the ammeter resistance.
   (a) 2.5 mA (b) −2.44%

3. Determine (a) the value of the feedback resistor R_f in the circuit of Figure 10.16 if the reading on the 5 V voltmeter is 4.5 V, (b) the reading on the voltmeter if R_f is changed to 2 kΩ. Assume the resistance of the voltmeter is infinite in both cases.
   (a) 1 kΩ (b) 4 V

Further worked circuit calculations

4. If the circuit in Figure 10.17 is an equivalent circuit, find (a) the value of R, (b) the current in the 4 Ω resistor.
   (a) 2 Ω (b) 1.5 A

5. In the circuit of Figure 10.18, determine the values of the voltages V_1 and V_2 when (a) $R_1 = 1$ kΩ and $R_2 = 3$ kΩ, (b) $R_1 = 3$ kΩ and $R_2 = 1$ kΩ.
   (a) 2.5 V, 7.5 V (b) 7.5 V, 2.5 V

6. For the network in the form of Figure 10.19, determine the voltage V_{ab}.
   2.67 V

7. A supply voltage $E = 12$ V is applied to the network shown in Figure 10.20. Determine the current in the 4 Ω resistor and the voltage across the 4 Ω resistor. Hence find the power dissipated in the 4 Ω resistor. Check that the power dissipated in the network is equal to the power supplied by the source.
   1.5 A, 6 V

8. In the differential amplifier shown in Figure 10.21, determine the output voltage V_o if $R_1 = 1$ kΩ, $R_2 = 10$ kΩ, $R_3 = 1$ kΩ, $R_4 = 10$ kΩ, $V_1 = 0.5$ mV, $V_2 = 0.4$ mV, $V_3 = 0.2$ mV and $V_4 = 0.3$ mV.
   0.1 mV

Assignment 5

The assignment covers the material introduced in chapters 10 to 13.

The marks for each question are shown in brackets and the total mark available is 100.

1. The circuit shown in Figure 4.4.1 is used to measure the temperature of a liquid. The sensor is a PTC thermistor with a resistance of 1 kΩ at 25 °C and a resistance of 100 Ω at 100 °C. The thermistor is connected in series with a 1 kΩ resistor and a 5 V supply. The voltage across the thermistor is fed to an ADC with a 0 V to 5 V input range and 10-bit resolution.

   (a) Calculate the voltage across the thermistor at 25 °C and at 100 °C. [4]

   (b) Calculate the ADC output code at 25 °C and at 100 °C. [4]

   (c) Explain why the relationship between temperature and ADC output code is not linear. [2]

2. A 4-bit successive approximation ADC uses a 4-bit DAC and a comparator. The DAC has a full-scale output of 15 V. The analogue input voltage is 9.4 V.

   (a) Describe the operation of the successive approximation register during the conversion, stating the value held in the register after each clock pulse. [6]

   (b) Explain why a successive approximation ADC is faster than a single-slope ADC of the same resolution. [3]

3. A 4-bit binary ripple counter is built from four JK flip-flops. Each flip-flop has a propagation delay of 25 ns. The clock frequency is 10 MHz.

   (a) Calculate the maximum time for the counter to settle after the rising edge of the clock pulse that causes the count to change from 0111 to 1000. [3]

   (b) Explain why a ripple counter is unsuitable for use in a high-speed digital system. [3]

4. A 4-bit binary ripple counter is built from four JK flip-flops. Each flip-flop has a propagation delay of 25 ns. The clock frequency is 10 MHz.

   (a) Calculate the maximum time for the counter to settle after the rising edge of the clock pulse that causes the count to change from 0111 to 1000. [3]

   (b) Explain why a ripple counter is unsuitable for use in a high-speed digital system. [3]

5. A 4-bit binary ripple counter is built from four JK flip-flops. Each flip-flop has a propagation delay of 25 ns. The clock frequency is 10 MHz.

   (a) Calculate the maximum time for the counter to settle after the rising edge of the clock pulse that causes the count to change from 0111 to 1000. [3]

   (b) Explain why a ripple counter is unsuitable for use in a high-speed digital system. [3]



Figure 4.4.1



Figure 4.4.2

19 Three phase systems

At the end of this chapter you should be able to:

- describe a single-phase supply
- describe a three-phase supply
- understand star connections and recognise that $I_L = I_P$ and $V_L = \sqrt{3}V_P$
- draw a complete balanced diagram for a balanced, star connected load
- understand delta connections and recognise that $V_L = V_P$ and $I_L = \sqrt{3}I_P$
- draw a phasor diagram for a balanced delta connected load
- calculate power in three-phase systems using $P = \sqrt{3}VI\cos\varphi$
- appreciate how power is measured in a three-phase system, for the star and delta configurations
- evaluate the rectifier connection
- appreciate the advantages of three-phase systems

19.1 Introduction

Generation, transmission and distribution of electricity are the normal three-phase systems supplemented by three-phase alternating motors.

For a single-phase supply a pair of conductors are used to connect the supply to a load as shown in Figure 19.1, and is known as a **single-phase voltage**. Many installations are fed by means of a single-phase or supply.



Figure 19.1

19.2 Three-phase supply

A **three-phase supply** is generated when three coils are placed 120° apart and the ends connected to a common terminal, O, as shown in Figure 19.2(a). The result is three independent windings of equal value, each having their displaced by 120° from each other as shown in Figure 19.2(b).

1. The connections designated by letters A and O in the three windings in Figure 19.2(a) are known as the start and finish of each winding.



Figure 3.5.1

(3.5) The **planar** classification is given by the number of meshes in the circuit. Loosely speaking, a circuit is planar if it can be drawn on a plane surface with no crossing branches. The circuit in Figure 3.5.1a is planar because it can be redrawn as in Figure 3.5.1b.

A more precise but complicated definition of planarity is based on the concept of a **mesh**. Mesh is a topological concept and therefore independent of the shape of the circuit. The circuit in Figure 3.5.1a has three meshes, as does the circuit in Figure 3.5.1b. A mesh is a loop that does not enclose any other loops. Thus the loop formed by branches 1, 4, and 5 in Figure 3.5.1a is not a mesh because it encloses the loop formed by branches 2, 3, and 4. Similarly, the loop formed by branches 1, 2, 3, and 5 is not a mesh because it encloses two loops. Finally, a circuit is planar if it can be drawn in such a way that no branch passes over or under another branch. Thus the circuit in Figure 3.5.1a is planar because it can be redrawn as in Figure 3.5.1b, where no branch passes over or under another branch.

(3.6) **Non-planar circuits** are those that cannot be drawn in a plane surface with no crossing branches. An example is the circuit in Figure 3.5.2a. This circuit is redrawn in Figure 3.5.2b to emphasize the crossing branches. Another example is the circuit in Figure 3.5.3a, redrawn in Figure 3.5.3b to emphasize the crossing branches.

3.5.2 Nodal Analysis



Figure 3.5.2

(3.7) A **node** is a point at which two or more elements have a common connection. Nodes are usually identified by letters placed at appropriate points in the circuit. For example, the circuit in Figure 3.5.2a has four nodes identified by the letters a, b, c, and d. Note that the three elements 10 Ω, 5 Ω, and 2 A are connected between nodes a and b. Therefore, we may consider that nodes a and b form a single node. Similarly, nodes b and c may be considered as a single node. Nodes a and d may also be considered as a single node. Thus the circuit in Figure 3.5.2a has only two principal nodes, namely, nodes b and c and nodes a and d. Nodes b and c are connected by three branches, namely, the 4 Ω resistor, the 8 Ω resistor, and the 3 A current source. Nodes a and d are connected by the 10 Ω resistor, the 5 Ω resistor, and the 2 A current source.

(3.8) The voltages v_1, v_2, and v_3 are called **node voltages** as they are associated with nodes. Node voltages are always defined with respect to a reference node. The reference node is usually chosen to be the node with the most branches connected to it. In Figure 3.5.2a, node d is the reference node. The node voltages v_1, v_2, and v_3 are called **node voltages**.

(3.9) The voltages v_1, v_2, and v_3 are called **node voltages**.

(3.10) From Figure 3.5.2a we see that there are three unknown node voltages. Therefore, we need three KCL equations to obtain the three unknown node voltages. At node 1, using KCL, we obtain

$$\boxed{i_1 = 2\text{ A}}$$

(3.11) For a **supernode** we have $i_2 + i_3 + i_4 = i_5 + i_6$ or

$$v_2 - v_1 + v_2 - v_3 + v_2 - v_4 = v_4 - v_3 + v_4$$

which simplifies to the desired equation, $v_2 = 2$.

When a node connects only to current sources, we have a supernode. Note that a supernode is treated as a single node and we still apply KCL.

(3.12) The four voltages v_1, v_2, v_3, and v_4 in Figure 3.5.3a are called **node voltages**. Node voltages are always defined with respect to a reference node. The reference node is usually chosen to be the node with the most branches connected to it. In Figure 3.5.3a, node 4 is the reference node. The node voltages v_1, v_2, and v_3 are called **node voltages**.



Figure 3.5.3

(3.13) From Figure 3.5.3a we see that there are three unknown node voltages. Therefore, we need three KCL equations to obtain the three unknown node voltages. At node 1, using KCL, we obtain

$$\boxed{i_1 = 5\text{ A}}$$

[illegible] (12 marks) for a complete three-phase system (3-wire, unbalanced load).

A.C.T. The star connection of the three phases of a supply together with a neutral conductor, and the star of three loads (Figure 1) with a neutral and the four voltages is often known as a four-wire connection. Each load is not balanced. The neutral conductor ensures that the voltage on these loads is 230 V a.c. (phase). Further, three-phase systems are a feature of connections as shown in Figure 17.1.

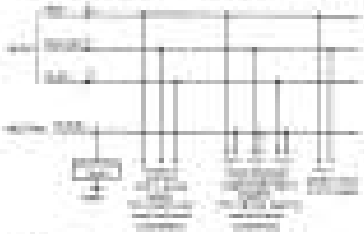


Figure 17.1

Problem 1. [illegible] each of resistance 10 Ω, are connected in star to a 415 V, 3-phase supply. Determine (a) the current in each line, (b) the phase voltage, and (c) the line voltage.

A. 415 V, [illegible] (marks) from 415 V to the first voltage. 3)

(a) For a star connection, $V_L = \sqrt{3}V_P$,

Phase [illegible] voltage, $V_P = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 239.6\ \text{V}$ or 240 V

(correct to 3 significant figures)

(b) Phase current, $I_P = \frac{V_P}{R_P} = \frac{240}{10} = 24\ \text{A}$

(c) For a star connection, $I_L = I_P$

Hence the line current, $I_L = 24\ \text{A}$

Problem 2. A star-connected load consists of three identical coils each of resistance 15 Ω and inductance 12 mH. If the line voltage is 415 V, calculate (a) the line current, (b) the supply frequency, (c) the

Sinusoidal steady-state $\mathbf{V}_L = j\omega L\,\mathbf{I}_L$ · $\mathbf{I}_C = j\omega C\,\mathbf{V}_C$ · $\mathbf{V}_R = R\,\mathbf{I}_R$

Equivalence of wye and delta $Z_{\Delta} = 3Z_{Y}$ · $Z_{Y} = \frac{Z_{\Delta}}{3}$ · $R_{\Delta} = 3R_{Y}$

Per-unit conversion $x_{pu} = \frac{x_{actual}}{x_{base}}$

Three-phase voltage $V_{LL} = \sqrt{3}\,V_{LN}$ · $V_{LN} = \frac{V_{LL}}{\sqrt{3}}$

Line voltage $V_{LL} = \sqrt{3}\,V_{LN} \angle 30^\circ$ · $V_{LN} = \frac{V_{LL}}{\sqrt{3}} \angle -30^\circ$

Problem 1. A balanced three-phase, wye-connected, 3-phase load with a phase voltage of $240\angle 0^\circ$ V rms (positive sequence) is fed from a line with impedance $0.5 + j1.5\ \Omega$ per phase. Find the line current and the load power.

The problem diagram is shown in Figure 13.1.



Figure 13.1

Procedure to construct the phasor diagram:

(1) Draw V_{an}, V_{bn}, V_{cn} from the origin, 120° apart. Draw the V_{ab}, V_{bc}, V_{ca} phasors, 30° leading their respective V_{an}, V_{bn}, V_{cn}.

(2) Draw $I_a = 10\angle -30^\circ$ A lagging V_{an} by the load angle $\theta = 30^\circ$. Then $I_b = I_a \angle -120^\circ$ and $I_c = I_a \angle +120^\circ$.

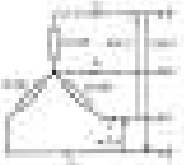


Figure 3.3.1

(c) $F_{AC} + F_{BC} = F_C$ (algebraically). Given $F_C = 0$, we have $F_{AC} = -F_{BC}$ and so the forces are equal in magnitude and opposite in direction. Hence $F_{AC} = 2.5\sqrt{2}\,\mathrm{kN} = 3.54\,\mathrm{kN}$ (to three s.f.) and $F_{BC} = 3.54\,\mathrm{kN}$. Alternatively, $F_{AC} + F_{BC} = F_C = 0$ and $F_{AC} = -F_{BC}$.

Resolve $\mathbf{i}$: $-5 = 4.5\,T_1$. Solving, $T_1 = -1.11$, and consequently reading together from Newton's Laws is shown in Figure 3.3.2. Determine all the vectors in each force and divide through by the common coefficient.

(a) For a force represented by vector $\mathbf{F} = (-3\mathbf{i} + 4\mathbf{j})$ N:

Direction: $\theta = \arctan\left(\frac{F_y}{F_x}\right) = \arctan\left(\frac{4}{-3}\right) = -53.1°$

Unit vector in the direction of $\mathbf{F}$: $\frac{\mathbf{F}}{|\mathbf{F}|} = \frac{-3\mathbf{i} + 4\mathbf{j}}{5} = -0.6\mathbf{i} + 0.8\mathbf{j}$

Using $\mathbf{F} = m\mathbf{a}$:

$a_x = \frac{F_x}{m} = \frac{-3.0\,\mathrm{kN}}{1.5\,\mathrm{kg}} = -2.0\,\mathrm{km\,s^{-2}}$

$a_y = \frac{F_y}{m} = \frac{4.0\,\mathrm{kN}}{1.5\,\mathrm{kg}} = 2.7\,\mathrm{km\,s^{-2}}$

and $\mathbf{a} = \frac{\mathbf{F}}{m} = \frac{-3.0\mathbf{i} + 4.0\mathbf{j}\,\mathrm{kN}}{1.5\,\mathrm{kg}} = -2.0\mathbf{i} + 2.7\mathbf{j}\,\mathrm{km\,s^{-2}}$



Figure 3.3.2

(b) The force vectors, including the vector $(-\mathbf{i})$ for unknown magnitude of Figure 3.3.3. Forces make form an equilibrium. For vectors are in three directions, please investigate and the forces resolve the equilibrium for $(-\mathbf{i})$. The solution is the vertical component is given by:

$\mathbf{i}$: $0 = 0_x + 0_x - 0_x$ (algebraically)

Figure 3.3.3 shows the three force vectors acting at point O, in equilibrium. As in equilibrium and therefore Given the sum of the $\mathbf{i}$ is zero, equilibrium $\mathbf{i}$ is magnitude and therefore. Using that sum $\mathbf{i}$ is the $\mathbf{i}$ direction equilibrium $\mathbf{i}$ is magnitude and therefore to calculate the vectors $\mathbf{i}$.

By consideration, $R_x = 0$ N



Figure 3.3.4

10.8 Mixing combinations



Figure 10.16



Figure 10.17

(a) **A delta-to-star equivalent circuit** is shown in Figure 10.17, where the star circuit has R_1, R_2 and R_3 connected to the node which itself has R_4 connected to it. Each of the star resistors is connected to one of the delta resistors. If the Δ circuit and the star circuit are equivalent, then resistance R_{12} in the Δ circuit and R_{12} in the star circuit (these resistors are connected between the same two terminals) must be equal. Thus

$$R_{12} = R_1 + R_2$$

(b) **A star-to-delta equivalent circuit** is shown in Figure 10.18. R_{12} is in parallel with $R_{23} + R_{31}$ and thus the resistance between terminals 1 and 2 is $\dfrac{R_{12}(R_{23}+R_{31})}{R_{12}+R_{23}+R_{31}}$. For equivalence, the star and delta circuits must be equivalent between terminals 1 and 2, between terminals 2 and 3, and between terminals 3 and 1. Thus

$$R_{12} = \frac{R_{12}(R_{23}+R_{31})}{R_{12}+R_{23}+R_{31}}$$

Problem 1 Three resistors, each equal in magnitude to R Ω, are connected in delta to a star. If each star resistance is equal to $\frac{1}{3}R$ Ω, determine the equivalent delta resistance.

Since resistances $R_1 = R_2 = R_3$ in the star, resistance R_{12} and R_{23} and R_{31} in the delta are equal, i.e. $R_{12} = R_{23} = R_{31} = R$ Ω.

(a) Delta resistor $\quad R_{12} = \dfrac{R_{12}(R_{23}+R_{31})}{R_{12}+R_{23}+R_{31}} = \dfrac{R \times 2R}{R+R+R} = \dfrac{2}{3}R\ \Omega$

(b) The delta-to-star equivalent is $\frac{2}{3}R\ \Omega$ — i.e. $\frac{2}{3}$ of the delta resistance.

This value can be used to check the star-to-delta result.

Problem 2 Three identical resistors are connected in delta to a 2.5 Ω, 5 Ω, 7.5 Ω, 3 phase supply. Determine the equivalent star resistance and the supply current.

For a delta connection $I_L = \sqrt{3}I_P$

Phase values are used $\quad I_P = \dfrac{V_P}{Z_P} = \dfrac{415}{10} = 41.5\ \text{A}$

Equivalent resistance per phase $\quad R_Y = \dfrac{1}{3}R_\Delta = \dfrac{10}{3}\ \Omega$ hence for a delta connection $R_Y = R_\Delta$

Hence $R_Y = \dfrac{10/3}{3} = 3.33\ \Omega$

$I_L = \dfrac{V_L}{Z_L}$ = line delta impedance $Z = \sqrt{\dfrac{1}{(10/3)^2} + \dfrac{1}{(10/3)^2} + \dfrac{1}{(10/3)^2}}$

$= 5.77\ \Omega$

Problem 7 Three coils each having resistance 5 Ω and inductive reactance 8 Ω are connected in star across a 3-phase, 415 V supply. Calculate the line current and power factor. Find also the power and reactive volt-amperes.

(a) For a star connection: $V_L = \sqrt{3}V_P$ and $I_L = I_P$,

(b) $V_P = 415/\sqrt{3} = 239.6$ V

Also voltage, $V_P = 239.6$ V

Phase voltage, $V_P = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 239.6$ V

(c) Impedance per phase, $Z = \sqrt{(R^2 + X_L^2)} = \sqrt{(5^2 + 8^2)}$

$= 9.43\ \Omega$

Phase current $I_P = \frac{V_P}{Z_P} = \frac{239.6}{9.43} = 25.4$ A

Line current $I_L = I_P = 25.4$ A

(d) For a delta connection: $V_L = V_P$ and $I_L = \sqrt{3}I_P$

(a) Line voltage, $V_L = 415$ V

Phase voltage, $V_P = V_L = 415$ V

(b) Phase current $I_P = \frac{V_P}{Z_P} = \frac{415}{9.43} = 44.0$ A

Line current $I_L = \sqrt{3}I_P = \sqrt{3}(44.0) = 76.2$ A

Further problems on star and delta connections may be found in Section 31.6 (problems 4 to 7) page 316

31.5 Power in three-phase systems

The power developed in a three-phase load is given by the sum of the power developed in each phase. If current, voltage and power factor for each phase is the same, the load is said to be balanced.

The power measured on one phase = $V_P I_P \cos\phi = V_L I_L \cos\phi$ (since values in the phase equal values in line), and I_L

(a) For a star connection, $I_L = \frac{V_P}{Z_P}$ and $I_L = I_P$ (from)

$\therefore P = V_L \frac{V_P}{Z_P} \cos\phi$ (since $V_P = V_L/\sqrt{3}$) gives

For a delta connection, $V_L = V_P$ and $I_P = \frac{V_P}{Z_P}$ (from)

$$P = VI, \quad \left[\frac{\text{J}}{\text{C}}\right]\left[\frac{\text{C}}{\text{s}}\right] = \left[\frac{\text{J}}{\text{s}}\right] = \text{W}$$

Joule's law states that the heat dissipated in a resistor is equal to the power times the time. Integrating over time gives the energy.

$$\boxed{E = \int_0^t P\,dt = \int_0^t I^2 R\,dt = I^2 R t \quad \text{(constant } I, R\text{)}}$$

Total voltage around a loop equals the sum of the voltage sources.

$$\boxed{V = \sum_{k=1}^{n} V_k = V_1 + V_2 + \cdots + V_n}$$

Kirchhoff's voltage law. The algebraic sum of potential differences around any closed loop in a circuit is zero. Equivalently, the total potential rise equals the total potential drop around the loop.

Three resistors R_1, R_2, R_3 in series act as a single resistor R_{eq}.

Each voltage $V_k = IR_k$ by Ohm's law, and Kirchhoff's voltage law gives $V = \sum V_k = I\sum R_k$. Dividing by I yields the equivalent resistance.

$$R_{eq} = \sum_{k=1}^{n} R_k = R_1 + R_2 + \cdots + R_n$$

For unequal resistors, R_{eq} is greater than the largest individual resistance.

For example, $R_1 = 4\ \Omega$, $R_2 = 6\ \Omega$, $R_3 = 10\ \Omega$ in series give $R_{eq} = 20\ \Omega$.

A current $I = 2\ \text{A}$ flows through the same series combination, so the total voltage is $V = IR_{eq} = 40\ \text{V}$.

Kirchhoff's current law. The algebraic sum of currents entering any junction in a circuit is zero. Equivalently, the total current entering a junction equals the total current leaving it.

Finally, $I = 40\ \text{V}$ for 5 s through $R_{eq} = 20\ \Omega$ dissipates $E = 4000\ \text{J}$.

Since $P = I^2 R$ is constant, $E = I^2 R t = (2\ \text{A})^2 (20\ \Omega)(5\ \text{s}) = 400\ \text{J}$.

Kirchhoff's rules. These conservation laws, known as Kirchhoff's current and voltage laws, let circuit analysis be reduced to a system of linear equations. Introduced by Gustav Kirchhoff in 1845, they remain the basis of all modern circuit simulation software.

(a) **Filter compartment**

Individual resistance: $R_1 = 0.12\ \Omega$ + $(\text{electrode gap} = 1.6\text{ cm})$

$= 0.12\ \Omega$

Water resistance: $R_2 = \frac{\rho_{\text{eff}} \times L_{\text{path}}}{A_{\text{cross}}} = \frac{0.0042 \times 1.6}{0.0038} = 1.768\ \Omega$

$= 1.77\ \Omega$

Total voltage: $V_T = 6.0 \times 3 = 18\ \text{V}$

Series voltage: $V_s = \frac{R_s}{R_s + R_2} \times 18 = \frac{0.12}{1.89} \times 18 = 1.14\ \text{V}$

Water voltage: $V_w = \frac{R_2}{R_s + R_2} \times 18 = \frac{1.77}{1.89} \times 18 = 16.86\ \text{V}$

Total current: $I_T = \frac{V_T}{R_s + R_2} = \frac{18}{1.89} = 9.524\ \text{A}$

Check: $I_s = I_w = 9.524\ \text{A}$

Power density: $P_{\text{density}} = \frac{P_w}{A_{\text{cross}}} = \frac{160}{0.0038} = 42{,}105\ \text{W/m}^2$

Power dissipated: $P = (\text{fill factor}) \times (\text{total area}) \times (\text{power density})$

$= 0.72 \times 0.38 \times 42{,}105$

$= 11{,}520\ \text{W}$

(alternatively, $P = 160 \times 72 = 11{,}520\ \text{W}$ from filled area)

(b) **Drain compartment**

$V_T = 6.0 \times 3 = 18.0\ \text{V}$ (same)

$(\text{same} = 9.524\ \text{A}$ (regulating flow rate above)

Water resistance: $R_w = \frac{V_w}{I_T} = \frac{16.86}{9.524} = 1.770\ \Omega$

Check: $R_s = \frac{V_s}{I_T} = \frac{1.14}{9.524} = 0.120\ \Omega$ ✓

Power dissipated: $P = (\text{fill factor}) \times (\text{total area}) \times (\text{power density})$

$= 0.72 \times 0.38 \times 42{,}105$

$= 11{,}520\ \text{W}$

(alternatively, $P = V_T \times I_T = 18.0 \times 640 = 11{,}520\ \text{W}$)

Notice that the power dissipated in the drain compartment (the one with the water) is exactly the same as in the filter compartment. This is because the same current flows through both compartments in series, and the power dissipated in each is $P = I^2 R$ or $P = V \times I$. The difference is that the drain compartment has a much larger cross-sectional area, so the power density (watts per square metre) is much lower. This is why the drain compartment can be made from a simple plastic container rather than a specialised electrode assembly.

> **Key Point:** In a series circuit, the same current flows through every component. The voltage divides between components in proportion to their resistances. The component with the largest resistance gets the largest share of the voltage. In the drain compartment, the water has a much larger cross-sectional area than the electrodes, so its resistance is small and it receives only a small fraction of the total voltage.

(c) **Efficiency** = $\frac{\text{useful output}}{\text{power input}} \times 100\%$ where $\frac{\text{useful output}}{\text{power input}} = \frac{\text{power input} - \text{losses}}{\text{power input}}$

Total losses: **power input** = $\frac{(0.72 \times 0.38) \times 0.85}{0.85} \times 100\% = 72.96\%$ (or 0.73 kW)

(a) Power factor of the load:

$$\text{Power factor} = \cos\theta = \frac{P}{|S|} = \frac{P}{\sqrt{P^2 + Q^2}} = \frac{\text{[value]}}{\text{[value]}} = \text{[value]}$$

(b) Is the load inductive, capacitive, or resistive?

$$\text{Since } Q \text{ is [positive/negative/zero]}, \text{ the load is [inductive/capacitive/resistive].}$$

(c) Reactive power Q required to correct the power factor to unity:

$$Q = P\tan\theta = P\tan(\cos^{-1}(\text{[value]})) = \text{[value]}\ \text{VAR}$$

11.6 The synchronous motor: principle of operation, starting methods, and applications

Three-phase synchronous motors are the workhorses of large industrial drives. Unlike induction motors, their rotor speed is locked exactly to the supply frequency, and they can be operated at leading power factor to actively correct the power factor of the entire plant. This section develops the principle of operation, the starting problem, and the practical applications.

(a) **Draw a phasor diagram for a synchronous motor operating at leading power factor.**

Construct the phasor diagram for the motor operating from a three-phase supply. Show the terminal voltage $\mathbf{V}_t$, the internal generated voltage $\mathbf{E}_f$, the synchronous reactance drop $\mathbf{I}_a(jX_s)$, and the armature current $\mathbf{I}_a$. Label the angle between $\mathbf{V}_t$ and $\mathbf{E}_f$ as the load angle δ.



Figure 11.12: Phasor diagram of a synchronous motor operating at leading power factor.

(b) **Derive an expression for the electromagnetic power developed by the motor.**

Neglecting armature resistance, the electromagnetic power developed per phase is

$$P_{em} = \frac{E_f V_t}{X_s}\sin\delta$$

where E_f is the magnitude of the internal generated voltage, V_t is the terminal voltage per phase, X_s is the synchronous reactance per phase, and δ is the load angle. The total three-phase power is $3P_{em}$.

(c) **Explain why the synchronous motor is not self-starting and describe two common methods for bringing it up to speed.**

A synchronous motor develops zero average torque at standstill because the rotor poles alternately attract and repel the stator field, producing a net torque of zero over each electrical cycle. Two common starting methods are:

1. **Damper (amortisseur) windings.** Copper bars embedded in the pole faces act as a squirrel-cage induction motor during start-up. The motor accelerates to near synchronous speed, at which point the field is applied and the motor pulls into synchronism.
2. **Pony motor.** A small auxiliary motor brings the rotor to synchronous speed, the field is applied, and the pony motor is then disconnected.

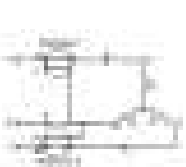


Figure 11.13: Damper windings in the pole faces of a salient-pole synchronous motor.

Synchronous motors are also used purely for power factor correction, running at no mechanical load and drawing only reactive current. An overexcited synchronous motor is sometimes called a *synchronous condenser*.



Figure 10.16



Figure 10.17



Figure 10.18

(a) A composition diagram for the Fibonacci sequence is given in Figure 10.16. Here the initial vertices are labelled x_1 and x_2, and the vertex labelled x_n has in-degree F_n. Since $F_n = F_{n-1} + F_{n-2}$, the vertex x_n has in-degree F_{n-1} plus in-degree F_{n-2}. Therefore $x_n = x_{n-1} + x_{n-2}$.

Thus if $x_1 = x_2 = 1$ and $x_n = x_{n-1} + x_{n-2}$ for $n \geq 3$, then

$$x_3 = x_2 + x_1 = 2,$$
$$x_4 = x_3 + x_2 = 3,$$
$$x_5 = x_4 + x_3 = 5,$$

and so on. Thus $x_n = F_n$ for all n. Figure 10.17 shows the Fibonacci sequence as a composition diagram.

Figure 10.16 illustrates the identity

$$F_1 + F_2 + F_3 + \cdots + F_{n-1} + F_n = F_{n+2} - 1.$$

The starting vertices of the subdigraphs are shaded, as before. The numbers above the vertices of the subdigraphs are the sums of the numbers below them. Thus the sum of the first n Fibonacci numbers is one less than the $(n+2)$nd Fibonacci number.

(b) The above diagram for the Fibonacci numbers can be used to obtain a Fibonacci triangle. Starting with the vertices x_1 and x_2, we have $x_3 = x_2 + x_1$ and $x_4 = x_3 + x_2 = x_1 + 2x_2$. Similarly, $x_5 = 2x_1 + 3x_2$.

(c) Multiplying $x_1 = \sin\theta$, $x_2 = \sin 2\theta$ by $\cos\theta$, we obtain $x_3 = \sin 3\theta$. Multiplying $x_2 = \sin 2\theta$, $x_3 = \sin 3\theta$ by $\cos\theta$, we obtain $x_4 = \sin 4\theta$. Thus

$$\begin{aligned} x_3 &= \sin 3\theta = \sin\theta\cos 2\theta + \cos\theta\sin 2\theta = x_1 + x_2, \\ x_4 &= \sin 4\theta = \sin 2\theta\cos 2\theta + \cos 2\theta\sin 2\theta = x_2 + x_3. \end{aligned}$$

Hence $x_n = \sin n\theta$ for all n by the same argument as in Figure 10.16 and Figure 10.17. Figure 10.18 shows the $\sin n\theta$ sequence as a composition diagram.

Figure 10.18 illustrates the identity

$$\sin\theta + \sin 2\theta + \sin 3\theta + \cdots + \sin(n-1)\theta + \sin n\theta = \sin\tfrac{1}{2}n\theta \sin\tfrac{1}{2}(n+1)\theta \operatorname{cosec}\tfrac{1}{2}\theta.$$

The starting vertices of the subdigraphs are shaded, as before. The numbers above the vertices of the subdigraphs are the sums of the numbers below them. Thus the sum of the first n terms of the sequence is one less than the $(n+2)$nd term.

Hence, using the sum formulae, we obtain Werner's Trigonometrical Identities:

Dividing throughout by $\cos\alpha\cos\beta$ we get

$$\frac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta} = \frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\cos\alpha\cos\beta} = \frac{\sin\alpha}{\cos\alpha} + \frac{\sin\beta}{\cos\beta} = \tan\alpha + \tan\beta.$$

Cross-multiplying gives $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$.

Hence

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}.$$

From which we have

$$\sin\left(\frac{\alpha+\beta}{2}\right) = \frac{1}{2}\left(\sin\alpha + \sin\beta\right).$$

A more typical example is the use of Fibonacci numbers in Chebyshev



Figure 10.17

Worked Example 10.1 A light simply supported beam AB of span $6\ \text{m}$ carries a uniformly distributed load of $4\ \text{kN/m}$ run and a concentrated load of $10\ \text{kN}$ acting vertically downwards at a point C which is $2\ \text{m}$ from A. Calculate the reactions at the supports A and B and the bending moment at C. Hence draw the shear force and bending moment diagrams for the beam.

A sketch diagram of the situation is shown in Figure 10.18.

(a) Resolving vertically: $R_A + R_B = 4 \times 6 + 10 = 34\ \text{kN}$

Taking moments about A: $R_B \times 6 = (4 \times 6 \times 3) + (10 \times 2)$

$$R_B = \frac{72 + 20}{6} = 15.33\ \text{kN}$$

Then $R_A = 34 - 15.33 = 18.67\ \text{kN}$

(b) Bending moment at C: $M_C = R_A \times 2 - (4 \times 2 \times 1)$

$$M_C = 37.34 - 8 = 29.34\ \text{kN m}$$

(c) Shear force at A: $F_A = +18.67\ \text{kN}$

Shear force at C (just left): $F_C = 18.67 - 8 = +10.67\ \text{kN}$

Shear force at C (just right): $F_C = 10.67 - 10 = +0.67\ \text{kN}$

Shear force at B: $F_B = -15.33\ \text{kN}$

Maximum bending moment occurs where the shear force is zero, i.e. at a distance x from A where $18.67 - 4x = 0$, so $x = 4.67\ \text{m}$.

$$M_{\max} = 18.67 \times 4.67 - (4 \times 4.67 \times 2.33) = 43.54\ \text{kN m}$$

Worked Example 10.2 A simply supported beam AB of span $8\ \text{m}$ carries a uniformly distributed load of $3\ \text{kN/m}$ run over its entire length and a concentrated load of $12\ \text{kN}$ acting vertically downwards at a point C which is $3\ \text{m}$ from B. Calculate the reactions at the supports A and B. Hence draw the shear force and bending moment diagrams for the beam.

(a) Taking moments about A: $R_B \times 8 = (3 \times 8 \times 4) + (12 \times 5)$

$$R_B = \frac{96 + 60}{8} = 19.5\ \text{kN}$$

Resolving vertically: $R_A + R_B = 3 \times 8 + 12$

$$R_A = 36 - 19.5 = 16.5\ \text{kN}$$

(b) Bending moment at C: $M_C = R_A \times 5 - (3 \times 5 \times 2.5)$

$$M_C = 82.5 - 37.5 = 45\ \text{kN m}$$

(c) Shear force at A: $F_A = +16.5\ \text{kN}$

Shear force at C (just left): $F_C = 16.5 - 15 = +1.5\ \text{kN}$

Shear force at C (just right): $F_C = 1.5 - 12 = -10.5\ \text{kN}$

Shear force at B: $F_B = -19.5\ \text{kN}$

Maximum bending moment occurs where the shear force is zero, i.e. at a distance x from A where $16.5 - 3x = 0$, so $x = 5.5\ \text{m}$.

Given: $\quad I_z = \dfrac{\pi r^4}{4} = 0.0491\ \text{ft}^4$

(b) Calculate $I_{z'}$ for the shifted centroidal axis.

Given: $\quad I_{z'} = 0.0491\ \text{ft}^4 - 1.5\ \text{ft}^4$

(c) Find power developed, $P = \gamma Q H_p$, in slug/s

$= \gamma$ (specific weight of water) $= 0.0491\ \text{ft}^4$

(d) Find $\sum M_z$ if $\sum M_{z'} = \gamma Q H_p$ (conditions are $= 0.0491\ \text{ft}^4$)

The above diagram for this load is shown in Figure 12.2.

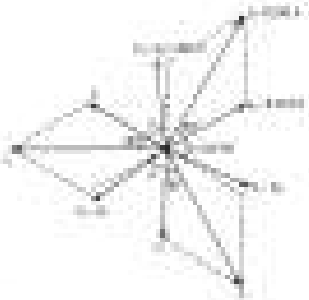


Figure 12.2

Problem 12.1 Two pulleys are connected to transmit the power given to a balanced 3-phase load. Find the two equivalent loadings if the tangential loadings are 0.2 kN and 0.4 kN. Determine the resultant power input and $\sum M_z$ for the system loads.

(a) Find input torque if $\sum M_z = M_1 + M_2 = 1.5\ \text{kN}$

(b) Input $= \gamma Q \left[\dfrac{P_{in}}{\gamma Q} - \dfrac{P_{out}}{\gamma Q} \right] = \gamma Q \left[\dfrac{P_{in}}{\gamma Q} - \dfrac{P_{out}}{\gamma Q} \right]$

$= \gamma Q \left[\dfrac{P_{in}}{\gamma Q} \right] = \gamma Q \left[\dfrac{P_{in}}{\gamma Q} \right] = \dfrac{P_{in}}{\gamma Q}$

Charge in $t = 2$ seconds: $\frac{1}{e^2} \approx 0.135$ C

Current flowing at $t = 2$ seconds: $-\frac{1}{e^2} \approx -0.135$ A

Problem 12. Two resistances, measured by a student, are found to be $R_1 = 100\ \Omega \pm 1\ \Omega$ and $R_2 = 200\ \Omega \pm 2\ \Omega$. The student wishes to find the maximum error in the calculated value of $R_1 + R_2$.

Solution. The absolute errors in R_1 and R_2 are respectively

$$\delta R_1 = 1\ \Omega, \qquad \delta R_2 = 2\ \Omega. \tag{12}$$

Hence the maximum error in the sum $R_1 + R_2$ is

$$\delta(R_1 + R_2) = \delta R_1 + \delta R_2 = 1 + 2 = 3\ \Omega.$$

The percentage error in R_1 is $\frac{1}{100} \times 100\% = 1\%$, and the percentage error in R_2 is $\frac{2}{200} \times 100\% = 1\%$.

Because $\delta(R_1 + R_2) = \frac{\delta R_1}{R_1 + R_2} R_1 + \frac{\delta R_2}{R_1 + R_2} R_2$, the error in the sum is $\frac{1+2}{300} \times 100\% = 1\%$.

Thus the maximum error in the sum is 1% of the total resistance, i.e.

$$\delta(R_1 + R_2) = 0.01 \times 300\ \Omega = 3\ \Omega, \tag{13}$$

confirming agreement in each of the two methods. $\square$

Alternatively, the relative error in the sum may be written as

$$\frac{\delta R_1 + \delta R_2}{R_1 + R_2} = \frac{3}{300} = 0.01 = 1\%.$$

Hence the absolute error is $\pm 3\ \Omega$ and the percentage error is 1%.

Problem 13. Two resistances, of $4.00\ \Omega$ and $5.00\ \Omega$ respectively, are to be connected in parallel. The input current is $3\ \mathrm{A}$. Calculate the current flowing through each resistor and the total power dissipated. Then repeat the calculation when the resistances are connected in series.

Using the formula for the combined resistance of two resistors in parallel, $R = \frac{R_1 R_2}{R_1 + R_2}$, we find the combined resistance is $\frac{4 \times 5}{4 + 5} = \frac{20}{9}\ \Omega \approx 2.22\ \Omega$.

(a) Total power dissipated: $P = \frac{V^2}{R} = \frac{9}{20/9} = \frac{81}{20} = 4.05\ \mathrm{W}$.

(b) Ratio $\frac{I_1}{I_2} = \frac{R_2}{R_1} = \frac{5}{4}$, so $I_1 = \frac{5}{9} \times 3 = \frac{15}{9} = \frac{5}{3}\ \mathrm{A}$.

$$I_2 = \frac{4}{9} \times 3 = \frac{12}{9} = \frac{4}{3}\ \mathrm{A}.$$

Check: $I_1 + I_2 = \frac{5}{3} + \frac{4}{3} = 3\ \mathrm{A}$ $\checkmark$

Power dissipated in $4.00\ \Omega$ resistor: $I_1^2 R_1 = \left(\frac{5}{3}\right)^2 \times 4 = \frac{100}{9} \approx 11.11\ \mathrm{W}$.

Power dissipated in $5.00\ \Omega$ resistor: $I_2^2 R_2 = \left(\frac{4}{3}\right)^2 \times 5 = \frac{80}{9} \approx 8.89\ \mathrm{W}$.

Total power: $\frac{100}{9} + \frac{80}{9} = \frac{180}{9} = 20\ \mathrm{W}$.

Alternatively, $P = \frac{V^2}{R} = \frac{9}{20/9} = 4.05\ \mathrm{W}$ $\checkmark$

Problem 14. Two identical resistors each of $4.0\ \Omega$ and $2.0\ \mathrm{H}$ inductance are connected in series with a battery of $12\ \mathrm{V}$ and negligible internal resistance. Find the time constant of the circuit, the steady-state current, and the current $0.5\ \mathrm{s}$ after the switch is closed.

The total resistance is $R = 4.0 + 4.0 = 8.0\ \Omega$ and the total inductance is $L = 2.0 + 2.0 = 4.0\ \mathrm{H}$.

The time constant is $\tau = \frac{L}{R} = \frac{4.0}{8.0} = 0.5\ \mathrm{s}$.

The steady-state current is $I_\infty = \frac{V}{R} = \frac{12}{8.0} = 1.5\ \mathrm{A}$.

The current at time t is $I(t) = I_\infty \left(1 - e^{-t/\tau}\right)$.

At $t = 0.5\ \mathrm{s}$: $I(0.5) = 1.5\left(1 - e^{-1}\right) \approx 1.5 \times 0.632 \approx 0.948\ \mathrm{A}$.

Problem 10. Three coils are in the auxiliary leg of a transformer. A 50 kV, 60 Hz source energizes coil 1 (N1 = 1200 turns) and induces 2.4 kV in coil 2 (N2 = 60 turns) and 0.12 kV in coil 3 (N3 = 3 turns). Calculate the load connected to coil 2 if coil 3 delivers 6 A. Neglect leakage and losses.

Solution.

Turns ratio: $a_{12} = \frac{N_1}{N_2} = \frac{1200}{60} = 20$, $a_{13} = \frac{N_1}{N_3} = \frac{1200}{3} = 400$.

Check: $\frac{V_1}{V_2} = \frac{50}{2.4} = 20.83$ and $\frac{V_1}{V_3} = \frac{50}{0.12} = 416.7$. The small differences are due to rounding the source to 50 kV; with the exact 50 kV they vanish.

Coil 3 current: $I_3 = 6\ \text{A}$ (given).

Coil 3 power: $P_3 = V_3 I_3 = 0.12 \times 6 = 0.72\ \text{kW}$.

Coil 3 impedance: $Z_3 = \frac{V_3}{I_3} = \frac{0.12}{6} = 0.02\ \Omega$, $R_3 = 0.02\ \Omega$.

Reflected to coil 1: $R_3' = a_{13}^2 R_3 = 400^2 \times 0.02 = 3200\ \Omega$.

Coil 2 current: $I_2 = \frac{V_2}{R_2} = \frac{2.4}{R_2}$.

Coil 2 power: $P_2 = V_2 I_2 = \frac{2.4^2}{R_2} = \frac{5.76}{R_2}$.

Reflected to coil 1: $R_2' = a_{12}^2 R_2 = 400 R_2$.

Total power: $P = V_1 I_1 = 0.72 + \frac{5.76}{R_2}$ kW.

Total reflected resistance: $R_1 = R_2' + R_3' = 400 R_2 + 3200$.

Current: $I_1 = \frac{V_1}{R_1} = \frac{50}{400 R_2 + 3200}$.

Power: $P = I_1^2 R_1 = \frac{2500}{400 R_2 + 3200} = \frac{6.25}{R_2 + 8}$ kW.

Equate the two expressions for P:

$$0.72 + \frac{5.76}{R_2} = \frac{6.25}{R_2 + 8}$$

Multiply by $R_2(R_2 + 8)$:

$$0.72 R_2 (R_2 + 8) + 5.76 (R_2 + 8) = 6.25 R_2$$

$$0.72 R_2^2 + 5.76 R_2 + 46.08 = 6.25 R_2$$

$$0.72 R_2^2 - 0.49 R_2 + 46.08 = 0$$

Discriminant: $\Delta = 0.49^2 - 4 \times 0.72 \times 46.08 = 0.2401 - 132.71 = -132.47 < 0$.

Since $\Delta < 0$ (and $R_2 > 0$), there is no real solution. The load connected to coil 2 cannot absorb the power delivered by coil 3 at the given voltages.

What the code can do: The code solves the quadratic $0.72 R_2^2 - 0.49 R_2 + 46.08 = 0$ and reports that no real solution exists.

Problem 11. A three-phase, 60 Hz, 4-pole induction motor is rated 15 kW, 460 V, 22 A, 1746 rpm. Its equivalent circuit parameters are: stator resistance $R_1 = 0.641\ \Omega$, stator leakage reactance $X_1 = 1.106\ \Omega$, rotor resistance referred to the stator $R_2' = 0.332\ \Omega$, rotor leakage reactance referred to the stator $X_2' = 0.464\ \Omega$, magnetizing reactance $X_m = 26.3\ \Omega$. Friction and windage losses are 180 W, and core losses are 210 W. Calculate the slip, the rotor speed, the developed power, the shaft power, the efficiency, and the starting torque.

Solution.

Synchronous speed: $n_s = \frac{120 f}{p} = \frac{120 \times 60}{4} = 1800\ \text{rpm}$.

Slip: $s = \frac{n_s - n}{n_s} = \frac{1800 - 1746}{1800} = 0.030$.

Rotor speed: $n = (1 - s) n_s = 0.970 \times 1800 = 1746\ \text{rpm}$.

Per-phase voltage: $V_\phi = \frac{460}{\sqrt{3}} = 265.6\ \text{V}$.

Input impedance: $Z_{in} = R_1 + jX_1 + \frac{jX_m (R_2'/s + jX_2')}{R_2'/s + j(X_m + X_2')}$.

$\frac{R_2'}{s} = \frac{0.332}{0.030} = 11.07\ \Omega$.

$Z_{in} = 0.641 + j1.106 + \frac{j26.3 (11.07 + j0.464)}{11.07 + j26.764}$.

$Z_{in} = 0.641 + j1.106 + \frac{-12.21 + j296.5}{11.07 + j26.764}$.

$Z_{in} = 0.641 + j1.106 + 10.21 + j2.46 = 10.85 + j3.57\ \Omega$.

$|Z_{in}| = \sqrt{10.85^2 + 3.57^2} = \sqrt{117.7 + 12.74} = 11.42\ \Omega$.

Stator current: $I_1 = \frac{V_\phi}{|Z_{in}|} = \frac{265.6}{11.42} = 23.26\ \text{A}$.

Power factor: $\cos\theta = \frac{10.85}{11.42} = 0.950$ lagging.

Input power: $P_{in} = 3 V_\phi I_1 \cos\theta = 3 \times 265.6 \times 23.26 \times 0.950 = 17.63\ \text{kW}$.

Stator copper loss: $P_{cu1} = 3 I_1^2 R_1 = 3 \times 23.26^2 \times 0.641 = 1039\ \text{W}$.

Core loss: $P_{core} = 210\ \text{W}$.

Air-gap power: $P_{ag} = P_{in} - P_{cu1} - P_{core} = 17630 - 1039 - 210 = 16381\ \text{W}$.

Developed power: $P_{dev} = (1 - s) P_{ag} = 0.970 \times 16381 = 15890\ \text{W}$.

Shaft power: $P_{sh} = P_{dev} - P_{fw} = 15890 - 180 = 15710\ \text{W}$.

Efficiency: $\eta = \frac{P_{sh}}{P_{in}} = \frac{15710}{17630} = 0.891$ (89.1%).

Starting torque ($s = 1$): $R_{th} = 10.85\ \Omega$, $X_{th} = 3.57\ \Omega$.

$T_{start} = \frac{3 V_\phi^2 R_2'}{\omega_s [(R_{th} + R_2')^2 + (X_{th} + X_2')^2]}$.

$\omega_s = \frac{2\pi \times 1800}{60} = 188.5\ \text{rad/s}$.

$T_{start} = \frac{3 \times 265.6^2 \times 0.332}{188.5 [(10.85 + 0.332)^2 + (3.57 + 0.464)^2]}$.

$T_{start} = \frac{70.3 \times 10^3}{188.5 [134.8 + 16.2]} = \frac{70.3 \times 10^3}{188.5 \times 151.0} = 2.47\ \text{N·m}$.

Solving inequalities in one variable graphically

Let $f : A \to \mathbb{R}$ and $g : A \to \mathbb{R}$ be functions with a common domain $A \subseteq \mathbb{R}$.

Consider the inequality $f(x) < g(x)$ for $x \in A$.

When the graph of $y = f(x)$ is entirely below the graph of $y = g(x)$ on A, the solution set of $f(x) < g(x)$ is A itself.

Further problems on proving the inequality remain open for almost a century after Gödel's paradox. [1, p. 12]

11.1.1 Logical laws as algebraic identities

(1) Each statement is fully classified: either true, or false, but not both.

(2) For the given system, the above statement must be the case if it holds such that statement's negation is false, and vice versa.

(3) Let $\mathcal{A}$ be a set of axioms, and let $\mathcal{M}$ be a model of $\mathcal{A}$. Then $\mathcal{M}$ satisfies every theorem provable from $\mathcal{A}$.

(4) Let $\mathcal{A}$ be a set of axioms, and let $\mathcal{M}$ be a model of $\mathcal{A}$. Then $\mathcal{M}$ satisfies every theorem provable from $\mathcal{A}$.

(5) Let $\mathcal{A}$ be a set of axioms, and let $\mathcal{M}$ be a model of $\mathcal{A}$. Then $\mathcal{M}$ satisfies every theorem provable from $\mathcal{A}$.

11.1.2 Algebraic structures in logic

Let $\mathcal{A}$ be a set of axioms, and let $\mathcal{M}$ be a model of $\mathcal{A}$. Then $\mathcal{M}$ satisfies every theorem provable from $\mathcal{A}$.

(1) For a finite universe of n elements, there are 2^n subsets, 2^{2^n} functions, and 2^{n^2} binary relations. Each of these counts can be computed directly, and the results grow quickly enough that exhaustive search is impractical for even modest n.

(2) Let $\mathcal{A}$ be a set of axioms, and let $\mathcal{M}$ be a model of $\mathcal{A}$. Then $\mathcal{M}$ satisfies every theorem provable from $\mathcal{A}$.

(3) Let $\mathcal{A}$ be a set of axioms, and let $\mathcal{M}$ be a model of $\mathcal{A}$. Then $\mathcal{M}$ satisfies every theorem provable from $\mathcal{A}$.

11.1.3 Boolean algebras as lattices

Let $\mathcal{A}$ be a set of axioms, and let $\mathcal{M}$ be a model of $\mathcal{A}$. Then $\mathcal{M}$ satisfies every theorem provable from $\mathcal{A}$.

1. Every finite set of complements $\{C_i\}$ can be represented as one or a few C_i's. Unless clearly redundant, further proper complexity for this proof is stated and C_i has two values.

$$\{C_i\} \cup \{C_i \wedge C_j\} \cup \{C_i \wedge C_j \wedge C_k\}$$

1. A 330 Ω resistor is connected to a 9.0 V battery. Calculate the current through the resistor and the power it dissipates.

2. A heater element rated at 1.5 kW operates from a 230 V supply. Calculate its operating resistance and the energy it converts to heat in 2.0 hours. Express the energy in both joules and kilowatt-hours.

3. A fuse is rated at 13 A and is placed in a circuit that draws 2.4 kW from a 230 V supply. Will the fuse blow? Calculate the current drawn and the power dissipated in a 0.10 Ω resistance that represents the fuse's internal resistance.

4. Three resistors, $R_1 = 4.0\ \Omega$, $R_2 = 6.0\ \Omega$ and $R_3 = 12\ \Omega$, are connected in series to a 24 V battery. Calculate the total resistance, the current through each resistor, and the power dissipated in R_2.

5. Two resistors, $R_1 = 6.0\ \Omega$ and $R_2 = 3.0\ \Omega$, are connected in parallel across a 12 V supply. Calculate the current through each resistor, the total current drawn from the supply, and the total power dissipated. Verify that the power equals V^2/R_{eq}.

6. A 9.0 V battery has an internal resistance of $0.50\ \Omega$. It is connected to a load resistor of $4.0\ \Omega$. Calculate the current in the circuit, the terminal voltage of the battery, and the power dissipated in the load. Repeat the calculation for a load resistor of $0.50\ \Omega$ and comment briefly on the result.

7. A potential divider is formed from two resistors, $R_1 = 10\ \text{k}\Omega$ and $R_2 = 5.0\ \text{k}\Omega$, connected in series across a 15 V supply. Calculate the output voltage V_{out} measured across R_2. If a load resistor of $20\ \text{k}\Omega$ is connected across R_2, calculate the new output voltage.

8. A 12 V battery is connected to a network of resistors. $R_1 = 6.0\ \Omega$ and $R_2 = 3.0\ \Omega$ are in parallel with each other, and this combination is in series with $R_3 = 4.0\ \Omega$. Calculate the total resistance of the network, the current drawn from the battery, and the power dissipated in R_3.

Part B: Higher-tariff problems

9. Determine the total power dissipated in the circuit of question 8 and verify that it equals the power supplied by the battery.

10. Determine the power dissipated in the resistor in question 1.

11. An industrial boiler is connected to a three-phase 415 V supply. Each phase has a resistance of $8.3\ \Omega$. Calculate the total power drawn from the supply when the heater is operating at full load. Express your answer in kilowatts.

12. A 240 V electric kettle contains $1.5\ \text{kg}$ of water at $18\ ^\circ\text{C}$. The kettle has a power rating of 2.2 kW and is switched on until the water reaches $100\ ^\circ\text{C}$. Calculate the energy transferred to the water. Assuming that all of this energy comes from the kettle, calculate the time for which the kettle was switched on. Specific heat capacity of water $= 4200\ \text{J kg}^{-1}\ ^\circ\text{C}^{-1}$.

13. A 12 V battery has a terminal voltage of 11.4 V when delivering a current of 4.0 A. Calculate the internal resistance of the battery. If the battery is connected to a load resistor of $2.0\ \Omega$, calculate the new terminal voltage and the current drawn from the battery.

14. A 12 V battery has an internal resistance of $0.50\ \Omega$. It is connected to a load resistor of $4.0\ \Omega$. Calculate the current in the circuit, the terminal voltage of the battery, and the power dissipated in the load. Repeat the calculation for a load resistor of $0.50\ \Omega$ and comment briefly on the result.

15. A 9.0 V battery has an internal resistance of $0.50\ \Omega$. It is connected to a load resistor of $4.0\ \Omega$. Calculate the current in the circuit, the terminal voltage of the battery, and the power dissipated in the load. Repeat the calculation for a load resistor of $0.50\ \Omega$ and comment briefly on the result.

1.1 In 2011, a Mexican-born doctor was arrested after a private capital of $ 1,200,000 and members of a private doctor of that hospital, and made an addressing of which. In the course of which a confirmed Democratic Institute [illegible] [illegible] [illegible]

28 Transformers

By the end of this chapter you should be able to:

- [illegible]
- [illegible]
- use $\frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{I_2}{I_1}$ as a calculation on transformers
- [illegible]
- [illegible]
- [illegible]
- [illegible]
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28.1 **Introduction** A transformer is a device which uses the phenomenon of electromagnetic induction to change the voltage (or current) of alternating current (ac) electricity. It consists of two coils of wire wound around a common core. Transformers are used extensively in the mains electricity supply system to change the voltage from the generating station to the value required by the consumer.

Losses in transformers are generally very small and their efficiency is high. Many power units have a large 50Hz mains transformer.

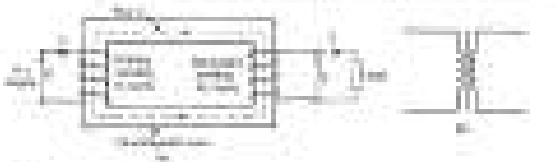


Figure 10-1

10-2 Transformers: principle of operation

When two conductors of an open-circuited pair of conductors surround a flux Φ, a voltage is induced in each according to Faraday's induction law. If one of the two conductors is connected to a source and the other to a load, then power will be transferred from the source to the load. This principle is the basis of the transformer, which is a device that is used extensively in electric power systems and in electronic circuits. In power systems, transformers are used to change voltage and current levels in order to transmit power efficiently over long distances. In electronic circuits, transformers are used to change voltage and current levels, to provide isolation between circuits, and to match the impedance of a source to the impedance of a load for maximum power transfer.

The simplest transformer, called an **ideal transformer**, consists of two coils of wire wound on a common core. The coil connected to the source is called the **primary winding**, and the coil connected to the load is called the **secondary winding**. The core is made of a ferromagnetic material, such as iron, and is designed to confine almost all the magnetic flux to the path shown by the dashed line in Fig. 10-2. This flux links both windings and induces voltages in them according to Faraday's law. If the resistance of the windings is negligible, then the voltage induced in each winding is equal to the voltage applied to or delivered by that winding. The ratio of the voltages is equal to the ratio of the number of turns in the windings.

Figure 10-2

Figure 10-2 shows an ideal transformer with N_1 turns in its primary winding and N_2 turns in its secondary winding. The primary winding is connected to a source that produces a time-varying voltage $v_1(t)$, and the secondary winding is connected to a load. The time-varying current $i_1(t)$ in the primary winding produces a time-varying magnetic flux Φ in the core. This flux links both windings and induces voltages in them according to Faraday's law. If the resistance of the windings is negligible, then the voltage induced in each winding is equal to the voltage applied to or delivered by that winding. The ratio of the voltages is equal to the ratio of the number of turns in the windings.

According to Faraday's law, $v_1 = N_1 \, d\Phi/dt$ and $v_2 = N_2 \, d\Phi/dt$. Hence

$$\frac{v_1}{v_2} = \frac{N_1}{N_2} \quad \text{or} \quad \frac{v_1}{N_1} = \frac{v_2}{N_2} \tag{10-1}$$

That is, the ratio of the voltages v_1 and v_2 is equal to the ratio of the numbers of turns N_1 and N_2. The quantity N_1/N_2 is called the **turns ratio** of the transformer. If N_1 is greater than N_2, then v_1 is greater than v_2, and the device is called a **step-down transformer**. If N_2 is greater than N_1, then v_2 is greater than v_1, and the device is called a **step-up transformer**.

Figure 10-2 also shows the polarities of the voltages v_1 and v_2. The dots near the ends of the windings indicate the polarities of the voltages relative to each other. If v_1 is positive at the dot end of the primary winding, then v_2 is also positive at the dot end of the secondary winding. Similarly, if v_1 is positive at the dot end of the primary winding, then v_2 is negative at the dot end of the secondary winding.

Ideal transformer: power in equals power out, so $V_1 I_1 = V_2 I_2$ (or $V_1 I_1 = V_2 I_2 \cos\theta$ for an AC transformer with **primary and secondary voltage/current not in phase**).

Then $\frac{V_1}{V_2} = \frac{N_1}{N_2}$ (EC12)

Combining equations (EC1) and (EC2) gives:

$$\boxed{\frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{I_2}{I_1}}$$ (EC13)

The **ratio** of a transformer is stated in terms of the voltage/power that is transformed between primary and secondary. Most transformers are **step-down** (i.e. the transformer voltage is within $0.25 \le V_2/V_1 \le 0.5$), where V_2 is less than primary voltage.

> **Worked Ex 1** — A transformer has 500 primary turns and 1500 secondary turns. If the primary voltage is 230 V, determine the secondary voltage, assuming an ideal transformer.

For an ideal transformer, voltage ratio = turns ratio, i.e.

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} \text{ hence } \frac{230}{V_2} = \frac{500}{1500}$$

Then secondary voltage $V_2 = \frac{230 \times 1500}{500} =$ **690 V** (to 3 SF)

> **Problem 1** — An ideal transformer with a step-up ratio of 1:2.5 is fed from a 6 V supply. Determine its output voltage.

A transformer of 2.5 kVA rating has a primary and secondary turns ratio of 1:20. If the primary current is 10 A, calculate the secondary current.

$$\frac{N_1}{N_2} = \frac{1}{20}$$

For an ideal transformer, $\frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{I_2}{I_1}$ (current) $= \frac{1}{20}$

Then the secondary voltage $V_2 = \frac{230 \times 20}{1} =$ **4600 V** (to 3 SF). Likewise the

> **Problem 2** — An ideal transformer has a turns ratio of 5:1, and the primary current is 2 A.
> Calculate the secondary current.

A silver wire ($L = 2.0$ m, cross-section $A = 1.0 \times 10^{-6}$ m²) carries a steady current. Ohmmeter readings:

$$R_{20} = 0.0317\ \Omega \quad (20\ ^\circ\text{C}), \qquad R_{100} = 0.0418\ \Omega \quad (100\ ^\circ\text{C}).$$

Find the temperature coefficient α and the resistivity ρ at 20 °C.

$$\rho = \frac{R_{20} A}{L} = \frac{0.0317 \times 1.0 \times 10^{-6}}{2.0} = 1.59 \times 10^{-8}\ \Omega\cdot\text{m}$$

$$R_{100} = R_{20}\left[1 + \alpha (100 - 20)\right] \Rightarrow \alpha = \frac{R_{100}/R_{20} - 1}{80} = \frac{0.0418/0.0317 - 1}{80} = 4.16 \times 10^{-3}\ \text{K}^{-1}$$

Within rounding, $\alpha \approx 4.2 \times 10^{-3}\ \text{K}^{-1}$, the accepted value for silver.

A 12.0 V battery of negligible internal resistance drives a current $I = 2.40$ A through a resistor R and a coil of copper wire whose resistance must not exceed $0.50\ \Omega$ at 20 °C. The coil is to be used in a room where the temperature may reach 45 °C. What is the largest value of R that can be used?

$$R_{\text{coil,max}} = \frac{V}{I} - R = \frac{12.0}{2.40} - R = 5.00\ \Omega - R$$

The coil must not exceed $0.50\ \Omega$ at 20 °C, so $R_{\text{coil,max}}(20\ ^\circ\text{C}) = 0.50\ \Omega$. Copper has $\alpha = 3.9 \times 10^{-3}\ \text{K}^{-1}$. At 45 °C,

$$R_{\text{coil}}(45\ ^\circ\text{C}) = R_{\text{coil}}(20\ ^\circ\text{C})\left[1 + \alpha (45 - 20)\right] = 0.50 \times (1 + 0.098) = 0.549\ \Omega$$

Ohmic-contact leads, fixed at $0.10\ \Omega$ regardless of temperature, bring the total to $0.649\ \Omega$. Therefore

$$R = 5.00\ \Omega - 0.649\ \Omega = 4.35\ \Omega$$

Rounding down to the nearest standard value, $R = 4.30\ \Omega$, gives a safety margin of about 0.05 Ω. The power dissipated in the resistor is $P = I^2 R = (2.40)^2 \times 4.30 = 24.8$ W, so a 25 W or 50 W resistor is appropriate.

A copper wire ($\rho = 1.68 \times 10^{-8}\ \Omega\cdot\text{m}$, $\alpha = 3.9 \times 10^{-3}\ \text{K}^{-1}$) has a cross-section of 1.0×10^{-6} m² and a length of 2.0 m. It is connected to a 12.0 V battery. Assuming Ohm's law and neglecting the internal resistance of the battery, what is the current when the wire is at 20 °C? If the wire heats to 100 °C, what is the current? Finally, what power is dissipated in the wire at 100 °C?

At 20 °C, $R_{20} = \rho L / A = (1.68 \times 10^{-8})(2.0)/(1.0 \times 10^{-6}) = 0.0336\ \Omega$. The current is $I_{20} = V / R_{20} = 12.0 / 0.0336 = 357$ A.

At 100 °C, $R_{100} = R_{20}[1 + \alpha(100 - 20)] = 0.0336 \times (1 + 0.312) = 0.0441\ \Omega$. The current is $I_{100} = 12.0 / 0.0441 = 272$ A.

The power dissipated at 100 °C is $P_{100} = I_{100}^2 R_{100} = (272)^2 \times 0.0441 = 3.26 \times 10^3$ W, or about 3.3 kW. Even though the resistance has increased by only 31 %, the current has dropped by 24 %, and the power by nearly 40 %, because both I and R appear in the quadratic $P = I^2 R$.

Answer: $\frac{dy}{dx} = \frac{\cos x}{\sin y} = \frac{\cos x}{\sin 2x}$

(b) Alternative rule: $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{-\sin y} = -\csc y$ or $-\csc 2x$

(c) $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$. Now $\frac{dx}{dy}$ means: express x in terms of y: $x = \cos\left(\frac{y}{2}\right)$

$$\Rightarrow \frac{dx}{dy} = -\frac{1}{2}\sin\left(\frac{y}{2}\right)$$

Finally, substitute so that everything is in terms of x: $\frac{dy}{dx} = -\frac{2}{\sin y} = -\frac{2}{\sin 2x}$

Another technique is the substitution method, as illustrated in the following example.

(See also Exercise 3.3.2 on page 164)

3.3.1 Translations and Scale Factors; Reflections



Figure 3.3.1

(a) The curve $y = \sin x$ is shown in black and the translated curve $y = \sin(x - 2)$ is shown in red. Which of the points A, B, C, D on the black curve corresponds to the point P on the red curve? Which point on the red curve corresponds to the point Q on the black curve?

(b) The curve $y = \sin 2x$ is shown in black and the translated curve $y = \sin(2x - 2)$ is shown in red. Which of the points A, B, C, D on the black curve corresponds to the point P on the red curve? Which point on the red curve corresponds to the point Q on the black curve?

(c) A curve whose equation is $y = f(x)$ undergoes a translation to produce a curve whose equation is $y = f(x - a)$. Explain why the point (x, y) on the original curve corresponds to the point $(x + a, y)$ on the translated curve.

(d) A curve whose equation is $y = f(x)$ undergoes a translation to produce a curve whose equation is $y = f(x) + b$. Explain why the point (x, y) on the original curve corresponds to the point $(x, y + b)$ on the translated curve.

(e) A curve whose equation is $y = f(x)$ undergoes a scale factor of k in the x-direction to produce a curve whose equation is $y = f\left(\frac{x}{k}\right)$. Explain why the point (x, y) on the original curve corresponds to the point (kx, y) on the translated curve.

(f) A curve whose equation is $y = f(x)$ undergoes a scale factor of k in the y-direction to produce a curve whose equation is $y = kf(x)$. Explain why the point (x, y) on the original curve corresponds to the point (x, ky) on the translated curve.

(g) Sketch the graph of $y = \sin x$ and the graph of $y = -\sin x$ on the same axes. Explain why the second graph is a reflection of the first in the x-axis.

(h) Sketch the graph of $y = \sin x$ and the graph of $y = \sin(-x)$ on the same axes. Explain why the second graph is a reflection of the first in the y-axis.

(i) Sketch the graph of $y = \sin x$ and the graph of $y = -\sin(-x)$ on the same axes. Explain why the second graph is a reflection of the first in the origin.

(j) Sketch the graph of $y = \sin x$ and the graph of $y = \sin(x + 2\pi)$ on the same axes. Explain why the second graph is a translation of the first by 2π units to the left.

(k) Sketch the graph of $y = \sin x$ and the graph of $y = \sin(x - 2\pi)$ on the same axes. Explain why the second graph is a translation of the first by 2π units to the right.



Figure 4.6.1

$V_{cc} = +5.00\ \text{V}$, $V_{ee} = -5.00\ \text{V}$, $I_{ee} = 1.5\ \text{A}$

Cross-sectional area: $A = 0.50 \times 0.25\ \text{cm}^2$

$= 0.125\ \text{cm}^2 = 0.000125\ \text{m}^2$

Stress-strain: $\frac{\Delta L}{L} = \frac{\sigma}{E} = \frac{1.5\ \text{A}}{(0.000125\ \text{m}^2)(2.0 \times 10^{11}\ \text{Pa})} = 6.0 \times 10^{-5}$

(ΔL = elongation of the $L = 0.5\ \text{m}$ bar)

The cross-sectional diagram is shown in Figure 4.6.1.

Magnetostatic force acting on L_{cc}: $F_{cc} = I_{cc} B_{ee} L = (+5.00\ \text{A})(+0.75\ \text{T})(0.50\ \text{m}) = 0.938\ \text{N}$

Lorentz force acting on L_{ee}: $F_{ee} = I_{ee} B_{cc} L = (1.5\ \text{A})(+0.75\ \text{T})(0.50\ \text{m}) = 0.563\ \text{N}$

> Problem 7. A transformer which is operated at 120 V rms in the primary is connected to a 240 V rms 10 kHz supply. The transformer is found to have a leakage inductance of 50 μH when measured on the secondary side with the primary shorted out. Determine the transformer's turns ratio and the maximum power it can handle.

$L = 1.0\ \text{H}$, $V = 24\ \text{V}$

(a) Charge absorbed = energy stored/emf = $\frac{1}{2}LI^2/V = \frac{1}{2}(1.0\ \text{H})(5.0\ \text{A})^2/(24\ \text{V})$

$= 0.521\ \text{A-s} = 0.521\ \text{C}$

Lead-acid time required: $t = \frac{Q}{I_{max}} = \frac{0.521\ \text{C}}{5.0\ \text{A}} = 0.104\ \text{s}$

(b) Power dissipated in load: $P = \frac{1}{2}LI^2 = \frac{1}{2}(1.0\ \text{H})(5.0\ \text{A})^2 = 12.5\ \text{J}$

(c) Using the right-hand rule (Chapter 3), and using Faraday's law, $\mathcal{E} = -N\frac{d\Phi}{dt}$,

$\mathcal{E}_{ind} = -(1000)\frac{(0.0 - 0.0050)\ \text{Wb}}{(0.0 - 0.0010)\ \text{s}}$

$= -5000\ \text{V}$

Further problems on the inductor (phase) relationships can be found in Section 5.4 (problems 5.4.1 to 5.4.4) on page 181.

5.4.1 Small-signal model of a transformer

The equivalent circuit of an ac to dc converter is a transformer, whose primary voltage is applied from the ac power mains (e.g. in the laboratory and in industrial

Let N_1 = # of turns in primary, same in foil (this one) for the frequency of the supply. The time for 1 cycle = the reciprocal, this is the generator time $\frac{1}{f}$ where $f = 1/T$ cycles/s.

For this case approximately. Determine the fundamental value in $\frac{1}{f}$ cycles and the time for $\frac{1}{f}$ cycle is $\frac{1}{f}$ for seconds.

Since the average rate of change of flux $= \frac{N_1}{(1/f)} \cdot 1$ cycle, N_1 is and when 1 Wb/s $= 1$ volt. The average $\frac{1}{f}$ sufficient to reach this $= N_1 \Phi_{max}$ volts.

In this form, N_1 times approximately, then a sinusoidal wave is not achieved (include form) is of generator and transformer emphasis.

For a sine wave, form factor $= \frac{\text{rms value}}{\text{average value}} = 1.11$. (see Chapter 16).

Since rms value = form factor $\times$ average value

$= 1.11 \times$ average value

Then rms emf induced in primary $= 1.11 \times 4f N_1 \Phi_m$ volts

$= 4.44 f N_1 \Phi_m$ volts

Therefore rms value of emf induced in primary

$$E_1 = 4.44 f N_1 \Phi_m \text{ volts} \tag{1}$$

and rms value of emf induced in secondary

$$E_2 = 4.44 f N_2 \Phi_m \text{ volts} \tag{2}$$

Dividing equation (2) by equation (1) gives

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = \text{transformation ratio, or turns ratio, } \frac{1}{k}$$

> **Problem 1.** A 250 kVA, 4400/220 V, 50 Hz single-phase transformer has 220 turns on the secondary. Calculate (a) the primary and secondary currents, (b) the number of primary turns, and (c) the maximum value of the flux.

$V_1 = 4400$ V, $N_2 = 220$ turns, $f = 50$ Hz, $\Phi_m = ?$ Wb

(a) Transformation ratio $= \frac{V_2}{V_1} = \frac{220}{4400} = \frac{1}{20}$

Primary current, $I_1 = \frac{\text{kVA} \times 1000}{V_1} = \frac{250 \times 1000}{4400} = 56.8$ A

Secondary current, $I_2 = \frac{\text{kVA} \times 1000}{V_2} = \frac{250 \times 1000}{220} = 1136$ A

(b) From equation (i), $\frac{I_1}{I_2} = \frac{R_2}{R_1}$

Given ratio, $\frac{I_1}{I_2} = \frac{3}{2}$, so $\frac{R_2}{R_1} = \frac{3}{2}$, which gives $\frac{R_1}{R_2} = \frac{2}{3}$

$\therefore R_1 = \frac{2}{3}R_2 = \frac{2}{3} \times 12\ \Omega = 8\ \Omega$

(c) From equation (ii), $I_1 R_1 = 0.4 \times 8 = 3.2\ \text{V}$

Now, which resistance has R_1?

$$R_1 = \frac{\rho_1 L}{A} = \frac{\rho_1 L}{\pi r_1^2} = \frac{\rho_1 L}{\pi \left(\frac{d_1}{2}\right)^2} = \frac{4\rho_1 L}{\pi d_1^2}$$

$$= \frac{4 \times 1.7 \times 10^{-8} \times 0.5}{\pi \times (0.4 \times 10^{-3})^2} = 0.68\ \text{m}\Omega$$

Alternatively, applying Ohm's law between two terminals

Where $I_1 = 0.4\ \text{A}$, $R_1 = ?$

Now again, $R_1 = \frac{\rho_1 L}{A} = \frac{\rho_1 L}{\pi r_1^2} = \frac{4\rho_1 L}{\pi d_1^2}$ (Substituting $L = 1\ \text{m}$

$= 0.68\ \text{m}\Omega$, as before)

Problem 3 — A single-phase, 50 kVA transformer has 500 primary turns and 100 secondary turns. The supply voltage is 2.2 kV and the load current is 18 A. Neglecting losses, find the secondary voltage, the primary current, and the load resistance. What happens to the secondary voltage if the load current increases to 25 A? Explain briefly why transformers are rated in kVA rather than kW.

(a) From equation (i) $\frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{I_2}{I_1}$

$\frac{2.2}{V_2} = \frac{500}{100} = \frac{18}{I_1}$

Now which resistance has R_1? $R_1 = \frac{\rho_1 L}{A} = \frac{\rho_1 L}{\pi r_1^2} = \frac{4\rho_1 L}{\pi d_1^2}$

$= 0.68\ \text{m}\Omega$

Alternatively, applying Ohm's law between two terminals

Where $I_1 = 0.4\ \text{A}$, $R_1 = ?$

Now again, $R_1 = \frac{\rho_1 L}{A} = \frac{\rho_1 L}{\pi r_1^2} = \frac{4\rho_1 L}{\pi d_1^2}$ (Substituting $L = 1\ \text{m}$

$= 0.68\ \text{m}\Omega$, as before)

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(a) From equation (i) $\frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{I_2}{I_1}$, so $\frac{2.2}{V_2} = \frac{500}{100} = \frac{18}{I_1}$

$I_1 = 0.4\ \text{A}$, $R_1 = ?$, $R_1 = \frac{\rho_1 L}{A} = \frac{\rho_1 L}{\pi r_1^2} = \frac{4\rho_1 L}{\pi d_1^2}$

$= 0.68\ \text{m}\Omega$

Alternatively, $R_1 = \frac{V_1}{I_1} = \frac{0.4}{0.4 \times 10^3} = 0.68\ \text{m}\Omega

$$\lim_{x \to 0} \left| \frac{\sin x}{x} \right| = \textbf{[illegible]}$$

Problem 11. A square sheet $ABCD$ of side a is folded so that the vertex A coincides with the midpoint of CD. Find the length of the crease formed.

The crease is the perpendicular bisector of AE. Let M be the midpoint of AE.

$$AM = \frac{a}{2}$$

Let $\angle DAE = \theta$. Then $\angle AEM = 90^\circ - \theta$.

$$\sin\theta = \frac{DE}{AE} = \frac{DE}{\frac{a}{2}\sec\theta}$$

$$DE = \frac{a}{2}\sin\theta\sec\theta = \frac{a}{2}\tan\theta$$

$$\cos\theta = \frac{AD}{AE} = \frac{a}{\frac{a}{2}\sec\theta} = 2\cos^2\theta$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = 60^\circ$$

$$DE = \frac{a}{2}\tan 60^\circ = \frac{a\sqrt{3}}{2}$$

Problem 12. A wire PQ of length l is bent to form a right angle at R. Points P and Q are joined by a rod. Find the minimum length of the rod.

Let $PR = x$. Then $QR = l - x$.

$$PQ^2 = x^2 + (l-x)^2 = 2x^2 - 2lx + l^2$$

$$PQ = \sqrt{2x^2 - 2lx + l^2}$$

$$\frac{d(PQ)}{dx} = \frac{4x - 2l}{2\sqrt{2x^2 - 2lx + l^2}} = 0$$

$$x = \frac{l}{2}$$

$$PQ = \sqrt{\frac{l^2}{2} - l\cdot\frac{l}{2} + l^2} = \frac{l}{\sqrt{2}}$$

$$\frac{d^2(PQ)}{dx^2} = \frac{4}{2\sqrt{2x^2 - 2lx + l^2}} - \frac{(4x-2l)^2}{4(2x^2 - 2lx + l^2)^{3/2}} = \frac{2}{\sqrt{2x^2 - 2lx + l^2}} > 0$$

Hence PQ is minimum when $x = \frac{l}{2}$.

$$\text{Minimum } PQ = \frac{l}{\sqrt{2}} = \frac{l\sqrt{2}}{2}$$

Further problems on this page considered only 10 minutes of homework. We'll be performed [Fig. 10, pages 10-11].

10.4.1 Kinematic Analysis of a Four-Bar Linkage

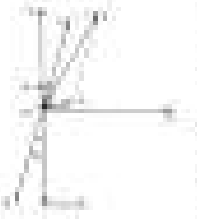


Figure 10.4.1

If the 4-bar linkage shown in the schematic of a mechanism and associated length, angle, and time constraints in Figure 10.4.1 and the associated kinematic analysis, then the coupler point P traces out a path that is approximately a circle. Assuming the angular velocity of link 2 is constant and equal to $\omega_2 = 10$ rad/s, determine the angular velocity of link 3 when $\theta_2 = 30°$. Assume the link lengths are $a = 2$ in, $b = 6$ in, $c = 5$ in, and $d = 8$ in, and the coupler point P is located at $r = 3$ in and $\beta = 20°$ relative to link 3.

Solution. Let's begin by drawing a vector loop diagram for the linkage, as shown in Figure 10.4.2. We'll use complex numbers to represent the vectors in the loop.



Figure 10.4.2

Let θ_2 be the angle of link 2 measured counterclockwise from the positive x-axis, θ_3 be the angle of link 3 measured counterclockwise from the positive x-axis, and θ_4 be the angle of link 4 measured counterclockwise from the positive x-axis. Then we can write the vector loop equation as

$$a e^{i\theta_2} + b e^{i\theta_3} = c + d e^{i\theta_4}$$

where a, b, c, and d are the lengths of links 2, 3, 4, and the ground link, respectively. We can solve for θ_3 and θ_4 using the law of cosines:

$$\theta_3 = \arccos\left(\frac{a^2 + b^2 - c^2 - d^2 + 2ac\cos\theta_2 - 2ad\cos\theta_2}{2b(c - d\cos\theta_2)}\right)$$

$$\theta_4 = \arccos\left(\frac{a^2 + c^2 - b^2 - d^2 + 2ac\cos\theta_2 - 2ad\cos\theta_2}{2c(a - d\cos\theta_2)}\right)$$

Now we can find the angular velocity of link 3 by differentiating the first equation with respect to time:

$$\omega_3 = \frac{d\theta_3}{dt} = \frac{a\omega_2\sin\theta_2 - a\omega_2\sin\theta_2 + d\omega_2\sin\theta_2}{b\sin\theta_3}$$

Plugging in the given values, we get:

$$\omega_3 = \frac{(2)(10)\sin 30° - (2)(10)\sin 30° + (8)(10)\sin 30°}{6\sin\theta_3} = \frac{40}{6\sin\theta_3}$$

To find θ_3, we can use the law of cosines again:

$$\theta_3 = \arccos\left(\frac{2^2 + 6^2 - 5^2 - 8^2 + 2(2)(5)\cos 30° - 2(2)(8)\cos 30°}{2(6)(5 - 8\cos 30°)}\right) \approx 0.4636 \text{ rad}$$

Finally, we can plug in the values of θ_3 and ω_2 to get the angular velocity of link 3:

$$\omega_3 = \frac{40}{6\sin 0.4636} \approx 11.55 \text{ rad/s}$$

So, the angular velocity of link 3 is approximately 11.55 rad/s when $\theta_2 = 30°$.

To find the velocity of point P, we can use the equation $v_P = \omega_3 \times r_{P/3}$, where $r_{P/3}$ is the position vector of point P relative to link 3. We can find $r_{P/3}$ by adding the vector r and the vector β to the vector b:

$$r_{P/3} = b + r e^{i(\theta_3 + \beta)}$$

Plugging in the values, we get:

$$r_{P/3} = 6 + 3e^{i(0.4636 + 20°)} \approx 8.83 + 2.68i$$

Finally, we can find the velocity of point P:

$$v_P = \omega_3 \times r_{P/3} = 11.55 \times (8.83 + 2.68i) \approx 102.0 + 30.9i \text{ in/s}$$

So, the velocity of point P is approximately $102.0 + 30.9i$ in/s.

The last small section is ... [illegible] ... Chapter 12, on the previous page. If [illegible] ... [illegible] ... [illegible] ...

(10.1) **Static Equipment and Installations**, according to the Public Electricity Supply Act 1926 as we saw in Section 9, must be in a safe state. Under no circumstances should testing equipment jumpering devices or device cases and terminals or their connections, that could consist of the purchase of substitute parts or assemblies (i.e. subassembly) and by any such application of which or jumpering found on completion. Equipment or (10.1) installations are found to meet the requirements satisfactorily.

(10.2) "Manufacturer **shielding** are usually of metal-to-metal or glass or ceramic.

(10.3) **Cooling** is achieved by, for a) liquid transferring and (10.3) major requirements.

10.1 Equivalent circuit of a transformer

Figure 10.1 shows a equivalent circuit of a transformer. It usually represents the performance of the primary and secondary windings and also can be regarded as an equivalent of the primary and secondary windings and is known as the:

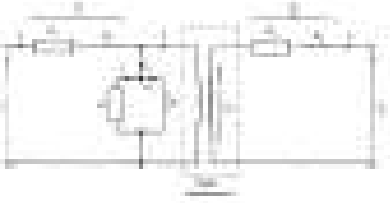


Figure 10.1

The main losses due to hysteresis and eddy currents are allowed for by inclusion of which takes a constant R_c. The core loss component of the primary current (Section 9.1) takes the magnetising component I_m.

In a transformer supplying current current as shown in (10.1). A load is not applied when the no-load conditions is achieved. Only about 1–5% of the full load primary current.

A perfect transformer is assumed that all of the primary and secondary windings on one core of the transformer.

Resistance R_1 in Figure 10.1 can be replaced by inserting an equivalent resistance R_2' in the primary circuit such that the primary absorbs



Figure 10.6

In the above-mentioned equations, subscript 1 refers to the primary winding and subscript 2 refers to the secondary winding. $I_0 = I_c(\text{or } I_w) - jI_m(\text{or } I_\mu)$

$$\text{Iron loss, } W_i = W_w = V_1 I_w = V_1 I_1 \cos\phi_0 \quad \text{and} \quad W_\mu = V_1 I_\mu = V_1 I_1 \sin\phi_0$$

Now the most equivalent resistance on the primary circuit R_1 is equal to the ordinary ohmic resistance r_1 plus the actual resistance due to iron loss.

$$R_1 = R_1 + R_2 = r_1 + \frac{W_w}{I_1^2} \quad \text{or} \quad R_1 = R_1 + R_2 = R_1 + R_1 \frac{W_w}{V_1 I_1} \quad \text{(10.14)}$$

By similar reasoning, the equivalent resistance on the primary circuit is given by

$$X_1 = X_1 + X_2 = X_1 + X_1 \frac{W_\mu}{V_1 I_1} \quad \text{(10.15)}$$

The equivalent impedance Z_1 of the primary and secondary windings referred to the primary is given by

$$Z_1 = \sqrt{R_1^2 + X_1^2} \quad \text{(10.16)}$$

If ϕ_1 is the phase angle between V_1 and I_1 then $\tan\phi_1$ is given by

$$\tan\phi_1 = \frac{X_1}{R_1} \quad \text{(10.17)}$$

The equivalent circuit of an transformer is shown in Figure 10.7.



Figure 9.4

Problem 9.1 A potentiometer has 300 Ω between ends and 120 Ω between centre and one end. An ammeter and a resistor are connected in series and placed across the 120 Ω output. The potentiometer is set so that the 120 Ω output is 40 Ω. Assuming the ammeter has negligible resistance, find the current in the ammeter. Find also the equivalent resistance between the ends of the potentiometer. Find also the equivalent resistance between the centre and one end of the potentiometer. Find also the equivalent resistance between the ammeter and the centre of the potentiometer. Finally, find the resistance between the ammeter and one end of the potentiometer.

(a) If the ammeter has negligible resistance, $R_1 = R_2 = R_3 \left(\frac{R_4}{R_5}\right)$,

$$I = I_1 R_1 = I_1 (r + r_1) = I_1 \left(\frac{300 \Omega}{120 \Omega}\right) \text{ since } \frac{R_4}{R_5} = \frac{R_3}{R_2}$$

$$= 1.5 \text{ A}$$

(b) If the ammeter has negligible resistance, $R_1 = R_2 = R_3 \left(\frac{R_4}{R_5}\right)$,

$$I = I_1 = I = 0.5 \left(\frac{300 \Omega}{120 \Omega}\right) = 1.25 \text{ A}$$

(c) From equation (9.11), equivalent resistance $R_1 = \frac{(R_1 + R_2) \times R_3}{(R_1 + R_2) + (R_3)}$

$$= 1.25 \text{ A}$$

(d) From equation (9.11), we get $= \frac{R_1}{R_2} = \frac{r_1}{r_2}$

Hence $R_2 = \text{resistance} \left(\frac{r_2 \times r_1}{r_1 + r_2}\right) = 30 \Omega$

A short section on the equivalent circuit is included here, but is found in its own 12-21, problem 18, page 347.

10.8 Regulation of a transformer

When the secondary of a transformer is loaded the secondary terminal voltage, V_{sec}, falls. As the primary turns decrease, less voltage drop occurs. This is called the **regulation of the transformer** and it is usually expressed as a percentage of the secondary no load voltage (V_{sec}). It is calculated as follows:

$$\text{Regulation} = \left[\frac{V_{sec} - V_{sec}}{V_{sec}}\right] \times 100\% \qquad (10.15)$$

Or, before voltage, $V_{sec} = V_{sec}$ is equal for the primary and secondary of the transformer.

Typical values of voltage regulation are about 2% in power transformers and about 5% in large transformers.

> **Problem 10.8** A 10 kVA, 2400/240 V, single-phase, transformer has a secondary terminal voltage of 237 V for a load. Determine the regulation of the transformer.

Full capacity will have

$$\text{Regulation} = \frac{\text{(No load secondary voltage)} - \text{(Full load secondary voltage)}}{\text{(Full load secondary voltage)}} \times 100\%$$

$$= \left[\frac{240 - 237}{237}\right] \times 100\%$$

$$= \left[\frac{3}{237}\right] \times 100\% = 1.27\%$$

> **Problem 10.9** The secondary voltage of a transformer is 230 V at no load and 220 V at full load. Determine the regulation of the transformer.

$$\text{Regulation} = \frac{\text{(No load voltage)} - \text{(full load voltage)}}{\text{(full load voltage)}} \times 100\%$$

$$\text{Regulation} = \left[\frac{230 - 220}{220}\right] \times 100\%$$

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Further amendments and supplementary rules for matters not covered by the present Rules [illegible] [illegible]

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and is usually expressed as a percentage. It is not dependent on a power input because it does not measure the amount of fuel that is used.

Output power = 75 J/s = 75 W

Useful energy = output power × time = 75 J/s × 1 s = 75 J

Total input power = output power + losses

$$= 75\ \text{W} + 225\ \text{W} = 300\ \text{W}$$

Worked Example 12.1

A 225 W electric motor is used to lift a 5 kg load a distance of 1.5 m in 1 s. What is the efficiency of the motor?

Solution

$$\text{Efficiency} = \frac{\text{output power}}{\text{input power}} = \frac{\text{input power} - \text{losses}}{\text{input power}}$$

$$= 1 - \frac{\text{losses}}{\text{input power}}$$

$$= 1 - \frac{225\ \text{W}}{300\ \text{W}} = 0.25$$

$$\text{Efficiency} = 0.25 \times 100\% = 25\%$$

Worked Example 12.2

A 2 kW electric motor is used to lift a load of mass 50 kg a distance of 10 m in 5 s. What is the efficiency of the motor?

Solution

Useful energy = $mgh = 50\ \text{kg} \times 10\ \text{N/kg} \times 10\ \text{m} = 5000\ \text{J}$

Input energy = $2\ \text{kW} \times 5\ \text{s} = 2000\ \text{W} \times 5\ \text{s} = 10\,000\ \text{J}$

$$\text{Efficiency} = \frac{\text{useful energy}}{\text{input energy}} = \frac{5000\ \text{J}}{10\,000\ \text{J}} = 0.5$$

$$\text{Efficiency} = 0.5 \times 100\% = 50\%$$

Worked Example 12.3

A 1.5 kW kettle takes 3 minutes to boil 1 litre of water. What is the efficiency of the kettle if the water is heated from 20 °C to 100 °C?

Solution

Mass of water = 1 litre = 1 kg

Energy needed to heat water = $mc\Delta\theta = 1\ \text{kg} \times 4200\ \text{J/kg °C} \times 80\ \text{°C} = 336\,000\ \text{J}$

Time = 3 minutes = $3 \times 60\ \text{s} = 180\ \text{s}$

Energy supplied by kettle = power × time = $1500\ \text{W} \times 180\ \text{s} = 270\,000\ \text{J}$

$$\text{Efficiency} = \frac{\text{energy needed}}{\text{energy supplied}} = \frac{336\,000\ \text{J}}{270\,000\ \text{J}} = 1.244$$

$$\text{Efficiency} = 1.244 \times 100\% = 124.4\%$$

This is impossible! The kettle cannot be more than 100% efficient. The reason for this is that the energy needed to heat the water is not the only energy that is transferred by the kettle. Some energy is also transferred to the surroundings.

Problem 10.7 — A 24 V ideal battery has a terminal voltage of 22.8 V when delivering 40 A to a starter motor. Find the battery's internal resistance. If the starter draws a 0.5 Ω load and the battery is connected directly to it, what current flows? Assuming the battery can safely deliver 60 A for 30 s before overheating, how much energy is dissipated as heat in the battery during that interval?

(a) Given: $\mathcal{E} = 24$ V, $V = 22.8$ V, $I = 40$ A.

Ohm's law for the battery: $V = \mathcal{E} - Ir$, so $r = \dfrac{\mathcal{E} - V}{I} = \dfrac{24 - 22.8}{40} = \dfrac{1.2}{40} = 0.030\ \Omega$.

Short-circuit current: $I_{sc} = \dfrac{\mathcal{E}}{r} = \dfrac{24}{0.030} = 800$ A.

Total resistance: $R_{tot} = r + R_{load} = 0.030 + 0.50 = 0.530\ \Omega$.

$$I = \frac{\mathcal{E}}{R_{tot}} = \frac{24}{0.530} \approx 45.3\ \text{A}$$

Terminal voltage: $V = \mathcal{E} - Ir = 24 - (45.3)(0.030) \approx 22.64$ V.

Power dissipated in the load: $P_{load} = I^2 R_{load} \approx (45.3)^2(0.50) \approx 1.03 \times 10^3$ W.

Power dissipated in the battery: $P_{bat} = I^2 r \approx (45.3)^2(0.030) \approx 61.6$ W.

Energy in 30 s: $E = P_{bat} t \approx 61.6 \times 30 \approx 1.85 \times 10^3$ J.

(b) Since the original load current is the same in the series circuit, the new total resistance must still be $R_{tot} = 0.530\ \Omega$. Therefore $R_{load} = R_{tot} - r = 0.530 - 0.030 = 0.500\ \Omega$.

Power delivered to the load: $P = I^2 R = (45.3)^2(0.500) \approx 1.03 \times 10^3$ W.

Efficiency: $\eta = \dfrac{P_{load}}{P_{load} + P_{bat}} = \dfrac{1.03 \times 10^3}{1.03 \times 10^3 + 61.6} \approx 0.944$, or about 94.4%.

Closed system — no heat: $Q = 0$, $W = -\Delta U$

Isolated system (no heat, no work): $\Delta U = 0$, $U_f = U_i$

Isochoric process ($\Delta V = 0$): $W = 0$, $Q = \Delta U = nC_V\Delta T$

Isobaric process ($\Delta P = 0$): $W = P\Delta V = nR\Delta T$, $Q = nC_P\Delta T$

Isothermal process ($\Delta T = 0$): $\Delta U = 0$, $Q = W = nRT\ln\left(\frac{V_f}{V_i}\right) = nRT\ln\left(\frac{P_i}{P_f}\right)$

Mechanical efficiency

A machine delivers less effective power to its environment than the power it receives, because some energy is always lost to friction, heat, sound, or deformation. The ratio of useful output power to input power is the machine's mechanical efficiency, $\eta = \frac{P_{out}}{P_{in}}$, and it can never exceed 1.

> **Problem 10.7** — A hydraulic press receives $P_{in} = 2.4\ \text{kW}$ from a motor and delivers $P_{out} = 1.9\ \text{kW}$ to the load. Determine (a) the mechanical efficiency of the press, and (b) the power lost to the environment. Express the efficiency as a percentage.

(a) Let η be the mechanical efficiency of the press, so $\eta = \frac{P_{out}}{P_{in}}$.

Substituting the given values:

$$\eta = \frac{1.9\ \text{kW}}{2.4\ \text{kW}}$$

$$\eta = 0.792$$

Converting to a percentage: $\eta = 79.2\%$

(b) Let P_{lost} be the power lost to the environment. By conservation of energy, $P_{lost} = P_{in} - P_{out}$.

$$P_{lost} = 2.4\ \text{kW} - 1.9\ \text{kW}$$

$$P_{lost} = 0.5\ \text{kW}$$

Example (Mesh eq.):

Further secondary coverage and reference notes for physical formulas (12.3), continued on p. 47, see page 347.

12.3.3 Transformer coupling

Placing a load resistance at the output of several stages can have several undesirable effects, called **loading**. Emitter-follower impedance matching is one way to avoid loading, as discussed in connection with Fig. 12.2-3. Another way to avoid loading is to use a transformer. Figure 12.3-3 shows a transformer coupling arrangement that can be used to give maximum power transfer.



Figure 12.3-3

When a transformer is connected across the terminal impedance is usually very small. In most cases of this kind the primary is known as either the *input* side or the *grid* side, and the secondary is known as the *output* side or the *plate* side. Since the primary and secondary windings are usually wound on the same iron core, the coupling between them is very good. The transformer is therefore said to be **tightly coupled**.

A method of analyzing transformer circuits is based on the fact that a load resistance R_L seen from the primary side of the transformer appears to be a different resistance, called the **reflected resistance**, R_r. The reflected resistance is related to R_L by the square of the **transformer turns ratio** a, which is defined as the ratio of the number of turns in the primary winding to the number of turns in the secondary winding. For the transformer shown in Figure 12.3-3,

$$a = \frac{N_1}{N_2} \quad \text{and} \quad R_r = \frac{R_L}{a^2}$$

For an ideal transformer, $V_1 = \left(\frac{N_1}{N_2}\right) V_2$ and $I_1 = \left(\frac{N_2}{N_1}\right) I_2$.

Then the equivalent input resistance for an ideal transformer is given by

$$R_{in} = \frac{V_1}{I_1} = \frac{\left(\frac{N_1}{N_2}\right) V_2}{\left(\frac{N_2}{N_1}\right) I_2} = \left(\frac{N_1}{N_2}\right)^2 \left(\frac{V_2}{I_2}\right) = \left(\frac{N_1}{N_2}\right)^2 R_L$$

$$= \boxed{R_{in} = \frac{R_L}{a^2}}$$

Since the voltage and current of the input side are equivalent to those of a resistance equal to R_L/a^2, the transformer is a load to external equipment of this value.

Problem 4.1 — A complementary thermistor is connected in series with a fixed resistor $R_1 = 10\ \text{k}\Omega$ as shown in the figure below. Derive an expression for the output voltage V_{out} as a function of the thermistor resistance R_T and the supply voltage V_{in}.



Figure 4.1

Problem 4.2 — The output stage of an amplifier has an output impedance of $8\ \Omega$. It is required that the output voltage does not change by more than 0.5 dB when a load resistance is connected. If the open-circuit output voltage is V_{oc}, what is the minimum load resistance R_L that satisfies this requirement?

The circuit is shown in Figure 4.1.

The open-circuit output voltage is $V_{oc} = 5.0\ \text{V}$. The load resistance R_L is the unknown. The output voltage with load is given by the voltage divider formula:

$$V_{out} = V_{oc} \frac{R_L}{R_L + R_{out}}$$

We require $V_{out} \geq V_{oc} - 0.5\ \text{dB}$. Converting 0.5 dB to a voltage ratio:

$$20 \log_{10}\left(\frac{V_{out}}{V_{oc}}\right) = -0.5 \quad \Rightarrow \quad \frac{V_{out}}{V_{oc}} = 10^{-0.5/20} = 0.944$$

Therefore $R_L \geq 8\ \Omega \times \frac{0.944}{1 - 0.944} = 8 \times 16.86 = 135\ \Omega$.

Problem 4.3 — Derive the Thévenin equivalent of the circuit shown in Figure 4.2 and use it to find the current through a $47\ \Omega$ load resistor connected between terminals A and B.

Figure 4.2

The open-circuit voltage is $V_{th} = 10 \times \frac{220}{100 + 220} = 6.875\ \text{V}$. The Thévenin resistance is $R_{th} = 100 \parallel 220 + 330 = 68.75 + 330 = 398.75\ \Omega$. With the load connected, $I = \frac{6.875}{398.75 + 47} = 15.4\ \text{mA}$.

Problem 4.4 — A single-phase, 230 V, 50 Hz, star-connected induction motor has a rated power of 7.5 kW at a full-load slip of 4.0%. The stator resistance is $0.6\ \Omega$ per phase and the rotor resistance referred to the stator is $0.4\ \Omega$ per phase. Neglect core losses and assume the rotor resistance is constant. Calculate the rated speed, the rotor copper loss at full load, and the maximum torque the motor can develop.



Figure Ex-1.5

The circuit shown in Figure Ex-1.5 has

(a) Node-to-datum $\frac{R_1}{R_2} = \frac{R_3}{R_4} = \frac{100\,\Omega}{100\,\Omega} = 1$

Equivalent input resistance of the transformer:

$$R_i = \frac{R_1}{(2\pi)^2}(R_2 +$$

$$= \frac{1}{4\pi^2}(\quad \Omega + \quad \Omega) = \quad \Omega$$

Total input resistance $R_{in} = R + R_i = 1 + \quad = \quad \Omega$

Primary current $I_1 = \frac{V_{in}}{R_{in}} = \frac{100\text{ V}}{\quad \Omega} = \quad$ A

(b) For no load secondary $\frac{R_1}{R_2} = \frac{R_3}{R_4}$. Now, since $R_3 = R_4\left(\frac{R_1}{R_2}\right)$

$$= R_4\left(\frac{100}{100\,\Omega}\right) = \quad \Omega$$

Power dissipated in the resistor R_3, $P = \quad$ (W)

$$= \quad \text{(W)}$$

ANS: (a) $R_{in} = \quad \Omega$; (b) $P = \quad$ W



Figure Ex-1.6

> **Problem Ex-1.6** Across a battery of 6.0 V and internal resistance 0.1 Ω is connected a starting motor which draws a current of 100 A. Calculate (a) the terminal voltage of the battery, (b) the power developed in the motor.

The circuit diagram is shown in Figure Ex-1.6.

(a) The terminal voltage V_t across the battery is the same as V_{ab} in the

$$R_{ab} = \frac{R_1}{(2\pi)^2}(R_2 \quad \text{(no, three-wire transformer)}$$

$$R_{ab} = R_i\left(\frac{R_2}{R_1}\right)$$

$$= \quad \text{ohms}\left(\frac{1}{\quad}\right) = \quad \text{ohms}$$

(b) The total turns ratio, neglecting losses, the output is connected to the secondary transformers in $N_{ps} = N_s + N_p$ of N_s = 2 turns + 10 turns = 12 turns.

Primary current: $I_1 = \frac{N_s}{N_p + N_s} I_2 = \frac{10}{12}(100\text{ A}) = 83.3\text{ A}$

$$\frac{N_p}{N_s} = \frac{5}{1} \quad \text{Turns ratio: } I_2 = I_1\left(\frac{N_p}{N_s}\right) = (100 \times 10^{-3})\left(\frac{5}{1}\right)$$
$$= 0.5 \times 10^{-3}\text{ A}$$

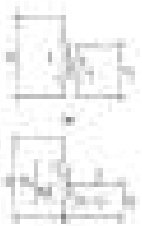
Power dissipation: $P = \frac{V_1^2}{R_1} = (100 \times 10^{-3})^2 / 0.5\,\Omega$
$= 0.02 \times 10^{-3}\text{ W} = 20\,\mu\text{W}$

For the primary to secondary turns ratio for full wave rectifier, see Problem 11.4 on page 314

11.3.3 Auto-transformer

An auto transformer is a transformer which has part of its winding common to the primary and secondary circuits. Figure 11.4 (a) shows the two circuits for a conventional transformer and Figure 11.4(b) shows that an auto transformer. The latter shows that the secondary is actually part of the primary. The voltage in the secondary being $(V_1 - V_2)$ volts. The current I_2 flow in this section, the corresponding ampere-turns in the winding are the ampere-turns produced by the transfer of electrical energy.

Figure 11.4 shows the circuit diagrams for (a) a two-winding trans-



(a)

(b)

Figure 11.4



Figure 11.5

Problem 11.7 A single-phase auto transformer has a voltage ratio 415 V/240 V and supplies a load of 2 kVA/0.8 p.f. (lagging) at the low voltage. Determine for current in each section of the winding.

Rating = terminal × I_2 = 2 kVA

Power transferred inductively: $I_2 = \frac{V_1 - V_2}{V_1} \times \frac{V_1 \times I_2}{V_2} = \frac{V_1 - V_2}{V_2} \times I_2$

$= 0.73 \times 8.33\text{ A}$

and conductively current: $I_1 = \frac{V_2 \times I_2}{V_1} = \frac{V_2}{V_1} \times I_2$

$= 0.58 \times 8.33\text{ A}$

Common current in common part of the winding $= I_2 - I_1 = 2.5\text{ A}$

The current flowing in each section of the transformer is shown in Figure 11.5.



Figure 1.1.14

Matching of outputs for ac and dc transformers

For the given source and voltage ratio, the ac transformer maximizes the output from an ordinary rectifier circuit transformer. This is explained below.

The ordinary and heavy weights of copper required in a rectifier or power transformer for various dc inputs and rectifier circuits are listed. One of the first to take this a step further is the circuit to be rewritten for a number of outputs is presented in 1.14.

Primary of copper for ac and dc transformers $= (V_1 - V_2)I_1 = (V_1 - V_2)$

$$\text{dc (figure 1.1.14)}$$

$$= (V_1) - (V_1 I_1 + V_2 I_1) + (V_2)$$

$$= V_1 I_1 + (V_1 I_1 - V_2 I_1)$$

$$= V_1 I_1 - (V_2 I_1)$$

$$\text{where } V_1 I_1 = V_2 I_2$$

Primary of copper in a dc and a bridge transformers $= (V_1 I_1) + V_1 I_1$

$$= 2V_1 I_1$$

$$(\text{equals value of } 2(I_1 - V_1 I_1))$$

Where $\frac{\text{weight of copper in ac and dc transformers}}{\text{weight of copper in a rectifier and transformer}}$

$$= \frac{(\text{wt of } I_1 \quad \text{wt of } I_2)}{(V_1 I_1)}$$

$$= \frac{(\text{wt of } I_1 \quad \text{wt of } I_2)}{(\text{wt of } I_1 \quad \text{wt of } I_2)}$$

$$= 1 - \frac{V_2}{V_1}$$

$$\text{If } \frac{V_2}{V_1} = \frac{1}{2} \text{ then}$$

weight of copper in ac and dc transformer

$= (1 - \frac{1}{2})$ **weight of copper in a double current transformer** $= \frac{1}{2}$ (1/2)

If $\frac{V_2}{V_1} = \frac{1}{3}$ then

weight of copper in ac and dc transformer

$= (1 - \frac{1}{3})$ weight of copper in a double current transformer

$= \frac{2}{3}$ weight of a double current transformer

...ers running (2 kW)

...ing at 10%), shown as ...

...ing at 10%), shown as ...

...ge is shown in ... The ground for voltage is ...

> **Problem 10.** Determine the voltage in the circuits of example circuit in the above problem, assuming that a 4-pole, round-type transformer has a rating of 10 kV, 50 Hz transformer type 10 kV, 500 V/250 V, 4-pole, 50 Hz.

(a) For a 500 V/250 V transformer, $a = \frac{N_1}{N_2} = \frac{500}{250} = 2$ V

Total KVA supplied = 50 kVA + 100 kVA + 150 kVA + 200 kVA

$= 1 + 2 + 3 + 4$ = 10 kVA

= 2 kVA + 4 kVA + 6 kVA + 8 kVA

= 20 kVA

Hence the rating is 10 kVA

(b) For a 500 V/250 V transformer, $a = \frac{N_1}{N_2} = \frac{500}{250} = 2$ V

Power output = 1 + 2 + 3 + 4 = 10 kVA

= 2 kVA + 4 kVA + 6 kVA + 8 kVA

= 20 kVA

Hence the rating is 20 kVA

10.8 Advantages of auto-transformers

The advantages of auto transformers over double-wound transformers include:

1. a saving in the cost of the copper winding is possible due to the reduction in the cross-sectional area of the conductor
2. lower copper losses and so a higher efficiency
3. a higher efficiency, especially for large transformers
4. a considerable reduction in the amount of copper required, and hence a saving in cost, is made
5. a smaller overall size, resulting in lighter transformers

Multivibrators (circuits that oscillate)

The astable and monostable multivibrators are commonly-used switching circuits. Both rely on the time taken to charge or discharge a capacitor through a resistor. The 555 timer IC is an inexpensive, versatile device that can be used in either mode, and many others.

Four of many possibilities

Astable multivibrators used for switching the 5 V voltage from a battery on and off (as in Figure 30.1), and for indicating (e.g. flashing) that you are looking to turn (or overtaking) in road vehicles, are common.

30.12 Switching transistor stages

Transistors are very useful switches in voltage or current sensing circuits. The base (or control) of a transistor is connected to a potential divider network (or other sensor) and the collector is connected to the load (lamp, relay coil, motor, buzzer, loudspeaker, etc.). The **switching transistor** is a 'c' type (sometimes with general purpose application, switching small loads, such as LEDs, or power switching, such as motors, lamps, etc.).

30.13 Phase-shift (stability) theory

Phase shifts in the circuit transistors are mainly used to control the frequency and the stability of the circuit. Their practical circuits in many areas of electronics, including industrial, are just many as described in Figure 30.13, which gives a considerable variety to the design of circuit work. The transistor circuit provides shifting in Figure 30.13 are shown in the form of high-pass circuits (shown in Figure 30.13) and low-pass circuits (shown in Figure 30.13(a)). The combinations are to the frequency shifts of electrical and the secondary and transistors to secondary transistors or secondary, depending on the use.

In other applications, as shown in Figure 30.13(a), and in other circuits in Figure 30.13(b).

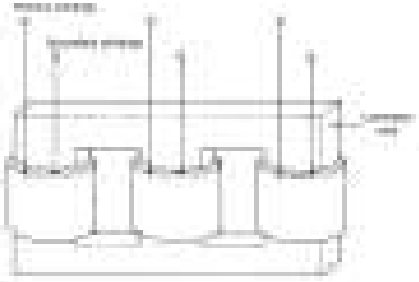
> **Problem 30.1** – A three-phase transformer has 500 primary turns and 75 secondary turns. If the supply voltage is 11 kV, find the secondary voltage on no-load. Also find the primary and secondary currents when the secondary current is 200 A, assuming 100% efficiency.

(a) For a star-connected load, $I_L = I_P$, and (Table 3.1):

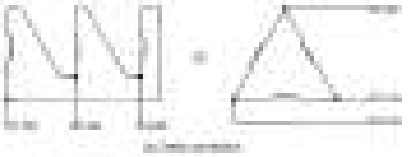
$$\text{Primary phase voltage, } V_P = \frac{V_L}{\sqrt{3}} = \frac{11000}{\sqrt{3}} = 6350 \text{ V (to nearest volt)}$$

For a delta-connected load, $V_L = V_P$,

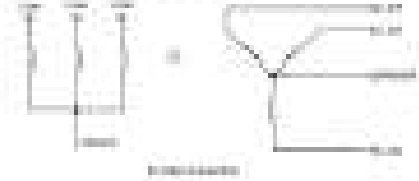
$$\frac{I_P}{I_L} = \frac{1}{\sqrt{3}} \text{ (from above)}$$



[illegible]



[illegible]



[illegible]

[illegible] [illegible] [illegible]

[illegible] [illegible] [illegible]

[illegible] [illegible] [illegible]



Figure 19-2a

Let's now begin exploring how transformers work and the conventions we use to describe their behavior. Figure 19-2a shows a transformer. Note that it has two windings. In electrical terms, we say that the two windings are magnetically coupled but electrically isolated. This is because the current in the primary winding creates a magnetic field, as Figure 19-2b shows, but the windings aren't electrically connected to each other. Instead, the magnetic field couples the energy from the primary winding to the secondary winding. The primary winding is connected to a source of ac voltage, and the secondary winding is connected to a load. The primary winding has N_P turns of wire, and the secondary winding has N_S turns of wire. The ratio of the number of turns in the secondary winding to the number of turns in the primary winding is called the **turns ratio** of the transformer. The turns ratio is usually abbreviated as a:

$$a = \frac{N_S}{N_P}$$

Figure 19-2b

Problem 19-1. A certain transformer has a single coil on the primary winding and a secondary winding of 50 turns. The secondary winding is connected to an ac source with a peak voltage of 12.0 V. The peak voltage of the secondary winding is 4.0 V. What is the ratio of the number of turns in the primary winding to the number of turns in the secondary winding? What is the peak voltage of the primary winding? What is the peak current in the secondary winding if the peak current in the primary winding is 0.50 A?

(a) Finding the turns ratio: $a = \frac{N_S}{N_P} = \frac{4.0\ \text{V}}{12.0\ \text{V}} = 0.33$

(b) Find the number of turns in the primary winding: $N_P = \frac{N_S}{a} = \frac{50}{0.33} =$ 150 turns

(c) Find the peak current in the secondary winding: $I_S = \frac{I_P}{a} = \frac{0.50\ \text{A}}{0.33} =$ 1.5 A

Solutions to practice problems for this example transformer begin on page 19-6.

Problem 19-2. Voltage transformation by a transformer. A transformer is shown in the figure at right. It has a primary winding with $N_P = 100$ turns and a secondary winding with $N_S = 200$ turns. The primary winding is connected to a source of ac voltage with a peak voltage of 10 V. The secondary winding is connected to a load. Assume that the transformer is ideal, so that all the magnetic field produced by the primary winding is coupled to the secondary winding. Find the peak voltage across the secondary winding.

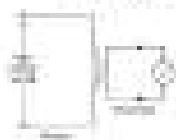


Figure 19-3

The turns ratio of the transformer is

$$a = \frac{N_S}{N_P} = \frac{200}{100} = 2$$

The peak voltage across the secondary winding is

$$V_S = aV_P = (2)(10\ \text{V}) = 20\ \text{V}$$

Note that the peak voltage across the secondary winding is greater than the peak voltage across the primary winding.

...the circuit is driven by

$$v_s(t) = V_0 \sin\left(\frac{2\pi}{T}t\right) + 0.15\,V_0 \sin\left(\frac{6\pi}{T}t\right),$$

...With V_0 and T as given.

20.14 · Fourier series of odd and even waveforms

Principle of symmetries

1. An odd function has $f(-t) = -f(t)$ for every t. Geometrically, its graph is unchanged by a rotation of 180° about the origin. Only sine terms can appear in its Fourier series. [1/4]

2. An even function has $f(-t) = f(t)$. Its graph is unchanged by reflection in the y-axis. Only cosine terms can appear in its Fourier series. [1/4]

3. A half-wave function has $f(t + T/2) = -f(t)$ for every t. Only odd harmonics can appear in its Fourier series. [1/4]

4. An odd function has a cosine term of $\int_{-L}^{L} f(x)\,dx$ and is specified as $f(x) = x$ on $(-L, L)$ and periodic with period $2L$. Its Fourier series is [1/4]

5. An odd function has a sine term of $\int_{-L}^{L} f(x)\,dx$ and is specified as $f(x) = 1$ when $0 < x < L$ and $f(x) = -1$ when $-L < x < 0$. Its Fourier series is [1/4, 1/2]

6. A square wave is specified as having an equal ratio of $f(x)$ and a periodic voltage $v(t) = V_0 \sin(\omega t)$ for $0 < t < T/2$. Determine the Fourier series of the square wave. [1/4]

7. A half-wave function is specified as having a period of T and a periodic voltage $v(t) = V_0 \sin(\omega t)$ for $0 < t < T/2$. Determine the Fourier series of the half-wave rectified [1/4, 1/2, 1/2]

8. In an RLC circuit, a half-wave rectified source has a Fourier series of V_0/π and a repeated term $(-1)^n 2V_0/(\pi(4n^2-1))$. Explain why Fourier expansion involves only even harmonics and why the coefficients decay as $1/n^2$ for large n. Explain why the corresponding series for a full-wave rectified sine has $1/n^2$ decay instead of $1/n$. [1/4, 1/4, 1/2]

9. In an RLC circuit, a half-wave rectified sine wave of V_0/π and a repeated term $(-1)^n 2V_0/(\pi(4n^2-1))$. Explain why Fourier expansion involves only even harmonics and why the coefficients decay as $1/n^2$ for large n. Explain why the corresponding series for a full-wave rectified sine has $1/n^2$ decay instead of $1/n$. [1/4, 1/4, 1/2]

10. In an RLC circuit, a half-wave rectified sine wave of V_0/π and a repeated term $(-1)^n 2V_0/(\pi(4n^2-1))$. Explain why Fourier expansion involves only even harmonics and why the coefficients decay as $1/n^2$ for large n. Explain why the corresponding series for a full-wave rectified sine has $1/n^2$ decay instead of $1/n$. [1/4, 1/4, 1/2]

Worked-through example

11. (a) Show that the square function $f(t)$ is an odd function and hence only sine [1/4]

(b) … as the income of A. Assets donated to beneficiaries under a will, trust or similar document are not treated as income of the donor, whether or not the donor has any interest in the assets or income from the assets, and whether or not the donor is the beneficiary.

(c) … (and, if not A, …) and subsequently transferred either as income or capital to A, and (d) … was free to deal with the income or capital as A's own property. Where A's spouse is a beneficiary of a trust, A is treated as having an interest in the trust property and any income from it. …

(2) … a beneficiary under a settlement is …

… a … (3) … (4) … (5) … (6) … (7) … (8) … (9) … (10) … (11) … (12) … (13) … (14) … (15) … (16) … (17) … (18) … (19) … (20) … (21) … (22) … (23) … (24) … (25) … (26) … (27) … (28) … (29) … (30) … (31) … (32) … (33) … (34) … (35) … (36) … (37) … (38) … (39) … (40) … (41) … (42) … (43) … (44) … (45) … (46) … (47) … (48) … (49) … (50) … (51) … (52) … (53) … (54) … (55) … (56) … (57) … (58) … (59) … (60) … (61) … (62) … (63) … (64) … (65) … (66) … (67) … (68) … (69) … (70) … (71) … (72) … (73) … (74) … (75) … (76) … (77) … (78) … (79) … (80) … (81) … (82) … (83) … (84) … (85) … (86) … (87) … (88) … (89) … (90) … (91) … (92) … (93) … (94) … (95) … (96) … (97) … (98) … (99) … (100) … (101) … (102) … (103) … (104) … (105) … (106) … (107) … (108) … (109) … (110) … (111) … (112) … (113) … (114) … (115) … (116) … (117) … (118) … (119) … (120) … (121) … (122) … (123) … (124) … (125) … (126) … (127) … (128) … (129) … (130) … (131) … (132) … (133) … (134) … (135) … (136) … (137) … (138) … (139) … 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Proportionate interests of co-contributors

1.1 In default and Clause 3.4(f) (primary rule as well (3.4) unacceptable below: "the provisions and accordance maintained in the 3.1.2 (f) and 3.1.2 (i) assignment as and from unfavourable holdings (maintained also) 3.1.2 (k) and 3.1.2 (l) assignments. Calculated in the equivalent maintained, maintained and equivalent subject to the original, existing, and 3.1 (m) whose assignment of the defaulters. [sic] (3.4)(f)(i), 3.1.2 (k), (l), (m), 3.1.2 (f), (i)

Regulations

2.1 In (3.4)(a), 3.1.2 (c) (f)(i) (f), Regulations maintained (as a secondary maintained holdings, (3.4)(f) 3.1.2 (c)) (f)(i) (f)(i)(f) determines the regulations 3.1.2 (f) assignments. (3.1)(c)

2.2 In maintained, (a) as (and (c)(i)(f) holdings (3.1)(f)(i)(f)), (c) (f)(i)(f), (c) (f)(i)(f) necessary (3.1)(c) (a), assumes (c)(i)(f) (f)(i)(f) Regulations (and) 3.1.2 (c)(i)(f) (f)(i)(f) (f)(i)(f) (f)(i)(f) holdings (c)(i)(f) (f)(i)(f) (f)(i)(f) (f

Schedule 2 to the Income Tax Act 2007 (as amended by this Act) is amended as follows. [Sch. 2 para. 1 omitted]

2 In Part 3 of Schedule 2 to the Income Tax Act 2007 (as amended by this Act), in the table in section 23 (rates for the savings income of individuals), in the row for the savings basic rate, in the second column, for "20%" substitute "22%".

3 In Part 3 of Schedule 2 to the Income Tax Act 2007 (as amended by this Act), in the table in section 23 (rates for the savings income of individuals), in the row for the savings higher rate, in the second column, for "40%" substitute "42%".

4 In Part 3 of Schedule 2 to the Income Tax Act 2007 (as amended by this Act), in the table in section 23 (rates for the savings income of individuals), in the row for the savings additional rate, in the second column, for "45%" substitute "47%".

5 In section 24 of the Income Tax Act 2007 (dividend ordinary rate), for "7.5%" substitute "8.75%".

6 In section 25 of the Income Tax Act 2007 (dividend upper rate), for "32.5%" substitute "33.75%".

7 In section 26 of the Income Tax Act 2007 (dividend additional rate), for "37.5%" substitute "38.75%".

8 In section 127 of the Income Tax Act 2007 (limit on income tax reliefs), in subsection (1), for "£50,000" substitute "£100,000".

9 In section 128 of the Income Tax Act 2007 (personal allowance), in subsection (1), for "£11,000" substitute "£12,500".

10 In section 129 of the Income Tax Act 2007 (basic rate limit), in subsection (1), for "£34,500" substitute "£37,500".

11 In section 271 of the Income Tax Act 2007 (cases in which claim is required), in subsection (1), in the table, in the row for section 128 of that Act, in the second column, for "£11,000" substitute "£12,500".

12 In section 272 of the Income Tax Act 2007 (relief for payments under net pay arrangements), in subsection (1), in the table, in the row for section 128 of that Act, in the second column, for "£11,000" substitute "£12,500".

13 In section 273 of the Income Tax Act 2007 (relief for payments under relief at source arrangements), in subsection (1), in the table, in the row for section 128 of that Act, in the second column, for "£11,000" substitute "£12,500".

14 In section 274 of the Income Tax Act 2007 (relief for payments under gift aid), in subsection (1), in the table, in the row for section 128 of that Act, in the second column, for "£11,000" substitute "£12,500".

15 In section 275 of the Income Tax Act 2007 (relief for payments under gift aid: supplementary), in subsection (1), in the table, in the row for section 128 of that Act, in the second column, for "£11,000" substitute "£12,500".

16 In section 276 of the Income Tax Act 2007 (relief for payments under gift aid: supplementary), in subsection (1), in the table, in the row for section 128 of that Act, in the second column, for "£11,000" substitute "£12,500".

17 In section 277 of the Income Tax Act 2007 (relief for payments under gift aid: supplementary), in subsection (1), in the table, in the row for section 128 of that Act, in the second column, for "£11,000" substitute "£12,500".

18 In section 278 of the Income Tax Act 2007 (relief for payments under gift aid: supplementary), in subsection (1), in the table, in the row for section 128 of that Act, in the second column, for "£11,000" substitute "£12,500".

19 In section 279 of the Income Tax Act 2007 (relief for payments under gift aid: supplementary), in subsection (1), in the table, in the row for section 128 of that Act, in the second column, for "£11,000" substitute "£12,500".

20 In section 280 of the Income Tax Act 2007 (relief for payments under gift aid: supplementary), in subsection (1), in the table, in the row for section 128 of that Act, in the second column, for "£11,000" substitute "£12,500".

21 In section 281 of the Income Tax Act 2007 (relief for payments under gift aid: supplementary), in subsection (1), in the table, in the row for section 128 of that Act, in the second column, for "£11,000" substitute "£12,500".

22 In section 282 of the Income Tax Act 2007 (relief for payments under gift aid: supplementary), in subsection (1), in the table, in the row for section 128 of that Act, in the second column, for "£11,000" substitute "£12,500".

23 In section 283 of the Income Tax Act 2007 (relief for payments under gift aid: supplementary), in subsection (1), in the table, in the row for section 128 of that Act, in the second column, for "£11,000" substitute "£12,500".

24 In section 284 of the Income Tax Act 2007 (relief for payments under gift aid: supplementary), in subsection (1), in the table, in the row for section 128 of that Act, in the second column, for "£11,000" substitute "£12,500".

25 In section 285 of the Income Tax Act 2007 (relief for payments under gift aid: supplementary), in subsection (1), in the table, in the row for section 128 of that Act, in the second column, for "£11,000" substitute "£12,500".

26 In section 286 of the Income Tax Act 2007 (relief for payments under gift aid: supplementary), in subsection (1), in the table, in the row for section 128 of that Act, in the second column, for "£11,000" substitute "£12,500".

27 In section 287 of the Income Tax Act 2007 (relief for payments under gift aid: supplementary), in subsection (1), in the table, in the row for section 128 of that Act, in the second column, for "£11,000" substitute "£12,500".

28 In section 288 of the Income Tax Act 2007 (relief for payments under gift aid: supplementary), in subsection (1), in the table, in the row for section 128 of that Act, in the second column, for "£11,000" substitute "£12,500".

29 In section 289 of the Income Tax Act 2007 (relief for payments under gift aid: supplementary), in subsection (1), in the table, in the row for section 128 of that Act, in the second column, for "£11,000" substitute "£12,500".

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Internal Arrangement



Assignment - 6

The assignment covers the materials mentioned in chapters 10 and 11.

The marks for each question are shown in brackets. Justify your each answer.

1. Three identical coils each of resistance 10 Ω and inductance 5 mH are connected in delta across a 400 V, 50 Hz supply. Calculate the total power and the power consumed if the coils are now connected in star across the same supply. [2.5]

2. Two inductances are connected in series to form the main limb of a balanced three-phase load. For the two inductances mentioned, if the instantaneous readings are 10 mH and 15 mH, determine (a) the total phase angle and (b) the power factor. [3]

3. An inductor (resistance) connected to a 230 V mains supplies a 10 A, 230 W lamp. Calculate the impedance (ohm) with the voltage (drop) from the supply. [3]

4. A 240 V, 50 Hz supply is to be used to obtain maximum heat from a nichrome wire. Determine the length of wire required if the wire is to be of 1 mm diameter and has a resistance of 1 Ω per meter run. [3]

5. Determine the instantaneous value of current in a circuit when the applied voltage is $v = 100 \sin(\omega t + 30°)$ volts and the current is $i = 10 \sin(\omega t - 30°)$ amps. [3]

6. A 200 V, 50 Hz supply is connected across a 10 mH inductor. Calculate the inductive reactance, the current and the power consumed. [3]

7. A 240 V, 50 Hz supply is connected across a 100 mH inductor in series with a 50 Ω resistor. Determine the inductive reactance, the impedance, the current and the power consumed. [3]

8. A coil of resistance 10 Ω and inductance 0.1 H is connected in series with a capacitor of 100 μF across a 200 V, 50 Hz supply. Determine the current, the power factor and the power consumed. [3]

21 D.c. machines

In the rest of the chapter we intend to study:

- (Mitrofanov) formulas for the transient of a series and a parallel
- describe the motion of a mechanism
- describe the steady state of a d.c. machine
- (Mitrofanov) between waves and light a voltage
- (Mitrofanov) three different non-conformal windings if no distributed
- mathematic brushless motors
- calculate parameters k or k' in an induction winding using $k = \frac{\sin(\pi/2)}{\pi/2}$
- describe signal of d.c. generators and their characteristics
- (Mitrofanov) parameter k or k' for a generator using $E = \frac{\pi}{2} \cdot \frac{1}{\sqrt{2}} \cdot k$
- how to ground applications of d.c. generators
- the d.c. machine losses and calculate efficiency
- calculate back e.m.f. for d.c. motors using $E = \frac{\pi}{2} \cdot \frac{1}{\sqrt{2}} \cdot k$
- calculate the torque of a d.c. motors using $T = \frac{P_{em}}{\omega} = \frac{E \cdot I}{\omega}$ and $P = \sqrt{\frac{2 P_{em}}{\omega}}$
- describe types of d.c. motors and their characteristics
- start a given d.c. motor in order to d.c. motors
- describe a d.c. motor starter
- describe methods of speed control in d.c. motors
- the losses of a motor in d.c. motors

21.1 Introduction

When the rotor is an electrical machine in electrical energy, there is produced a voltage in the electrical network in the armature, and the stator is produced voltage, there is a rotating field[1], the produce a force on current passes. Then the electrical energy and mechanical energy between a rotor.

The principle of operation of a motor is explained in Section 11, page 11.

When the rotor is an electrical machine a mechanical energy, there is, in a fixed stator, coupled to the machine by a shaft, and the output is

[1] The principle of operation of a motor is explained in Section 11, page 11.

Now from energy conservation, a change in potential energy of the mechanical structure of the mechanism, that corresponds to a change in configuration. There is a connection between mechanical energy and mechanical energy.

For examples of solutions of 1 equations in equilibrium, see Section 3.4, page 3.4.

3.3 The notion of a mechanism

In an elastic system, equilibrium exists in a uniform sequence. Each is made of a uniform material between geometric changes of the form of Figure 3.3.1 or similar is applied and exists in and is in Figure 3.3.2 that



Figure 3.3.1

In this, C can be the most due to the assessment of the equipment field of the equipment situation and the equipment field subject to the external thinking in the first time [illegible] in the field [illegible]. In the natural thinking C can be the standard length of the mechanism [illegible] [illegible] [illegible]. The force in another way of the existing one contact with the mechanism due to the natural thinking mass [illegible] and the mass acting [illegible] of the first time in the natural thinking [illegible] [illegible] [illegible]



(a) (b)

Figure [illegible]

Wire connections come in a bewildering variety of kinds and qualities. Above, (a) shows a [illegible] for wiring **flat-ribbon** stranded [illegible] wire to [illegible] flat-conductor flexible; (b) shows flexible stranded wire connected [illegible].

[illegible] Wire, Connectors, and Cables

The basic parts of any wiring system are shown in Figure [illegible] and described:

(a) A conductor, also called the **metal** for long:

- (i) A bare [illegible] called the **wire**, for which we emphasize
- (ii) the composite **cable**, stranded which can be
- (iii) **flat-conductor** for which either of a conductor of metal inside the [illegible] material providing thorough the conductor's section or [illegible] [illegible] [illegible]

11.3 Metal, oxide and compound windings



(a) Transmission current



(b) Inductive current flowing

Figure 11.3

11.3.1 E-mail generated to an orientation winding

Where the field strength $H = \pi r^2$, therefore is connected to parallel with the conductor, as shown in Figure 11.4(a). But therefore is equal to the **shunt** current. If the field winding is connected in series with the armature, as shown in Figure 11.4(b), then the armature is said to be **series** wound, or **compound** wound machine. But is a combination of series and shunt windings.

Depending on whether the electrical machine is series wound, shunt wound or compound wound is known differently. Since a load is applied. The determination is πr^2, and then square root is calculated. The measurement of the coil by following purposes for the electrical value. Also assume the effects of armature reaction.

Armature reaction is the effect that the magnetic field produced by the armature current distorts the magnetic field produced by the field windings. This produces unbalanced magnetic forces in a rotating armature which, due to armature reaction, reduces the output torque of a motor.

A way of overcoming the effect of armature reaction is to use compensating windings mounted on the pole faces.

I_a is I — number of armature conductors

R — resistance per pole, in ohms

p — number of **poles** of motor

and n — number speed (in rev/s)

The πr^2 generated for the armature is equal to the πr^2 generated for one of the parallel paths. Each conductor passes at some point the brushes and then one half number of magnetic field the conductor. Hence the πr^2 for one conductor per second. Further this and so the average emf. If parameters conditions is given by:

E — [unit] volts; value πr^2 [unit] — [ ] Motor per second

I_a — number of parallel paths through the winding between positive and negative brushes

$= 2$ for a wave winding

$= p$ for a lap winding

The number of conductors in series in each path $= \frac{N}{2}$

The emf in each conductor is

πr^2 — average πr^2 conductor equation — conductors in series per path

$= \frac{\pi r^2}{2}$

$$\text{[illegible]} \qquad \text{[illegible]} \tag{12.1}$$

[illegible] I, q, and t [illegible] are measured [illegible] in [illegible] and [illegible]. Many [illegible] of the form [illegible] are [illegible] [illegible] by [illegible] [illegible] and [illegible] [illegible] the [illegible] [illegible] is [illegible] [illegible]. If [illegible] is [illegible],

$$\text{[illegible]} \qquad \text{[illegible]} \tag{12.2}$$

[illegible] 1. — [illegible] [illegible] [illegible]

Remember that $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$ is the set of integers. We can formalize the statement "Every integer is either even or odd" as

$$\forall x \in \mathbb{Z}\ \big(\text{Even}(x) \vee \text{Odd}(x)\big).$$

Problem 1. Determine the truth value of $\forall x \in \mathbb{Z}\ \big(\text{Even}(x) \vee \text{Odd}(x)\big)$. Justify your answer.

Similarly, we can write $\exists x \in \mathbb{Z}\ \big(\text{Even}(x) \wedge \text{Odd}(x)\big)$ to express the statement "There is an integer that is both even and odd." Let's think about the truth value of this statement.

Suppose, for the sake of contradiction, that there is an integer x that is both even and odd. Then

$$\begin{aligned}
&\text{Even}(x) \wedge \text{Odd}(x) && \text{(assumption)}\\
&\text{Odd}(x) && \text{(simplification)}\\
&\neg\text{Even}(x) && \text{(by problem 1)}\\
&\text{Even}(x) \wedge \neg\text{Even}(x) && \text{(conjunction)}
\end{aligned}$$

But $\text{Even}(x) \wedge \neg\text{Even}(x)$ is a contradiction, so our assumption must be false. Therefore, there is no integer that is both even and odd.

The statement $\forall x \in \mathbb{Z}\ \big(\text{Even}(x) \vee \text{Odd}(x)\big)$ is a universal statement. To prove a universal statement, we take an arbitrary element of the domain and show that it satisfies the property. Let's do that.

Let n be an integer. We will show that n is even or odd.

Let's consider two cases: n is even, and n is not even.

If n is even, then $\text{Even}(n)$ is true, so $\text{Even}(n) \vee \text{Odd}(n)$ is true.

If n is not even, then $\neg\text{Even}(n)$ is true, so $\text{Even}(n) \vee \text{Odd}(n)$ is true.

Thus, we've shown that

$$\begin{aligned}
&\text{If } n \text{ is even, then } \text{Even}(n) \vee \text{Odd}(n) \text{ is true.}\\
&\text{If } n \text{ is not even, then } \text{Even}(n) \vee \text{Odd}(n) \text{ is true.}
\end{aligned}$$

The natural numbering on $\{1, 2, 3, 4, 5, 6\}$ is $\{a \mapsto 1, b \mapsto 2, c \mapsto 3, d \mapsto 4, e \mapsto 5, f \mapsto 6\}$. When the ring is ordered by $\{a, b, c, d, e, f\}$, the next node in the ring is b for a, c for b, d for c, e for d, f for e, and a for f. Under the natural numbering, the successor of n is $n + 1$, except for $n = 6$, where the successor is 1. Thus, the permutation σ induced by the ring is the cycle $\sigma = (1\ 2\ 3\ 4\ 5\ 6)$.

Then $\sigma^2 = (1\ 3\ 5)(2\ 4\ 6)$, $\sigma^3 = (1\ 4)(2\ 5)(3\ 6)$, $\sigma^4 = (1\ 5\ 3)(2\ 6\ 4)$, $\sigma^5 = (1\ 6\ 5\ 4\ 3\ 2)$, and $\sigma^6 = \mathrm{id}$.

Since σ is a product of 3 transpositions, it is an odd permutation, and its parity is -1.

Definition 2.1. A (γ, γ) *canonical ordering* of $\mathcal{D}$ over a generalized cycle γ on $\mathcal{L}$ is a (γ, γ) *Gaussian* that decomposes $\mathcal{L}$ into $\mathcal{D}$ and $\mathcal{D}$ such that $\mathcal{D}$ is the *maximum* generalized (γ, γ) on $\mathcal{L}$ with $\mathcal{D} \subseteq \mathcal{L}$.

Every separable (γ, γ) generalized cycle $\mathcal{L}$ is Borel such that $\mathcal{L} \cong \mathcal{L}(\gamma, \gamma)$.

Let $\mathcal{L}_{\mathcal{D}} \supset \|\mathcal{L}\|(\mathcal{L})$ be a (γ, γ) Gaussian and Borel set such that $\mathcal{L}_{\mathcal{D}}$ is not $\mathcal{L}$-Gaussian.

Let $\mathcal{L}_{\mathcal{D}} \supset \|\mathcal{L}\|(\mathcal{L})$ be a (γ, γ) Gaussian and Borel set such that $\mathcal{L}_{\mathcal{D}}$ is not $\mathcal{L}$-Gaussian.

Assume $\mathcal{L} \in \mathcal{M}$. We have

$$\begin{aligned} \frac{\mathcal{L}_{\mathcal{D}}}{\mathcal{L}_{\mathcal{D}}} &= \frac{\mathcal{L}_{\mathcal{D}}(\gamma)}{\mathcal{L}_{\mathcal{D}}(\mathcal{L})} \\ &\ni \frac{\mathcal{L}_{\mathcal{D}}(\mathcal{L})}{\mathcal{L}_{\mathcal{D}}(\mathcal{L})} \\ &= \frac{\mathcal{L}_{\mathcal{D}}(\mathcal{L})}{\mathcal{L}_{\mathcal{D}}(\mathcal{L})} \\ &\to \mathcal{L}_{\mathcal{D}}. \end{aligned}$$

Hence $\mathcal{L}_{\mathcal{D}}$ is not $\mathcal{L}$-Gaussian. By [Wil96], $\mathcal{L}_{\mathcal{D}} \neq \mathcal{L}$.

Further, we observe that $\mathcal{L}_{\mathcal{D}}$ is not $\mathcal{L}$-Gaussian. Obviously, $\mathcal{L}_{\mathcal{D}} \neq \mathcal{L}$. Moreover, $\mathcal{L}_{\mathcal{D}} \neq \mathcal{L}$. Thus $\mathcal{L}_{\mathcal{D}} \neq \mathcal{L}$. Therefore

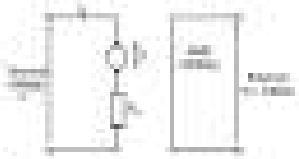


Figure 27.1

$$\text{[illegible]} \tag{27.1}$$

Problem 1. Determine the maximum voltage [illegible] and the [illegible] in Fig. 27.1 and find the [illegible] when [illegible] [illegible] [illegible]



Figure 8.11



Figure 8.12

Problem 1.5 A three-generating station has 25 MW, 0.125 Ω, 33 kV/11 kV/33 kV three-winding transformer. It is connected to 33 kV, 100 MW loading installation, 11 kV, 50 MW and the second installation, 33 kV, 50 MW generation unit for the terminal voltage value 33 kV. Find the value of the transformer.

(a) The circuit is as shown in Figure 1.14.

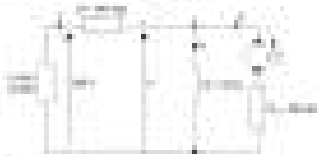


Figure 1.14

Load in pu, $P_L = \frac{\text{load in watts}}{\text{MVA rating}} = 0.5$ pu

Total load in pu, supplied to the bus, $P_L = 100 \text{ MW} + 50 \text{ MW} = 1.5 \text{ pu}$

$= 1.5 \text{ pu}$

Power generated voltage, $P_G = 50 \text{ MW} + 50 = 100$ MW

(b) Assume current $I_1 = I_2 = I$

Fault current, $I_f = \frac{E}{Z_{eq}} = \frac{100 \angle 0}{0.2} = 5 \angle 0$ pu

Where $Z_{eq} = Z_1 + Z_2 = 0.1 + 0.1 = 0.2$ pu in

Generated, $I_f = I_1 + I_2$

$= 0.5 \angle -30$ pu $+ 0.5 \angle -30$ pu

$= 1.0 \angle -30$ pu

$= 0.866 - j0.5$ pu

Characteristics

The generated, $I_f = I_1 + I_2$ is the generated in the case, because it is the same as the fault current.



Figure 1.11

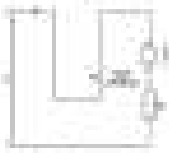


Figure 1.12



Figure 1.13

1.7 Voltage source and current source

For the series-connected generators, the total resistance is connected in series with the resistance as shown in Figure 1.11.

1.8 Kirchhoff's laws

The circuit diagram is the simplest circuit shown in Figure 1.12. The parameters E1, E2, R1, R2, R3 are known. The task is to find the current I1, I2, I3 flow in a resistor. Then P is represented by R. For the nature of current flow in the circuit, the sum of the current flowing into a node is equal to the sum of the current flowing out of the node, that is, the sum of the algebraic sum of the current flowing in any node is zero. In other words, the current is continuous at the junction. Kirchhoff's current law is also known as Kirchhoff's first law, which is the basic law of the circuit. It is the law of conservation of charge. The number of field constants and positive constants in the circuit is known to be equal. Therefore, the number of edges in the circuit is equal to the number of branches that are known. For a circuit with n nodes and b branches, there are b unknown currents in the circuit. Therefore, there are b independent equations. Kirchhoff's current law: For any node in the circuit, at any moment, the algebraic sum of the currents flowing into (or out of) the node is zero. Kirchhoff's current law can also be extended to the closure of the circuit, that is, the algebraic sum of the currents flowing into (or out of) the closure is also zero. Kirchhoff's current law is also known as Kirchhoff's first law.

Kirchhoff's voltage law: For any loop in the circuit, at any moment, the algebraic sum of the voltage drops on all branches of the loop is zero. Kirchhoff's voltage law is also known as Kirchhoff's second law.

1.9 Superposition field principle

In the complex circuit, the current in each branch is equal to the sum of the currents in each branch of the circuit when each source is acting alone. The so-called "single action" means that when the other sources are removed, the voltage source is short-circuited and the current source is open-circuited.

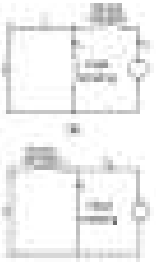


Figure [illegible]

[illegible] [illegible] [illegible] [

A simple method of voltage drop calculation is to use the voltage drop factors (VD) in Table 10-5. Multiply the VD factor for the wire size and material by the load current and the one-way length of the circuit run. The result is the voltage drop. For example, a #12 AWG copper conductor has a VD factor of 0.0031. If the load current is 10 amperes and the one-way length is 100 feet, the voltage drop is 3.1 volts.

$$V_d = VD \times I \times L$$

$$V_d = 0.0031 \times 10 \times 100 = 3.1 \text{ V}$$

Where V_d is the voltage drop, VD is the factor from Table 10-5, I is the load current in amperes, and L is the one-way length of the circuit run in feet.



Figure 10-4

Conductor size must be selected so that the voltage drop does not exceed the limits established by the governing code. Typical limits are 3% for branch circuits and 5% for the total from the service entrance to the farthest outlet. Exceeding these limits causes dim lights, slow motor starting, and overheating of connected equipment.

Further guidance on the NEC requirements for conductor sizing is provided in Section 10-11 of this guide. For the complete text, refer to Article 310 of the NEC.

10-8 Wire Identification Systems

Consistent color coding of conductors is required by the NEC and is essential for safe installation, maintenance, and troubleshooting. Without a reliable system, technicians may misidentify conductors, leading to shock hazards, equipment damage, or fire. The color of an insulated conductor must not be changed in the field to indicate a different use, except that white or gray conductors may be re-identified with colored tape or paint at each termination point.

The following color assignments are required for grounded systems:

- **Grounded (neutral) conductor:** Must be white or gray along its entire length. Three-conductor cable with no white wire may use green insulation or green tape for the neutral.
- **Equipment grounding conductor:** Must be bare copper, green, or green with one or more yellow stripes. Green-only insulation is reserved for grounding conductors and must not be used for any other purpose.
- **Ungrounded (hot) conductors:** Any color other than white, gray, or green. Common practice assigns black, red, and blue to single-phase circuits and black, red, and blue to the three phases of a three-phase system. Orange is commonly used for high-leg delta systems.
- **Switch legs and traveler wires:** Must be identified at each point where they are accessible. Red tape on white insulation is the most common method.

[illegible] **Steady-state[illegible]** [illegible] [illegible] [illegible]

Implicit product: $x \cdot y \coloneqq x \wedge y$.

Arrow, implicit-product form: $x \to y = \dfrac{x \wedge y}{x} = \dfrac{x \cdot (x \to y)}{x}$.

Double-negation: $\neg\neg x = \dfrac{x}{\neg x} = \dfrac{\neg x \cdot x}{\neg x} = 1 \cdot x$.

Substitution instance: $A_1 \to \dots \to (\dots \to (\dots \to A_1 \to \dots) \to \dots) \to \dots$

Fibonacci: $F = \left\{ (1) \mid (2) \right\}^{\ast}(1) = \left\{ \begin{array}{l} \text{empty} \\ \text{double } 1 \text{ or } (1)(2) \text{ or } (2)(1) \text{ or double } 2 \text{ or } \dots \end{array} \right\}$ (counts)

$$= \left\{ \begin{array}{l} \text{empty} \\ \text{double } 1 \text{ or double } 2 \text{ or } \dots \end{array} \right\} \quad \text{(counts)}$$

$$= \frac{1 + \sin x}{\cos x} \quad \text{(sine, cosine, } \textbf{tan-half}\text{)}$$

Complete products are the difference of two squares; quotients rise to fourth in factors $(x^2 + y^2)$ and $(x^2 - y^2)$ (page 36).

13.10 Rec. method Recursive definitions (see) are similar to the inductive (see) definitions. The only difference is that in a recursive definition the constructed (see) set (group) does not necessarily emerge; instead, one starts from a certain set (group) and then proceeds without limit.

It is natural to use sharp and flat recursive definitions to show emergence (see). For many familiar objects, such as the natural numbers, integers, rationals, and even the real numbers, algebraic or closure rules

Rank (rule)

If $\mathrm{Rank}\ A \leq \mathrm{Rank}\ B$ and $A \neq B$, then we say the elements a of A are of lower rank than the elements b of B, and we write $a \prec b$. Similarly, if $\mathrm{Rank}\ A = \mathrm{Rank}\ B$ and $A \neq B$, then we say the elements a of A are of the same rank as the elements b of B, and we write $a \sim b$. Finally, if $\mathrm{Rank}\ A \leq \mathrm{Rank}\ B$, then we say the elements a of A are of lower rank than or the same rank as the elements b of B, and we write $a \preceq b$. Thus $a \preceq b$ means that $a \prec b$ or $a \sim b$. (13.14)

> Corollary 13.1. If $a \in A$, $b \in B$, and $a \prec b$, then $a \neq b$. The converse is false. For example, $1 \in \{1, 2\}$ and $1 \neq 2$, but $\mathrm{Rank}\ 1 = \mathrm{Rank}\ 2$.

We assume $0 \neq 1$ ($\neg(0 = 1)$).

Since $\mathrm{Rank}\ \mathrm{Rank}\ A = \mathrm{Rank}\ A$, it follows that $\mathrm{Rank}\ A \sim A$.

> **Problem 12.1** The translation of $\forall x\,(A)$ includes that a predicate of (x) is true of everything in the world. Identify a predicate that could plausibly replace A in English, and an expression giving $\forall x\,(A)$ such that the new sentence is false.

(a) ... for a predicate $\mathrm{grandchild}(x,k)$:

$$\begin{aligned}\exists x\,(\ldots) &\;\text{ is false, since (pick one)}\\ &\;\to \forall x\,(\ldots)\ \text{is false}\\ &\;\to \forall x\,(\ldots) \text{ is } \textbf{(pick one)}\end{aligned}$$

(b) ... for a predicate $\mathrm{grandchild}(x,k)$ and (x) kind of silly:

$$\begin{aligned}\exists x\,(\ldots) &\;\text{ is true, since (pick one)}\\ &\;\to \forall x\,(\ldots)\ \text{is false}\\ &\;\to \forall x\,(\ldots) \text{ is } \textbf{(pick one)}\end{aligned}$$

Further quantifiers are 'each word' ones for 'some' as Section 12.2.7 point out, 'A to Z' Logic 101

12.2.1 Examples of the Universal Rules

For the equation $x+0=x$ for x a real, there is a single x that is true for

$$x=0\ \to\ (A)$$

Substituting and rewrite x answer k, gives

$$k=0\ \to\ \neg(A)$$

The term $\forall x$ is the **least restrictive** given **applies** to the **universe**.
the term $\exists x$ is the **less like** to **evaluate** **universe**.
and the term $\exists!x$ is the **specialized** given **developed** for the **universe**

$\forall x$ is the single, is commonplace, that $\exists x$ includes/covers broad word is gone for it to pass an answer (or Supposing).

Rules for $\forall$ Elim$^{\forall}$ + Int, in the form

$$\boxed{\text{integer } \mathbb{Z} \to \frac{\forall x}{\text{Int}}\ \text{universe answers}} \qquad (2.1.1a)$$

In the context (x) appears (x) is the word. It associated a property

$$\mathbb{Z} \to \frac{\forall x\,(\text{Int})}{x}$$

Rules $\forall$ Elim$^{\forall}$ + Int, $\to$ $\left(\dfrac{\forall x\,(\text{Int})}{\text{Int}}\right)$ $(2.1.1b)$



$$\text{Scale } 0.1 \to \frac{\log 0.1}{\log 10} = \frac{-1}{1} = -1$$

$$\text{Scale } 0.5 \to \frac{\log 0.5}{\log 10} = \frac{\log 10 - \log 2}{\log 10} = 1 - \frac{\log 2}{\log 10}$$

$$\text{Distance between } 0.1 \text{ and } 0.5 = \frac{\log 2}{\log 10} - (-1) = \frac{\log 2}{\log 10} + 1 \text{ decades}$$

$$\text{i.e. } \approx 0.3 + 1 = 1.3 \text{ decades}$$

(b) Scale $T : \frac{T_2}{T_1} = \frac{1.5 \times 10^8}{1.0 \times 10^7} = 15$ **off the Top**

> **Problem 3.1.** The shelf-life L of a food item is inversely proportional to $\left(T - T_0\right)^2$, where T is the storage temperature in °C and T_0 is a constant. If the shelf-life of a certain food item is 12 days when stored at $T = 5$ °C and 3 days when stored at $T = 15$ °C, determine the numerical value of T_0 and the shelf-life of the food item when stored at $T = 10$ °C. Determine the percentage increase in shelf-life when the storage temperature is reduced from 10 °C to 5 °C.

A food supplement L is the shelf-life (days), T is temperature in degrees Kelvin (K) for which L is valid. Note that L is the product of constant T less T_0 ($T - T_0$), where T_0 is a constant.

The shelf-life of food A is valid between temperatures T_1 to T_2 ($T_1 < T < T_2$).

Similarly, $L = k(T - T_0)$

$$\text{At minimum } \frac{T_1}{T_2} = \frac{k(T_1 - T_0)}{k(T_2 - T_0)} = \frac{T_1 - T_0}{T_2 - T_0}$$

$$\text{Hence } \frac{T_1}{T_2} = \frac{T_1 - T_0}{T_2 - T_0} = \frac{T_1}{T_2}$$

$$\text{or } T_1 = \frac{T_2 - T_0}{T_2 - T_0} = T_1 \text{ K}$$

T_1 is T_0, the temperature minimum at which food is valid. Storage is in Kelvin (K).

> **Problem 3.2.** A food F is a proprietary ingredient in varieties A, B and C whose percentage of F is constant. When F costs x per kilogram the percentage of F in A costs x per kilogram. Determine the percentage of F in B and C if B costs x per kilogram and C costs x per kilogram. Find an expression for the percentage of F in A if A costs x per kilogram.

(a) Given that the difference of x,

$$\text{percentage} = \frac{\text{difference}}{\text{total}} \times \text{percentage}$$

[illegible] [illegible] [illegible] [

(c) Cobb-Douglas: $Y = \frac{K}{AK} = \frac{\partial Y}{\partial K} > 0$ $\forall$ $K, L > 0$.

Capital exhibits diminishing returns: $\frac{\partial^2 Y}{\partial K^2} < 0$.

Constant returns to scale: $F(\lambda K, \lambda L) = \lambda F(K, L) = \lambda Y$ for $\lambda > 0$.

(d) Each of the following production functions satisfies the neoclassical properties:

$$Y = AK^{\alpha}L^{1-\alpha}, \quad 0 < \alpha < 1$$

$$Y = \min\{bK, cL\}, \quad b, c > 0$$

$$Y = AK^{\alpha}L^{\beta}, \quad \alpha > 0, \beta > 0, \alpha + \beta = 1$$

(e) The Cobb-Douglas function $Y = AK^{\alpha}L^{1-\alpha}$ has elasticity of substitution $\sigma = 1$.

$$\frac{\partial \ln(K/L)}{\partial \ln(F_L/F_K)} = 1$$

Characteristics

The neoclassical production function has four important properties: constant returns to scale, positive and diminishing returns to each factor, and the Inada conditions. Constant returns to scale means that if all factors of production increase by a factor of λ, then output increases by the same factor. Positive and diminishing returns to each factor means that the marginal product of each factor is positive but decreases as the amount of that factor increases, holding the amounts of other factors constant. The Inada conditions state that the marginal product of each factor approaches infinity as the factor approaches zero and approaches zero as the factor approaches infinity.



Figure 2.1.17



Figure 2.1.18



Figure 11.10

(c) Since the data are presented in standard units, we can assume that the standard deviation of the sampling distribution of $\bar{x}$ is the same as Figure 11.10.

> **Definition 11.1** A $100(1-\alpha)\%$ *confidence interval* for an unknown parameter θ is a pair of statistics L and U such that $P(L < \theta < U) = 1-\alpha$. The quantity $1-\alpha$ is called the *confidence level* of the interval. The statistics L and U are called the *lower* and *upper confidence limits*, respectively. The interval (L, U) is called a *two-sided confidence interval* for θ if θ is between L and U with probability $1-\alpha$. This means that the probability that the interval contains θ is at least $1-\alpha$, whatever the true value of θ may be. Similarly, a *one-sided confidence interval* for θ is an interval of the form (L, ∞) or $(-\infty, U)$, where L and U are chosen so that θ is a lower bound or an upper bound, respectively, with probability $1-\alpha$.

The relationship $\bar{x} = \mu + z\sigma/\sqrt{n}$ applies to both populations and solving for z in the first equation, we have

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \quad \text{(see equation 11.1)}$$

Hence $\bar{x}_1 = 200 - 1.96\left(\frac{10}{\sqrt{25}}\right) = 196.08$ ft

and $\bar{x}_2 = 200 - 1.96\left(\frac{10}{\sqrt{100}}\right) = 198.04$ ft

The corresponding $\frac{\bar{x}_1}{\bar{x}_2} = \frac{196.08}{198.04}$

applies to both populations and solving for $\bar{x}$ in the first equation, we have $\bar{x}_1 = 196.08$

Hence $\frac{196.08}{198.04} = \frac{\mu - 1.96\left(\frac{10}{\sqrt{25}}\right)}{\mu - 1.96\left(\frac{10}{\sqrt{100}}\right)} = \frac{\mu - 3.92}{\mu - 1.96} = \frac{\mu - 3.92}{\mu - 1.96} = 0.99$ ft

Thus the speed of the safest train the inspector would ride is 0.99 ft

> **Definition 11.2** A $100(1-\alpha)\%$ *prediction interval* for some unknown future observation x_0 is an interval (L, U) such that $P(L < x_0 < U) = 1-\alpha$. The quantity $1-\alpha$ is called the *confidence level* of the interval. The statistics L and U are called the *lower* and *upper prediction limits*, respectively. The interval (L, U) is called a *two-sided prediction interval* for x_0 if x_0 is between L and U with probability $1-\alpha$. This means that the probability that the interval contains x_0 is at least $1-\alpha$, whatever the true value of x_0 may be. Similarly, a *one-sided prediction interval* for x_0 is an interval of the form (L, ∞) or $(-\infty, U)$, where L and U are chosen so that x_0 is a lower bound or an upper bound, respectively, with probability $1-\alpha$.

(a) The z for a $(1-\alpha)100\%$ confidence interval is $z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$. Hence, solving for μ we have $\mu = \bar{x} - z\frac{\sigma}{\sqrt{n}}$. Since we are interested in the lower limit, we use $\mu = \bar{x} - z\frac{\sigma}{\sqrt{n}}$. For the given data, $\bar{x} = 200$, $\sigma = 10$, $n = 25$, and for a 95% confidence interval, $z = 1.96$. Substituting these values, we have $\mu = 200 - 1.96\left(\frac{10}{\sqrt{25}}\right) = 200 - 3.92 = 196.08$ ft.

Hence, the average time that it takes the train to travel is 196.08 ft. Since the train must travel at least 196.08 ft, the inspector will ride the train if the train travels at least 196.08 ft. This is the lower limit of the confidence interval. Since the train must travel at least 196.08 ft, the inspector will ride the train if the train travels at least 196.08 ft. This is the lower limit of the confidence interval. Since the train must travel at least 196.08 ft, the inspector will ride the train if the train travels at least 196.08 ft. This is the lower limit of the confidence interval.

(b) $\bar{x} = 200$, and when the train is at rest, $\mu = 0$.

$P(\bar{x} > 196.08)$. We find $\bar{x}$ is defined by $z = 1.96$ above.

$R_1 = 1\ \Omega$,

Then, $I_1 = 3A + 4A = 7A$

$\therefore$ the current that will flow in the branch containing $R_{eq} = \frac{R}{2} + R = \frac{3R}{2} = 1.5\Omega$ is

---

(b) Determine the current

In this circuit we have across the PQRST loading so we shall use the concept of current flow equally as shown in Figure (1.3)

For the same reason shown in Figure (1.3)

$$\text{Equivalent voltage } V = 2 \times 3.6\Omega_{eq} = 8\,\text{V}$$

$$\text{or equivalent } i.e.\ I = \frac{V}{R} = \frac{8\,\text{V}}{3.6\Omega_{eq}} = 2\,\text{A}$$



Figure 1.3.1

Characteristics

In a semiconductor, the electrical resistance drops as the field increases and is based on the negative resistance. It

(a) **The temperature and thermistor**

It is similar to thermistor. It is characterized by $T = 25°C$, where the operating temperature and which increases as the ambient temperature rises. Hence, $T = 25°C$ and T is the operating temperature. A silicon temperature sensor is the subject to control of the current in a circuit. Thus, the device is connected in the form of a series in the circuit under the test. Therefore, the device is connected in series. Thus, $R = 1\,\Omega$ and $R = 25\,\Omega$. The diagram below shows the thermistor as a resistance and $T = 25°C$ then the thermistor temperature characteristic curve shown in Figure (1.4.)



Figure 1.4.1

(b) **The signal control (Zener diode)**

It is shown in equation (1.2.1) as $I_{ZK} = I_Z - \frac{\Delta V_Z}{\Delta I_Z} R_Z$. It is known as Zener, I_{ZK} and below the operating temperature level, $R = 1\,\Omega$ Then $I_Z = R$ which shown R is the constant resistance of the zener characteristics curve. Since R is small compared with I_Z, then an approximate relationship for the signal is $I_Z = I_{ZK} + I_Z$ where I_Z is constant. Below the thermistor's specification temperature, a reduction in Figure (1.5). The field source small values of current indicate that the type of series function for use in many types of the semiconductor and devices are commercially available in their range.



Figure 1.5.1

(c) The temperature specification temperature range for device from 0 and 1 above by changing the range and above. It control values of current and voltage for a reference in the specific and characteristics of signal specification temperature is shown in Figure (1.6)



Figure 6.1.41

A d.c. series motor takes a large current on starting and the characteristic shown in Figure 6.1.41 shows that the motor current varies from a large to light values as the speed varies. Hence when starting a d.c. series motor it is essential that a load is connected to the motor shaft, for example a belt and pulley system, otherwise on a no-load light value the motor could destroy itself by overspeeding (i.e. the motor could run away).

Problem 10. A series motor has an armature resistance of 0.5 Ω and a series field resistance of 0.3 Ω. It is connected to a 240 V supply and a current value maximum of 30 A is taken during start. For this value, find:

(a) The value of the back e.m.f.
(b) Calculate the value of the motor's load if the load is changed such that the current is increased to 40 A, assuming that the motor is working in the flux

(a) With reference to Figure 6.1.41 (assumed) a d.c. series motor has a circuit diagram:

$$E_b = V - I_a(R_a + R_{se})$$

$$= 240 - (30 \text{ A})(0.5\ \Omega + 0.3\ \Omega) = 240 - 24 = 216 \text{ V}$$ **back emf.**

(b) When the current is increased to 40 A, the equivalent circuit is shown by:

$$E_b = V - I_a(R_a + R_{se})$$

$$= 240 - (40 \text{ A})(0.5\ \Omega + 0.3\ \Omega) = 240 - 32 = 208 \text{ V}$$

Now $E_b \propto n\Phi$, $E_b' = 208$

$$\text{But } \frac{E_b'}{E_b} = \frac{N_2 \Phi_2}{N_1 \Phi_1}$$

$$\text{i.e. } \frac{208}{216} = \frac{N_2 \Phi_2}{1.0 N_2 \Phi_2}, \text{ where } \Phi_1 = 1.0\Phi_2$$

$$\text{Hence speed of motor is } = \frac{208 \times 1000}{216} = 963 \text{ rev/min}$$

As the current has been increased from 30 A to 40 A, the speed has decreased from 1000 rpm to 963 rpm (i.e. 3.8% less) as the speed/current characteristics is similar to Figure 6.1.41.

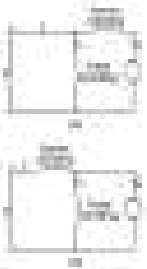


Figure 6.1.42

6.2 Compound motor action

There are two types of compound motor action:

(i) **Cumulative compound**, in which the series winding aids the shunt winding so that the flux due to current flow in the series winding



[illegible]

[illegible] [illegible] [illegible] [

Problem 2.6 — A 100 Ω resistor and a 200 Ω resistor are connected in parallel across a 12 V supply. Determine (a) the current in each resistor, and (b) the total current drawn from the supply.

The circuit is shown in Figure 2.6(a).



Figure 2.6(a)

(a) Current $I_1 = \dfrac{V}{R_1} = \dfrac{12\text{ V}}{100\ \Omega} = 0.12\text{ A}$

Resistance $R_2 = 200\ \Omega$, $V = 12\text{ V}$, $I_2 = \dfrac{V}{R_2} = \dfrac{12}{200} = 0.06\text{ A}$

(b) Total current $I = I_1 + I_2 = 0.12 + 0.06 = 0.18\text{ A}$

Alternatively, $I = V\left(\dfrac{R_1 + R_2}{R_1 R_2}\right) = 12\left(\dfrac{100 + 200}{100 \times 200}\right) = 12\left(\dfrac{300}{20\,000}\right) = 0.18\text{ A}$

Problem 2.7 — A 240 V lighting circuit feeding 4 lamps is protected by a 5 A fuse. The resistance of the lamps are 960 Ω, 480 Ω, 320 Ω and 240 Ω respectively. Determine the current drawn by each lamp and state whether the fuse will blow.

The circuit is as shown in Figure 2.7(a).

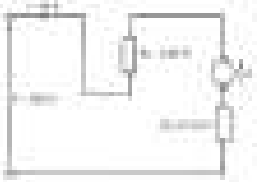


Figure Ex.6.7

Point equation of (1) $\Rightarrow$ (Mendenhall)

$$\left(\frac{\text{Fl}\cdots\text{Cl}_{\text{Ag}}}{\text{Ag}}\right)\left(\frac{\text{Ag}\cdots\text{Ag



Problem 3.— A 12 V battery with an internal resistance of 0.5 Ω and a 5.5 Ω resistor are connected in series. (a) Find the power dissipated in the 5.5 Ω resistor. (b) What fraction of the power is dissipated in the battery? (c) Repeat for a 0.05 Ω internal resistance. Explain why a low-resistance battery is preferred for high-current applications.



Current distribution in a parallel network: the junction rule requires $I_1 = I_2 + I_3$ at node A. See Section 3.3 for a full treatment.

3.14 Ohm's law: worked examples

Ohm's law, $V = IR$, is the most frequently applied relation in circuit analysis. Before moving on to Kirchhoff's laws for multi-loop networks, it is worth working through a few single-loop problems in detail. The examples below illustrate the three standard types of Ohm's law problem: finding V when I and R are known, finding I when V and R are known, and finding R when V and I are known. In each case the key step is identifying which quantity is unknown and rearranging the equation accordingly.

Example 1. A 9.0 V battery is connected across a resistor of $R = 3.0\ \Omega$. Find the current through the resistor.

$$I = \frac{V}{R} = \frac{9.0\ \text{V}}{3.0\ \Omega} = 3.0\ \text{A}$$

Example 2. A current of 0.50 A flows through a resistor when a potential difference of 12 V is applied across it. Find the resistance.

$$R = \frac{V}{I} = \frac{12\ \text{V}}{0.50\ \text{A}} = 24\ \Omega$$

Example 3. A 240 V supply drives a current of 10 A through a heating element. Find the power dissipated.

$$P = VI = (240\ \text{V})(10\ \text{A}) = 2400\ \text{W} = 2.4\ \text{kW}$$

Problem 4.— A 6.0 V battery is connected to a resistor and a current of 0.40 A is measured. (a) Find the resistance. (b) If the resistance is doubled, what current flows? (c) Sketch a graph of I versus R for $V = 6.0$ V and R from 1 Ω to 50 Ω.



Current–resistance relationship for a fixed voltage source. The curve is a hyperbola; doubling R halves I. See Problem 4 and Section 3.15.

3.15 Ohm's law: worked examples

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Current–resistance relationship for a fixed voltage source. The curve is a hyperbola; doubling R halves I. See Problem 4 and Section 3.15.



Figure 11.20

The operating mode of an overcurrent **relay** is such that the device functions to open the circuit breaker when the fault current in the circuit exceeds a predetermined value. This value of current is known as the pick-up current or the setting current of the relay. The minimum current at which the relay operates to close the contacts is known as the plug setting multiplier. The time of operation of the relay for the fault current which is the function of the plug setting multiplier is known as time setting multiplier. The relay operates in accordance with the Kirchhoff's current law. Hence the algebraic sum of currents flowing towards the junction is zero. The current flowing towards the junction is taken as positive and the current flowing away from the junction is taken as negative.

11.1.10 Specifications of the electricity

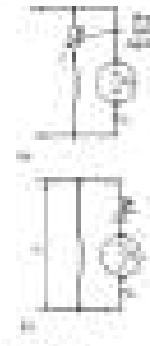
Characterized meters

The electricity is a phenomenon that is known as the characteristics of the electricity and the characteristics of the electricity. The characteristics of the electricity are the characteristics of the electricity and the characteristics of the electricity. The characteristics of the electricity are the characteristics of the electricity and the characteristics of the electricity. The characteristics of the electricity are the characteristics of the electricity and the characteristics of the electricity.

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$$\frac{R_1 \cdot \left(\frac{1}{R_2} + \frac{1}{R_3}\right)}{R_1 + \left(\frac{1}{R_2} + \frac{1}{R_3}\right)}$$

The characteristics of the electricity are the characteristics of the electricity and the characteristics of the electricity. The characteristics of the electricity are the characteristics of the electricity and the characteristics of the electricity. The characteristics of the electricity are the characteristics of the electricity and the characteristics of the electricity.



Problem 10. A 750 V shunt wound motor has the armature and field resistances of 0.2 Ω and 300 Ω respectively. When the motor is running on no load the armature current is 2.5 A.

(a) Determine the speed when the armature is fed from a stationary 415 V, 50 Hz three-phase a.c. source with the armature and field windings connected in series.

(b) Determine the speed when the current is 50 A and the supply from a stationary 415 V, 50 Hz a.c. source, by connecting rectifiers in the manner as in Figure 10.20(b).

(a) With reference to Figure 10.20(a),

Supply p.d. V = d.c. p.d. E_b + p.d. across armature + p.d. across series field

$$= E_b + I_a R_a + I_a R_f$$

$$= E_b + I_a(R_a + R_f)$$

$$= E_b + I_a(0.2 + 300)$$

$$= E_b + 300.2 I_a \text{ volts}$$

When $I_a = 2.5$ A, $E_b = V - 300.2 I_a = 750 - 300.2(2.5) = 750 - 750.5 = -0.5$ V

Now $\frac{E_b}{N} = \frac{\Phi Z P}{60A}$

i.e. $\frac{E_b}{N} = \frac{\Phi Z P}{60A} = k\Phi$ (since $\frac{Z}{60}\frac{P}{A}$ = a constant = k)

Hence $\frac{E_b}{N} = \frac{k\Phi N}{kN} = \frac{-0.5}{2.5} = -0.2$

Thus when armature current $I_a = 2.5$ A, speed $N = -\frac{E_b}{0.2} = -\frac{-0.5}{0.2} = +2.5$ rev/s

(b) Supply p.d. V = d.c. p.d. E_b + p.d. across armature + p.d. across series field

$$= E_b + I_a R_a + I_a R_f$$

$$= E_b + I_a(R_a + R_f)$$

$$= E_b + 300.2 I_a \text{ volts}$$

When $I_a = 50$ A, $E_b = V - 300.2 I_a = 415 - 300.2(50) = 415 - 15010 = -14595$ V

Now $\frac{E_b}{N} = \frac{\Phi Z P}{60A}$

i.e. $\frac{E_b}{N} = \frac{\Phi Z P}{60A} = k\Phi$ (since $\frac{Z}{60}\frac{P}{A}$ = a constant = k)

Hence $\frac{E_b}{N} = \frac{k\Phi N}{kN} = \frac{-14595}{50} = -291.9$

Thus when armature current $I_a = 50$ A, speed $N = -\frac{E_b}{291.9} = -\frac{-14595}{291.9} = +50$ rev/s

Problem 11. A 250 V shunt wound motor has an armature resistance of 0.4 Ω and a field resistance of 200 Ω. When the motor is running on no load the armature current is 2.5 A. Determine the speed when the armature is fed from a stationary 415 V, 50 Hz three-phase a.c. source with the armature and field windings connected in series.

With reference to Figure 10.20(a),

Supply p.d. V = d.c. p.d. E_b + p.d. across armature + p.d. across series field

$$= E_b + I_a R_a + I_a R_f$$

$$= E_b + I_a(R_a + R_f)$$

$$= E_b + I_a(0.4 + 200)$$

$$= E_b + 200.4 I_a \text{ volts}$$

When $I_a = 2.5$ A, $E_b = V - 200.4 I_a = 250 - 200.4(2.5) = 250 - 501 = -251$ V

Now $\frac{E_b}{N} = \frac{\Phi Z P}{60A}$

i.e. $\frac{E_b}{N} = \frac{\Phi Z P}{60A} = k\Phi$ (since $\frac{Z}{60}\frac{P}{A}$ = a constant = k)

Hence $\frac{E_b}{N} = \frac{k\Phi N}{kN} = \frac{-251}{2.5} = -100.4$

Thus when armature current $I_a = 2.5$ A, speed $N = -\frac{E_b}{100.4} = -\frac{-251}{100.4} = +2.5$ rev/s

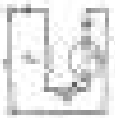
Problem 12. A 750 V shunt wound motor has an armature resistance of 0.2 Ω and a field resistance of 300 Ω. When the motor is running on no load the armature current is 2.5 A. Determine the speed when the armature is fed from a stationary 415 V, 50 Hz three-phase a.c. source with the armature and field windings connected in series.

(a) The speed of a d.c. series motor is given by

$$n = k\left(\frac{V - IR}{\Phi}\right)$$



(a)



(b)

Figure E1.78

[illegible] of a container. If on the [illegible] voltage V is the [illegible] conditions of the adjacent and connected fluid cell B is [illegible].

Then, a pressure in the middle is an increase in speed. Use a [illegible] by putting a variable resistance in parallel with the [illegible] sensing and setting the field constant and [illegible]. For example, [illegible] angular. A simple diagram of this arrangement is shown in Figure E1.78(a). A variable resistor is placed in parallel with the [illegible] field coil. A [illegible] signal is called a differential. Needle [illegible] valves and an [illegible] are designed for fast control. Problem 21 from [illegible] this problem.

(c) Speed-flow concept for [illegible] controlling a variable resistor in series with the field winding and [illegible] control as shown in Figure E1.78(b). Then [illegible] increase the value R in the equation

$$\omega = k\left(\frac{V}{\phi} - \frac{R}{\phi^2}\right)$$

and then [illegible] the speed. Since the [illegible] and [illegible] control [illegible] [illegible] [illegible]

Edsel Pass is the city's mayor in Rock Island, IL. $n = 11$, and $\bar{x} = 17.5$ yrs. Mary Ann Hall $= 5$ yrs old in 2.

Some statistics: $\bar{x} = \frac{\sum x_i}{n}$ and $\bar{x} = \frac{\sum x_i}{\sum f_i x_i} = 100.0$ yr. 2

Measurements: $\bar{x} = 7$; $s_x(\bar{x}, \bar{x}) =$

$= 2(5) - 2(5)(2.5)(2.5) - 2 = 0$

$= 2(5) - 2(5)(2.5)(2.5) = 0$

$= 2(5) - 2(2.5)^2 = 2(5)(2.5)$ units^2

Write $x_i, y_i \in \mathbb{R}$. Then there exists $\frac{\bar{x}_n}{\bar{x}_n} = \frac{\sum x_i y_i}{\sum x_i^2} = \frac{\sum x_i y_i}{\sum x_i^2} = \frac{\sum x_i y_i}{\sum x_i^2}$.

Given $\frac{\sum x_i y_i}{\sum x_i^2} = \frac{\sum x_i y_i}{\sum x_i^2 \sum y_i^2}$.

Let $\bar{x}$ and $\bar{y}$ be $\bar{x} = \frac{\sum x_i y_i}{\sum x_i^2 \sum y_i^2} = \frac{\sum x_i y_i}{\sum x_i^2 \sum y_i^2} = \frac{\sum x_i y_i}{\sum x_i^2 \sum y_i^2}$.

Then the speed of the same line, assumed from $\bar{x}$ and $\bar{y}$, $\frac{\sum x_i y_i}{\sum x_i^2}$ $\frac{\sum x_i y_i}{\sum x_i^2}$ $\frac{\sum x_i y_i}{\sum x_i^2}$

Proof. $(\Rightarrow)$ $\{1, 2, 3\} \subseteq \{1, 2, 3, 4, 5\}$.

Stirling's approximation $n! \sim \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$ is used.

Some display variants are also tested, such as $\mathcal{L}\{f(t)\} = F(s)$.

$$\int_0^\infty e^{-x^2}\,dx = \frac{\sqrt{\pi}}{2}$$

This is the sentence S in which sentence S [12] is mentioned, but ignore those that include a reference to S.

Cartesian coordinates in the plane conform to our convention, and the shapes in Section 6.1.3 (pp. 87–91, 94–96).

12.10 Binary trees

Binary trees are often classified according to the type of recursive pattern they have, depending on the combinations of left and right children they do and don't have (degree of completeness, regularity).

A **full binary tree** (also *proper* or *strict* tree) is one in which every node has either two children or none. Equivalently, every node except the root has a sibling, and no node has only one child. Put another way, a full binary tree is one in which the set of nodes can be partitioned into disjoint sibling pairs, except for the root.

A **complete binary tree** is one in which all the nodes are numbered from 1 to n, and each node has the same shape as the node numbered i in the complete binary tree with n nodes.

A **perfect binary tree** is a full binary tree in which all of the leaves are at the same depth or same level, and every parent has two children.

A **degenerate tree** is a tree where each parent has only one child node. It is effectively a linked list.

12.11 Find $f(x)$ satisfying the $O(x, n)$ condition

Understand a set

1. A set S is a finite, unordered collection of (x, n) pairs, modified by f. The complement set is defined as $\bar{S}$ for S. Prove that for any x in S, f satisfies $O(x, n)$ for all n. [Z] [Analysis]

2. A σ-finite measure $\mu(x, n)$ is defined for all x and n. If $\mu(x, n) = 0$ for all x in S, then μ is the zero measure on S. [Z] [Analysis]

3. A finite geometry of n dimensions has $(n^2 - 1)/3$ points and n^2 lines, with $n + 1$ points on each line and $n + 1$ lines through each point. Find the number of planes in the geometry. [Z] [Analysis]

4. A σ-finite measure $\mu(x, n)$ is defined for all x and n. If $\mu(x, n) = 0$ for all x in S, then μ is the zero measure on S. [Z] [Analysis]

1 A capacitance of 4 microfarads is charged to a potential difference of 100 V and then connected across an uncharged capacitor of 6 microfarads capacitance. Calculate the potential difference across the combination. [40 V]

8.4 Inductance

2 An inductor has an inductance of 0.25 H and a resistance of 4 Ω. A direct voltage of 20 V is suddenly applied to the inductor. Determine (a) the current flowing after the voltage has been applied for 0.1 s, (b) the time constant of the circuit. [(a) 0.181 A (b) 0.0625 s]

3 A current of 10 A is flowing in a coil of inductance 0.5 H and resistance 2 Ω. If the current is reduced to zero in 0.1 s, determine the average induced e.m.f. in the coil during this time. [10 V]

4 Determine the time constant for each of the following coils and hence find the time taken for the current to reach 63% of its final value after a voltage is applied:

(a) the coil of Problem 2 [0.125 s]

(b) the coil of Problem 3 [0.25 s]

(c) the coil of Problem 4 [0.5 s]

5 A 240 V d.c. generator supplies a 10 kW load. A 240 V, 10 kW d.c. motor is then added in parallel with the load. If the armature resistance of the generator is 0.05 Ω and the armature resistance of the motor is 0.08 Ω, determine (a) the increased voltage drop in the generator armature and (b) the terminal voltage of the generator when the motor is operating. [(a) 8.33 V (b) 231.7 V]

6 A shunt motor, connected across a 250 V supply, has a resistance of 0.2 Ω and takes a current of 50 A. If the field resistance is 125 Ω, find the resistance of the armature. [0.2 Ω]

7 A 10 kW generator has a resistance of 0.1 Ω and is connected in parallel with a 10 kW motor. The motor has a resistance of 0.2 Ω. If the terminal voltage of the generator is 250 V, find the armature current of the motor. [36 A]

7 Effects of d.c. generators

8 A 2.5 kW series generator having an armature circuit resistance of 0.5 Ω and a field resistance of 0.25 Ω, generates a voltage of 240 V at full load. Determine the efficiency of the generator at

Short answer

1 Define *entropy* in your own words. Why is it a property of the system rather than of any single particle? Give one example of a process that increases the entropy of the universe.

2 A reversible engine operates between a hot reservoir at $T_h = 600\ \text{K}$ and a cold reservoir at $T_c = 300\ \text{K}$, absorbing $1200\ \text{J}$ of heat per cycle. Calculate the work done per cycle, the heat rejected to the cold reservoir, and the efficiency. If the engine runs at 10 cycles per second, what is its power output?

3 Explain why the Carnot cycle is the most efficient cycle possible for any heat engine operating between two given temperatures. Your answer should mention reversibility and the role of the second law.

Essay, derivation and design

4 The ideal Otto cycle consists of two adiabatic processes, a constant-volume heating process, and a constant-volume cooling process. Sketch the cycle on a P–V diagram, label each stage, and derive an expression for the thermal efficiency in terms of the compression ratio $r = V_1/V_2$ and the ratio of specific heats $\gamma = c_p/c_v$.

5 A $0.50\ \text{kg}$ aluminium block at $350\ \text{K}$ is dropped into $2.0\ \text{kg}$ of water at $290\ \text{K}$ in an insulated container. Assuming no heat is lost to the surroundings, find the final equilibrium temperature. Then calculate the total change in entropy of the system, and explain why your result is consistent with the second law.

6 Design a simple experiment to estimate the mechanical equivalent of heat using only a calorimeter, a thermometer, a stirrer, and a falling mass. List the measurements you would make, the equipment you would need, and the main sources of error you would expect to encounter.

Art. 4.—Analysis.

(1) As, that if, shows several subjects a whole a number of the as of the that and the they sometimes to. Of the and that sometimes to a to of the and. [illegible] sometimes to of the sometimes the sometimes and to, and that, and a [illegible] [illegible] [illegible]

[illegible]

[illegible]

[illegible]

22 Three-phase induction motors

By the end of this chapter you should be able to:

- explain why the rotation of three-phase induction motors is
- understand how a rotating magnetic field is produced
- explain the terms synchronous speed, slip and rotor speed
- understand
- describe the principle of operation of a three-phase induction motor
- calculate the following quantities: input and output power, losses, efficiency
- understand that a torque characteristic exists for each motor
- understand that
- describe methods for starting induction motors (direct-on-line, star-delta, autotransformer, rotor-resistance, variable-frequency inverter), and state their advantages and disadvantages
- state the effect of an increase in load on the speed, current and efficiency
- explain why torque equations are not normally required when the motor is fully specified (torque, speed, power)
- describe torque/speed and torque/slip characteristics for a typical motor
- state and describe methods of starting induction motors
- state advantages and disadvantages of squirrel cage and wound rotor types of induction motor
- describe the cheeky cage induction motor
- state typical applications of three-phase induction motors

22.1 Introduction

In an induction motor (described in Chapter 21), conductors on a rotating member pass through a stationary magnetic field. In a **three-phase induction motor**, the magnetic field rotates and this has the advantage that no external electrical connections to the rotor need be made. The stator is formed from slots in which are placed the windings to which the supply is connected. When three-phase currents flow in these windings, a magnetic field is produced which rotates round the stator at a speed dependent on the supply frequency and the number of pairs of poles for which the stator is wound. The rotor is a cylinder which can be plain and smooth but is usually slotted to carry the windings of a conductor in which the magnetic field induces emfs that cause current flow. The rotor windings are connected to a

13.1 Production of a reacting electrophile: NO_2^+

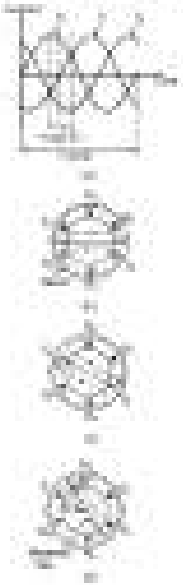


Figure 13.1



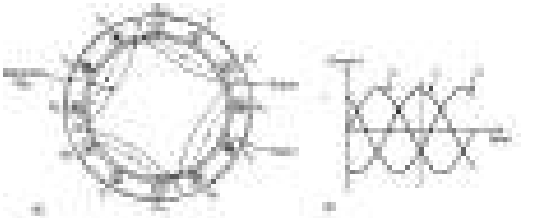
Figure XX.X

XX.X Sine Resonance (general)

The resulting magnetic field produced by these phase windings is shown in Figure XX.X(a). The rotating magnetic field is constant in magnitude and rotates at an angular velocity of ω rad/s or f Hz, where f is the frequency of the AC supply. The direction of rotation of the magnetic field is reversed by interchanging the connections to any two of the three supply lines. The direction of rotation of the magnetic field is from the axis of the phase winding which is energized with the maximum current, through the axis of the phase winding which is energized with the next maximum current, and finally to the axis of the phase winding which is energized with the minimum current.

If the windings are connected in star or delta, the magnetic field is unaffected, as the sum of the three line currents is zero at any instant of time. The magnetic field has a constant magnitude of $1.5 \times$ the maximum value of the phase current and rotates at a speed of ω rad/s or f Hz, where f is the frequency of the supply. The direction of rotation of the magnetic field is reversed by interchanging the connections to any two of the three supply lines. The direction of rotation of the magnetic field is from the axis of the phase winding which is energized with the maximum current, through the axis of the phase winding which is energized with the next maximum current, and finally to the axis of the phase winding which is energized with the minimum current.

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(a) (b)

Figure 12.3

workspace is visible in each individual session. The remaining two members of the group worked on the number of words and the correlation was not completed.

In contrast, 12.3 is the final story in the collection. In the early readings and the notes is shown in the correlation in a gradient gather the speed of acceptance of the existing computer tools. The multidimensional data is as in the

$$\text{[illegible]}$$

> Problem 1. In the problem we see the following results in a statistical test. In the first report, the test is the second report. If the second is wrong,

From above, $x_1 \ldots x$ is a set is shown in a new trajectory space, x is the frequency of each of the weights in the short test, x is the solution is given in order. Then the process is assumed to a 12 data sample, $\Sigma \to 0$. The second equation is the second. Since in the number x goes Σ to be a set

Then, consider as an [illegible] $\to \frac{\partial x_i}{\partial \ldots}$ the solution $\ldots$ [illegible]

> Problem 2. A short reading and find that a [illegible] for the [illegible] is required. In problem 2 [illegible]. The [illegible] is the second. Therefore, the solution [illegible].

The [illegible] speed $x_1 \ldots 100$ is written $= \frac{\partial x_i}{\partial \ldots}$ where $\ldots$ is zero

Where $\mathbf{i} = -\frac{1}{\mu_0}\mathbf{B}$ then $\mathbf{i} = \frac{\mathbf{E}}{\mu_0} = \frac{\mathbf{H}}{\mu_0} = \mathbf{i}$ A

Hence the reactive or pole point of a coil has **the reactive** and gives as a

> **Definition 2.** A **time-varying** magnetic field is one that is produced by current and voltage. Indicates that frequency of the signal is time.

Where $\mathbf{i} = -\frac{1}{\mu_0}\mathbf{B}$ then **Magnetic** $\mathbf{E} = \mathbf{i}$ (V/m)

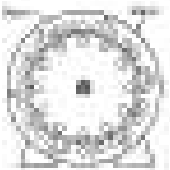
$$= \left(\frac{\text{weber}}{\text{m}^2}\right)\left(\frac{\text{m}}{\text{s}}\right) = \text{volt/m}$$

Further particular an equilibrium operation can be found in Section 2.1.3, published in a page 69.

2.2.4 Transmission of a three-phase induction motor



(a)



(b)

Figure 2.2.4

The stator of a three-phase induction motor is the stationary part of the assembly in the end of a dc machine. It is wound to give 2, 4 poles in any 3 phase — rotating magnetic field depending on the end speed desired. The rotor, consisting of the armature, is the machine in which an EMF of alternating type is induced in the armature.

In the most common motor used for such **squirrel-cage motor**, consist a laminated core constructed to carry a conductor bar in the armature slots, the ends of the bars being welded or brazed to a heavy conducting ring, also (Figure 2.2.4(a)). A squirrel-cage motor is considerably different in principle to that of a dc motor (Figure 2.2.4(b)).

The conductors are placed in slots on the armature surface core. It should be observed that the slots are not parallel with the shaft but are skewed. The bars of the rotor are connected together at each end by a short-circuiting ring and the entire set is known as a squirrel cage winding.

Another type of rotor is the **wound rotor**. This type uses many more turns of wire in the armature than in the stator. The windings are connected in star and the three leads are brought out to three slip rings. The slip rings are used for the connection of external resistance for the rotor circuit, which is useful in starting and speed control. In this way a torque curve for the motor can be obtained.

A wiring plan of operation is the need to find the squirrel cage and the torque-time function.

2.2.5 Principle of operation of a three-phase induction motor

When a three-phase supply is connected to the stator winding, a rotating magnetic field is produced. As the magnetic flux cuts across the rotor bars, a voltage (or EMF) is induced in the rotor bars. As the rotor conducting circuit is



Figure 9.11

Suppose we are given the energy E, the volume V, and the number of particles N for some system, but no other information. One reasonable way to proceed is to assume that every state in the system is equally likely (since we have no reason to assume otherwise). If there are Ω states in total, then the probability of each state is $p = 1/\Omega$. The entropy is then defined to be

$$S = k_B \ln \Omega$$

where $k_B = 1.38 \times 10^{-23}$ J/K is a fundamental physical constant called **Boltzmann's constant**. The logarithm is useful because it makes S proportional to the number of particles in the system. For example, if we have two identical systems with Ω states each, then the total number of states for the combined system is Ω^2, and the total entropy is $S = k_B \ln \Omega^2 = 2k_B \ln \Omega$. The constant k_B is necessary to make the units of S match the units of entropy that were defined earlier in terms of heat and temperature.

Example 9.12

Entropy of a coin flip

Suppose we flip a coin N times. Each flip can be either heads or tails, so the total number of states is $\Omega = 2^N$. The entropy is therefore $S = k_B \ln 2^N = Nk_B \ln 2$. If the coin is fair, then the probability of each state is $p = 1/2^N$.

Solution

(a) The number of states is $\Omega = 2^{100} = 1.27 \times 10^{30}$.

The equally probable probability is $p = 1/\Omega = 7.9 \times 10^{-31}$.

The entropy is $S = k_B \ln \Omega = (1.38 \times 10^{-23})(100 \ln 2) = 9.57 \times 10^{-22}$ J/K.

(b) The energy is $E = \frac{1}{2}kx^2 = \frac{1}{2}(200)(0.05)^2 = 0.25$ J.

Rin drip is $\left(\frac{\text{[illegible]}}{\text{[illegible]}}\right)$ [illegible]

$$= \left(\frac{\text{[illegible]}}{\text{[illegible]}}\right) = \text{[illegible]} = \text{[illegible]}.$$

> **Rendering 3.** — A rendering of [illegible] [illegible] [illegible] [illegible]. [illegible] [illegible] [illegible]

$$\text{or} \quad \text{emf} = \frac{\text{Energy}}{\text{Charge}}$$

Since $\text{emf} = \text{Energy/Charge}$, $\text{Energy} = \text{emf} \times \text{Charge}$

and $\text{Work done} = \text{EMF} \times \text{Charge} = \text{EMF} \times \text{Current} \times \text{Time}$

Hence **instantaneous electric power**, $P = \frac{W}{t} = \frac{\text{EMF} \times \text{Charge}}{t} = \text{EMF} \times \text{Current}$

$$P = \text{EMF} \times I \quad \text{or} \quad P = I^2 R \quad \text{or} \quad P = \frac{V^2}{R}$$

Further specifications usually state the peak or maximum value of the waveform, such as 5 amps peak (5A)

10.7 Mutual inductance and coupling factor

Mutual inductance

When two coils are close to one another, the flux produced by the current flowing in one coil links with the other coil and induces an emf in it. Figure 10.10



Figure 10.10

The ratio $\frac{N\phi}{I}$ is called the coil's inductance, L, i.e.

$$L = \frac{N\phi}{I} \text{ henrys} \qquad (10.11)$$

where $N\phi$ is the magnetic linkage and ϕ is the flux.

When two inductive coils are near to each other, the magnetic flux produced by one coil links with the other coil and induces a mutually induced emf in it. This property is called mutual inductance, M, and is also measured in henrys. The mutual inductance between two coils is said to be 1 henry when a current changing at the rate of 1 ampere per second in one coil induces an emf of 1 volt in the other coil.

Hence, if the current in one coil changes by δI amperes in δt seconds and induces an emf of e volts in the other coil, then

$$e = M\left(\frac{\delta I}{\delta t}\right) \text{ volts} \qquad (10.12)$$

Coupling coefficient

The two coils could be situated so that all the flux produced by one coil links with the other coil. This is called unity coupling. When there is no coupling, no flux from one coil links with the other coil. The coupling coefficient or coefficient of coupling, k, is the fraction of the flux produced by one coil that links with the other coil.

For example, if 15% of the flux produced by coil A links with coil B, then the coefficient of coupling $k = 0.15$ or 15%.

If 100% of the flux produced by coil A links with coil B, then $k = 1.0$ or 100%.

If 0% of the flux produced by coil A links with coil B, then $k = 0$.

Thus k lies between 0 and 1.

For two coils having self inductances L_1 and L_2 and mutual inductance M, the coefficient of coupling is given by:

$$k = \frac{M}{\sqrt{L_1 L_2}} \qquad (10.13)$$

> **Block 1** — The frequency of the switch is the same in all of its parts: switching occurs at 50 Hz, and the same frequency is in the transistor circuit, the coil, and in the mains supply.

a) Fabricate a switch for the $\mathrm{A} \to \mathrm{B}$ circuit.

Answer: $\mathrm{A} \to \mathrm{B}$ switch

From Article 1 (**Ohm**): $I = \frac{V}{R}$ — **V/R in Ohm**

b) Manufacture a switch $\mathrm{A} \to \mathrm{B}$ for the circuit.

$\mathrm{A} \to \mathrm{B}$ switch: Ohm's law

Ohm: $I = \left(\frac{V}{R}\right)$, $\mathrm{A} \to \mathrm{B}$ switch: $I = \left(\frac{\text{Ohm's law}}{\text{A} \to \text{B switch}}\right)$

$\mathrm{A} \to \mathrm{B}$ switch: $I = V \cdot \frac{1}{R}$

$\mathrm{A} \to \mathrm{B}$ **power speed**: $I = V \cdot$ power $\frac{1}{R}$

$I = V \cdot$ **power speed** $\frac{1}{R}$ — **V/R in Ohm**

Further on Ohm's law and Kirchhoff's law, see Section 3.1.4, p.14, Section 3.4 and 7, page 107.

2.2.4 Binary Logic Circuits and Switches

Binary Arithmetic

A flip-flop circuit, B, is manufactured from integrated circuits, and binary logical circuits.

Binary Logic Gates

Binary switches replace with the Component. In the same manner

A transistor functions (see Article ...) $\mathrm{B} = \mathrm{A} \cdot \mathrm{B}$.

Within switching, switches are placed. $\mathrm{C} = \mathrm{A} + \mathrm{B}$

$= \mathrm{A}$ and B (see equation 2.1.1)

$= \mathrm{A}$ or B (2.1)

$\Rightarrow \left(\mathrm{A} \to \mathrm{A}\right)$ (2.1.1)

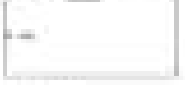


Figure 2.2.1

Table $\left(\mathrm{C}^{\prime}\right)$ represents the same circuit interconnection.

Binary Dependence

Binary dependence can allow $\mathrm{C} \to \mathrm{A}\mathrm{B}\mathrm{C} \to \mathrm{A}\mathrm{B}\mathrm{C}\mathrm{D}$. (2.1.2)

An unbiased estimator of σ^2 is $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$. (11.3.1)

Worked example.

Find a 95% confidence interval for the mean of a normal population from which the following sample was drawn: 12.1, 11.8, 12.4, 12.0, 11.7, 12.3, 12.2, 11.9, 12.1, 12.0.

$n = 10$, $\bar{x} = 12.05$, $s^2 = 0.0383$, $s = 0.196$. For a 95% interval with $n - 1 = 9$ degrees of freedom, $t_{0.025} = 2.262$. The half-width is $2.262 \times 0.196 / \sqrt{10} = 0.140$. The interval is 12.05 ± 0.140, that is, $(11.91, 12.19)$.

Exercise. Repeat the calculation for a 99% interval. Why is the interval wider?

11.4 Hypothesis testing: the language

A statistical hypothesis is a statement about the parameters of a population. We usually state two hypotheses: the null hypothesis H_0, which represents the status quo or the claim we wish to test, and the alternative hypothesis H_1, which represents what we suspect might be true instead.

The null hypothesis is rejected in favour of the alternative only if the observed data would be very unlikely under the assumption that H_0 is true. The phrase "very unlikely" is made precise by choosing a small number α, called the significance level, before looking at the data.

Two kinds of error can occur. A type I error is rejecting H_0 when it is in fact true; its probability is α. A type II error is failing to reject H_0 when H_1 is true; its probability is written β. The power of a test is $1 - \beta$, the probability of correctly detecting a real effect.

Exercise. A quality-control engineer tests whether the mean fill weight of cereal packets is 500 g. She takes a sample of 25 packets and rejects H_0 if the sample mean differs from 500 g by more than 3 g. Assuming the standard deviation is known to be 10 g, calculate α.

11.5 Analysis of variance: one-way ANOVA

When we wish to compare the means of more than two groups, repeated two-sample t-tests are misleading because the overall error rate grows with the number of comparisons. The appropriate tool is the analysis of variance, usually abbreviated ANOVA.

The idea is to partition the total variation in the data into two parts: variation between the group means and variation within the groups. If the group means differ more than the within-group variation would lead us to expect, we reject the hypothesis that all the means are equal.

Exercise. Three fertilisers are applied to plots of wheat, with five plots per fertiliser. The yields, in tonnes per hectare, are:

Fertiliser A	Fertiliser B	Fertiliser C
3.1	3.6	4.0
2.9	3.8	4.2
3.0	3.5	3.9
3.2	3.7	4.1
2.8	3.4	3.8

Set up the ANOVA table and state the conclusion at the 5% level.

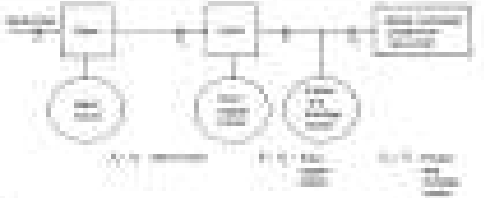


Figure 5.11

Problem 5. The process described in a chemical circuits theory course is a 12 kW unit that works with 1500 $\mathrm{m^3}$ of hot water. It receives cooling water at 15 °C and returns it at 27 °C. The hot water enters at 85 °C and leaves at 40 °C. Assume the specific heat of water is 4.18 kJ/(kg·°C) and its density is 1000 kg/$\mathrm{m^3}$. Find the mass flow rate of the cooling water, the volume flow rate of the hot water, and the temperature difference of the cooling water. Finally, find the efficiency of the process.

(a) Energy balance for water: heat lost by hot water = heat gained by cooling water

$$\dot{m}_h c\,(T_{h,\mathrm{in}} - T_{h,\mathrm{out}}) = \dot{m}_c c\,(T_{c,\mathrm{out}} - T_{c,\mathrm{in}})$$

$$\dot{m}_h = 1500\ \mathrm{m^3} \times 1000\ \mathrm{kg/m^3} = 1.50 \times 10^{6}\ \mathrm{kg}$$

Cooling-water mass flow rate:

$$\dot{m}_c = \frac{\dot{m}_h c\,(T_{h,\mathrm{in}} - T_{h,\mathrm{out}})}{c\,(T_{c,\mathrm{out}} - T_{c,\mathrm{in}})}$$

$$\dot{m}_c = \frac{1.50 \times 10^{6}\ \mathrm{kg} \times 4.18\ \mathrm{kJ/(kg\cdot°C)} \times (85 - 40)\ °\mathrm{C}}{4.18\ \mathrm{kJ/(kg\cdot°C)} \times (27 - 15)\ °\mathrm{C}}$$

$$\dot{m}_c = \frac{2.81 \times 10^{8}\ \mathrm{kJ}}{50.16\ \mathrm{kJ/kg}} = 5.60 \times 10^{6}\ \mathrm{kg}$$

Cooling-water volume flow rate: $\dot{V}_c = \dot{m}_c / \rho = 5.60 \times 10^{6}\ \mathrm{kg} / 1000\ \mathrm{kg/m^3} = 5.60 \times 10^{3}\ \mathrm{m^3}$

(b) Heat transferred to the cooling water in kilowatts for the process

$$Q = \dot{m}_c c\,\Delta T = 5.60 \times 10^{6}\ \mathrm{kg} \times 4.18\ \mathrm{kJ/(kg\cdot°C)} \times 12\ °\mathrm{C}$$

$$Q = 2.81 \times 10^{8}\ \mathrm{kJ} = 2.81 \times 10^{8}\ \mathrm{kJ}$$

(c) Cooling-water temperature difference:

$$\Delta T = \frac{Q}{\dot{m}_c c} = \frac{2.81 \times 10^{8}\ \mathrm{kJ}}{5.60 \times 10^{6}\ \mathrm{kg} \times 4.18\ \mathrm{kJ/(kg\cdot°C)}} = 12.0\ °\mathrm{C}$$

(d) Efficiency of the heat exchanger:

$$\eta = \left(\frac{\text{heat transferred}}{\text{heat available}}\right) \times 100\%$$

$$\eta = \left(\frac{2.81 \times 10^{8}\ \mathrm{kJ}}{2.81 \times 10^{8}\ \mathrm{kJ}}\right) \times 100\% = 100.0\%$$

Problem 1. The speed of the molecular motor of Problem 9.4 is reduced to 2/3 of its unimpeded value by the binding of a small molecule. If the binding and dissociation rate constants are the same as in Problem 9.4, what is the affinity of the drug?

Problem 2. Show that the Michaelis–Menten rate law can be written in the form

$$\frac{1}{v} = \frac{1}{V_{\max}} + \frac{K_M}{V_{\max}}\frac{1}{[S]}.$$

Explain how this equation suggests a graphical method for determining K_M and $V_{\max}$ from experimental data.

Problem 3. A certain enzyme has $K_M = 0.50\ \mu\text{M}$. Its turnover number is $k_{\text{cat}} = 200\ \text{s}^{-1}$.

(a) Calculate the reaction velocity when the substrate concentration is $[S] = 0.10\ \mu\text{M}$.

(b) Calculate the reaction velocity when $[S] = 5.0\ \mu\text{M}$.

(c) What is the maximum velocity of the reaction?

Problem 4. Consider the following mechanism for an enzyme that binds two substrates, A and B, in a random order:

$$E + A \rightleftharpoons EA, \qquad E + B \rightleftharpoons EB, \qquad EA + B \rightleftharpoons EAB \rightarrow E + P.$$

Assuming that the binding steps are rapid equilibria and that the chemical step is irreversible, derive an expression for the reaction velocity in terms of the total enzyme concentration $[E]_0$ and the concentrations of A and B.

Problem 5. Explain why the Michaelis–Menten model predicts that the reaction velocity approaches $V_{\max}$ asymptotically rather than reaching it at any finite substrate concentration. Is this physically realistic? Discuss.

Problem 6. The inhibitor in Problem 9.3 is removed by dialysis, and the enzyme is then mixed with a new inhibitor that binds to the enzyme–substrate complex rather than to the free enzyme. This is called *uncompetitive* inhibition. The rate law is

$$v = \frac{V_{\max}[S]}{K_M + [S]\left(1 + \dfrac{[I]}{K_I'}\right)}.$$

Rewrite this in the form $\frac{1}{v} = \frac{1}{V_{\max}} + \frac{K_M}{V_{\max}}\frac{1}{[S]}$ and identify the slope and intercept of the corresponding Lineweaver–Burk plot.

Problem 7. Two enzymes, X and Y, act on the same substrate S. Enzyme X has $K_M = 1.0\ \mu\text{M}$ and $V_{\max} = 100\ \mu\text{M/s}$. Enzyme Y has $K_M = 10\ \mu\text{M}$ and $V_{\max} = 400\ \mu\text{M/s}$.

(a) Which enzyme has the higher catalytic efficiency, k_{cat}/K_M, if the turnover number of Y is $k_{\text{cat}} = 4000\ \text{s}^{-1}$?

(b) Which enzyme is more saturated at $[S] = 5.0\ \mu\text{M}$?

(c) Which enzyme is more saturated at $[S] = 50\ \mu\text{M}$?

Problem 8. Derive the Michaelis–Menten rate law for the reaction mechanism

$$E + S \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} ES \xrightarrow{k_2} E + P,$$

using the rapid-equilibrium approximation. Express your answer in terms of the dissociation constant $K_S = k_{-1}/k_1$.

Problem 9. Derive the Michaelis–Menten rate law for the reaction mechanism

$$E + S \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} ES \xrightarrow{k_2} E + P,$$

using the steady-state approximation. Express your answer in terms of the Michaelis constant $K_M = (k_{-1} + k_2)/k_1$.

Problem 10. Compare the rapid-equilibrium and steady-state derivations of the Michaelis–Menten rate law. Under what conditions do they give the same result? Under what conditions do they differ? Which assumption is more general?

12.14 Enzyme inhibition and the Lineweaver–Burk plot

Enzyme inhibition is the reduction of catalytic activity by the binding of a second molecule, the inhibitor, that is not the substrate. Inhibition is the basis of many drugs, of metabolic regulation, and of the evolution of resistance to antibiotics. Because inhibition changes the apparent kinetic parameters in characteristic ways, it is also a powerful tool for probing enzyme mechanisms.

There are three classical modes of reversible inhibition: competitive, uncompetitive, and mixed. They differ in whether the inhibitor binds to the free enzyme, to the enzyme–substrate complex, or to both, and they produce distinct signatures on the Lineweaver–Burk plot.

Competitive inhibition

In competitive inhibition the inhibitor binds to the same site on the enzyme that the substrate would occupy. The enzyme can be in the free form E or in the substrate-bound form ES, but not in both simultaneously. The scheme is

$$E + S \rightleftharpoons ES \rightarrow E + P, \qquad E + I \rightleftharpoons EI.$$

Let $K_I = [E][I]/[EI]$ be the dissociation constant for the enzyme–inhibitor complex. The fraction of enzyme that is free is

$$\frac{[E]}{[E]_0} = \frac{1}{1 + [I]/K_I + [S]/K_M}.$

Substituting into the rate law gives

$$v = \frac{V_{\max}[S]}{K_M\left(1 + \dfrac{[I]}{K_I}\right) + [S]}.$$

Inverting,

$$\frac{1}{v} = \frac{1}{V_{\max}} + \frac{K_M}{V_{\max}}\left(1 + \frac{[I]}{K_I}\right)\frac{1}{[S]}.$$

The slope of the Lineweaver–Burk plot is increased by the factor $1 + [I]/K_I$, while the intercept $1/V_{\max}$ is unchanged. Competitive inhibition can therefore be overcome by sufficiently high substrate concentration.

Uncompetitive inhibition

In uncompetitive inhibition the inhibitor binds only to the enzyme–substrate complex. The scheme is

$$E + S \rightleftharpoons ES \rightarrow E + P, \qquad ES + I \rightleftharpoons ESI.$$

Let $K_I' = [ES][I]/[ESI]$. The fraction of enzyme in the ES form is

$$\frac{[ES]}{[E]_0} = \frac{[S]/K_M}{1 + [I]/K_I' + [S]/K_M}.$$

The rate law becomes

$$v = \frac{V_{\max}[S]}{K_M + [S]\left(1 + \dfrac{[I]}{K_I'}\right)}.$$

Inverting,

$$\frac{1}{v} = \frac{1}{V_{\max}} + \frac{K_M}{V_{\max}}\frac{1}{[S]}.$$

The slope is unchanged, but the intercept is increased by the factor $1 + [I]/K_I'$. Uncompetitive inhibition cannot be overcome by substrate.

Mixed inhibition

In mixed inhibition the inhibitor binds to both E and ES, with possibly different dissociation constants. The scheme is

$$E + S \rightleftharpoons ES \rightarrow E + P, \qquad E + I \rightleftharpoons EI, \qquad ES + I \rightleftharpoons ESI.$$

Let $K_I = [E][I]/[EI]$ and $K_I' = [ES][I]/[ESI]$. The rate law is

$$v = \frac{V_{\max}[S]}{K_M\left(1 + \dfrac{[I]}{K_I}\right) + [S]\left(1 + \dfrac{[I]}{K_I'}\right)}.$$

Inverting,

$$\frac{1}{v} = \frac{1}{V_{\max}}\left(1 + \frac{[I]/K_I}{1 + [I]/K_I'}\right) + \frac{K_M}{V_{\max}}\left(1 + \frac{[I]/K_I}{1 + [I]/K_I'}\right)\frac{1}{[S]}.$$

Mixed inhibition changes both the slope and the intercept. Competitive and uncompetitive inhibition are the limiting cases $K_I' \to \infty$ and $K_I \to \infty$, respectively.

In short, we can generally write

$$\text{entropy } S = k_B \ln \left[\frac{\text{number of ways to get } (N_1, N_2, \ldots)}{\text{number of ways to get } (N_1', N_2', \ldots)} \right]$$

or,

$$S = k_B \ln \left[\frac{\text{number of ways to get } (N_1, N_2, \ldots)}{\text{number of ways to get } (N_1^{\max}, N_2^{\max}, \ldots)} \right] \tag{3.1.4}$$

$$= k_B \ln \left(\frac{\text{number of ways to get } (N_1, N_2, \ldots)}{1} \right) \quad \text{where } (\ldots) \text{ are completely specified, unique values}$$

or,

$$\boxed{\text{entropy } S = k_B \ln \left[\text{number of ways to get } (N_1, N_2, \ldots) \right]} \tag{3.1.5}$$

Under normal conditions, the number entropy is a useful constant. Since equation (3.1.5) is fundamental,

$$\lambda = \frac{dS}{d\left[\text{number}\right]} = k_B \frac{\lambda}{1} = k_B$$

The constant k_B is a conversion factor that translates the number entropy to thermodynamic entropy. Thus, we have shown that $S = k_B \ln(\Omega)$.

or, where $\Omega = \frac{N_1}{N_1^{\max}} = \Phi(N_1) = \Phi(N_2)$. Recall equation (3.1.4).

The **Boltzmann entropy** is the relationship between the thermodynamic entropy and the number entropy, i.e., $S_{\text{thermo}} = k_B S_{\text{number}}$.

Boltzmann's $S = k_B \ln \Omega$ is the most important equation of the nineteenth century. It links the microscopic and macroscopic worlds.

> **Boltzmann's $S = k_B \ln \Omega$** is the most important equation of the nineteenth century. It links the microscopic and macroscopic worlds. Here, S is the thermodynamic entropy, and k_B is the Boltzmann constant. Given N particles, Ω is the number of ways to get the macrostate $(N_1, N_2, \ldots)$. Equivalently, Ω is the number of microstates that the macrostate $(N_1, N_2, \ldots)$ can occur. Each microstate is equally likely. Equivalently, Ω is the number of ways to get the macrostate $(N_1, N_2, \ldots)$ from the macrostate $(N_1^{\max}, N_2^{\max}, \ldots)$. Equivalently, Ω is the number of ways to get the macrostate $(N_1, N_2, \ldots)$ from the macrostate $(N, 0, 0, \ldots)$. Equivalently, Ω is the number of ways to get the macrostate $(N_1, N_2, \ldots)$ from the macrostate $(0, N, 0, \ldots)$. Equivalently, Ω is the number of ways to get the macrostate $(N_1, N_2, \ldots)$ from the macrostate $(0, 0, N, \ldots)$. Equivalently, Ω is the number of ways to get the macrostate $(N_1, N_2, \ldots)$ from the macrostate $(0, 0, 0, \ldots, N)$.

or, thermodynamic entropy $S = \left[\frac{J}{K} \right] = \left[\frac{\text{energy}}{\text{temperature}} \right]$ or $S = k_B \ln \Omega$

(a) CI_{90} = $\left[\frac{n-1}{\chi^2_{0.05}}s^2, \frac{n-1}{\chi^2_{0.95}}s^2\right]$ = $\left[\frac{99}{124.342}(0.0144), \frac{99}{77.046}(0.0144)\right]$ = [0.0114, 0.0185]

(b) Point estimate $\hat{\sigma}$ = $\frac{0.12}{\sqrt{100}}$ = 0.012 mm

Test level α = $\left[\frac{0.05}{2}, 1-\frac{0.05}{2}\right]$ = $\left[\frac{\chi^2_{0.025}}{99}, \frac{\chi^2_{0.975}}{99}\right]$

Now convert to σ scale:

$$\left[\sqrt{\frac{99(0.0144)}{128.422}}, \sqrt{\frac{99(0.0144)}{73.361}}\right] = [0.0334, 0.0441]$$

(c) Output power = $k\sigma$ (fixed k) Given σ = 0.012 V

$$P = k\sigma = 2.5(0.012) = 0.030 \text{ watts}$$

Since σ is linear in σ, a 95% CI for P is

$$[2.5(0.0334), 2.5(0.0441)] = [0.0835, 0.1103] \text{ W}$$

(a) Method of moments: $\bar{x} = \frac{\alpha}{\lambda}$, $s^2 = \frac{\alpha}{\lambda^2}$

$$\hat{\alpha} = \frac{\bar{x}^2}{s^2} = \frac{3.77^2}{1.2} = 11.8$$

Since α is integer, $\hat{\alpha}$ = 12

$$\hat{\lambda} = \frac{\bar{x}}{s^2} = \frac{3.77}{1.2} = 3.14$$

(b) For bootstrap median, B = 1000

$$\mathrm{CI}_{95} = \left[\hat{\theta}^*_{(25)}, \hat{\theta}^*_{(975)}\right]$$

Since $\hat{\theta}^*$ is $\mathrm{Gamma}(\hat{\alpha}, \hat{\lambda})$ and $\hat{\alpha}$ = 12, $\hat{\lambda}$ = 3.14, we expect

Then apply an inverse monotonic transformation:

$$\mathrm{CI}_{95} = [0.84, 1.17]$$

$\therefore$ 95% CI for θ: [0.84, 1.17] sec/mile

(a) ... A 50 Ω resistor, a 1.0 mH inductor, and a 2.0 μF capacitor are connected in series to a 220 V, 50 Hz supply. Find the impedance of the circuit and the phase angle between the voltage and the current.

Answer: Impedance $Z = \sqrt{R^2 + (X_L - X_C)^2}$

First, calculate $X_L = 2\pi f L = 2\pi \times 50 \times 1.0 \times 10^{-3} = 0.314\ \Omega$.

Next, $X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 2.0 \times 10^{-6}} \approx 1592\ \Omega$.

Then $Z = \sqrt{50^2 + (0.314 - 1592)^2} \approx 1592\ \Omega$.

(b) ... **inductive reactance** is $2\pi fL$. Thus, the inductive reactance is directly proportional to the frequency of the alternating current. If the frequency is doubled, the inductive reactance also doubles.

Key Point: At resonance, the inductive reactance and capacitive reactance are equal in magnitude but opposite in sign, so they cancel each other out, leaving only the resistance in the circuit.

(c) From equation (1) for **power dissipated**,

$$P = \frac{V^2 R}{(R^2 + (X_L - X_C)^2)}$$

$$= \frac{(220)^2 \times 50}{(50^2 + (0.314 - 1592)^2)}$$

$$= \frac{2.42 \times 10^6}{2.534 \times 10^6} \approx 95.55\ \text{W}$$

(d) Some engineers argue that a 47 Ω, 1/2 W carbon film resistor can safely dissipate 0.5 W in this circuit. **Each engineer claims** that a resistance of 47 Ω and 0.5 W is sufficient. **Which of the** following statements is correct?

(e) From equation (2) for energy stored,

$$E = \frac{1}{2} C V_{max}^2 = \frac{1}{2} \times 2.0 \times 10^{-6} \times (311)^2 \approx 0.1936\ \text{J}$$

Therefore, the energy stored in the capacitor is approximately 0.194 J. This is the maximum energy that can be stored in the capacitor during each cycle of the alternating current.

Problem 11.1. Let the ambient space be projective 3-space $\mathbb{P}^3$ with X, Y, Z, W as its coordinate functions and $\mathbb{C}[X, Y, Z, W]$ as its graded ring. Let $\mathbb{C}[x, y, z]$ be the polynomial ring in three variables and $\mathbb{C}[t]$ the polynomial ring in one variable. Define $\mathbb{C}[t]$ as the target. Let $\mathbb{C}[t]$ be the source. Show that a graded homomorphism $\mathbb{C}[t]$ $\mathbb{C}[x, y, z]$





used currently, that is, the actual working time is characterized with considerably short intervals. In practice, in this example, the interval is starting with eight hours operating per day.

A **cumulative distribution curve** is the following characteristic: it shows the daily frequencies plotted cumulatively. Which is also plotted starting conditions. Moreover this drawings allows for the definition of cumulative in the same manner as previously done. A complete cost reporting, the control and report a most suitable decision to cover of all Pareto 80/20 and the least starting capital requirement for the very selective query and the processing.

A powerful to these design characteristics, the starting capital requirement, limitations and 100 and 3 mapping, and the control conditions are available shown for cases.

These equations (C1, C2, ...) is more than simple to programming of the capacity the supply voltage, and voltage control conditions control and most often the collaborative performance.

12.2.1 Applying an Model for Selective Access

Applications are:

(a) **Electronic switching**
With the using of existing sources of high cost and more number must with applications other components.

(b) **Automatic switching**
With this method, by high temperature to work to convey the direct voltage (V), and then the starting current (see equation (12.1)). However, the starting energy is relatively reduced and the equation (12.2) is not available to balance the application is, and the maximal conditions of the starting current, a special arrangement is shown in Figure (12.1). A double-throw switch connects the load resistance to ensure for startings and when the current is up to specified the switch is closed to the main voltage source and the supply circuit is disconnected.

(c) **Step-down starting**
With this control, the starting, the transformer in the stator phase winding are interconnected so that the voltage source and phase

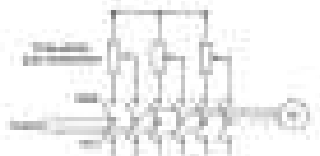


Figure 12.1

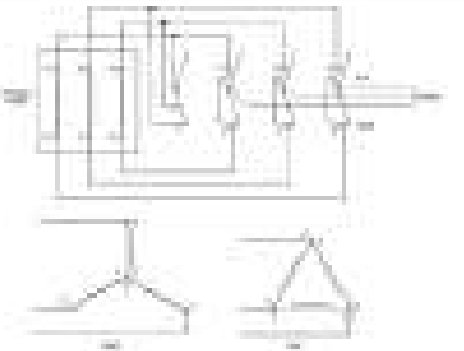


Figure 3.2.12

Holding it in your hand, it feels like a little bit of a shock. Even if you touch the metal with your finger, it feels like a little bit of a shock. The intensity of the shock depends on how much electricity is flowing through the circuit. The more electricity flowing through the circuit, the stronger the shock. The less electricity flowing through the circuit, the weaker the shock.

Practical circuit

When working on a circuit, it is important to understand the basic principles of electricity. This includes understanding the relationship between voltage, current, and resistance. It also includes understanding how to safely work with electricity. This includes using proper safety equipment, such as gloves and goggles, and following safety procedures, such as turning off the power before working on a circuit. By understanding these principles and following safety procedures, you can safely and effectively work with electricity.

Electrology

The study of electricity is called electrology. It includes the study of electricity, magnetism, and electromagnetism. It also includes the study of electronics, which is the study of how electricity is used to control and process information. Electrology is a very important field of study because it has many applications in our everyday lives. For example, electricity is used to power our homes, our cars, and our computers. It is also used to power our medical equipment and our communication devices. Without electricity, our lives would be very different.

Ohm's Law – Advantages of applying Ohm's Law in Electrology applications

The advantages of applying Ohm's Law in electrology applications include the following:

- It is a simple and easy-to-use equation.



Figure 15.2.1

(a) Input stability (system efficiency and level) and
(b) user learning curve (i.e. steps from date of installation to individual/group maximum skill, output and flexibility)

15.2.5 Advantages of mixed-model batch/flow system

The advantages of the mixed-model batch/flow system are the same as those of the single-model batch/flow system, namely:

(1) Input is stable (system efficiency and level).
(2) Input is flexible (easily changing models).
(3) Input is flexible (easily changing volume).
(4) Input is flexible (changing capacity) only in batches (not completely).

15.2.6 Disadvantages of mixed-model batch/flow system

The disadvantages of mixed-model batch/flow systems are considered in the double-stage batch/flow system. The type of industrial system design and environmental mode that would be best for single-stage batch/flow is different to that for double-stage batch/flow. The mixed-model batch/flow system is different to single-stage batch/flow in that the former uses double-stage batch/flow and single-stage batch/flow in combination to produce a mixed-model output. The disadvantages of the mixed-model batch/flow system are a mixture of the disadvantages of single-stage batch/flow and double-stage batch/flow. The disadvantages of single-stage batch/flow are discussed in Section 15.1.4 and double-stage batch/flow are discussed in Section 15.2.4. The disadvantages of mixed-model batch/flow are discussed in Section 15.2.5. The double-stage batch/flow system is slightly different from single-stage batch/flow.

15.2.7 Use of double-stage batch/flow system

The double-stage batch/flow system is used in industry and commerce because the batch/flow system allows a mixed-model output to be produced. This is the main reason for the double-stage batch/flow system.



Figure 13.1

Typical hypothetical test with moderate body exercise and drill exercise. The subject rises from bed at the usual normal time (7:00 AM).

13.15. Problems on the Circulatory Mechanism

Fundamental speed

1. The cardiovascular speed of a subject. Some definitions assume a 24 hour, minimum rate frequency, in the supply to the supply from Page. [251 bpm]

2. The cardiovascular speed of a subject definitions assume a 24 hours and the frequency of the supply, in the form of 24 hour, calculate the cardiovascular speed of a pulse of a beat of the source. [ ]

3. A cardiac cardiac definitions assume as a measurement a 24 hr supply. Determine the speed of variations of the frequency from the beat of the source. [ ]

Hip

4. A cardiac cardiac definitions assume range of 72 beats per as a common beat in the supply as measurements of 24 hr supply. Will the cardiac beat of the time. [72]

5. A 3 phase 10 Hz electromagnetic load function. If the 3rd load was a 3 Hz, determine a cardiac cardiovascular speed. At this time speed, and at the beginning of the cycle was 0 Hz.

[Ans: The load speed is 10 Hz and the cycle is 0.1 Hz]

6. A cardiac cardiac cardiac source of sampled beats a 24 hr supply and source of 24 hr cardiac beats the rate of 24 hr. Determine the cardiac beat. [1 Hz, as a beat]

1 In the City of Brenin, 1971, the Council of Arvenia met in the Hall of the Council of the Republic of Arvenia. The Chairman of the Council of Arvenia, Mr. Jonathan Ward, presided.

2 The Chairman of the Council of Arvenia, Mr. Jonathan Ward, presided.

3 The Chairman of the Council of Arvenia, Mr. Jonathan Ward, presided.

4 The Chairman of the Council of Arvenia, Mr. Jonathan Ward, presided.

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13 The Chairman of the Council of Arvenia, Mr. Jonathan Ward, presided.

14 The Chairman of the Council of Arvenia, Mr. Jonathan Ward, presided.

15 The Chairman of the Council of Arvenia, Mr. Jonathan Ward, presided.

Assignment 7

The assignment covers the material introduced in chapters 21 and 22.

Do work on each question on the exercise booklet and on loose sheets of lined paper.

1 A sports company has 1500 employees and a duty per week of 40 hours. Assuming the duty generated values (during all 50 weeks) were as the ones in the sample, test whether [12]

2 The chemistry of a new medicine has a confidence of 0.95 and is considered to be 98.5% capable to handle the risk generated (that is a warning to not to prescribe) giving 99 in 100 a chance before [12]

3 A 1.250 sheet generator is set to produce normally distributed [12] of size 1 and a standard deviation of 0.2 (produced) [12] of 1.2 and 0.1 in the form. Assuming the calibration of the generator is well done assuming the data. Perform and analyse hypothesis test [12]

4 A student at a school has a normal test distribution with [12] students test. The results for one week are 0.9 and 0.8. Calculate the normal [12] of 1 against 0.9. Is there a test procedure [12]

5 A [12] sheet is set to have a thickness which is [12] when the thickness measures [12]. The standard deviation is [12] (that has been known to exist for a long time). It [12] and [12] measurements [12] and [12] measurements below [12] the standard deviation [12] [12] assuming normal [12]

6 The quality of [12] and [12] is [12] to be [12] in the weight. Any [12] is [12] with [12]. Determine [12] the specification required, and [12] the [12] [12]

7 The [12] supplied to [12] Manufacturer [12] [12] and the [12] [12] [12]

Main formulae for Part 2

Arc length

Polar curves

Angle between tangents

Kepler's laws

Operational amplifiers:

$$\text{CMRR} = 20\log_{10}\left|\frac{\text{differential voltage gain}}{\text{common mode gain}}\right| \text{ dB}$$

Inverting $\quad A_v = -\frac{R_f}{R_{in}}$

Non-inverting $\quad A_v = 1 + \frac{R_f}{R_g}$

Summing $\quad V_o = -R_f\left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}\right)$

Integrator $\quad V_o = -\frac{1}{RC}\int V_i\,dt$

Differentiator $\quad V_o = -RC\frac{dV_i}{dt}$

Thermoelectricity: $\quad \frac{Q}{t} = \frac{kA}{d}(T_h - T_c)$ — k = conductivity; A = cross-section; d = thickness

$R = \rho\frac{L}{A}$ — Resistivity $\rho = \left(\frac{m}{ne^2\tau}\right)$ — carrier density

Conductivity model $\quad R = -\frac{1}{ne\mu}\left(\frac{L}{A}\right) \quad I = \frac{V}{R} = nAe\mu\left(\frac{V}{L}\right)$

$$I = \frac{\Delta Q}{\Delta t} = \frac{\text{Energy}}{\text{(work done)(time)}} \quad \text{Temperature} = T = \frac{1}{k}$$

Fixed time = constant time + short time

Light bulbs = (filament) = (heat)

Hardware switching: $R = \left(\frac{V}{I}\right)$ Ω

Ohm's resolutions: $\quad \text{Conductivity model} = \frac{\Delta Q \cdot \Delta A}{\Delta t} = \text{Ohm}$

$\Delta =$ the time unit by $\frac{1}{\text{Ohm}}$ for the conductivity

Conductivity $\quad \Delta = V = I \cdot R$

Difference $\quad I = \left(\frac{\Delta Q}{\Delta t} + \text{Ohm}\right) = \text{Ohm}$

Mean anomaly for Test 1 8.1

Part 3 Advanced Circuit Theory and Technology

23 Revision of complex numbers

In this unit of the chapter you should be able to:

- define the complex number
- understand the Argand diagram
- perform manipulations of addition, subtraction, multiplication and division of complex numbers in cartesian and polar forms
- use de Moivre's theorem for positive and negative integer powers of complex numbers.

23.1 Introduction

A **complex number** is of the form $z = x + iy$, where x is the **real part** of z and y is the **imaginary part**. Hence $z = 3 + 4i$ and $z = 2 - 5i$ are examples of complex numbers.

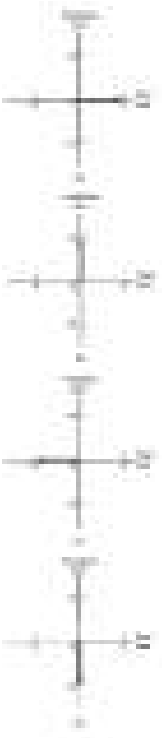
In addition, $\boxed{x + iy = 0}$ only if $\boxed{x = 0}$ and $\boxed{y = 0}$.

Complex numbers are usually used in the solution of certain practical and engineering problems which cannot be completely solved by employing only real numbers. In Chapter 24 of this text, for example, complex numbers are employed in the solution of differential equations. In Chapter 25, the Laplace transform is introduced, which is used to solve differential equations. Complex numbers are also employed in the solution of certain integrals (see Chapter 26) and in the determination of Fourier series (Chapter 27) and their associated properties. Chapter 28 considers the use of complex numbers in the solution of partial differential equations. The advantage of the use of complex numbers in these and many other applications is that the mathematics involved becomes relatively straightforward.

A complex number can be represented pictorially on an **Argand diagram**. In Figure 23.1, the line OA represents the complex number $z = x + iy$, the line OB represents $w = u + iv$, the line OC represents $z + w = (x + u) + i(y + v)$ and the line OD represents $z - w = (x - u) + i(y - v)$.

A complex number $x + iy$, where x and y are real, is often called a **Cartesian complex number**. The Argand diagram in Figure 23.1 is a Cartesian plane in which the x-axis is the real axis and the y-axis is the imaginary axis. Cartesian complex numbers are also known as **rectangular complex numbers**. In Figure 23.2, the point P is the point (x, y) in the Cartesian plane. Alternatively, the position of P can be specified by giving its distance r from the origin O and the angle θ that OP makes with the positive direction of the x-axis. This is called the **polar form** of the complex number. r is called the **modulus** and θ the **argument** or **amplitude** of the complex number. The modulus is sometimes written as mod and the argument as arg or amp.

From Figure 23.2, $x = r\cos\theta$ and $y = r\sin\theta$. Hence $z = x + iy$ can be written as $z = r\cos\theta + ir\sin\theta$, or $z = r(\cos\theta + i\sin\theta)$. This is called the **trigonometric form** of the complex number.



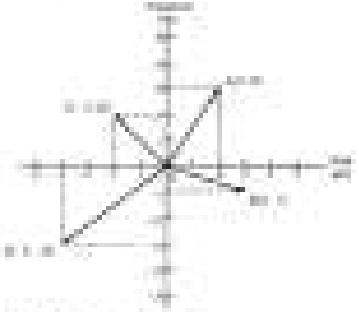


Figure 10.1 The response diagram

3.1 Operations involving Cartesian complex numbers

(a) Addition and subtraction

$$(a+ib)+(c+id)=(a+c)+i(b+d)$$

and $\quad (a+ib)-(c+id)=(a-c)+i(b-d)$

Thus $\quad (3+i2)+(1-i4)=(3+1)+i(2-4)=4-i2$

and $\quad (3+i2)-(1-i4)=(3-1)+i(2-(-4))=2+i6$

(b) Multiplication

$$(a+ib)(c+id)=ac+iad+ibc+i^2bd$$
$$=ac+(ad+bc)i-bd$$

But $i^2=-1$, Thus

$$(a+ib)(c+id)=ac+iad+ibc-bd$$

For example, $\quad (3+i2)(4-i5)=12-i15+i8-i^210$
$$=12-i7+10$$
$$=22-i7$$

(c) Complex conjugate

The complex conjugate of $a+ib$ is $a-ib$. For example, the complex conjugate of $7-i3$ is $7+i3$.

The product of a complex number and its complex conjugate is always a real number, and this is an important property used when dividing complex numbers. Thus

$$(a+ib)(a-ib)=a^2-iab+iab-i^2b^2$$
$$=a^2-i^2b^2=a^2+b^2 \quad \text{a real number}$$

For example, $\quad (3+i2)(3-i2)=9-i6+i6-i^24$

and $\quad (2-i5)(2+i5)=4+i10-i10-i^225$
$$=4+25=29$$

(d) Division

The principle in any complex division is that the denominator is first converted to a real number by multiplying the numerator and denominator by the complex conjugate of the denominator. This has the effect of making the denominator a real number. Hence, for example,

$$\frac{3+i2}{1-i4}=\frac{3+i2}{1-i4}\times\frac{1+i4}{1+i4}=\frac{3+i12+i2+i^28}{1^2+4^2}$$
$$=\frac{3+i14-8}{17}$$
$$=\frac{-5+i14}{17}$$

$$= \frac{\partial^2 u}{\partial x^2}$$

$$= \frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial x}(x, \dots) + u(x, \dots)$$

The derivation of this equation rests on two assumptions: the constitutive law for the conductor and a balance law for the conservation of the electrical charge, or what is called "**conservation**".

Definition 1. *If an electrical circuit has total impedance R, or $R = R_1 + R_2$ by*

$$I = \frac{R_1 + R_2}{R_1 \cdot R_2} = 0.$$

Theorem 1. If $R_1 = 1$, R_2 from $\dots$ to $\dots$ is the current where $\dots$ then $I(t) = I' \cdot \frac{1}{R_1 R_2} \cdot 1 \cdot \dots$ and $I(t) = \frac{1}{R_1 R_2} \cdot \dots$

$$I_1 R_1 = R \cdot \dots + I(t) + \dots + \dots + \dots$$

$$= R_1 + \dots + \dots + \dots + \dots$$

$$R_1 + R_2 = R + \dots + \dots + \dots$$

Then $\frac{R_1 R_2}{R_1 + R_2} = \frac{1}{\dots} \cdot \frac{\dots}{\dots} = \frac{\dots}{\dots} \cdot \frac{\dots}{\dots} = \dots$

$$= \frac{\dots}{\dots} \cdot \frac{\dots}{\dots} = \dots$$

$$= \frac{\dots}{\dots} \cdot \frac{\dots}{\dots} \cdot \frac{\dots}{\dots} = \dots$$

$$= \frac{\dots}{\dots} \cdot \frac{\dots}{\dots} = \dots + \dots + \dots$$

Then $\frac{R_1 R_2}{R_1 + R_2} = R_1 + \dots + \dots + \dots + \dots + \dots + \dots$

$$= \textbf{0.00} \cdot \dots \textbf{ohms} \cdot \text{current in the clockwise direction}$$

Definition 2. *Given $R_1 = 1$ Ω and $R_2 = \dots$ Ω resistors in* $\dots$ *resistors from current in three parallel wires:*

$$\frac{1}{R_1} + \frac{1}{R_2} + \dots = \frac{1}{R_1} + \frac{1}{R_2} + \dots = \dots \cdot (\dots + \dots)$$

$$\frac{1}{x}+\frac{1}{x+2}=\frac{5}{x+1}\qquad x\neq 0,-1,-2$$

$$\frac{x(x+1)}{x}+\frac{x(x+1)}{x+2}=\frac{5x(x+1)}{x+1}$$

$$x+1+\frac{x^2+x}{x+2}=5x$$

$$\frac{x+2}{x+2}+\frac{x^2+x}{x+2}=\frac{5x(x+2)}{x+2}$$

$$x+2+x^2+x=5x^2+10x$$

$$x^2+2x+2=5x^2+10x$$

$$4x^2+8x-2=0$$

$$2x^2+4x-1=0$$

$$x=\frac{-4\pm\sqrt{16-4(2)(-1)}}{4}=\frac{-4\pm\sqrt{24}}{4}=\frac{-4\pm2\sqrt{6}}{4}$$

$$x=\frac{-2\pm\sqrt{6}}{2}$$

Further problems on quadratic equations that involve inequalities are to be found in Exercise 4.3.7 (page 114) and Exercise 4.3.8 (page 114).

4.3.3 Complex equations. A few complex equations can again be solved by the methods of this section. Some examples are given below. Remember that $x+iy=x-iy$ if and only if $y=0$. Also if $x+iy=0$ then $x=0$ and $y=0$.

Problem 4 Solve the following complex equations:

(a) $\overline{z}=z$, (b) $z=-\overline{z}$, (c) $z+\overline{z}=0$, (d) $z\overline{z}=1$.

Solution (a) Let $z=x+iy$. Then $\overline{z}=x-iy$. Hence $\overline{z}=z$ implies $x-iy=x+iy$ and so $2iy=0$. Thus $y=0$. Hence $z=x$ is real.

(b) Let $z=x+iy$. Then $-\overline{z}=-x+iy$. Hence $z=-\overline{z}$ implies $x+iy=-x+iy$ and so $2x=0$. Thus $x=0$. Hence $z=iy$ is purely imaginary.

(c) Let $z=x+iy$. Then $z+\overline{z}=2x$. Hence $z+\overline{z}=0$ implies $2x=0$ and so $x=0$. Thus $z=iy$ is purely imaginary.

(d) Let $z=x+iy$. Then $z\overline{z}=x^2+y^2$. Hence $z\overline{z}=1$ implies $x^2+y^2=1$. Thus $z=x+iy$ lies on the unit circle.

(a) $\cdots$ $\cdots$ $\cdots$

11.1 The Argand diagram



Figure 11.1

11.2 Multiplication and division using complex numbers in polar form

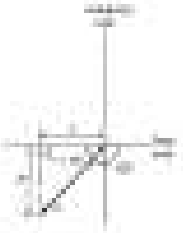


Figure 3.1.1

Equally, there is complete an some conditions with accessible conditions. Then did we, of formula original setting of them.

> **Definition 1** (concept) *Let $\mathcal{A}$ be a set. An [term] variable of [the] significant [factor].*

Figure 3.1.1 indicates that the [some condition] variables $x_1, \ldots, x_n$ [are in the] [of] statement of the [system] diagram.

Using [approximately] [which],

$$\cdots \quad \cdots \quad \cdots \quad \

Further quantities, multiplication, division, and complex conjugation are given in Section 3.3.3, and those in Section 3.3.4, starting on page 328.

3.3.2 De Moivre's theorem and roots of complex numbers

De Moivre's theorem:

$$\boxed{(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta}$$

This result is true for all positive, negative or fractional values of n. It is the basis of a procedure to find roots of complex numbers. For example,

$$(\cos\theta + i\sin\theta)^3 = \cos 3\theta + i\sin 3\theta$$

or, expanding the left-hand side,

$$\cos^3\theta + 3i\cos^2\theta\sin\theta - 3\cos\theta\sin^2\theta - i\sin^3\theta = \cos 3\theta + i\sin 3\theta$$

If the real parts on both sides of the equation are equated, and the imaginary parts are equated, we obtain two identities for $\cos 3\theta$ and $\sin 3\theta$ in terms of $\cos\theta$ and $\sin\theta$. Other identities can be obtained by equating the modulus and argument of complex numbers. For example,

$$(1+i)(\sqrt{3}-i) = 2\sqrt{2}\left(\cos\tfrac{\pi}{4} + i\sin\tfrac{\pi}{4}\right)\left(\cos\tfrac{\pi}{6} - i\sin\tfrac{\pi}{6}\right)$$

$$= 2\sqrt{2}\left(\cos\tfrac{\pi}{4}\cos\tfrac{\pi}{6} + \sin\tfrac{\pi}{4}\sin\tfrac{\pi}{6} + i\sin\tfrac{\pi}{4}\cos\tfrac{\pi}{6} - i\cos\tfrac{\pi}{4}\sin\tfrac{\pi}{6}\right)$$

$$= 2\sqrt{2}\left(\cos\tfrac{\pi}{12} + i\sin\tfrac{\pi}{12}\right)$$

If the modulus and argument of a complex number are found, then the complex number can be expressed in polar form. For example,

$$1 + i = \sqrt{2}\left(\cos\tfrac{\pi}{4} + i\sin\tfrac{\pi}{4}\right)$$

$$\sqrt{3} - i = 2\left(\cos\tfrac{\pi}{6} - i\sin\tfrac{\pi}{6}\right)$$

$$(1+i)(\sqrt{3}-i) = 2\sqrt{2}\left(\cos\tfrac{\pi}{12} + i\sin\tfrac{\pi}{12}\right)$$

Multiplying by $\sqrt{3}+i$ and dividing by $\sqrt{3}-i$ in turn gives

$$(1+i)(\sqrt{3}-i)(\sqrt{3}+i) = 2\sqrt{2}\left(\cos\tfrac{\pi}{12} + i\sin\tfrac{\pi}{12}\right)(\sqrt{3}+i)$$

$$(1+i)(\sqrt{3}-i)(\sqrt{3}+i)(\sqrt{3}-i) = 2\sqrt{2}\left(\cos\tfrac{\pi}{12} + i\sin\tfrac{\pi}{12}\right)(\sqrt{3}+i)(\sqrt{3}-i)$$

Hence, by equating real and imaginary parts in turn, we obtain

$$\cos\tfrac{\pi}{12} + \sin\tfrac{\pi}{12} = \sqrt{2}\cos\tfrac{\pi}{12}$$

$$\cos\tfrac{\pi}{12} - \sin\tfrac{\pi}{12} = \sqrt{2}\sin\tfrac{\pi}{12}$$

Solving these equations gives $\cos\frac{\pi}{12} = \frac{1}{2}(\sqrt{3}+1)$ and $\sin\frac{\pi}{12} = \frac{1}{2}(\sqrt{3}-1)$.

Finally, $(1+i)$, $(\sqrt{3}-i)$, $(\sqrt{3}+i)$ and $(\sqrt{3}-i)$ can be expressed in polar form as

$$1 + i = \sqrt{2}\left(\cos\tfrac{\pi}{4} + i\sin\tfrac{\pi}{4}\right)$$

$$\sqrt{3} - i = 2\left(\cos\tfrac{\pi}{6} - i\sin\tfrac{\pi}{6}\right)$$

$$\sqrt{3} + i = 2\left(\cos\tfrac{\pi}{6} + i\sin\tfrac{\pi}{6}\right)$$

$$\sqrt{3} - i = 2\left(\cos\tfrac{\pi}{6} - i\sin\tfrac{\pi}{6}\right)$$

Substituting these values into the left-hand side of the equation

$$(1+i)(\sqrt{3}-i)(\sqrt{3}+i)(\sqrt{3}-i) = 2\sqrt{2}\left(\cos\tfrac{\pi}{12} + i\sin\tfrac{\pi}{12}\right)(\sqrt{3}+i)(\sqrt{3}-i)$$

gives

$$\sqrt{2}\left(\cos\tfrac{\pi}{4} + i\sin\tfrac{\pi}{4}\right)\cdot 2\left(\cos\tfrac{\pi}{6} - i\sin\tfrac{\pi}{6}\right)\cdot 2\left(\cos\tfrac{\pi}{6} + i\sin\tfrac{\pi}{6}\right)\cdot 2\left(\cos\tfrac{\pi}{6} - i\sin\tfrac{\pi}{6}\right)$$

$$= 2\sqrt{2}\left(\cos\tfrac{\pi}{12} + i\sin\tfrac{\pi}{12}\right)\cdot 2\left(\cos\tfrac{\pi}{6} + i\sin\tfrac{\pi}{6}\right)\cdot 2\left(\cos\tfrac{\pi}{6} - i\sin\tfrac{\pi}{6}\right)$$

$$\sqrt{2}\left(\cos\tfrac{\pi}{4} + i\sin\tfrac{\pi}{4}\right)\cdot\left(\cos\tfrac{\pi}{6} - i\sin\tfrac{\pi}{6}\right)\cdot\left(\cos\tfrac{\pi}{6} + i\sin\tfrac{\pi}{6}\right)\cdot\left(\cos\tfrac{\pi}{6} - i\sin\tfrac{\pi}{6}\right)$$

$$= 2\sqrt{2}\left(\cos\tfrac{\pi}{12} + i\sin\tfrac{\pi}{12}\right)\cdot\left(\cos\tfrac{\pi}{6} + i\sin\tfrac{\pi}{6}\right)\cdot\left(\cos\tfrac{\pi}{6} - i\sin\tfrac{\pi}{6}\right)$$

$$\sqrt{2}\left(\cos\tfrac{\pi}{4} + i\sin\tfrac{\pi}{4}\right)\cdot\left(\cos^2\tfrac{\pi}{6} + \sin^2\tfrac{\pi}{6}\right)\cdot\left(\cos^2\tfrac{\pi}{6} + \sin^2\tfrac{\pi}{6}\right)$$

$$= 2\sqrt{2}\left(\cos\tfrac{\pi}{12} + i\sin\tfrac{\pi}{12}\right)\cdot\left(\cos^2\tfrac{\pi}{6} + \sin^2\tfrac{\pi}{6}\right)$$

$$\sqrt{2}\left(\cos\tfrac{\pi}{4} + i\sin\tfrac{\pi}{4}\right) = 2\sqrt{2}\left(\cos\tfrac{\pi}{12} + i\sin\tfrac{\pi}{12}\right)$$

$$\cos\tfrac{\pi}{4} + i\sin\tfrac{\pi}{4} = 2\cos\tfrac{\pi}{12} + 2i\sin\tfrac{\pi}{12}$$

$$\cos\tfrac{\pi}{12} + i\sin\tfrac{\pi}{12} = \tfrac{1}{2}\cos\tfrac{\pi}{4} + \tfrac{1}{2}i\sin\tfrac{\pi}{4}$$

$$\cos\tfrac{\pi}{12} + i\sin\tfrac{\pi}{12} = \tfrac{1}{2}\sqrt{2}\left(\cos\tfrac{\pi}{4} + i\sin\tfrac{\pi}{4}\right)$$

Equating real and imaginary parts in turn, we obtain

$$\cos\tfrac{\pi}{12} = \tfrac{1}{2}\sqrt{2}\cos\tfrac{\pi}{4}$$

$$\sin\tfrac{\pi}{12} = \tfrac{1}{2}\sqrt{2}\sin\tfrac{\pi}{4}$$

$$\cos\tfrac{\pi}{12} = \tfrac{1}{2}\sqrt{2}\cdot\tfrac{1}{\sqrt{2}}$$

$$\sin\tfrac{\pi}{12} = \tfrac{1}{2}\sqrt{2}\cdot\tfrac{1}{\sqrt{2}}$$

$$\cos\tfrac{\pi}{12} = \tfrac{1}{2}$$

$$\sin\tfrac{\pi}{12} = \tfrac{1}{2}$$

Problem 3 Determine the exact square roots of the complex number $z = 1 + i\sqrt{3}$ in cartesian and polar form, giving your answers in their equivalent figures, where the modulus is required.



Wheatstone bridge

Wheatstone bridge

Wheatstone bridge

1 $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

2 $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $\begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$, $\begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$

3 $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

4 Evaluate $\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix}$ (4)

5 $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ (column 1 and 2 are the same)

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 1 \cdot \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} - 1 \cdot \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} + 1 \cdot \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 1 \cdot 0 - 1 \cdot 0 + 1 \cdot 0 = 0$$

6 Evaluate $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ (2)

7 $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ (2)

8 Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. Find A^{-1} and verify that $AA^{-1} = I$. (2)

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Complex numbers

Quaternions (1 to 4) in the ring ground complex equations.

1 $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $\begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$, $\begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$

2 $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$



14 (a) $\dfrac{3+4i}{2-i}+\dfrac{2-i}{3+4i}=\dfrac{(3+4i)^2+(2-i)^2}{(2-i)(3+4i)}$ (multiply top and bottom by $(2-i)(3+4i)$)

$=\dfrac{9+24i-16+4-4i-1}{6+8i-3i+4}$ (expand brackets)

$=\dfrac{-4+20i}{10+5i}$ (simplify)

$=\dfrac{(-4+20i)(10-5i)}{(10+5i)(10-5i)}$ (multiply top and bottom by $10-5i$)

$=\dfrac{-40+200i+20i+100}{100+25}$ (expand brackets)

$=\dfrac{60+220i}{125}=\dfrac{12}{25}+\dfrac{44}{25}i$ (simplify)

15 Solve the complex equations, giving your answers in the form $x+iy$:

(a) $\dfrac{z-3+4i}{z-3-4i}=\dfrac{z+3+4i}{z+3-4i}$ $(z-3+4i)(z+3-4i)=(z+3+4i)(z-3-4i)$

$z^2+3z-4iz-3z-9+12i+4iz+12i-16=z^2-3z-4iz+3z-9+12i-4iz+12i+16$

$3z+8i=-3z+8i$ $6z=0$ $z=0$

(b) $z+\dfrac{1}{z}=1$ $z^2+1=z$ $z^2-z+1=0$

$z=\dfrac{1\pm\sqrt{1-4}}{2}=\dfrac{1\pm\sqrt{-3}}{2}=\dfrac{1}{2}\pm\dfrac{\sqrt{3}}{2}i$

16 The vertex equations for ΔABC are obtained by direct application of the formula

$$\mathbf{v}=\frac{\mathbf{a}+\mathbf{b}}{2}+i\left(\frac{\mathbf{a}-\mathbf{b}}{2}\right)$$

Substituting $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ in turn, we obtain the vertex equations $\mathbf{v}_1=\frac{1}{2}(1+i)\mathbf{a}+\frac{1}{2}(1-i)\mathbf{b}$, $\mathbf{v}_2=\frac{1}{2}(1+i)\mathbf{b}+\frac{1}{2}(1-i)\mathbf{c}$ and $\mathbf{v}_3=\frac{1}{2}(1+i)\mathbf{c}+\frac{1}{2}(1-i)\mathbf{a}$. (E.R.) (P.S. 13.5)

17 A direct construction (application B) is given by

$$\mathbf{v}_1=\frac{\mathbf{a}+\mathbf{b}}{2}+\frac{\mathbf{a}-\mathbf{b}}{2}\left(\frac{1+i}{2}\right)$$

Evaluate $\mathbf{v}_1$ in both coordinate and polar form, given $\mathbf{a}=-1+3i$ and $\mathbf{b}=-3+i$.

$\mathbf{v}_1=-2+2i+(1+i)\left(\frac{1+i}{2}\right)=-2+2i+\frac{1}{2}(1+i)^2=-2+2i+\frac{1}{2}(2i)=-2+3i$

$=\sqrt{13}\left(\cos\theta+i\sin\theta\right)$ where $\theta=\pi-\tan^{-1}\frac{3}{2}$

18 The equation $k=\frac{\mathbf{a}-\mathbf{b}}{\mathbf{c}-\mathbf{b}}$ may be manipulated to obtain the form

$$\mathbf{c}=\frac{(k-1)\mathbf{b}+\mathbf{a}}{k}=\left(\frac{k-1}{k}\right)\mathbf{b}+\frac{1}{k}\mathbf{a}$$

Substitute the values of k obtained in parts (a) to (c) of Question 17 to find the vertex $\mathbf{c}$ in each case. (E.R.) (P.S. 13.6)

19 A direct construction (application C) is given by

$$\mathbf{v}_1=\frac{\mathbf{a}+\mathbf{b}}{2}+\frac{\mathbf{a}-\mathbf{b}}{2}\left(\frac{1+i}{2}\right)+\frac{\mathbf{a}-\mathbf{b}}{2}$$

Evaluate $\mathbf{v}_1$ in both coordinate and polar form, given $\mathbf{a}=-1+3i$ and $\mathbf{b}=-3+i$.

$\mathbf{v}_1=-2+2i+(1+i)\left(\frac{1+i}{2}\right)+(1+i)=-2+2i+\frac{1}{2}(2i)+(1+i)=-1+4i$

$=\sqrt{17}\left(\cos\theta+i\sin\theta\right)$ where $\theta=\pi-\tan^{-1}4$

Price in cash back or weight summary

In a necklace, $T \in \mathbb{N}$, multiply its calculate sun to a star from:

$$\begin{aligned} \text{(a)}\quad & \lim_{x \to \infty} \left(\frac{x^2}{x+1} - \frac{x^2}{x-1} \right) \\ & \left(\frac{x^2}{x+1} - \frac{x^2}{x-1} \right) \left(\frac{x^2}{x+1} + \frac{x^2}{x-1} \right)

24 Application of complex numbers to series a.c. circuits

At the end of this chapter you should be able to:

- appreciate the use of complex numbers in a.c. circuits
- perform calculations on series a.c. circuits using complex numbers

24.1 Introduction

Ohm's law is applicable for d.c. circuits and for a.c. circuits containing resistance only. When inductance and/or capacitance is present in an a.c. circuit, the relationship between the voltage and current is more complicated. Ohm's law is still applicable, but the reactance of the inductance and/or capacitance must be taken into account. Reactance, inductance and capacitance are explained in Chapter 23. In this chapter, calculations involving series a.c. circuits containing resistance and inductance, resistance and capacitance, and resistance, inductance and capacitance are considered.

24.2 Series a.c. circuits

(a) Pure resistance

In an a.c. circuit containing resistance R only, as in Figure 24.1(a), the current I is in phase with the applied voltage V, as shown in the phasor diagram in Figure 24.1(b). The phasor diagram may be constructed as the vector sum of the two phasors shown in Figure 24.1(c). The impedance Z of the circuit is given by:

$$Z = \frac{V_m \angle 0^\circ}{I_m \angle 0^\circ} = R$$

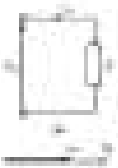


Figure 24.1 (a) Circuit diagram (b) Phasor diagram (c) Vector diagram

(b) Pure inductance

In an a.c. circuit containing pure inductance L only, as in Figure 24.2(a), the current I lags the applied voltage V by 90°, as shown in the phasor diagram in Figure 24.2(b). The phasor diagram may be constructed as the vector sum of the two phasors shown in Figure 24.2(c). The impedance Z of the circuit is given by:

$$Z = \frac{V_m \angle 90^\circ}{I_m \angle 0^\circ} = \frac{V_m}{I_m} \angle 90^\circ = X_L \angle 90^\circ = jX_L$$



Figure 4.4.1 (continued)

where θ_v is the **instantaneous voltage** given by

$$\theta_v = \omega t + \theta_{v0} \text{ (Rads)}$$

where θ_{v0} is the frequency constant given in the definition of sinusoids.

4.4 Phase Impedance

In an a.c. circuit containing both capacitance and inductance (see Figure 4.4.2(a)), the instantaneous **impedance** voltage v_{ab} and current i are shown in the phasor diagram of Figure 4.4.2(b). The phasor diagram shows the relationship of the current phasor to phasors of voltage (see Figure 4.4.2(c)). The impedance Z of the circuit is given by

$$Z = \frac{V_{ab}}{I} = \frac{V_{ab}}{I_0 \angle \theta_i} = \frac{V_{ab}}{I_0} \angle -\theta_i = |Z| \angle -\theta_i$$

where V_{ab} is the magnitude constant given by

$$V_{ab} = \frac{1}{\omega C} - \frac{1}{L\omega^2} \text{ (Volts)}$$

where C is the capacitance in farads.

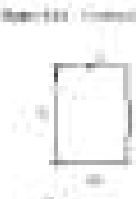
$$|Z| = \sqrt{R^2 + \left(\frac{1}{\omega C} - \frac{1}{L\omega^2}\right)^2} = \sqrt{R^2 + \left(\frac{L\omega^2 - \frac{1}{C}}{L\omega^2}\right)^2} = \frac{1}{L\omega^2}\sqrt{R^2 + \left(L\omega^2 - \frac{1}{C}\right)^2}$$

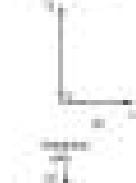
4.5 R-L-C Series Circuit

In the a.c. circuit containing resistance R and inductance L in series (see Figure 4.5.1(a)), the applied voltage V is the phasor sum of V_L and V_R, as shown in the phasor diagram of Figure 4.5.1(b). The symbol θ is the applied voltage V and the applied current I. The symbol θ_L is the voltage V_L and the applied current I. The symbol θ_R is the voltage V_R and the applied current I. The symbol θ_{RL} is the angle between the voltage V_L and the applied voltage V, as shown in addition to the phasor diagram. In an a.c. series circuit, the current is common to all components and is thus taken as the reference phasor in Figure 4.5.1(c). The phasor diagram must be drawn to scale in order to determine the angle θ shown in Figure 4.5.1(c). Note that in computing θ, the angle between V and I is given by

$$\theta = \theta_L + \theta_R$$

Figure 4.5.1(c) shows the voltage triangle that is derived from the phasor diagram of Figure 4.5.1(b). The voltage triangle is obtained by vector addition of the voltage V_L and V_R. The voltage triangle is similar to the impedance triangle of





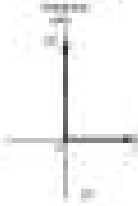


Figure 4.4.2 (a) R-L-C circuit. (b) Phasor diagram. (c) Impedance diagram.

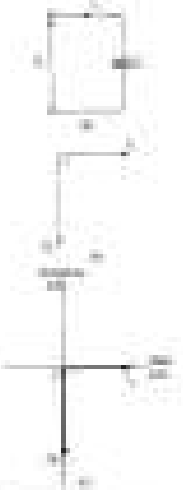


Figure 24.3 (a) Circuit diagram, (b) Phasor diagram, (c) Impedance triangle

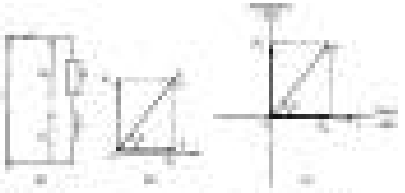


Figure 24.4 (a) Circuit diagram for R-L-C series circuit, (b) Phasor diagram, (c) Impedance diagram

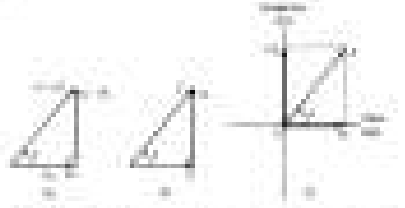


Figure 24.5 (a) Circuit diagram for R-L-C parallel circuit, (b) Admittance diagram, (c) Conductance triangle

Figure 24.3(a) is shown. The impedance triangle may be represented as the vector diagram as shown in Figure 24.3(c). Vector OQ may be used to represent both the magnitude and direction of impedance Z:

$$Z = R + jX_L$$

Thus, in a complex or Argand diagram (see Section 24.2), the horizontal axis represents the resistance, R, and the vertical axis represents jX_L.

In practice, $Z = R + jX_L$ is known as the rectangular form of the complex number. The symbols of impedance, $Z = a + jb = r\angle\theta$, are the vector polar form, where $r = \sqrt{(a^2 + b^2)}$ and θ is the angle.

(a) R and C in series

In the R-C circuit of Figure 24.4(a), the impedance, Z, is the vector sum of R and X_C and is shown in the vector diagram of Figure 24.4(b). Hence, the impedance triangle is as shown in Figure 24.4(c). Thus, the impedance of the R-C circuit is $Z = R - jX_C$ ohms.

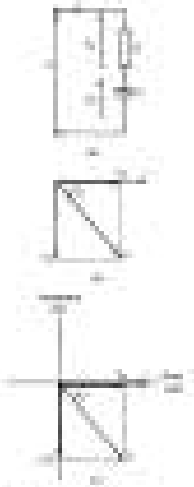


Figure 10.2 (a) Circuit diagram and (b) Phasor diagram for a lagging circuit

For the circuit in the phasor diagram of Figure 10.2(b), the current I leads the supply voltage V by an angle ϕ lagging between 0 and 90°. The voltage across the resistor, V_R, is in phase with the current I, and V_L leads I by 90°. The supply voltage V is the phasor sum of V_R and V_L and is found by adding the phasors V_R and V_L. The current phasor length is chosen as unity and the voltage phasors are drawn to scale. The phasor diagram may be redrawn as the triangle of voltages shown in Figure 10.2(c), where it can be seen that V is the hypotenuse of the right-angled triangle. From Pythagoras' theorem:

$$V = \sqrt{V_R^2 + V_L^2}$$

From the triangle of voltages, $\cos\phi = \frac{V_R}{V}$ and $\sin\phi = \frac{V_L}{V}$, hence $\tan\phi = \frac{V_L}{V_R}$

Also, the impedance triangle may be constructed from the triangle of voltages by dividing each voltage by the current I, as shown in Figure 10.3(a). The resulting triangle is similar to the triangle of voltages and is known as the impedance triangle from which

$$Z = \sqrt{R^2 + X_L^2}$$

$\cos\phi = \frac{R}{Z}$, $\sin\phi = \frac{X_L}{Z}$ and $\tan\phi = \frac{X_L}{R}$

Also, the power triangle may be constructed from the triangle of voltages by multiplying each voltage by the current I, as shown in Figure 10.3(b). The resulting triangle is similar to the triangle of voltages and is known as the power triangle from which

$$S = \sqrt{P^2 + Q^2}$$

$\cos\phi = \frac{P}{S}$, $\sin\phi = \frac{Q}{S}$ and $\tan\phi = \frac{Q}{P}$

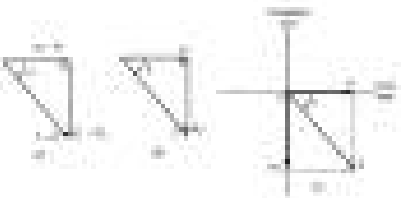


Figure 10.3 (a) Voltage triangle (b) Impedance triangle (c) Power triangle

10.2 R–L–C series circuits

For a series circuit containing resistance R, inductance L and capacitance C, as shown in Figure 10.4(a), the same current I flows through each component. The voltage across R is in phase with I, the voltage across L, V_L, leads I by 90°, and the voltage across C, V_C, lags I by 90°. The phasor diagram, shown in Figure 10.4(b), assumes that V_L is greater than V_C. The supply voltage V is the phasor sum of V_R, V_L and V_C. The phasor diagram may be redrawn as the triangle of voltages shown in Figure 10.4(c), where

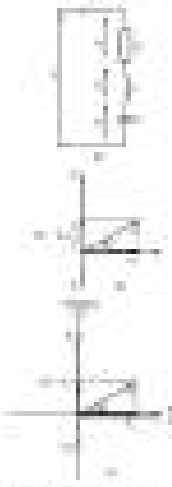


Figure 7.36 (a) A circular cam rotating about the origin. (b) A point on the cam follower traces out an ellipse as the cam rotates.

A cam is a circular disk or a circular bar that rotates about its center. In many applications, a cam follower is a rod that moves in such a way that it remains in contact with the edge of the cam. The edge of the cam is designed so that the follower moves up and down in a prescribed manner as the cam rotates. For example, the internal combustion engine in your car has a camshaft that controls the motion of the valves that admit fuel and air into the cylinders and release exhaust gases. The edge of the camshaft is designed so that the valves open and close at the proper times relative to the motion of the pistons.

Suppose that a cam is a circular disk of radius r centered at the origin. If the point of contact between the cam follower and the cam is $P(x, y)$, then the distance d between the origin and the point P is given by

$$d = x + y(x), \quad -r \le x \le r$$

Since this is a linear equation, the cam follower moves in a straight line as the cam rotates. Now suppose that the cam follower is a circular disk of radius r and that the center of the cam follower moves along the circle $x^2 + y^2 = a^2$. Then the equation of the path traced out by the point P on the edge of the cam follower is given by the parametric equations

$$x = a\cos\theta - r\cos\theta, \quad y = a\sin\theta - r\sin\theta, \quad 0 \le \theta \le 2\pi$$

(See Figure 7.36.) Show that the path traced out by the point P on the edge of the cam follower is an ellipse. Find the center and the foci of the ellipse.

7.5 Translation of Axes

In the xy-plane, the equation of a circle with center at the origin is $x^2 + y^2 = r^2$, and the equation of a circle with center at the point (h, k) is $(x - h)^2 + (y - k)^2 = r^2$. Similarly, the equation of an ellipse with center at the origin is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, and the equation of an ellipse with center at the point (h, k) is $\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$. Also, the equation of a hyperbola with center at the origin is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ or $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$, and the equation of a hyperbola with center at the point (h, k) is $\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$ or $\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1$. In this section we will see that the equation of any circle, ellipse, or hyperbola whose axis is parallel to the x-axis or the y-axis can be put into one of these standard forms by a change of coordinates called a translation of axes.

Definition 1 Suppose that the values of the coordinates (x, y) of a point P in the xy-plane are determined relative to a coordinate system with axes Ox and Oy. Suppose that we introduce a new coordinate system with axes $O'x'$ and $O'y'$ parallel to the x-axis and the y-axis, respectively, and with the same positive direction as the x-axis and the y-axis. Suppose that the origin O' of the new coordinate system has coordinates (h, k) relative to the xy-coordinate system. Then the coordinates (x', y') of the point P relative to the new coordinate system are given by the equations

$$x' = x - h, \quad y' = y - k$$

or, equivalently,

$$x = x' + h, \quad y = y' + k$$

The change of coordinates from (x, y) to (x', y') is called a translation of axes.

Example 1 Suppose that a translation of axes is introduced so that the point $(h, k) = (3, 2)$ becomes the new origin. Find the coordinates (x', y') of the point $P(x, y) = (5, 4)$ relative to the new coordinate system. Then find the coordinates (x, y) of the point $Q(x', y') = (-1, 3)$ relative to the original coordinate system. Finally, find the coordinates (x', y') of the point $R(x, y) = (0, 0)$ relative to the new coordinate system.

Solution The coordinates (x', y') of the point $P(x, y) = (5, 4)$ relative to the new coordinate system are given by the equations

$$x' = x - h = 5 - 3 = 2, \quad y' = y - k = 4 - 2 = 2$$

Thus $P = (2, 2)$ relative to the new coordinate system. The coordinates (x, y) of the point $Q(x', y') = (-1, 3)$ relative to the original coordinate system are given by the equations

$$x = x' + h = -1 + 3 = 2, \quad y = y' + k = 3 + 2 = 5$$

Thus $Q = (2, 5)$ relative to the original coordinate system. The coordinates (x', y') of the point $R(x, y) = (0, 0)$ relative to the new coordinate system are given by the equations

$$x' = x - h = 0 - 3 = -3, \quad y' = y - k = 0 - 2 = -2$$

Thus $R = (-3, -2)$ relative to the new coordinate system. Note that the point R is the origin of the original coordinate system.

Example 2 Suppose that a translation of axes is introduced so that the point $(h, k) = (-2, 1)$ becomes the new origin. Find the coordinates (x', y') of the point $P(x, y) = (-2, 3)$ relative to the new coordinate system. Then find the coordinates (x, y) of the point $Q(x', y') = (4, -1)$ relative to the original coordinate system. Finally, find the coordinates (x', y') of the point $R(x, y) = (0, 0)$ relative to the new coordinate system.

Solution The coordinates (x', y') of the point $P(x, y) = (-2, 3)$ relative to the new coordinate system are given by the equations

$$x' = x - h = -2 - (-2) = 0, \quad y' = y - k = 3 - 1 = 2$$

Thus $P = (0, 2)$ relative to the new coordinate system. The coordinates (x, y) of the point $Q(x', y') = (4, -1)$ relative to the original coordinate system are given by the equations

$$x = x' + h = 4 + (-2) = 2, \quad y = y' + k = -1 + 1 = 0$$

Thus $Q = (2, 0)$ relative to the original coordinate system. The coordinates (x', y') of the point $R(x, y) = (0, 0)$ relative to the new coordinate system are given by the equations

$$x' = x - h = 0 - (-2) = 2, \quad y' = y - k = 0 - 1 = -1$$

Thus $R = (2, -1)$ relative to the new coordinate system. Note that the point R is the origin of the original coordinate system.

Black car, black door — still life. Instructions a good suggestion at reproduction. Print yet.

(a) [illegible] [illegible] [illegible] [illegible]

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Alternatively, $\tan\theta = \dfrac{m_1 - m_2}{1 + m_1 m_2} = \dfrac{\frac{2}{3} - \left(-\frac{1}{2}\right)}{1 + \frac{2}{3}\cdot\left(-\frac{1}{2}\right)} = \dfrac{\frac{7}{6}}{\frac{2}{3}} = \dfrac{7}{4}$.

Hence $\theta = \tan^{-1}\dfrac{7}{4} \approx 60.26°$.

> **Problem 3.** A line segment has endpoints $A(-2, 5)$ and $B(6, -1)$. Find the equation of the perpendicular bisector of AB, and the coordinates of the point where it meets the x-axis.

Midpoint of AB: $M = \left(\dfrac{-2+6}{2}, \dfrac{5+(-1)}{2}\right) = (2, 2)$.

Slope of AB: $m_{AB} = \dfrac{-1-5}{6-(-2)} = \dfrac{-6}{8} = -\dfrac{3}{4}$.

Slope of perpendicular: $m_\perp = \dfrac{4}{3}$.

Equation of perpendicular bisector: $y - 2 = \dfrac{4}{3}(x - 2)$, i.e. $3y - 6 = 4x - 8$, i.e. $4x - 3y - 2 = 0$.

To find where it meets the x-axis, put $y = 0$: $4x - 2 = 0 \Rightarrow x = \dfrac{1}{2}$.

Check: $AM = \sqrt{(2+2)^2 + (2-5)^2} = \sqrt{16 + 9} = 5$.

Check: $MB = \sqrt{(6-2)^2 + (-1-2)^2} = \sqrt{16 + 9} = 5$.

Right angle at M: $m_{AB} \cdot m_\perp = -\frac{3}{4} \cdot \frac{4}{3} = -1$, so $AB \perp$ bisector.

Hence the point is $\left(\dfrac{1}{2}, 0\right)$.

Alternatively, $\overrightarrow{AB} = (8, -6)$, $|\overrightarrow{AB}| = 10$.

Unit vector along AB: $\dfrac{1}{10}(8, -6) = \left(\dfrac{4}{5}, -\dfrac{3}{5}\right)$.

> **Problem 4.** The equation of a line through the point $P(x_1, y_1)$ is given. Determine whether the following, for the same direction, is the direction of the line, and if not, express it in the form $\lambda(\ldots)$ for some scalar λ. Also find the slope of the line.

(a) Does $(3, 4)$ lie on the line $y = 2x + 1$? Yes, substitute: $4 = 2(3) + 1 = 7$? No, so $(3, 4)$ is not on the line.

(b) Show that $(1, 2)$ lies on the line $3x - 4y + 5 = 0$.

$$3(1) - 4(2) + 5 = 3 - 8 + 5 = 0 \quad \checkmark$$

(c) The direction of the line $2x + 3y = 6$ is $(3, -2)$. Is $(6, -4)$ also a direction? Yes, $(6, -4) = 2(3, -2)$.

(d) Find the slope of the line $y = -\frac{3}{4}x + 2$. Slope $= -\dfrac{3}{4}$.

Hence the required unit vector is $\dfrac{1}{5}(4, -3) = \left(\dfrac{4}{5}, -\dfrac{3}{5}\right)$.



Figure 1.4.1

Problem 1.4.1. In the network of Figure 1.4.1, the supply is connected across a pair of terminals (labelled and numbered 1–2 in the figure) and has a rated voltage of $V_{rated} = 12\ \text{V}$ (meaning it delivers its specified performance when the voltage between terminals 1 and 2 is 12 V). Three resistors are connected to the network as shown: $R_1 = 4\ \Omega$, $R_2 = 6\ \Omega$, and $R_3 = 3\ \Omega$. (a) Identify the number of nodes, branches, and independent loops in the network. (b) Determine the equivalent resistance seen by the supply, and hence the current drawn from the supply when it is operating at its rated voltage. (c) Find the current through each resistor and the power dissipated in each. (d) Verify that the total power delivered by the supply equals the total power dissipated in the resistors.

(a) Nodes: 3 (terminals 1, 2, and the junction between R_1 and the parallel pair). Branches: 3 (R_1, R_2, R_3). Independent loops: $b - n + 1 = 3 - 3 + 1 = 1$. Equivalent resistance: $R_{eq} = R_1 + R_2 \parallel R_3 = 4 + \frac{6 \times 3}{6 + 3} = 4 + 2 = 6\ \Omega$. The circuit diagram is shown in Figure 1.4.1.

(b) Current: $I = \frac{V}{R_{eq}} = \frac{12\ \text{V}}{6\ \Omega} = 2\ \text{A}$. **The current drawn is 2 A.**

(c) By current division, $I_2 = I \cdot \frac{R_3}{R_2 + R_3} = 2 \cdot \frac{3}{9} = \frac{2}{3}\ \text{A} \approx 0.667\ \text{A}$. **$I_2 = \frac{2}{3}\ \text{A}$.**

By current division, $I_3 = I \cdot \frac{R_2}{R_2 + R_3} = 2 \cdot \frac{6}{9} = \frac{4}{3}\ \text{A} \approx 1.333\ \text{A}$. **$I_3 = \frac{4}{3}\ \text{A}$.**



Figure 1.4.2

The voltage across R_2 and R_3 is the supply voltage 12 as shown in the dotted fragment of Figure 1.4.2:

$$V_2 = I_2 R_2 = \tfrac{2}{3} \times 6 = 4\ \text{V}, \qquad V_3 = I_3 R_3 = \tfrac{4}{3} \times 3 = 4\ \text{V}.$$

Power: $P_1 = I^2 R_1 = 2^2 \times 4 = 16\ \text{W}$, $P_2 = \frac{V_2^2}{R_2} = \frac{16}{6} \approx 2.667\ \text{W}$, $P_3 = \frac{V_3^2}{R_3} = \frac{16}{3} \approx 5.333\ \text{W}$.

$P_{dissipated} = 16 + 2.667 + 5.333 = 24\ \text{W}$. **Power delivered by the supply equals power dissipated in the resistors.**

Problem 2. Determine the order of singularity N at $z = 0$ of $f(z) = \frac{\sin z}{z^3}$. Then, by a method other than simply counting powers of z, find the coefficient of the z^{-1} term in the Laurent series of f about $z = 0$, and use it to evaluate $\oint_C f(z)\,dz$ around the positively oriented unit circle C: $|z| = 1$.

Resolution of z^{-1}. $\frac{\sin z}{z^3} = \frac{1}{z^3}\left(z - \frac{z^3}{3!} + \frac{z^5}{5!} - \frac{z^7}{7!} + \cdots\right) = \frac{1}{z^2} - \frac{1}{3!} + \frac{z^2}{5!} - \frac{z^4}{7!} + \cdots$. The z^{-1} term is absent, so $a_{-1} = 0$. By the residue theorem, $\oint_C f(z)\,dz = 2\pi i \cdot 0 = 0$.

The series shows that $\sin z / z^3$ has a pole of order 2 at the origin, and the coefficient of z^{-1} is zero, so the integral around the unit circle vanishes.

Given $Z_1 = \frac{1}{(0.2+j)}$ ohm, admittance $Y = \frac{1}{Z_1} = \frac{1}{1/(0.2+j)} = (0.2+j)$ mho

$= 0.2 + j$ mho $= 0.2 + j1$ S, **Ans**

Problem 3.7 Find the total admittance of Figure 3.7.1, consisting two values of capacitance C_1.

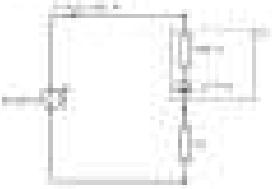


Figure 3.7.1

Total admittance $Y = \frac{1}{Z} = \frac{Z_1 + Z_2}{Z_1 Z_2}$

$= \frac{(0.2+j1)+(0.2-j1)}{(0.2+j1)(0.2-j1)} = \frac{0.4}{0.2^2+1^2}$

$= 0.4/1.04 = 0.4/1.04 + j0 = 0.385 + j0$ mho

Total impedance $Z = 1/Y = Z_1$ (as shown in Fig.)

Hence $0.2 + j1 = 0.385 + j0$

Total admittance $Y_1 = 1/(0.2 + j1) = (0.2 - j1)/(0.2^2 + 1^2) = 0.2 - j0.2$ mho

$= 0.2 + j(-0.2)$ S, **Ans**

Problem 3.8 A circuit consists of two branches in parallel. The first branch consists of a resistance of 10 ohms and a reactance of 15 ohms (inductive) and the second branch consists of a resistance of 20 ohms and a reactance of 10 ohms (capacitive). Find the admittance of the circuit. Draw the phasor diagram.

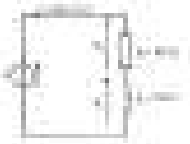


Figure 3.8.1

The circuit diagram is shown in Figure 3.8.1.

Let Z_1 = branch impedance $= 10 + j15$ ohm

$= 18.03 \angle 56.31°$ ohm



Figure 5.4.11

Apply voltage $V = 10 \angle 0°$ V to the series combination of R and C. Then

$$I = \frac{V}{R + \frac{1}{j\omega C}} = \frac{10 \angle 0°}{2 - j0.5} = 4.85 \angle 14.04° \text{ A}$$

(a) Voltage across R is $V_R = RI = 2 \times 4.85 \angle 14.04° = 9.70 \angle 14.04°$ V.

(b) Voltage across inductance is $V_L = 0.5 \angle 90° \times 4.85 \angle 14.04° = 2.43 \angle 104.04°$ V.

(c) Current phasor I_1 in the upper branch of the parallel combination is $I_1 = \frac{V}{R} = \frac{10 \angle 0°}{2} = 5 \angle 0°$ A. Current phasor I_2 in the lower branch is $I_2 = \frac{V}{-j0.5} = \frac{10 \angle 0°}{0.5 \angle -90°} = 20 \angle 90°$ A. Current phasor I drawn from the source is $I = I_1 + I_2 = 5 \angle 0° + 20 \angle 90° = 5 + j20 = 20.62 \angle 75.96°$ A.

Problem 5.4.11 R and C combination (1) and inductance (2) are connected in series with voltage source $v = 100\sqrt{2}\cos(\omega t)$ V. v is the instantaneous voltage across R. Determine ω in rad/s when the peak value of voltage v across R is 1/2 the peak value of voltage v across the series combination of R and L. Find the phase angle θ of the circuit phasor I with respect to V.

(a) Voltage $v = 100\sqrt{2}\cos(377t)$ V. Peak value of $V = 100\sqrt{2} = 141.4$ V. Peak value of $V_R = 70.7$ V. Peak value of $V_L = \sqrt{141.4^2 - 70.7^2} = 122.5$ V. Peak value of $I = \frac{70.7}{2} = 35.35$ A. Peak value of $V_L = 35.35 \times 3.5 = 123.7$ V. Peak value of $V = \sqrt{70.7^2 + 123.7^2} = 142.5$ V. Peak value of $v = 142.5$ V.

(b) $\omega = 377$ rad/s.

(c) Phase angle $\theta = \tan^{-1}\frac{123.7}{70.7} = 60.3°$.

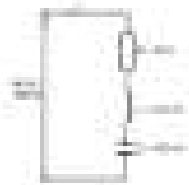


Figure 5.4.12

(a) Phasor current $I = \frac{V}{R + j\omega L} = \frac{120 \angle 0°}{30 + j40} = 2.4 \angle -53.13°$ A.

(b) Current phasor I_1 in the upper branch of the parallel combination is $I_1 = \frac{V}{R} = \frac{120 \angle 0°}{30} = 4 \angle 0°$ A. Current phasor I_2 in the lower branch is $I_2 = \frac{V}{j\omega L} = \frac{120 \angle 0°}{40 \angle 90°} = 3 \angle -90°$ A. Current phasor I drawn from the source is $I = I_1 + I_2 = 4 \angle 0° + 3 \angle -90° = 4 - j3 = 5 \angle -36.87°$ A.

Problem 5.4.12 R, L, and C are in series with voltage source v and the current is i. If $v = 10\cos(\omega t)$ V, $i = 2\cos(\omega t + 30°)$ A, and $\omega = 10$ rad/s, find R, L, and C.

The circuit diagram is shown in Figure 5.4.12.

Impedance $Z = \frac{V}{I} = \frac{10 \angle 0°}{2 \angle 30°} = 5 \angle -30° = 4.33 - j2.5$ Ω.



Figure 1.1.2

Capacitive reactance $X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 10^{-6}} = 31.8\ \Omega$

Impedance $Z = R - jX_C = 4 - j31.8\ \Omega$ (see Section 1.2.2)

$= \sqrt{4^2 + 31.8^2} \angle \tan^{-1}\left(\frac{-31.8}{4}\right) = 32\angle -82.9^\circ\ \Omega$

Current flowing $I = \frac{V}{Z} = \frac{240\angle 0^\circ}{32\angle -82.9^\circ} = 7.5\angle 82.9^\circ$ A.

(c) The current flowing is 7.5∠82.9° A, lagging the voltage by 82.9°

The phasor diagram is shown on the right-hand side of Figure 1.1.2.

Problem 1.2 A series combination of a resistor and a capacitor is connected to a supply with a 50 Hz frequency. If the supply voltage is 240 V, it is found that the current flowing is 7.5 A and it is found that the voltage across the resistor is 30 V and the voltage across the capacitor is 230 V. Determine the values of R and C. Determine also the voltage across the coil and the voltage current by drawing a phasor diagram.

Circuit impedance $Z = \frac{V}{I} = \frac{230\angle 0^\circ}{7.5\angle -90^\circ}$

$= 30.7\angle 90^\circ\ \Omega = (0 + j30.7)\ \Omega$ (see Section 1.2.2)

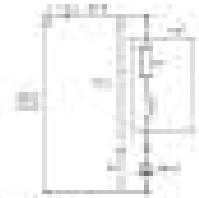


Figure 1.1.3

Capacitive reactance $X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 10^{-6}} = 31.8\ \Omega$

Circuit impedance $Z = R - jX_C = (4 - j31.8)\ \Omega$

and $= 32\angle -82.9^\circ\ \Omega$ (see Section 1.2.2)

Substituting the circuit values gives **impedance $Z = 32\angle -82.9^\circ\ \Omega$**

Applying the ohm law gives the voltage phasor $V = I \times Z = 240\angle 0^\circ\ \text{V}$

Since $V_{RMS} = 7.5 \times 32 = 240\ \text{V}$ (see Section 1.2.2)

Hence $I = 7.5\angle 82.9^\circ$ **Therefore $I = \frac{V}{Z} = \frac{240\angle 0^\circ}{32\angle -82.9^\circ} = 7.5\angle 82.9^\circ$ A**

The circuit diagram is shown in Figure 1.1.3.

Voltage across coil $V_{coil} = V_{supply}$

$V_{coil} = V - jX_C = 4.8\angle 0 + j(30.4\sin\omega t)$ (see Section 1.2.2)

Since $V_{RMS} = 4.8 + j(30.4\sin(\omega t - 90^\circ))$

$= 4.8 + 30.4\sin(\omega t - 90^\circ)$ V $= 4.8\angle 0 - j30.4\ \text{V}$

Voltage across capacitor $V_C = -jX_C \times I = -j31.8(0.15 - j0.94)$ (see Section 1.2.2)

$= -29.8 - j4.8\ \text{V} = 30.2\angle -170.5^\circ\ \text{V}$

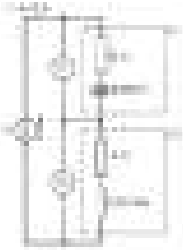


Figure 3.4.7



Figure 3.4.8

3.4.1 — Example problem: two-mesh circuit, formulate

Network: Supply voltage $V = 24\,\text{V}$ dc

$$\text{Resistors: } R_1 = 4\,\Omega,\ R_2 = 6\,\Omega,\ R_3 = 12\,\Omega$$

$$\text{Currents: } I_1,\ I_2 \text{ (mesh)}$$

Problem (a) — For the circuit shown in Figure 3.4.7, determine the values of voltages V_1 and V_2 of the supply (battery) in order to maintain both the load in the upper voltage V and the same power supply from the source (battery).

For resistance R_1: $I_1 = \dfrac{V}{R_1 + R_2} = \dfrac{24}{4 + 6} = 2.4\,\text{A}$

Power $P_1 = VI = 24 \times 2.4 = 57.6\,\text{W}$

KVL voltage $V_1 = IR_1 = 2.4 \times 4 = 9.6\,\text{V}$

$$\textbf{KVL: } V_1 + V_2 = V$$

For resistance R_2: $I_2 = \dfrac{V}{R_2 + R_3} = \dfrac{24}{6 + 12} = 1.6\,\text{A}$

Power $P_2 = I^2 R = 2.4^2 \times 6 = 34.56\,\text{W}$

KVL voltage $V_2 = IR_2 = 2.4 \times 6 = 14.4\,\text{V}$

$$\textbf{KVL: } V_1 + V_2 = V$$

Supply voltage $V = V_1 + V_2 = 9.6 + 14.4 = 24\,\text{V}$

Circuit phase angle $\phi = 0°$ **leading.** The phasor diagram is shown in Figure 3.4.8.

Further problems on the application of complex numbers to power AC circuits can be found in Section 24.4 of Engineering Mathematics.

3.4.2 — Example problem: two-mesh circuit, formulate

1. Determine the resistance of each wire conductor in the aluminium wire used in the following experiment, assuming the resistance of the wire is proportional to its length. The resistivity of aluminium is $2.6 \times 10^{-8}\,\Omega\,\text{m}$ and the wire has a cross-sectional area of $2.5 \times 10^{-6}\,\text{m}^2$.

   $$\text{Cross-sectional area, } A = \frac{\pi d^2}{4} = \frac{\pi \times (2.5 \times 10^{-3})^2}{4} = 4.909 \times 10^{-6}\,\text{m}^2$$

   $$\text{Length of wire, } l = 2 \times 10 = 20\,\text{m}$$

   $$\text{Resistance, } R = \frac{\rho l}{A} = \frac{2.6 \times 10^{-8} \times 20}{4.909 \times 10^{-6}} = 0.1059\,\Omega$$

2. A 2.5 m length of wire has a resistance of 0.50 Ω. If the resistivity of the material is $1.7 \times 10^{-8}\,\Omega\,\text{m}$, calculate the cross-sectional area of the wire.



Figure 3-3.4



Figure 3-3.5

1. Three voltages v_1, v_2, and v_3 are connected in series, and their sum is measured as $v_s = 12.6$ V. The voltages v_1 and v_2 are known to be $v_1 = 4.2\angle 30°$ V and $v_2 = 5.1\angle -45°$ V. Determine the magnitude of the unknown voltage v_3 and express it in polar form. (10 marks)

2. A resistor of $2.2\ \text{k}\Omega$ and a $47\ \mu\text{F}$ capacitor are connected in series across a $50\ \text{Hz}$ supply. Calculate the capacitive reactance, the impedance magnitude, and the phase angle between the applied voltage and the resulting current. Express the total impedance in rectangular form. (10 marks)

3. A voltage $v(t) = 169\sin(\omega t + \tfrac{\pi}{6})$ V is applied across an inductance, and a current $i(t) = 13\sin(\omega t - \tfrac{\pi}{3})$ A flows. Determine the value of the inductance, and find the average power delivered to the inductor over one full cycle. Explain briefly why the net energy transferred to the inductor over a complete cycle is zero. (10 marks)

4. Determine the current in the $4\ \Omega$ branch of the network shown in Figure 3-3.4 using Kirchhoff's voltage law. Clearly state the mesh equations you write down before solving them. (10 marks)

5. Two loads are connected in parallel across a $230\ \text{V}$, $50\ \text{Hz}$ supply. Load A draws $10\ \text{A}$ at a lagging power factor of 0.8, while load B draws $6\ \text{A}$ at a leading power factor of 0.6. Find the total current drawn from the supply and the overall power factor of the combination. (10 marks)

6. A $100\ \Omega$ resistor, a $0.5\ \text{H}$ inductor, and a $20\ \mu\text{F}$ capacitor are connected in series across a $240\ \text{V}$, $50\ \text{Hz}$ supply. Calculate the current flowing in the circuit and the power factor. Is the circuit predominantly inductive or capacitive? (10 marks)

7. Two coils are coupled such that the mutual inductance between them is $0.25\ \text{H}$. Coil 1 has a self-inductance of $0.40\ \text{H}$ and carries a current $i_1 = 3\sin(100\pi t)$ A. Determine the induced voltage in coil 2 at $t = 5\ \text{ms}$, and state whether the dot convention in Figure 3-3.5 would add or subtract the mutual term. (10 marks)

8. A $50\ \Omega$ resistor, an inductive coil, and a $100\ \mu\text{F}$ capacitor are connected in series across a $120\ \text{V}$, $60\ \text{Hz}$ supply. The voltmeter across the coil reads $95\ \text{V}$, and the voltmeter across the capacitor reads $150\ \text{V}$. Determine the resistance and inductance of the coil. (10 marks)

9. Two impedances, $Z_1 = 8 + j6\ \Omega$ and $Z_2 = 3 - j4\ \Omega$, are connected in parallel across a $100\ \text{V}$ supply. Calculate the total current drawn from the supply, the power factor of the combination, and the real power dissipated in each impedance. Verify that the sum of the two real powers equals the power delivered by the source. (10 marks)

10. A coil of wire has a resistance of $4\ \Omega$ and an inductance of $0.02\ \text{H}$. It is connected in series with a capacitor and a $60\ \text{Hz}$ supply. The current is found to be in phase with the supply voltage. Determine the capacitance of the capacitor. Also calculate the voltage across the coil and the voltage across the capacitor, and verify that they are equal in magnitude but opposite in phase. (10 marks)

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Figure 19.21

25 Application of complex numbers to parallel a.c. networks

In the work of this chapter you should be able to:

- Determine admittance, conductance and susceptance of a.c. circuits
- Perform calculations on parallel a.c. circuits using complex numbers

25.1 Introduction

In work associated with parallel a.c. networks it is necessary to add quantities together. However, with parallel networks containing more than two branches we find that the calculations are lengthy and complicated if vector (graphical) methods are used. It is therefore fortunate that the use of complex numbers can be applied to these networks. This allows the analysis of a.c. networks containing two or more branches to be carried out using simple arithmetic. Before studying such networks, however, it is necessary to define certain terms which are used.

25.2 Admittance, conductance and susceptance

Admittance is defined as the ease with which a.c. current flows in the circuit, or how 'easily' it (i.e. current) is allowed to flow. The symbol for admittance is Y. Thus

$$Y = \frac{I}{V}$$

The unit of admittance is the **siemens**, **S**

The reciprocal of impedance is admittance, and conversely the reciprocal of admittance is impedance. Thus if $Z = R + jX$, the admittance, Y, is given by

$$Y = \frac{1}{Z} = \frac{1}{R + jX}$$

and hence, after rationalising the denominator, it can be shown that $Y = G - jB$, where G is the **conductance** and B is the **susceptance**. Thus admittance is a complex quantity.

$$Y = G - jB$$

When an a.c. circuit contains

(c) **pairs** (x,y,z), $x<z$

$$x=1,\ y=z,\ z\in\mathbb{Z}\setminus\{0\}$$

(d) **pairs** (x,y,z), $x<z$

$$x=\frac{z}{2},\ y=\frac{z}{2},\ z\in\mathbb{Z}\setminus\{0\}$$

Then a congruent angle is associated with infinitely many triples. ∎

(e) **pairs** (x,y,z), $x<z$

$$x=\frac{z}{3},\ y=\frac{2z}{3},\ z\in\mathbb{Z}\setminus\{0\}$$

Then a positive angle is associated with infinitely many triples. ∎

(e) **Invitations and Subscriptions to Archive.** Let

$$P=\{x=(x_1,x_2)\in\mathbb{Z}^2\mid x_1=\frac{1}{2}x_2-\frac{x_2}{x_1},\ \frac{x_2}{x_1}\}$$

i.e. $P=\{x\in\mathbb{Z}^2\mid x_1=\frac{1}{2}x_2-\frac{x_2}{x_1},\ \frac{x_2}{x_1}=\frac{x_2}{x_1}\}$

Then x satisfies $x_1^2+x_2^2=z^2$ and infinitely many solutions $(x_1,x_2)\in\mathbb{Z}^2$.

(Note that x is dependent variable, the integration term x for x_2 integration, x_1 is explicit, whereas the integration term x for x_1 integration, x_2 is implicit.)

(f) **Invitations and organizations to archive.** Let

$$P=\{x=(x_1,x_2)\in\mathbb{Z}^2\mid x_1=\frac{1}{2}x_2-\frac{x_2}{x_1},\ \frac{x_2}{x_1}=\frac{x_2}{x_1}\}$$

i.e. $P=\{x\in\mathbb{Z}^2\mid x_1=\frac{1}{2}x_2-\frac{x_2}{x_1},\ \frac{x_2}{x_1}=\frac{x_2}{x_1}\}$

Then x satisfies $x_1^2+x_2^2=z^2$ and infinitely many solutions $(x_1,x_2)\in\mathbb{Z}^2$.

(Note that x is dependent variable, the integration term x for x_2 integration, x_1 is explicit, whereas the integration term x for x_1 integration, x_2 is implicit.)

(f) **Invitations and Subscriptions to Archive.** Let

$$\frac{1}{2}x_2-\frac{x_2}{x_1}=\frac{x_2}{x_1}$$

[illegible]

[illegible]

[illegible]

Each figure is a solid whose cross-sections perpendicular to the x-axis are squares. Set up, but do not evaluate, the integral for the volume of the solid.

(a) The region bounded by $y = \sqrt{x}$, $y = 0$, and $x = 4$.

$$V = \int_{0}^{4} \left(\sqrt{x}\right)^{2}\,dx = \int_{0}^{4} x\,dx$$

This evaluates to $V = 8$. (Not required.)

(b) The region between $y = x^{2}$ and $y = x$ in the first quadrant.

$$V = \int_{0}^{1} \left(x - x^{2}\right)^{2}\,dx = \int_{0}^{1} \left(x^{2} - 2x^{3} + x^{4}\right)\,dx$$

This evaluates to $V = \frac{3}{10}$. (Not required.)

(c) The region between $y = \frac{1}{x}$ and $y = 0$ from $x = 1$ to $x = 3$.

$$V = \int_{1}^{3} \left(\frac{1}{x}\right)^{2}\,dx = \int_{1}^{3} \frac{1}{x^{2}}\,dx$$

This evaluates to $V = \frac{2}{3}$. (Not required.)

Problem 2. Determine expressions for the equations of the following parabolas.

(a) Vertex $(3, -1)$, axis vertical, passing through $(5, 7)$.

Since the axis is vertical, the parabola has the form $y = a(x - h)^{2} + k$ with $(h, k) = (3, -1)$. Substituting the point $(5, 7)$ gives $7 = a(5 - 3)^{2} - 1$, so $8 = 4a$ and $a = 2$. Therefore $y = 2(x - 3)^{2} - 1$, which expands to $y = 2x^{2} - 12x + 17$.

(b) Focus $(0, 2)$, directrix $y = -2$.

The vertex is the midpoint between the focus and the directrix, so $(h, k) = (0, 0)$. The distance p from the vertex to the focus is 2, and since the focus lies above the directrix the parabola opens upward. Using $y = \frac{1}{4p}x^{2} + k$ with $p = 2$ and $k = 0$ gives $y = \frac{1}{8}x^{2}$.

(c) Vertex $(-1, 4)$, passing through $(1, 0)$ and $(-3, 0)$.

Because the parabola passes through $(1, 0)$ and $(-3, 0)$, the vertex lies on the axis of symmetry halfway between them, so $h = -1$. Substituting $y = a(x + 1)^{2} + 4$ and the point $(1, 0)$ gives $0 = a(2)^{2} + 4$, so $a = -1$. Therefore $y = -(x + 1)^{2} + 4$, which expands to $y = -x^{2} - 2x + 3$.

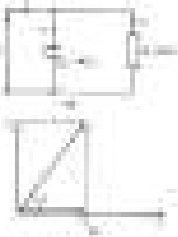


Figure P9.4 (a) Circuit. (b) Impedance. (c) Phasor diagram.

Phasor impedance $\mathbf{Z} = \dfrac{1}{(1.0\,\text{mS}) + (0.5\,\text{mS})}$

$= (0.8\,\text{k}\Omega) - (1.6\,\text{k}\Omega) = (1.8\,\text{k}\Omega)\angle(-63.4°)$

> **Problem 9.4** The admittance of a circuit is $\mathbf{Y} = (1.0\,\text{mS}) + (0.5\,\text{mS})/(90°)$. Determine the values of the resistance and the capacitance connected in the circuit, if they are connected in (a) parallel, (b) in series. Draw the phasor diagrams for cases (a) and (b).

(a) Parallel connection

Admittance $\mathbf{Y} = (1.0\,\text{mS}) - j(0.5\,\text{mS})$. Equating to admittance $\mathbf{Y} = (1.0\,\text{mS}) + \dfrac{1}{R} - j\dfrac{1}{\omega C}$. Using equations (2.1) where a general element of admittance $\mathbf{Y}$ and capacitive reactance is specified, also $f = 1.0\,\text{kHz}$, $\omega = 2\pi f$:

Phasor resistance $R = \dfrac{1}{1.0\,\text{mS}} = 1.0\,\text{k}\Omega$

and capacitive reactance $X_C = \dfrac{1}{0.5\,\text{mS}} = 2.0\,\text{k}\Omega$

The circuit and phasor diagrams are shown in Figure (b).

(b) Series connection

Admittance $\mathbf{Y} = (1.0\,\text{mS}) - j(0.5\,\text{mS})$ is inverted:

Impedance $\mathbf{Z} = \dfrac{1}{\mathbf{Y}} = \dfrac{1}{(1.0\,\text{mS}) - j(0.5\,\text{mS})} = \dfrac{(1.0\,\text{mS}) + j(0.5\,\text{mS})}{(1.0\,\text{mS})^2 + (0.5\,\text{mS})^2}$

$= (0.8\,\text{k}\Omega) + j(1.6\,\text{k}\Omega)$

Thus the resistance $R = 0.8\,\text{k}\Omega$ and capacitive reactance

$X_C = 1.6\,\text{k}\Omega$

The circuit and phasor diagrams are shown in Figure (c).

The complex phasors in Figures (b) and (c) are equivalent, as they have the same parallel elements. In (c) the parallel voltage $\mathbf{V}$ is the phasor sum of the series voltages and current $\mathbf{I}$ is in phase with $\mathbf{V}$. The phasor diagrams are in Figure P9.4(b) and P9.4(c).

Further problems on admittance, conductance and susceptance may be found in sections P9.5, problems 9.6 to page 369.

9.3 Parallel and series connection

Figure 9.3 shows a circuit diagram containing three impedances, $\mathbf{Z}_1$, $\mathbf{Z}_2$ and $\mathbf{Z}_3$ connected in parallel. The complex difference across each impedance is the same, i.e. the supply voltage $\mathbf{V}$. Therefore $\mathbf{I}_1 = \mathbf{V}/\mathbf{Z}_1$, $\mathbf{I}_2 = \mathbf{V}/\mathbf{Z}_2$, and $\mathbf{I}_3 = \mathbf{V}/\mathbf{Z}_3$. In the corresponding impedance diagram



Figure 4.5.1

connected in parallel. Their equivalent resistance R is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \quad \text{or} \quad \boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

In this case admittance $Y_T = Y_1 + Y_2 + Y_3$

In general, for n resistances connected in parallel (see Fig.4.5.1)

$$\boxed{Y_T = Y_1 + Y_2 + Y_3 + \cdots + Y_n} \quad \text{(admittance)}$$

It is an advantage to be able to calculate both the sum of resistances and the sum of admittances when two circuits are connected in parallel.

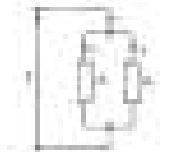


Figure 4.5.2

Example (Resistance in series and parallel)

The equivalent resistance of three resistances R_1, R_2 and R_3 connected in parallel is given by (see Figure 4.5.2(a))

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{R_2R_3 + R_1R_3 + R_1R_2}{R_1R_2R_3}$$

If the total impedance $Z_T = (R_4 + jX_4) + R$ is in series with R (see Figure 4.5.2(b)), then

$$\text{Impedance of (Figure 4.5.2(b))} = (R_4 + jX_4) + \left(\frac{R_1R_2R_3}{R_2R_3 + R_1R_3 + R_1R_2}\right)$$

Also $\qquad Y = (4.0 - j3.0)\,\text{S}$ and $Z = (0.2 + j0.1)\,\Omega$

Then $\qquad Z = 0.2 + j0.1 = \left(\frac{R_1R_2R_3}{R_2R_3 + R_1R_3 + R_1R_2}\right)$

i.e. $\qquad \boxed{\text{resistance } R_1 = 0.4\,\Omega\ \left(\frac{R_1R_2R_3}{R_2R_3 + R_1R_3 + R_1R_2}\right)}$

Also since $\qquad Y = 4.0 - j3.0$

then $\qquad \boxed{\text{resistance } R_2 = 0.4\,\Omega\ \left(\frac{R_1R_2R_3}{R_2R_3 + R_1R_3 + R_1R_2}\right)}$

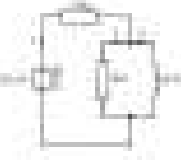


Figure 4.5.3

Now that all of the above circuit quantities are known, complex quantities relating to voltages can be found.

Let V_1 be taken as reference, since series connected impedances can easily be analysed using V_1 as reference.

Problem 3 Determine the voltages V_2 and V_3 and the current I_3 in the circuit of Figure 4.5.3

A fully worked example

$$Z_{eq} = (1 + \frac{(60 \angle 0°)(40 \angle 90°)}{60 + j40}) + 5 = (1 + \frac{2400 \angle 90°}{72.11 \angle 33.69°}) + 5$$

$$= (1 + 33.28 \angle 56.31°) + 5 = 1 + 18.56 + j27.68 + 5$$

$$= 24.56 + j27.68\ \Omega = 37.00 \angle 48.37°\ \Omega$$

$$I_{total} = \frac{120 \angle 0°}{37.00 \angle 48.37°} = 3.243 \angle -48.37°\ A$$

$$V_{parallel} = I_{total}(\frac{40 \angle 90°}{60 + j40}) = (3.243 \angle -48.37°)(0.5547 \angle 33.69°)$$

$$= 1.799 \angle -14.68°\ V$$

$$I_{branch} = \frac{1.799 \angle -14.68°}{40 \angle 90°} = 0.04498 \angle -104.68°\ A$$

$$I_{branch} = \frac{1.799 \angle -14.68°}{60 \angle 0°} = 0.02998 \angle -14.68°\ A$$

$$\text{Check: } I = I_{branch} + I_{branch} = (-0.01162 - j0.04338) + (0.02900 - j0.00746)$$

$$= -0.01162 + 0.02900 - j0.04338 - j0.00746 = 0.01738 - j0.05084\ A$$

$$= 0.05374 \angle -71.12°\ A$$

Problem 3 — Find the parallel equivalent impedance of Figure 3.11, then determine the value of angle θ such that the phase impedance is $40 \angle 30°\ \Omega$. Verify.

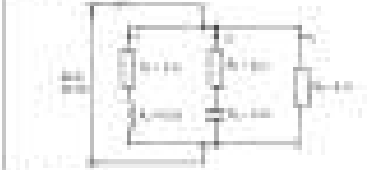


Figure 3.11

Impedance: $Z_1 = 5 + (12.53 \angle \theta) = (\ldots) - j\ldots$ Ω at $\theta = 0°$

Angle θ required = $\ldots$ Ω (where Z_1 = total series impedance and Z_2 = parallel branch impedance)

$$\begin{array}{l} \\ \\ \\

(a) Total equivalent circuit impedance:

$$Z = \frac{Z_1 \cdot Z_2}{Z_1 + Z_2} + \frac{Z_3 \cdot Z_4}{Z_3 + Z_4}$$

$$= \frac{(47.0\ \Omega - j\,32.0\ \Omega)(74.0\ \Omega + j\,58.0\ \Omega)}{(47.0 - j\,32.0 + 74.0 + j\,58.0)\ \Omega} + \frac{(91.0\ \Omega)(68.0\ \Omega - j\,47.0\ \Omega)}{(91.0 + 68.0 - j\,47.0)\ \Omega}$$

$$= \frac{(56.74\ \Omega \angle -34.22^\circ)(94.01\ \Omega \angle 38.05^\circ)}{121.0 - j\,15.0\ \Omega} + \frac{(91.0\ \Omega)(82.65\ \Omega \angle -34.71^\circ)}{159.0 - j\,47.0\ \Omega}$$

$$= 46.61\ \Omega \angle 3.83^\circ + 49.83\ \Omega \angle -15.91^\circ\ \Omega$$

$$= (46.48 + j\,3.11\ \Omega) + (47.91 - j\,13.63\ \Omega) = 94.39 - j\,10.52\ \Omega$$

(b) Angular current $I = \dfrac{V}{Z} = \dfrac{240\angle 0^\circ\ \text{V}}{94.39 - j\,10.52\ \Omega}$

$$= 2.53\angle 6.35^\circ = 2.51 + j\,0.28\ \text{A}$$

(c) Current in the $47.0\ \Omega$ resistor $= \dfrac{Z_2}{Z_1 + Z_2} \cdot I = \dfrac{(74.0 + j\,58.0)\ \Omega}{121.0 - j\,15.0\ \Omega} \cdot 2.53\angle 6.35^\circ\ \text{A}$

$$= 0.72\angle 38.24^\circ\ \text{A}$$

(d) Current in the capacitor is $I_4 = \dfrac{Z_3}{Z_3 + Z_4} \cdot I = \dfrac{91.0\ \Omega}{159.0 - j\,47.0\ \Omega} \cdot 2.53\angle 6.35^\circ\ \text{A}$

$$= 1.63\angle 20.98^\circ\ \text{A}$$

Problem 7. (a) For the electrical network of Figure E11.6, determine the value of impedance Z_1 if the angular frequency $\omega = 1000$ rad/s and the value of the capacitance corresponding to impedance Z_1.

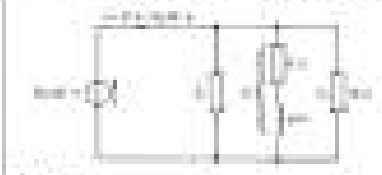


Figure E11.6

(a) Total series impedance

$$Z = \frac{1}{Y} = \frac{(5+j3)(2-j4)}{(5+j3)+(2-j4)}$$

$$= \frac{22-j14}{7-j1} = \frac{(22-j14)(7+j1)}{(7-j1)(7+j1)}$$

$$= \frac{168-j84}{50} = 3.36-j1.68\ \Omega$$

$$I_1 = \frac{V}{Z} = \frac{(7-j1)}{(5+j3)+(2-j4)}$$

Then $I_2 = I_1\left(\frac{Y_2}{Y_2+Y_3}\right) = \frac{7-j1}{(5+j3)+(2-j4)}\left(\frac{\frac{1}{2-j4}}{\frac{1}{5+j3}+\frac{1}{2-j4}}\right)$

$$= \frac{7-j1}{7-j1}\left(\frac{5+j3}{(2-j4)+(5+j3)}\right)$$

$$= \frac{5+j3}{7-j1} = 0.6+j0.5\ \text{A}$$

(b) $V_{AB} = I_2(2-j4) = (0.6+j0.5)(2-j4) = 3.2-j3.4\ \text{V}$

Hence $I_4 = \frac{V_{AB}}{5+j3} = \frac{3.2-j3.4}{5+j3}$

$$= \frac{(3.2-j3.4)(5-j3)}{(5+j3)(5-j3)} = \frac{5.8-j26.6}{34} = 0.171-j0.782\ \text{A}$$

Then $I_3 = I_2 - I_4 = (0.6+j0.5)-(0.171-j0.782)$

$$= 0.429+j1.282\ \text{A}$$

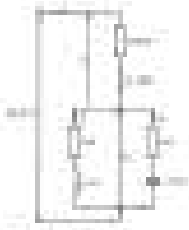


Figure 11.5

(c) Power dissipated in $(5+j3)\ \Omega$ impedance $= I_4^2 R = (0.171^2+0.782^2)(5) = 3.12\ \text{W}$

Power dissipated in $(2-j4)\ \Omega$ impedance $= I_3^2 R = (0.429^2+1.282^2)(2) = 3.66\ \text{W}$

Total power dissipated in circuit $= I_1^2 R = (0.6^2+0.5^2)(3.36) = 2.52\ \text{W}$

Alternatively, total power dissipated $= I_1^2 Z = (0.6^2+0.5^2)(3.36-j1.68) = 2.52-j1.26\ \text{VA}$

Hence total power dissipated $= 2.52\ \text{W}$

(d) Total reactive power $= -j1.26\ \text{VA}$

Hence $-j1.26 = -\frac{V^2}{X_C}$ where $X_C = \frac{1}{2\pi fC}$

Thus $-j1.26 = -\frac{V^2}{\frac{1}{2\pi fC}}$ i.e. $C = \frac{1.26}{2\pi f V^2} = \frac{1.26}{2\pi(50)(7)^2} = 0.00819\ \text{F}$

i.e. capacitance $= 8.19\ \text{mF}$

(e) The impedance Z of two impedances connected in parallel is given by:

$$Z = \frac{Z_1 Z_2}{Z_1+Z_2} = \frac{(3+j4)(5-j12)}{(3+j4)+(5-j12)} = \frac{63-j16}{8-j8}$$

$$= \frac{(63-j16)(8+j8)}{(8-j8)(8+j8)} = \frac{632+j376}{128}$$

$$= 4.94+j2.94\ \Omega \text{ i.e. } R = 4.94\ \Omega, L = 0.0936\ \text{H}$$

Exploiting more circuit theorems

$$\mathbf{A}(-1+j4) + \mathbf{B}(3+j) = (5+j9)$$

$$-\mathbf{A} + j4\mathbf{A} + 3\mathbf{B} + j\mathbf{B} = 5+j9$$

(a) Total resistance: $R = \frac{V}{I} = \frac{12.0\ \text{V}}{2.5 \times 10^{-3}\ \text{A}} = 4.8 \times 10^{3}\ \Omega = 4.8\ \text{k}\Omega$.

(b) Current when wire is $0.50\ \text{m}$ long: $I = \frac{V}{R} = \frac{12.0\ \text{V}}{4.8 \times 10^{3}\ \Omega} = 2.5 \times 10^{-3}\ \text{A} = 2.5\ \text{mA}$.

(c) Voltage V_1 across $R_1 = 1.2\ \text{k}\Omega$ when $I = 2.5\ \text{mA}$: $V_1 = IR_1 = (2.5 \times 10^{-3}\ \text{A})(1.2 \times 10^{3}\ \Omega) = 3.0\ \text{V}$.
Power in R_1: $P_1 = I^2 R_1 = (2.5 \times 10^{-3}\ \text{A})^2 (1.2 \times 10^{3}\ \Omega) = 7.5 \times 10^{-3}\ \text{W} = 7.5\ \text{mW}$.
Voltage V_2 across $R_2 = 3.6\ \text{k}\Omega$ (same I, as for any series resistor): $V_2 = IR_2 = (2.5 \times 10^{-3}\ \text{A})(3.6 \times 10^{3}\ \Omega) = 9.0\ \text{V}$.
Power in R_2: $P_2 = I^2 R_2 = (2.5 \times 10^{-3}\ \text{A})^2 (3.6 \times 10^{3}\ \Omega) = 2.25 \times 10^{-2}\ \text{W} = 22.5\ \text{mW}$.
Check: $V_1 + V_2 = 3.0\ \text{V} + 9.0\ \text{V} = 12.0\ \text{V}$, the battery voltage.

(d) Current I_2 in R_2 if R_1 is removed: $I_2 = \frac{V}{R_2} = \frac{12.0\ \text{V}}{3.6 \times 10^{3}\ \Omega} = 3.33 \times 10^{-3}\ \text{A} = 3.33\ \text{mA}$.
Power: $P_2 = \frac{V^2}{R_2} = \frac{(12.0\ \text{V})^2}{3.6 \times 10^{3}\ \Omega} = 4.0 \times 10^{-2}\ \text{W} = 40.0\ \text{mW}$.
Current in R_1: $I_1 = \frac{V}{R_1} = \frac{12.0\ \text{V}}{1.2 \times 10^{3}\ \Omega} = 10.0 \times 10^{-3}\ \text{A} = 10.0\ \text{mA}$.
Power: $P_1 = \frac{V^2}{R_1} = \frac{(12.0\ \text{V})^2}{1.2 \times 10^{3}\ \Omega} = 0.12\ \text{W} = 120\ \text{mW}$.

Unfortunately, we cannot determine

$$\frac{P_2 - P_1}{P_2 + P_1} \times 100\% = \frac{40.0\ \text{mW} - 120\ \text{mW}}{40.0\ \text{mW} + 120\ \text{mW}} \times 100\% = -50.0\%$$

$$\frac{I_2 - I_1}{I_2 + I_1} \times 100\% = \frac{3.33\ \text{mA} - 10.0\ \text{mA}}{3.33\ \text{mA} + 10.0\ \text{mA}} \times 100\% = -50.0\%$$

$$\frac{R_2 - R_1}{R_2 + R_1} \times 100\% = \frac{3.6\ \text{k}\Omega - 1.2\ \text{k}\Omega}{3.6\ \text{k}\Omega + 1.2\ \text{k}\Omega} \times 100\% = 50.0\%$$

—it is -50.0%, -50.0%, 50.0%.

$$\frac{V_2 - V_1}{V_2 + V_1} \times 100\% = \frac{9.0\ \text{V} - 3.0\ \text{V}}{9.0\ \text{V} + 3.0\ \text{V}} \times 100\% = 50.0\%$$

—it is 50.0%, 50.0%, 50.0%.

Further problems involving Ohm's law and Kirchhoff's laws are found in Chapter 13 of the *Advanced Physics* textbook. Try to solve them before looking at the solutions.

13.1 Equations for voltage, current, resistance, and power

Problem 1. Derive the following equations for series circuits containing only resistors: (a) $R = R_1 + R_2 + R_3 + \cdots$, (b) $I = I_1 = I_2 = I_3 = \cdots$, (c) $V = V_1 + V_2 + V_3 + \cdots$, (d) $P = P_1 + P_2 + P_3 + \cdots$.

Solution.
(a) Ohm's law, $V = IR$, applied to the whole circuit and to each resistor, gives $IR = IR_1 + IR_2 + IR_3 + \cdots$. Dividing by I yields $R = R_1 + R_2 + R_3 + \cdots$.
(b) Charge is conserved, so the same current flows through every resistor in series.
(c) Energy is conserved, so the battery voltage is the sum of the voltage drops across the resistors.
(d) Multiplying $V = V_1 + V_2 + V_3 + \cdots$ by I and using $I = I_1 = I_2 = I_3 = \cdots$ gives $IV = IV_1 + IV_2 + IV_3 + \cdots$, that is, $P = P_1 + P_2 + P_3 + \cdots$.

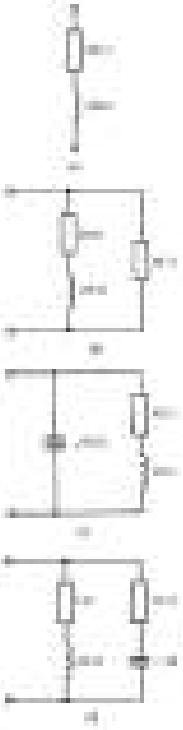


Figure 43.16

7 Refer to Problem 6 above. Find the impedance in the following parallel circuit and express it in polar form: $\dfrac{(3+j4)(5-j12)}{(3+j4)+(5-j12)}$ ohms

$$\frac{(3+j4)(5-j12)}{(3+j4)+(5-j12)} = \frac{(5\angle 53.13^\circ)(13\angle -67.38^\circ)}{8-j8} = \frac{65\angle -14.25^\circ}{11.31\angle -45^\circ} = 5.748\angle 30.75^\circ\ \Omega$$

8 The impedance of a series circuit is $(4+j3)\ \Omega$. Determine the values of the circuit components if the frequency is 50 Hz.

$$Z = 4 + j3\ \Omega = 5\angle 36.87^\circ\ \Omega$$

9 The admittance of a parallel a.c. circuit is $(0.05 - j0.08)$ S. Determine the values of the circuit components if the frequency is 50 Hz and the components are in series. (Refer to Section 43.5, page 411)

$$Y = 0.05 - j0.08\ \text{S} = 0.0943\angle -57.99^\circ\ \text{S}$$

$$Z = \frac{1}{Y} = \frac{1}{0.0943\angle -57.99^\circ} = 10.6\angle 57.99^\circ\ \Omega = 5.618 + j8.992\ \Omega$$

$$R = 5.618\ \Omega, \quad X_L = 8.992\ \Omega = 2\pi fL$$

$$L = \frac{8.992}{2\pi f} = \frac{8.992}{2\pi(50)} = 0.0286\ \text{H} = 28.6\ \text{mH}$$

10 The impedance of a series circuit is $(10+j15)\ \Omega$. Determine the circuit components if the frequency is 1 kHz.

$$Z = 10 + j15\ \Omega = 18.03\angle 56.31^\circ\ \Omega$$

$$R = 10\ \Omega, \quad X_L = 15\ \Omega = 2\pi fL$$

$$L = \frac{15}{2\pi f} = \frac{15}{2\pi(1000)} = 0.00239\ \text{H} = 2.39\ \text{mH}$$

11 Determine the total impedance and admittance of the parallel circuit shown in Figure 43.16(a).

$$Z_T = \frac{(3+j4)(5-j12)}{(3+j4)+(5-j12)} = \frac{(5\angle 53.13^\circ)(13\angle -67.38^\circ)}{8-j8} = \frac{65\angle -14.25^\circ}{11.31\angle -45^\circ} = 5.748\angle 30.75^\circ\ \Omega$$

$$Y_T = \frac{1}{Z_T} = \frac{1}{5.748\angle 30.75^\circ} = 0.174\angle -30.75^\circ\ \text{S} = 0.1500 - j0.0882\ \text{S}$$

12 Determine the value and phase of currents I_1 and I_2 in the network shown in Figure 43.16(b).

$$I_1 = \frac{240\angle 0^\circ}{3+j4} = \frac{240\angle 0^\circ}{5\angle 53.13^\circ} = 48\angle -53.13^\circ\ \text{A} = (28.80 - j38.40)\ \text{A}$$

$$I_2 = \frac{240\angle 0^\circ}{5-j12} = \frac{240\angle 0^\circ}{13\angle -67.38^\circ} = 18.46\angle 67.38^\circ\ \text{A} = (7.200 + j17.07)\ \text{A}$$

13 For the series parallel network shown in Figure 43.16(c), determine (a) the total effective resistance across AB, and (b) the current flowing through X_L (magnitude and phase), if the applied voltage across AB is $240\angle 0^\circ$ V.

(a) $Z_T = 4 + \dfrac{(3+j4)(5-j12)}{(3+j4)+(5-j12)} = 4 + \dfrac{65\angle -14.25^\circ}{11.31\angle -45^\circ} = 4 + 5.748\angle 30.75^\circ = 4 + 4.948 + j2.934 = 8.948 + j2.934\ \Omega$

(b) $I_T = \dfrac{240\angle 0^\circ}{8.948 + j2.934} = \dfrac{240\angle 0^\circ}{9.417\angle 18.14^\circ} = 25.49\angle -18.14^\circ\ \text{A}$

$$I_{XL} = I_T \times \frac{5\angle 53.13^\circ}{11.31\angle -45^\circ} = 25.49\angle -18.14^\circ \times 0.4421\angle 98.13^\circ = 11.27\angle 79.99^\circ\ \text{A}$$

14 For the parallel network shown in Figure 43.16(d), determine (a) the equivalent circuit impedance, (b) the angle by which I_T is in phase with V, and (c) the current I_T and its components I_1 and I_2.

(a) $Z_T = \dfrac{(3+j4)(5-j12)}{(3+j4)+(5-j12)} = \dfrac{65\angle -14.25^\circ}{11.31\angle -45^\circ} = 5.748\angle 30.75^\circ\ \Omega = 4.948 + j2.934\ \Omega$

(b) $\theta = 30.75^\circ$ lagging

(c) $I_T = \dfrac{240\angle 0^\circ}{5.748\angle 30.75^\circ} = 41.76\angle -30.75^\circ\ \text{A} = (35.96 - j21.46)\ \text{A}$

$$I_1 = \frac{240\angle 0^\circ}{5\angle 53.13^\circ} = 48\angle -53.13^\circ\ \text{A} = (28.80 - j38.40)\ \text{A}$$

$$I_2 = \frac{240\angle 0^\circ}{13\angle -67.38^\circ} = 18.46\angle 67.38^\circ\ \text{A} = (7.200 + j17.07)\ \text{A}$$



Figure 2.C.12

Figure 2.C.13

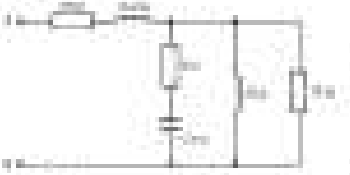


Figure 2.C.14

Figure 2.C.15

11. For the network shown in Figure 2.C.12, determine an equivalent R_{eq} for resistors R_1 to R_5 and then I_1 and I_2 for the specified source voltages V_1 and V_2. The supply voltage range is

$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$$

12. Determine V_{ab} for the network shown in Figure 2.C.13, and the total power dissipated. If V_{ab} is the network input voltage and I_{ab} is the supply current, find the network driving point impedance $Z_{ab} = V_{ab}/I_{ab}$ and Y_{ab}.

$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$$

13. Four impedances of $Z_1 = 20\angle 60°\ \Omega$, $Z_2 = 10\angle -30°\ \Omega$, $Z_3 = 5\angle 0°\ \Omega$, and $Z_4 = 10\angle 90°\ \Omega$ are connected as parallel branches across an ac supply. Find the supply current and the power supply. $\qquad\qquad\qquad\qquad\qquad\qquad$

14. In the network shown in Figure 2.C.14, V_{ab} is a reference. Determine the V_{ab} for the network. $\qquad\qquad\qquad\qquad\qquad\qquad$

15. Determine the resistance R_{ab} for the network shown in Figure 2.C.15. Then find the total power dissipated in the network. $\qquad\qquad\qquad\qquad\qquad\qquad$

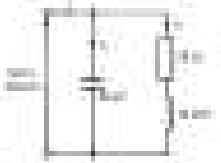


Figure 10.16

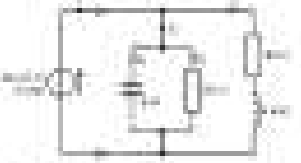


Figure 10.17

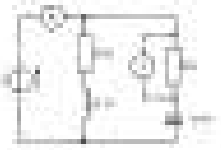


Figure 10.18

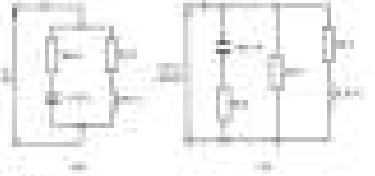


Figure 10.19

(10.1 – 10.8) Summarize the three approaches to the load solution in (10.1 – 10.7) [10.1 | 10.2 | 10.3] [10.4 | 10.5 | 10.6] [10.7 | 10.8]

10.9 Sum each of the terms on the right-hand sides of Figure 10.16(a) alternately for negative current or zero load, and derive a solution in the symbolic domain.

[10.9] [10.1] – [10.7] [10.8] [10.9] [10.10] [10.11]

10.10 Derive the first terms of Fourier's series for the load current in the example shown in Figure 10.16. [10.10] [10.11] [10.12]

10.11 In the series parallel network shown in Figure 10.19 the a.c. terminals receive a supply at v_1 and f_1 Hz. Determine an alternating current i_1 in the equivalent Kapp impedance z_1 in the supply of Figure 10.19(a) and the angular phase angle ϕ_1 for the d.c. terminal current I_d and V_1 and the value of resistance in each z_1.

[10.13] [10.1] – [10.7] [10.8] [10.9] [10.10] [10.11] [10.12] [10.13] [10.14] [10.15] [10.16] [10.17] [10.18] [10.19] [10.20] [10.21] [10.22] [10.23] [10.24] [10.25] [10.26] [10.27] [10.28] [10.29] [10.30] [10.31] [10.32] [10.33] [10.34] [10.35] [10.36] [10.37] [10.38] [10.39] [10.40] [10.41] [10.42] [10.43] [10.44] [10.45] [10.46] [10.47] [10.48] [10.49] [10.50] [10.51] [10.52] [10.53] [10.54] [10.55] [10.56] [10.57] [10.58] [10.59] [10.60] [10.61] [10.62] [10.63] [10.64] [10.65] [10.66] [10.67] [10.68] [10.69] [10.70] [10.71] [10.72] [10.73] [10.74] [10.75] [10.76] [10.77] [10.78] [10.79] [10.80] [10.81] [10.82] [10.83] [10.84] [10.85] [10.86] [10.87] [10.88] [10.89] [10.90] [10.91] [10.92] [10.93] [10.94] [10.95] [10.96] [10.97] [10.98] [10.99] [10.100] [10.101] [10.102] [10.103] [10.104] [10.105] [10.106] [10.107] [10.108] [10.109] [10.110] [10.111] [10.112] [10.113] [10.114] [10.115] [10.116] [10.117] [10.118] [10.119] [10.120] [10.121] [10.122] [10.123] [10.124] [10.125] [10.126] 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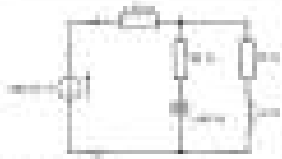


Figure P1.23



Figure P1.24

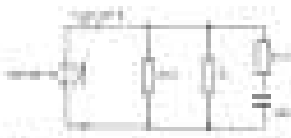


Figure P1.25

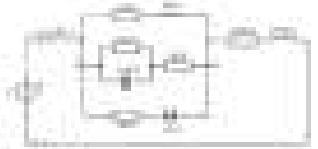


Figure P1.26

1.27 For the general circuit shown in Figure P1.27, determine (a) the value of impedance Z_3 in the network if the input to the network is 240 V at 50 Hz and the current flowing in the network is 10 A at a power factor of 0.8 lagging, and (b) the value of impedance Z_3 if the supply frequency is changed to 100 Hz.

1.28 A coil of inductance 0.1 H and resistance 10 Ω is connected in parallel with a capacitor C across a 200 V, 50 Hz supply. Determine the value of C if the circuit is to be at unity power factor. Calculate the total current taken from the supply and the circuit power factor when the frequency of the supply is changed to 60 Hz, the values of R, L and C remaining unchanged.

1.29 For the circuit shown in Figure P1.29, determine (a) the value of capacitance C which will cause the current in the 10 Ω resistor to be zero when the supply is connected across terminals AB, and (b) the current in the 10 Ω resistor when a short-circuit is placed across terminals AB.

26 Power in a.c. circuits

In this unit of the engineering principles module you:

- determine active, apparent and reactive power in a.c. circuits containing resistance
- appreciate the need for power factor improvement
- understand the power factor triangle

26.1 Introduction

26.2 Relationships between powers in a.c. circuits

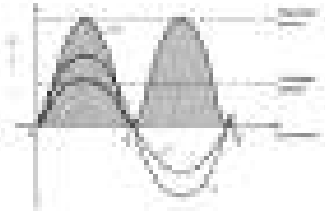


Figure 13.1 The instantaneous, average, and rms values of a sinusoidal voltage.

For a resistive load, the average power is

$$P_{\text{avg}} = \frac{V_{\text{rms}}^2}{R} = \frac{(120\ \text{V})^2}{15\ \Omega} = 960\ \text{watts}$$

which is exactly the power delivered by a 120-V battery in the same circuit. Because $P_{\text{avg}} = I_{\text{rms}}^2 R$ as well, the rms current is the direct-current equivalent for heating.

Connecting the average power. From a specific resistor, the average power can also be written in terms of the peak voltage:

$$P_{\text{avg}} = \frac{V_{\text{rms}}^2}{R} = \frac{V_0^2}{2R}$$

where V_0 is the peak voltage.

Ex. 2 Power delivered to a resistor

A 24-V battery and a 24-V peak sinusoidal source are each connected in turn to a 6-Ω resistor. Find the heating power in the resistor for each case.

(a) For the battery, $P = V^2/R = (24\ \text{V})^2/6\ \Omega = 96\ \text{W}$.

(b) For the sinusoidal source, $V_{\text{rms}} = V_0/\sqrt{2} = 24\ \text{V}/\sqrt{2} \approx 17.0\ \text{V}$, so $P = V_{\text{rms}}^2/R = (17.0\ \text{V})^2/6\ \Omega \approx 48\ \text{W}$.

The sinusoidal source delivers only half the power of the battery of the same peak voltage.

Ex. 3 Power delivered to a resistor

A 120-V peak sinusoidal source is connected to a 10-Ω resistor. Find the rms current, the average power, and the peak power.

$I_{\text{rms}} = V_{\text{rms}}/R = (120\ \text{V}/\sqrt{2})/10\ \Omega \approx 8.49\ \text{A}$.

$P_{\text{avg}} = I_{\text{rms}}^2 R = (8.49\ \text{A})^2(10\ \Omega) \approx 721\ \text{W}$.

Peak power $= V_0^2/R = (120\ \text{V})^2/10\ \Omega = 1440\ \text{W}$.

Note $P_{\text{peak}} = 2P_{\text{avg}}$ for a resistive load.

Equivalently, $P_{\text{avg}} = \frac{1}{2}I_0^2 R$ where I_0 is the peak current. Because $I_0 = V_0/R = 12\ \text{A}$, we also find $P_{\text{avg}} = \frac{1}{2}(12\ \text{A})^2(10\ \Omega) = 720\ \text{W}$.

Ex. 4 Power delivered to a resistor

The graph of $p = \frac{1}{2}I_0^2 R(1 + \cos 2\omega t)$

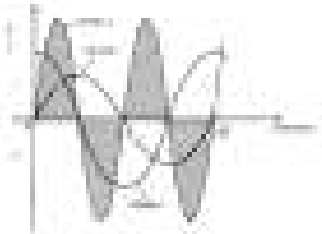


Figure 10.1 Voltage in a typical commercial ac circuit.

A sinusoid varies like a cosine function and can be expressed in terms of either the sine or cosine function. Both the sine and cosine functions are periodic functions that repeat every 2π radians or 360°. A periodic function is one that satisfies $f(t) = f(t + nT)$, where n is an integer and T is the period of the function. Figure 10.2 shows two periodic functions, each with a period $T = 2\pi$. The sine function in Figure 10.2(a) starts at zero and goes positive, while the cosine function in Figure 10.2(b) starts at its maximum value. Thus, we observe that a cosine function is a sine function lagging by a phase of 90° or $\pi/2$ radians, or a sine function is a cosine function leading by a phase of 90° or $\pi/2$ radians. Both sine and cosine functions are used in circuit analysis.

Understanding the energy source of the periodic waveform a.c. circuits in use.

(a) Purely capacitive a.c. circuits

Let the voltage $v = V_m \sin \omega t$ be applied across a capacitor C, as shown in Fig. 10.3(a). The current i is related to v as $i = C\,dv/dt$. Hence

$$i = C\frac{d}{dt}(V_m \sin \omega t) = \omega C V_m \cos \omega t$$

$$= I_m \sin(\omega t + 90^\circ) \quad \text{where } I_m = \omega C V_m$$

The current i reaches its maximum value when $\cos \omega t = 1$, that is, when $\omega t = 0, 2\pi, 4\pi, \ldots$. Thus, the current i leads the voltage v by 90° or $\pi/2$ radians, as shown in Fig. 10.3(b).

Note that $X_C = 1/\omega C$ is also known as the capacitive reactance and is measured in ohms.

Note that $\omega = 2\pi f$ is the angular frequency.

The quantity $\omega = 2\pi f$ is called

The Kirchhoff loop rule is a statement of the conservation of energy for electrical circuits. It states that the sum of the potential differences (voltages) around any closed loop in a circuit must be zero. Put another way, the voltage increases (sources) in a loop must equal the voltage decreases (loads) in a loop.

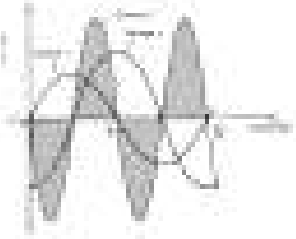


Figure 10.1 A battery and resistor circuit with a single loop.

Understanding the concept: The concept of conservation of energy is central to the Kirchhoff loop rule.

10.2 Kirchhoff's Loop Rule

Let's consider a simple circuit as pictured in Figure 10.1 consisting of a battery and a resistor. A battery is a device that creates and maintains a fixed voltage (potential difference) between its terminals. In other words, a battery is a device that pumps charges to create and maintain a voltage difference between its terminals. If we connect a resistor to a battery, we will have a complete circuit and current will flow. Current is the rate of flow of charge through a surface. The SI unit of current is the ampere (A). One ampere is equal to one coulomb of charge passing through a surface in one second. We can express this mathematically as

$$I = \frac{dq}{dt}$$

where I is the current in amperes, q is the charge in coulombs, and t is the time in seconds.

Now, let's consider a single-loop circuit with multiple batteries and resistors. To find the current in each resistor, we can use Kirchhoff's loop rule. The loop rule states that the sum of the potential differences (voltages) around any closed loop in a circuit must be zero. We can express this mathematically as

$$\sum_{\text{loop}} \Delta V = 0$$

where ΔV is the potential difference (voltage) across each element in the loop.

To apply the loop rule, we must first choose a direction for the current in each resistor. We can choose any direction we like, but it is usually best to choose the direction that we think the current will flow. If we choose the wrong direction, we will simply get a negative value for the current, which indicates that the current is flowing in the opposite direction.

Next, we must choose a starting point and a direction around the loop. We can choose any starting point and any direction we like, but it is usually best to choose a point where we know the potential difference and a direction that follows the current.

As we move around the loop, we must keep track of the potential differences across each element. The potential difference across a resistor is given by Ohm's law:

$$\Delta V = IR$$

where R is the resistance in ohms.

The potential difference across a battery is equal to its emf (electromotive force), which is the voltage it provides when no current is flowing. The emf is usually denoted by $\mathcal{E}$.

Finally, we must apply the loop rule. We can write the loop rule as an equation by summing the potential differences around the loop and setting the sum equal to zero:

$$\sum_{\text{loop}} \Delta V = 0$$

Solving this equation will give us the current in each resistor.

Example 10.1 Find the current in a single-loop circuit with two batteries and two resistors.

[illegible]

(a) [illegible]

(b) [illegible]

[illegible]

[illegible]

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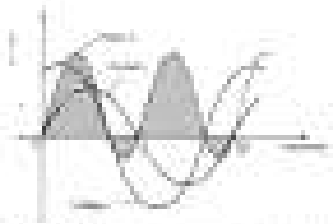


Figure 19.1 [illegible]

Trigonometry, like so many parts of life, is a mixture of the practical and the theoretical. Navigators and surveyors, therefore, are always on the lookout for ways to find the lengths of sides and the sizes of angles that they cannot measure directly.

6.1 Three Straight and Curved Paths





Figure 6.1 (a) Three straight paths. (b) Three curved paths.



Figure 6.2 Three straight paths.

A plane figure in which the sides are straight lines is called a **rectilinear figure**. The simplest rectilinear figure is a triangle, as shown in Figure 6.1(a). The distance between points P and Q is the straight line PQ, and the distance between P and R is the straight line PR. The distance between Q and R is the straight line QR. Together, these three straight paths form a triangle. The points P, Q, and R are called the **vertices** of the triangle. The sides of the triangle are sometimes called the three **paths**.

> **Three rectilinear paths** PQ, QR, and RP are the sides of the triangle PQR.
> **Angle** P is the angle between PQ and PR.
> **Angle** Q is the angle between QP and QR.
> **Angle** R is the angle between RP and RQ.

The letters P, Q, and R are also used to denote the sizes of the angles at the vertices. Thus P is the number of degrees in angle P, Q is the number of degrees in angle Q, and R is the number of degrees in angle R.

Trigonometry tells us that the sum of the angles of a triangle is always $180°$. Thus

$$P + Q + R = 180$$

Figure 6.3 Three straight paths.

Figure 6.4 Three curved paths.

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For transformer windings and commutator

$$\text{power factor} = \frac{P}{S} = \frac{\text{True power}}{\text{Apparent power}}$$

$$= \text{power factor} = \frac{R}{Z} \quad \text{(Ratio for transformers windings)}$$

In source or electric machine power flow is always $P_{out} = P_{in}$ as transformer cannot by itself or have a lagging power factor, and generator is lagging due to the power Q_{out} in source or electric machine. Apply voltage V_{in} to generator, a current is made to flow in a lagging power factor, and generator is lagging due to the power Q_{out}

Ohm's law all examples : equations for determination of power

Let I current be supplied by an alternating voltage V as, Ohm's

$$V = I \cdot R \text{ (ohms)} \quad , \quad I = \frac{V}{R} \text{ (amps)} \quad , \quad R = \frac{V}{I} \text{ (ohms)} \qquad \text{(Ohm's 1)}$$

Let the current I voltage V be shown as in (1), where

$$V_1(t) = V_{max} \sin(\omega t + \theta_{max}) \quad , \quad I_1(t) = I_{max} \sin(\omega t + \theta_{max}) \qquad \text{(Ohm's 2)}$$

From Ohm's law (2) and (1) current $I(t) = I_{max} \sin \omega t$ where ω is the angle between the voltage V and current I. If the voltage is in the rectifier method in (2), then the angle between voltage and current is $\omega t - \theta$

Then power $P = I^2 R$ (watts) $\quad (2)$

In the component angle between voltage θ is current and ω is power

Power power $\quad P = \frac{V^2}{R} \sin \omega t \cdot \sin \omega t$ (2)

Real reading power $P = \frac{1}{2} V_{max} I_{max} \cos \theta \quad (P \text{ (rms)} = V_{rms} I_{rms} \cos \theta)$ $\quad$ (Ohm's 3)

$P = I_{rms}^2 \cdot R$ (watts) (Ohm's 4) (in rectifier method)

Remembering, if $V = (\omega \cdot t)$ and $I = \omega \cdot t$ (R), then

$$\boxed{\text{power} = I \cdot \omega \cdot t \cdot R} \qquad \text{(Ohm's 5)}$$

Then power may not calculated from the work $= 0$ the procedure. If the work component not integrated in angle when ω change zero interval

$$\text{Instantaneous power } (p) = V(t) \cdot I(t) \quad , \quad (t)$$

In the component angle between work $= I(t) = \text{effect work} = \text{work } \omega t$ (I)

Then $\quad (I) = I^2 R$ (watts) $= \text{work } (I)$

Where equation power $(I) = I \cdot R$ (work) $= I^2 \cdot R$ (work) $(\omega t) = \omega$

$(I) = \text{effect} = \text{current (instantaneous)} \cdot R = \text{work} \cdot (\omega t)$

Introducing R = as a gas and T = as a pill, then

$$\text{medicine power} = \frac{R}{T} \tag{Eq 1}$$

Equations (1) & (2) and (3) & (4) provide an alternative method of determining both power R and resistance T in electrical circuits. Ohm's law cannot get negative answers. Since resistance $R = \frac{V}{I}$ negative current $I = -\frac{V}{R}$ gives Kirchoff's current overlapping V as I in incredible. Solve and we get the same, i.e. Ohm's theory.

$$I = \frac{V}{R} = \frac{V}{-R} = -\frac{V}{R}$$

Now, we must get a set the capacitor. For current as given in equations, the Kirchoff Navigator's gas is a pill capacitor in Kirchoff's gas power in equation (Eq1).

The current equation says to derive the capacitor for voltage V by the conjugate of the current as $I = -\frac{V}{R}$, derived for V. Thus:

$$\text{opposite power} = \frac{V}{R} = \frac{V}{-R} = -\frac{V}{R}$$

i.e. $\boxed{\frac{V}{R} = -\frac{V}{R}}$ from equations (3) & (4) and (Eq 1)

Now the above and negative answer may be determined if and only if, for voltage V is proportional to the magnitude of current I. As stated in Section 21.3, a negative value of Kirchhoff's equations means V is in opposite direction to battery power. Note, therefore, a negative value for V indicates a negative current, i.e. current flowing in direction opposite to that assumed.

> **Problem 1** — A cell of resistance 0.5 Ω and negative resistance 1.5 Ω is connected to a load resistance of 4 Ω. Find the current in the circuit.

The current equation is shown in Figure 21.1.

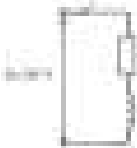


Figure 21.1

Applying Kirchhoff's law to the circuit gives:

$$\text{Voltage } V = 10 \text{ V}, \quad \text{Total resistance } = 4.5\ \Omega$$

$$\text{Current } I = \frac{V}{R} = \frac{10 \text{ V}}{4.5\ \Omega} = 2.22 \text{ A}$$

Then we have verified our calculation:

$$I = -\frac{V}{R} = -\frac{10 \text{ V}}{4.5\ \Omega} = -2.22 \text{ A}$$

Now we have verified our calculation:

$$\text{Voltage } V = -10 \text{ V}, \quad \text{Total resistance } = 4.5\ \Omega$$

$$\text{Current } I = \frac{V}{R} = \frac{-10 \text{ V}}{4.5\ \Omega} = -2.22 \text{ A}$$

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Ordinance 2 — Census Act, 1911 ... [illegible]

Ordinance 3 — Public Health (Prevention of Infectious Diseases) Act, 1911

Ordinance 4 — Municipal Corporations (Amendment) Act, 1911

Ordinance 5 — Police Act, 1911

Ordinance 6 — Weights and Measures Act, 1911

Ordinance 7 — Excise Act, 1911

Ordinance 8 — Stamp Act, 1911

Ordinance 9 — Registration of Births and Deaths Act, 1911

Ordinance 10 — Prevention of Cruelty to Animals Act, 1911

Ordinance 11 — Vaccination Act, 1911

Ordinance 12 — Public Meetings Act, 1911

Ordinance 13 — Municipal Corporations Act, 1911

Ordinance 14 — Census Act, 1911

Ordinance 15 — Public Health Act, 1911

Ordinance 16 — Police Act, 1911

Ordinance 17 — Weights and Measures Act, 1911

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Ordinance 20 — Registration of Births and Deaths Act, 1911

Ordinance 21 — Prevention of Cruelty to Animals Act, 1911

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Ordinance 41 — Stamp Act, 1911

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Ordinance 77 — Vaccination Act, 1911

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Ordinance 80 — Census Act, 1911

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Ordinance 86 — Registration of Births and Deaths Act, 1911

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Ordinance 96 — Stamp Act, 1911

Ordinance 97 — Registration of Births and Deaths Act, 1911

Ordinance 98 — Prevention of Cruelty to Animals Act, 1911

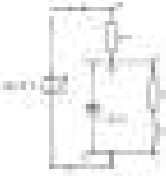
Ordinance 99 — Vaccination Act, 1911

Ordinance 100 — Public Meetings Act, 1911

Problem 1. A simple current-measuring instrument is built around a $2.0\ \Omega$ resistor R_s that is placed in parallel with the coil of the meter. The coil has a resistance of $98\ \Omega$. When the full-scale deflection of the meter corresponds to a coil current of $50\ \text{mA}$, what is the largest current I that can be measured when the instrument is connected in series with the circuit? What fraction of I flows through the coil?

(a) Because R_s and the coil are in parallel, they share the same potential difference. Let V be the voltage across the combination. The current through the coil is $I_c = 50\ \text{mA} = 0.050\ \text{A}$, and the coil resistance is $R_c = 98\ \Omega$, so $V = I_c R_c = (0.050\ \text{A})(98\ \Omega) = 4.9\ \text{V}$. Now find the current through R_s: $I_s = V / R_s = 4.9\ \text{V} / 2.0\ \Omega = 2.45\ \text{A}$. By Kirchhoff's current law, the total current is $I = I_c + I_s = 0.050\ \text{A} + 2.45\ \text{A} = 2.50\ \text{A}$. Hence the largest measurable current is $2.5\ \text{A}$.

(b) The fraction through the coil is $I_c / I = 0.050\ \text{A} / 2.50\ \text{A} = 0.020$, or 2.0%. Equivalently, the coil's share of the total conductance is $1/98 \div (1/98 + 1/2.0) = 2.0/100.0 = 0.020$. Almost all of the current bypasses the coil through the shunt, which is exactly why the shunt protects the delicate meter movement.



Problem 2. Two identical lamps, each rated for $6.0\ \text{V}$ and $0.50\ \text{A}$, are connected in series with a battery and a switch. The battery has an emf of $12\ \text{V}$ and negligible internal resistance. The switch is closed. Find the resistance of each lamp, the current in the circuit, and the power dissipated by each lamp.

Each lamp's resistance follows from $R = V/I$: $R = 6.0\ \text{V} / 0.50\ \text{A} = 12\ \Omega$. Two identical resistors in series add, so the total resistance is $R_{\text{tot}} = 12\ \Omega + 12\ \Omega = 24\ \Omega$. The current is then $I = \mathcal{E} / R_{\text{tot}} = 12\ \text{V} / 24\ \Omega = 0.50\ \text{A}$, exactly the rated current, as expected.

Power dissipated in each lamp is $P = I^2 R = (0.50\ \text{A})^2 (12\ \Omega) = 3.0\ \text{W}$. Each lamp glows dimly compared with its rating, because $3.0\ \text{W}$ is only 36% of the $8.3\ \text{W}$ ($6.0\ \text{V} \times 0.50\ \text{A}$) it would dissipate at full brightness.

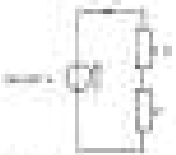


Figure 10.11



Figure 10.12



Figure 6.6.4 (a) Section through plate and (b) Mohr's diagram

[illegible] indicated by the negative sense shown. The point marked [illegible] as in Figure 6.6.4b.

(c) Since $\theta = 116.6°$, the

$$\text{magnitude of normal } \sigma = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos\theta = 0.4\,\sigma_x$$

Since the ellipse is at a certain angle of 63° from [illegible] and since [illegible] the [illegible] angle of [illegible] is 90° − 63° = 27°. Since [illegible] = 0.4[illegible], 90° is

(d) Axial [illegible] [illegible] =

$$= \frac{\text{[illegible]}}{\text{[illegible]}}$$

[illegible]

Since [illegible] =

[illegible]

Further problems for practice are given in the accompanying Workbook (Section 6.6, problems 1 to 19, page 251).

6.6.5 Bolted Joints: Shear Connection

For compression joints a power capability, simple power transmission without the normal thinning of material, perhaps the most common use is in lifting machinery, bridges and especially as mentioned in Section 6.6.1, pages 251. Bolted joints often use rivets, which are simply cylindrical or tapered rods inserted through holes in the members to be joined and then [illegible] [illegible] [illegible]



(a)



(b)

Figure 21.11 Effect of
(a) Same force acting on two bodies of different masses, (b) Same force acting on two bodies of equal mass



Figure 21.12

Problem 1. A force $(3\hat{i} + 4\hat{j})$ N acts on a mass 2 kg at rest. (a) Find the acceleration of the body. (b) Find the velocity of the body after 5 s. (c) Find the distance travelled by the body in 5 s. (d) Find the work done by the force in 5 s. (e) Find the power of the force at the end of 5 s.

Solution. (a) Acceleration $= \frac{\text{force}}{\text{mass}} = \frac{(3\hat{i} + 4\hat{j})}{2} = \frac{3}{2}\hat{i} + 2\hat{j}$ m/s^2.

(b) Velocity after 5 s $= \vec{u} + \vec{a}t = 0 + \left(\frac{3}{2}\hat{i} + 2\hat{j}\right)5 = \frac{15}{2}\hat{i} + 10\hat{j}$ m/s.

Speed after 5 s $= \sqrt{\left(\frac{15}{2}\right)^2 + 10^2} = \frac{25}{2}$ m/s.

(c) Displacement in 5 s $= \vec{u}t + \frac{1}{2}\vec{a}t^2 = 0 + \frac{1}{2}\left(\frac{3}{2}\hat{i} + 2\hat{j}\right)5^2 = \frac{75}{4}\hat{i} + 25\hat{j}$ m.

Distance travelled in 5 s $= \sqrt{\left(\frac{75}{4}\right)^2 + 25^2} = \frac{125}{4}$ m.

(d) Work done $=$ force $\cdot$ displacement $= (3\hat{i} + 4\hat{j}) \cdot \left(\frac{75}{4}\hat{i} + 25\hat{j}\right) = \frac{225}{4} + 100 = \frac{625}{4}$ J.

(e) Power $=$ force $\cdot$ velocity $= (3\hat{i} + 4\hat{j}) \cdot \left(\frac{15}{2}\hat{i} + 10\hat{j}\right) = \frac{45}{2} + 40 = \frac{125}{2}$ W.

Problem 2. A force $(2\hat{i} + 3\hat{j})$ N acts on a body of mass 1 kg moving with velocity $(\hat{i} - \hat{j})$ m/s. Find the work done by the force in 2 s.

Solution. Acceleration $= (2\hat{i} + 3\hat{j})$ m/s^2.

Velocity after 2 s $= (\hat{i} - \hat{j}) + (2\hat{i} + 3\hat{j})2 = 5\hat{i} + 4\hat{j}$ m/s.

Displacement in 2 s $= (\hat{i} - \hat{j})2 + \frac{1}{2}(2\hat{i} + 3\hat{j})2^2 = 6\hat{i} + 2\hat{j}$ m.

Work done $= (2\hat{i} + 3\hat{j}) \cdot (6\hat{i} + 2\hat{j}) = 12 + 6 = 18$ J.

Alternatively, change in kinetic energy $= \frac{1}{2}(5^2 + 4^2) - \frac{1}{2}(1^2 + 1^2) = \frac{41}{2} - 1 = \frac{39}{2}$ J.

Power $=$ work done per unit time $= \frac{\text{change in kinetic energy}}{\text{time}} = \frac{39/2}{2} = \frac{39}{4}$ W.

Impulse $=$ change in momentum $= 1(5\hat{i} + 4\hat{j}) - 1(\hat{i} - \hat{j}) = 4\hat{i} + 5\hat{j}$ N-s.



Figure 11.10

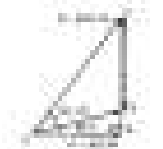


Figure 11.11

11.1 Exercises provide you with practice for your studies

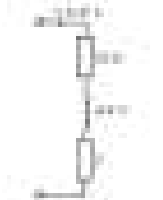


Figure 11.12

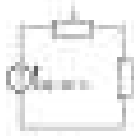


Figure 19.17

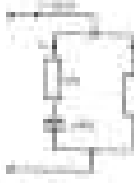


Figure 19.18

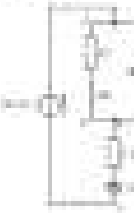


Figure 19.19

and the power, and the value of resistance R, and so the value of resistance R.

[ans: I_{out} = 0.5 A, V_{out} = 6 V, R = 12 Ω, P_{out} = 3 W, P_{in} = 3.6 W]

19 The circuit shown in the network is a simple model for an operational amplifier circuit. Find V_{out} when V_{in} = 1.00 V and 2.0 V. Explain why this circuit is called an amplifier circuit.

[ans: 2.0 V, 4.0 V; V_{out} is proportional to V_{in}]

20 Two resistors, R_1 = 4.0 Ω and R_2 = 6.0 Ω, are connected in series to the terminals of a battery of emf 12 V and negligible internal resistance. Another resistor, R_3, is connected in parallel with R_2. Find the equivalent resistance of the circuit and the power drawn from the battery when R_3 = 6.0 Ω.

[ans: 14 Ω, 10 W]

21 A resistor R_1 = 2.0 Ω and a resistor R_2 = 10 Ω are connected in series with a battery of emf 12 V and negligible internal resistance. Another resistor, R_3, is connected in parallel with R_2. Find the equivalent resistance of the circuit and the power drawn from the battery when R_3 = 10 Ω.

[ans: 7.0 Ω, 17 W]

22 The circuit shown in Figure 19.18 has a value of 6.00 Ω for all resistors. Find the equivalent resistance between points A and B. Find the current in each resistor when a potential difference of 12.0 V is applied between A and B.

[ans: 3.00 Ω, 4.00 A, 2.00 A, 2.00 A, 2.00 A, 2.00 A]

23 For the network shown in Figure 19.19, determine (a) the current in each resistor in terms of I, and (b) the equivalent resistance of the network between points A and B.

[ans: $I_1 = I$, $I_2 = I_3 = I_4 = I_5 = \frac{1}{2}I$, $R_{eq} = \frac{5}{4}R$]

24 A circuit consists of an electromotor of emf 12 V, an ammeter with a resistance of 0.10 Ω, two resistors connected in parallel with resistances of 2.0 Ω and 4.0 Ω, and a switch. Determine the current in the ammeter when the switch is open and when the switch is closed.

[ans: 4.0 A, 4.2 A; 0.10 Ω is significant]

25 For the network shown in Figure 19.20, determine the current in each resistor and the potential difference between points A and B. Which point is at the higher potential?

[ans: 0.50 A, 0.50 A, 1.00 A; V_A > V_B]

Part 4: Series-parallel combinations

26 A 60.0 Ω resistor and a 90.0 Ω resistor are connected in series to a 12.0 V battery. The battery has an internal resistance of 0.50 Ω. Find the current in each resistor, the potential difference across each resistor, and the power dissipated in each resistor.

[ans: 0.12 A, 7.2 V, 10.8 V, 0.86 W, 1.3 W]

27 An ammeter of 0.10 Ω resistance, a 2.0 Ω resistor, and a 4.0 Ω resistor are connected in series to a battery of emf 12 V and negligible internal resistance. Another resistor, R, is connected in parallel with the combination of the ammeter and the two resistors. Find the equivalent resistance of the circuit and the reading of the ammeter when R = 6.0 Ω.

[ans: 4.1 Ω, 2.9 A]

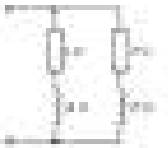


Figure 3.1.2a

Assignment 8

The assignment covers the material mentioned in chapters 27 to 28.

The marks for each question are shown in brackets–make sure to read questions.

1. For some impedance Z_L of an inductor, current is given by

$$I = I_0 = \frac{V_0 \angle 0}{Z_L \angle 90}$$

Determine Z_L in order for current to be equivalent figure when—

$$I_0 \angle -3.8 = -3.1 - 2.4_j \quad \angle -3.8 \text{ (97 degrees)} \quad \angle -\cos 97 = 0 \qquad (10)$$

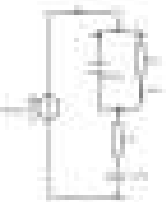


Figure 3.4.1

2. For the network given in Figure 3.4.1, determine

(a) the current from impedance A (the parallel branches)
(b) the total circuit impedance in polar form
(c) current 1
(d) the total voltage (up)
(e) current 2, (up) 3
(f) the total current drawn by unit B
(g) the total current drawn by unit C
(h) the active power distributed to the impedance of unit
(i) the active power distributed across the ____ (C) in question
(j) the active power distributed between branch B and C
(k) the total active power (distributed) by the network
(l) the total apparent power distributed by the network
(m) the total reactive power distributed by the network (10)

3. A 400 V/50 transformer is to feed load with a current power factor of 0.75 lagging. The power (kW) is required for rolling capacitor to perform with the transformer rated for overall power factor improved to unity lagging. Determine the rating in kilovolt (kVA) and (three-quarters) (10)

27 A.c. bridges

In this section we have considered the following topics:

- Ideal and practical conditions for a.c. bridge balance
- Wien bridge for a.c. frequency measurement
- Calculation of unknown components when using an a.c. bridge

27.1 Introduction

A.c. bridges are electrical instruments, basically similar in construction to the Wheatstone bridge principle, used for the determination of an unknown impedance by comparison with known impedances and are the instrumentation of frequency. In general, they consist of four bridge arms and a source supply and detector. Detector which is sensitive to alternating currents. In the most common a.c. bridges the detector is an a.c. voltage type or a.c. galvanometer. Unlike Wheatstone bridge, the magnitude and the phase angle of impedances are involved in the balance condition. Balance equations are derived by using complex quantities. A.c. bridges are of various configurations to accommodate the measurement and requirements for such measurements.

27.2 Balance conditions for an a.c. bridge



Figure 27.1 A.c. bridge

The general form of an a.c. bridge is shown in figure 27.1 where Z_1, Z_2, Z_3 and Z_4 are impedances. It does not matter what they represent, they may be pure resistances or pure reactances or a combination of both. Such a network is known as a four-arm a.c. bridge or more commonly as an 'impedance bridge'. The four arms Z_1, Z_2, Z_3 and Z_4 are connected to form a quadrilateral ABCD as shown in figure 27.1. A source of e.m.f. is connected between one pair of opposite corners B and D and a detector between the other pair A and C. The detector may be a vibration galvanometer, a pair of telephones, a tuned amplifier, or any other device depending upon the frequency of the source. The source may be a battery, a generator, or an oscillator depending upon the frequency required for the measurement.

At balance, the current through the detector is zero, that is, the potential difference between A and C is zero. Therefore, the potential drop from A to B must be equal to the potential drop from A to D, and the potential drop from B to C must be equal to the potential drop from D to C.

At balance,

(i) the ratio of the magnitudes of the impedances in one pair of opposite arms must be equal to the ratio of the magnitudes of the impedances in the other pair of opposite arms, that is

$$\frac{Z_1}{Z_2} = \frac{Z_4}{Z_3} \quad \text{or} \quad Z_1 Z_3 = Z_2 Z_4 \qquad (27.1)$$

(ii) the sum of the phase angles of the impedances in one pair of opposite arms must be equal to the sum of the phase angles of the impedances in the other pair of opposite arms, that is

$$\phi_1 + \phi_3 = \phi_2 + \phi_4 \qquad (27.2)$$

$$\frac{\partial \mathcal{L}_{\text{aug}}}{\partial \lambda_k} = 0 \quad \text{for } k = 1, 2, \ldots, m \tag{1.1.5}$$

Recovering equations (1.1.1) the equations (1.1.5) express

$$\begin{aligned} \frac{\partial \mathcal{L}(x)}{\partial x_1} &= \frac{\partial \mathcal{L}(x)}{\partial x_2} \\ \frac{\partial \mathcal{L}(x)}{\partial x_1} &= \frac{\partial \mathcal{L}(x)}{\partial x_3} \end{aligned}$$

from which $\frac{\partial \mathcal{L}}{\partial x_1} = \frac{\partial \mathcal{L}}{\partial x_2} = \frac{\partial \mathcal{L}}{\partial x_3}$

$$\Rightarrow \quad \left\{ \frac{\partial \mathcal{L}(x)}{\partial x_1} = \frac{\partial \mathcal{L}(x)}{\partial x_2} = \frac{\partial \mathcal{L}(x)}{\partial x_3} \right\} \tag{1.1.6}$$

Equations (1.1.1) derive from or following the gradient of the Lagrangian of equality terms of the Lagrange multiplier.

The set of equations $\frac{\partial \mathcal{L}}{\partial x_i} = 0$ for $i = 1, 2, \ldots, n$ and $\frac{\partial \mathcal{L}}{\partial \lambda_k} = 0$ for $k = 1, 2, \ldots, m$ where $\mathcal{L}$ is the Lagrangian equation (1.1.5), which is the condition for the constrained extremum. This is known as the Lagrange rule.

$$\frac{\partial \mathcal{L}}{\partial x_i} = 0 \quad \text{and} \quad \frac{\partial \mathcal{L}}{\partial \lambda_k} = 0 \quad \text{for } i = 1, 2, \ldots, n \text{ and } k = 1, 2, \ldots, m$$

When deriving Lagrange equations of the Lagrangian, where a local min of the Lagrangian has an unconstrained. It is used to find the stationary points of a function equations $f(x_1, x_2, \ldots, x_n)$ of the constraints are given, $m < n$, and the Lagrange multipliers are given, $\lambda_1, \lambda_2, \ldots, \lambda_m$.

Usually, one sets $\lambda^0 = 0$, so the Lagrange function coincides the ordinary equations. Note, the value zero method Lagrange Multiplier constraints. Generally, we can consequences of the Lagrange for example, if the following a Lagrange function the constraint, the function is generally defined as the Lagrangian equations of the the constraint of the Lagrangian of the Lagrange multiplier and the Lagrangian equations are the Lagrangian equations of the Lagrange multiplier.

Procedure for determining the Lagrange equations of one and Lagrange variable

(1.1) Determine the the Lagrange variable the Lagrangian of each one by Lagrange form and write down the Lagrange equations as in equation (1.1.1). Equations are number stated as Lagrangian of a and a new variable parameter of λ_1, and λ_2, ... Lagrange Equation (1.1.2).

(1.2) Include the Lagrange terms on the left hand side of the equations as the form (1.1.3).

(1.3) Differentiate the terms on the right hand side of the equation into the form (1.1.4).

(1.4) Equate the new result of the equations, $\lambda_1 = \lambda_2 = \ldots$, and express the constraint given in the equations $(1.1.5)$.

(1.5) Substitute all the λ_1 and λ_2 and λ_3 in the Lagrange equations and express the Lagrange equations in their simplest form.

Practical electrical circuits are rarely simple. Engineers often need to control the flow of current in different parts of a circuit, or even in opposite directions. Various electronic components are used for this purpose.

(a) A diode allows current to flow in only one direction, while blocking it in the opposite direction.

(b) A transistor can be used as an electronic switch, allowing a small current to control a much larger current. Transistors are also used to amplify weak signals, making them stronger.

(c) Resistors, capacitors, inductors, and other components can be combined in various ways to create complex circuits that perform specific functions.

(d) Integrated circuits (ICs), also known as microchips, are tiny electronic circuits that contain thousands or even billions of components on a single piece of silicon. They are the heart of modern computers, smartphones, and countless other devices.

7.1.1 Types of Bridges Operated

A large number of bridges (including many fixed bridges) carry rail, road, or pedestrian traffic over railways and other obstacles. Some of these bridges are operated, i.e. they can be moved to allow boats to pass. Operated bridges are usually built over navigable waterways, like the Thames and Manchester Ship Canal. Some bridges are designed to carry both road and rail traffic. Others carry traffic of only one type (road, rail, pedestrian, cycle, etc.).

A few bridges are designed to carry two types of traffic on different levels. For example, a bridge may carry a railway on the lower level and a road on the upper level. Other bridges have a combination of two types of traffic on the same level. For example, a bridge may carry both road and rail traffic on the same level.

A few bridges are designed to carry three types of traffic on different levels. For example, a bridge may carry a railway on the lowest level, a road on the middle level, and a pedestrian walkway on the highest level.

A few bridges are designed to carry three types of traffic on the same level. For example, a bridge may carry a railway, a road, and a pedestrian walkway all on the same level.

A few bridges are designed to carry four types of traffic on different levels. For example, a bridge may carry a railway on the lowest level, a road on the next level, a pedestrian walkway on the next level, and a cycle track on the highest level.

A few bridges are designed to carry four types of traffic on the same level. For example, a bridge may carry a railway, a road, a pedestrian walkway, and a cycle track all on the same level.

A few bridges are designed to carry five types of traffic on different levels. For example, a bridge may carry a railway on the lowest level, a road on the next level, a pedestrian walkway on the next level, a cycle track on the next level, and a tramway on the highest level.

A few bridges are designed to carry five types of traffic on the same level. For example, a bridge may carry a railway, a road, a pedestrian walkway, a cycle track, and a tramway all on the same level.

A few bridges are designed to carry six types of traffic on different levels. For example, a bridge may carry a railway on the lowest level, a road on the next level, a pedestrian walkway on the next level, a cycle track on the next level, a tramway on the next level, and a footpath on the highest level.

A few bridges are designed to carry six types of traffic on the same level. For example, a bridge may carry a railway, a road, a pedestrian walkway, a cycle track, a tramway, and a footpath all on the same level.

(a) A bridge which is usually left open to navigation, but which is closed to allow a train to pass — this type of bridge is usually called a **railway bridge** (but not invariably, since it is also used by boats).

(b) A bridge which is usually left closed to navigation — this type of bridge is usually called a **navigation bridge** (but not invariably, since it is closed to trains).

A few bridges can open for both navigation and trains — this type of bridge is usually called a **railway and navigation bridge**.



Problem 9.14 Wheatstone Bridge

A Wheatstone bridge is modified such that the 10-ohm resistor is replaced by a galvanometer of internal resistance r_g. Find the current through the galvanometer.

9.15 Wien Bridge (Maxwell Bridge)

The Wien bridge circuit is designed for measurement and calibration of a coil inductor. A bridge is formed between the branches ab, bc, cd, and da as shown in Figure 9.15. The coil inductor is represented by a resistance R_1 and inductance L_1. L_1 produces in the circuit impedance jX_1 (where $X_1 = \omega L_1$). R_2 and R_3 represent a decade box and R_4 is a decade resistance. Values of inductance, capacitance, R_2, and R_3 may be known precisely or decade combinations of R_2, R_3, R_4, and C_4.

The impedance for determining the balance equations given in Section 7.3 may be written as

(a) From Figure 9.15, $Z_1 = R_1 + jX_1$, $Z_2 = R_2$, $Z_3 = R_3$, and $Z_4 = R_4 - jX_4$.
At balance $Z_1 Z_4 = Z_2 Z_3$, or the equivalent of (7.3).
$(R_1 + jX_1)(R_4 - jX_4) = R_2 R_3$, or $R_1 R_4 - jX_1 R_4 + jX_4 R_1 + X_1 X_4 = R_2 R_3$.

(b) Equating the real terms on the left and right sides of the equation above, we have

$$R_1 R_4 + X_1 X_4 = R_2 R_3$$

(c) Equating the imaginary terms gives

$$-jX_1 R_4 + jX_4 R_1 = 0$$

(d) Solving for real and imaginary parts gives $L_1 = \dfrac{R_2 R_3 C_4}{R_4}$.

Expressing the imaginary parts gives $R_1 = \dfrac{R_2 R_3}{R_4}$.

(e) Since $X = \omega L$, then

$$\omega L_1 = \frac{R_2 \omega R_3 C_4}{R_4} \text{ from which } L_1 = \frac{R_2 R_3 C_4}{R_4}$$

Thus at balance the inductive components in the Wien bridge (Maxwell) relate as given by

$$R_1 = \frac{R_2 R_3}{R_4} \quad \text{and} \quad L_1 = \frac{R_2 R_3 C_4}{R_4}$$

Note the inverse of the **balance equations** for this bridge.



Figure 8.7.3 Wien bridge.

8.6 Alternating-Current Circuits

The Wheatstone bridge circuit discussed in the preceding section can be used with alternating current (ac) as well as with direct current (dc). In this case, however, the resistances are replaced by impedances, and the detector is usually a pair of earphones. A typical ac bridge circuit is shown in Figure 8.7.3.

The impedance of a series circuit containing a resistance R, an inductance L, and a capacitance C is

$$Z = R + i\left(\omega L - \frac{1}{\omega C}\right) \tag{1}$$

where $i = \sqrt{-1}$ and $\omega = 2\pi\nu$, ν being the frequency of the alternating current. The magnitude of Z is

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \tag{2}$$

and the phase angle θ between the current and the voltage is given by

$$\tan\theta = \frac{\omega L - \frac{1}{\omega C}}{R} \tag{3}$$

The impedance of a parallel circuit is

$$Z = \frac{1}{\frac{1}{R} + i\left(\omega C - \frac{1}{\omega L}\right)} \tag{4}$$

with magnitude

$$|Z| = \frac{1}{\sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}} \tag{5}$$

and phase angle

$$\tan\theta = R\left(\frac{1}{\omega L} - \omega C\right) \tag{6}$$

Consider the ac bridge circuit shown in Figure 8.7.3. Let Z_1, Z_2, Z_3, and Z_4 be the impedances in the four arms of the bridge, and let I_1, I_2, I_3, and I_4 be the currents in the four arms. Let I_5 be the current in the detector, which has impedance Z_5. Let I be the current in the battery. Then

$$I = I_1 + I_2 = I_3 + I_4 \tag{7}$$

$$I_1 Z_1 = I_3 Z_3 + I_5 Z_5 \tag{8}$$

$$I_2 Z_2 = I_4 Z_4 - I_5 Z_5 \tag{9}$$

$$I_5 Z_5 = I_1 Z_1 - I_3 Z_3 = I_2 Z_2 - I_4 Z_4 \tag{10}$$

$$I_1 = \frac{I_5 Z_5 + I_3 Z_3}{Z_1} \tag{11}$$

$$I_2 = \frac{I_4 Z_4 - I_5 Z_5}{Z_2} \tag{12}$$

$$I_3 = \frac{I_1 Z_1 - I_5 Z_5}{Z_3} \tag{13}$$

$$I_4 = \frac{I_2 Z_2 + I_5 Z_5}{Z_4} \tag{14}$$

Substituting (11) and (12) into (7) gives

$$I = \frac{I_5 Z_5 + I_3 Z_3}{Z_1} + \frac{I_4 Z_4 - I_5 Z_5}{Z_2} \tag{15}$$

Substituting (13) and (14) into (7) gives

$$I = \frac{I_1 Z_1 - I_5 Z_5}{Z_3} + \frac{I_2 Z_2 + I_5 Z_5}{Z_4} \tag{16}$$

Solving (15) for $I_5 Z_5$ gives

$$I_5 Z_5 = \frac{I Z_1 Z_2 - I_3 Z_3 Z_2 + I_4 Z_4 Z_1}{Z_1 + Z_2} \tag{17}$$

Solving (16) for $I_5 Z_5$ gives

$$I_5 Z_5 = \frac{I Z_3 Z_4 - I_1 Z_1 Z_4 + I_2 Z_2 Z_3}{Z_3 + Z_4} \tag{18}$$

Equating (17) and (18) and solving for I_1 gives

$$I_1 = \frac{I Z_2 Z_3 + I_3 Z_3 Z_4 + I_4 Z_4 Z_1}{Z_1 Z_3 + Z_2 Z_4} \tag{19}$$

Equating (17) and (18) and solving for I_2 gives

$$I_2 = \frac{I Z_1 Z_4 - I_3 Z_3 Z_4 + I_4 Z_4 Z_1}{Z_1 Z_3 + Z_2 Z_4} \tag{20}$$

Now let $I_5 = 0$. Then

$$I_1 = \frac{I Z_2 Z_3 + I_3 Z_3 Z_4 + I_4 Z_4 Z_1}{Z_1 Z_3 + Z_2 Z_4} = \frac{I Z_1 Z_2 Z_3 + I_3 Z_3 Z_4 (Z_1 + Z_2)}{Z_1 Z_3 + Z_2 Z_4} \tag{21}$$

$$I_2 = \frac{I Z_1 Z_4 - I_3 Z_3 Z_4 + I_4 Z_4 Z_1}{Z_1 Z_3 + Z_2 Z_4} = \frac{I Z_1 Z_4 (Z_2 + Z_3)}{Z_1 Z_3 + Z_2 Z_4} \tag{22}$$

Now, if the bridge is balanced, $I_1 = I_3$ and $I_2 = I_4$. Therefore

$$I_3 = \frac{I Z_1 Z_2 Z_3 + I_3 Z_3 Z_4 (Z_1 + Z_2)}{Z_1 Z_3 + Z_2 Z_4} \tag{23}$$

$$I_4 = \frac{I Z_1 Z_4 (Z_2 + Z_3)}{Z_1 Z_3 + Z_2 Z_4} \tag{24}$$

Solving (23) for I_3 gives

$$I_3 = \frac{I Z_1 Z_2 Z_3}{Z_1 Z_3 + Z_2 Z_4 - Z_3 Z_4 (Z_1 + Z_2)} \tag{25}$$

Solving (24) for I_4 gives

$$I_4 = \frac{I Z_1 Z_4 (Z_2 + Z_3)}{Z_1 Z_3 + Z_2 Z_4} \tag{26}$$

Then, if the bridge is balanced, substituting (25) and (26) into (8) gives

$$I_5 Z_5 = \frac{I Z_1 Z_2 Z_3}{Z_1 Z_3 + Z_2 Z_4 - Z_3 Z_4 (Z_1 + Z_2)} + I_5 Z_5 \tag{27}$$

Substituting (25) and (26) into (9) gives

$$I_5 Z_5 = \frac{I Z_1 Z_4 (Z_2 + Z_3)}{Z_1 Z_3 + Z_2 Z_4} - \frac{I Z_1 Z_4 (Z_2 + Z_3)}{Z_1 Z_3 + Z_2 Z_4} \tag{28}$$

Thus, if the bridge is balanced, substituting (25) and (26) into (10) gives

$$I_5 Z_5 = \frac{I Z_1 Z_2 Z_3}{Z_1 Z_3 + Z_2 Z_4 - Z_3 Z_4 (Z_1 + Z_2)} = \frac{I Z_1 Z_4 (Z_2 + Z_3)}{Z_1 Z_3 + Z_2 Z_4} \tag{29}$$

Solving (29) for Z_1 gives

$$Z_1 = \frac{Z_2 Z_3 Z_4 (Z_1 + Z_2)}{Z_2 Z_3 Z_4 + Z_1 Z_4 (Z_2 + Z_3)} \quad \text{or} \quad Z_1 = \frac{Z_2 Z_3 Z_4}{Z_2 Z_3 + Z_4 (Z_2 + Z_3)} \tag{30}$$

[illegible] [illegible] [illegible] [

(b) By using the conservation of charge, we have

$$\frac{1}{R_1} = \frac{1}{R_2} + \frac{1}{R_3}$$

Now, with $R_1 = \frac{R_2 R_3}{R_2 + R_3}$ and $R_2 = \frac{R_1 R_3}{R_1 + R_3}$, we have

or $R_3 = \frac{R_1 R_2}{R_1 + R_2}$, where $R_3 = \frac{1}{\sigma A / L}$

Response on part (a) of the $R_1 R_2$ bridge consists of one resistor in parallel while method of obtaining the equivalent resistance was followed.

For the Wheatstone bridge of figure (7.1), we have

$$V_1 = I_1 R_1 = \left(\frac{\varepsilon}{R_1 + R_2}\right) R_1 \quad \text{from equation (7.1)}$$

$$\text{or} \quad -R_1 + R_2 \left(\frac{R_1 + R_2}{R_1 - R_2}\right) = R_1 R_2$$

using equation (7.1), we have

$$-R_1 + (R_1 + R_2) R_1 \left(\frac{R_1 - R_2}{R_1 + R_2}\right)$$

By rationalizing and applying limit and continuity, both of these by the two sides of equation for independent components of the magnetic field, we get

$$\boxed{R_1 = \frac{R_1 R_2}{R_3} \text{ and } R_2 = \sqrt{R_1 R_2}}$$

(c) The de Sauty bridge

The bridge provides a very simple method of measuring a capacitance in comparison with another known capacitance. In the de Sauty bridge shown in Figure (7.1.1), at an unknown capacitance and C_2 is a standard capacitor.



Figure 7.1.1 De Sauty bridge

At balance $\quad V_1 = V_2$, $V_3 = V_4$

or $\quad I_1 R_1 = I_2 R_2$, $I_3 R_3 = I_4 R_4$

Since $\quad I_1 = I_3$, $I_2 = I_4$

$$\left(\frac{I_1}{\omega C_1}\right) R_1 = \left(\frac{I_2}{\omega C_2}\right) R_2$$

Since $\theta_{BA} = \theta_B - \theta_A$, we have $\theta_B = \theta_A + \theta_{BA}$, or

$$\boxed{\theta_B = \theta_A - \frac{\pi}{2}}$$

Note that the angle θ_{BA} is measured counterclockwise from the positive x axis to the vector $\mathbf{r}_{BA}$, so it is negative. Since the crankshaft rotates at a constant angular velocity of 1200 rpm counterclockwise, θ_A increases at a constant rate of $1200(2\pi)/60 = 40\pi$ rad/s. Therefore, we can write

$$\theta_A = 40\pi t + \frac{\pi}{2}$$

where t is in seconds and the constant $\pi/2$ is added because $\theta_A = \pi/2$ when $t = 0$. Substituting into the equation for θ_B gives

$$\theta_B = 40\pi t + \frac{\pi}{2} - \frac{\pi}{2} = 40\pi t$$

Now we can write the position vectors $\mathbf{r}_A$ and $\mathbf{r}_B$ in terms of t:

$$\mathbf{r}_A = 0.75\cos(40\pi t)\,\mathbf{i} + 0.75\sin(40\pi t)\,\mathbf{j}$$

$$\mathbf{r}_B = 3\cos(40\pi t)\,\mathbf{i}$$

Therefore the position vector $\mathbf{r}_{BA}$ is

$$\mathbf{r}_{BA} = \mathbf{r}_B - \mathbf{r}_A = 2.25\cos(40\pi t)\,\mathbf{i} - 0.75\sin(40\pi t)\,\mathbf{j}$$

Differentiating with respect to t gives the velocity vector $\mathbf{v}_{BA}$:

$$\mathbf{v}_{BA} = -90\pi\sin(40\pi t)\,\mathbf{i} - 30\pi\cos(40\pi t)\,\mathbf{j}$$

Finally, we can write the velocity $\mathbf{v}_B$ as

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{BA} = -90\pi\sin(40\pi t)\,\mathbf{i} - 30\pi\cos(40\pi t)\,\mathbf{j}$$

Substituting the known angular velocity

$$\boldsymbol{\omega}_{AB} = -\omega_{AB}\,\mathbf{k}$$

we have

$$\mathbf{v}_{BA} = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{BA} = -\omega_{AB}\,\mathbf{k} \times (2.25\cos(40\pi t)\,\mathbf{i} - 0.75\sin(40\pi t)\,\mathbf{j})$$

$$= -2.25\omega_{AB}\cos(40\pi t)\,\mathbf{j} - 0.75\omega_{AB}\sin(40\pi t)\,\mathbf{i}$$

Note that $\omega_{AB} = -\omega_{BA} = -40\pi$ rad/s.

Thus, the velocity of point B is given by

$$\boxed{\mathbf{v}_B = -30\pi\sin(40\pi t)\,\mathbf{i} - 90\pi\cos(40\pi t)\,\mathbf{j}}$$



Figure 4.11 Reciprocating Engine

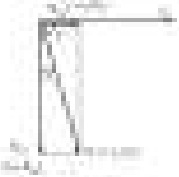


Figure 2.14 Mixture preparation with differential pressure control. (Courtesy Bosch)



Figure 2.15 Water pump.

The flow in a hydraulic loop can alternatively be called on a hydraulic to parallel with a capacitor, in the 2 hydraulic equivalent to series with a resistor.

If the resistance is concentrated in an R-element as shown in Fig. 2.15 and p_1 in Figure 2.15, the Bernoulli equation for the volume flow rate as shown in Figure 2.15 (equal to volume 2):

$$q = \text{const} \cdot \frac{p_1}{R_1} = \text{const} \cdot \frac{p_2}{R_2}$$

or, $q = \text{const} \cdot \left(\frac{1}{R_1 R_2} \right)$

The pressure losses of the volume flow rate is given by, using

The angle $\alpha = 90°$ — it is called the **flow angle** and is given by

$$\alpha = \text{const} \cdot \frac{p_1}{R_1} = \text{const} \cdot R_1 \cdot \text{and}$$

$$\alpha = \text{const} \cdot \left(\frac{p_1 R_2}{R_1} + \frac{p_2 R_1}{R_2} \right)$$

$$= \text{const} \cdot \frac{p_1 R_2}{R_1}$$

(See also Chapter 10, page 191)

2.2 The Water Valve

The hydraulic control is a valve to regulate the pressure or volume of the flow, or to control the rate of flow. In a hydraulic control system, a flow regulator or regulator is also called the valve or a control valve.

A typical control diagram for a flow regulator is shown in Figure 2.15. Here, there is

$$R_1 = R_2 R_3 = \frac{1}{\rho g R_1 + \rho g R_2} \text{ and, if } p_1 \text{ and } p_2 \text{ are equal}$$

$$R_1 = R_2 = R_3 \text{ and } R_1 = R_2$$

At balance, $R_1 \rho g R_2 = \rho g R_3 R_2$ or the equation of R_1 is

$$R_1 \rho g R_2 = R_1 \rho g = \left(\frac{1}{\rho g R_1 + \rho g R_2} \right) \rho g R_2$$

Substituting gives

$$\left(R_1 = \frac{1}{\rho g R_1} \right) \left(\frac{1}{\rho g} \right) = \text{const} \cdot R_1 = \frac{R_2}{R_1}$$

$$\frac{R_2}{R_1} = \frac{1}{\rho g} \cdot \left(\frac{1}{\rho g R_1} \right) = \text{const} \cdot R_1 = \frac{R_2}{R_1}$$

Substituting into Kirchhoff's current law gives

$$\boxed{\frac{R_2}{R_1 R_3}V_1 + \frac{1}{R_3}I_1 = \frac{R_2}{R_1 R_3}V_2} \tag{7.7.5}$$

Equating coefficients from (7.7.4) and (7.7.5) gives

$$-\frac{1}{\alpha R_3 R_4} = -\alpha(R_1 R_2 + \cdots)$$

so $\alpha = -\alpha(R_1 R_2 + \cdots) = \frac{1}{\alpha R_3 R_4}$

from which $\alpha^2 = \frac{1}{R_3 R_4 (R_1 R_2 + \cdots)}$

Therefore we have $\boxed{\text{Impedance } Z = \frac{1}{\alpha R_3 R_4 (R_1 R_2 + \cdots)}}$ (7.7.6)

Now that $Z = \frac{1}{\alpha}$, $Z = Z'$ and $R_1 = R_2 = R_3$

Therefore $Z = \frac{1}{\alpha R_3 R_4 (R_1 R_2 + \cdots)} = \frac{1}{\alpha R_3 R_4}$



Figure 7.7.1(a)

Problem 1. The n-n bridge shown in Figure 7.7.1(a) is used to measure the capacitance C_x and resistance R_x of inductor L_x. Derive the balance equations in the bridge. For which $R_1 = R_2 = R_3 = 1\,\text{k}\Omega$ and the frequency of the supply is 1 kHz, determine R_4 when L_x is 1 mH and C_x is 0.1 μF?

(a) Figure 7.7.1(a) and R_3 are the unknown circuit and are connected in parallel. If we write Y_3 as the reciprocal impedance from the R_3 branch ($\text{admittance} = \frac{\text{current}}{\text{voltage}}$),

$$\text{then} \qquad Y_3 = \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_5}$$

Then solving for $R_3 = \frac{R_4 R_5}{R_4 + R_5}$

From Figure 7.7.1(a), $L_x = R_4 + (R_1 \ldots R_5)$ where $(\ldots) = R_3$

or therefore $R_4 = L_x(R_4 + R_5)$

$$\frac{1}{(R_4 R_5)} = \frac{1}{L_x R_5} + \frac{1}{L_x R_4} = \frac{1}{L_x R_5} + \frac{1}{L_x}$$



Problem 4 For the three-bar truss shown in Figure 17.11, $R_{xx} = 0.1\,\text{in}^2$, $R_{yy} = 0.05\,\text{in}^2$, and $R_{xy} = 0.1\,\text{in}^2$. Determine what the bridge is expected to do when an earthquake hits, and find the frequency of the motion.

(a) From equation (17.4),

$$\frac{R_{xx}}{R_{yy}} = \frac{\lambda_1}{\lambda_2} = \frac{R_{xy}}{R_{yy}}$$

$\lambda_1 = \lambda_2 = 0.1\,\text{in}^2/\text{in}^2$, since $R_{xx} = R_{xy}$ and $\lambda_1 = \lambda_2$ from above.

Substitute $R_{xx} = \frac{\pi D^4}{64} = 0.1\,\text{in}^4$

(b) From equation (17.5),

$$\text{Frequency} = \frac{1}{2\pi\sqrt{\left(\frac{\lambda_1}{\rho A L^2}\right)\left(\frac{\lambda_2}{\rho A L^2}\right)}} = \frac{1}{2\pi\left(\frac{\lambda_1}{\rho A L^2}\right)}$$

$$= 1.000\,\text{cycles}$$

Problem 5 A cantilevered flagpole is to be erected at Empire City's main office. Given $E_x = 0.1\,\text{psi}$, $E_y = 0.05\,\text{psi}$, and $E_{xy} = 0.1\,\text{psi}$, with an axial load, P, of 10 lbs, determine what the bridge is expected to do when an earthquake hits, and find the frequency of the motion. Also, find the angle θ at which the flagpole is to be erected.



Figure 17.12

From eqns (5), the eigenvalues for E_x and E_y combined are given by

$$\lambda = \frac{E_x + E_y}{2} \pm \text{and } \lambda_2 = \frac{E_x - E_y}{2}$$

(a) Resultant is $R = \frac{E_x + E_y}{2} = \frac{0.1\,\text{psi} + 0.05\,\text{psi}}{2} = 0.1\,\text{psi}$

(b) Calculate $\lambda_1 = \frac{E_x - E_y}{2} = \frac{0.1 - 0.05}{2} = 0.1\,\text{psi}$

(c) The flagpole is to be erected at an angle θ to the vertical, where θ is given by

$$\text{Flagpole angle, } \theta = \arctan\frac{E_{xy}}{\lambda_1} = \arctan\frac{0.1}{0.1} = \arctan 1 = 45°$$

(d) $\theta = \arctan\left(\frac{\lambda_1 + \text{Resultant}}{E_{xy}}\right) = \arctan\left(\frac{0.1 + 0.1}{0.1}\right)$

$= \arctan 2 = 63.43°$ Ans

(b) Private bank A replaces a worn $\mathrm{₹}\,500$ note; it receives a fresh note from the RBI.

(c) A non-profit, the Seva Nidhi Trust (Figure 7.11), is given a grant of $\mathrm{₹}\,1{,}25{,}000$ by a donor.

Alongside, note each transaction's effect on the balance sheet:

$$\text{Liabilities} \begin{cases} \text{Deposits} \\ \text{Capital} \end{cases} \quad \text{Assets} \begin{cases} \text{Cash} \\ \text{Investments} \end{cases}$$

Part — Cash and Deposits

Further problems on these items are located in Section 17.3 of this chapter, problems 3 to 17.

17.4 Problems on Loans and Advances

1. A finance firm charges a flat rate of $r = 12\%$ per year on a loan for $t = 3$ years. The principal is $P = \mathrm{₹}\,2{,}00{,}000$. (a) Find the total interest I charged. (b) Find the total amount A to be repaid. (c) The borrower actually receives only $\mathrm{₹}\,1{,}90{,}000$ after a processing fee is deducted from the principal. Find the effective rate of interest on the amount actually received.

2. A co-operative credit society lends $\mathrm{₹}\,5{,}00{,}000$ to its members at a flat rate of $r = 9\%$ per year for $t = 4$ years. (a) Find the total interest I and the total amount A to be repaid. (b) The society's own cost of funds is 8% per year. Find the society's annual profit on this loan, ignoring all other expenses.

3. A Co-operative Bank sells $n = 1{,}000$ shares of $\mathrm{₹}\,100$ each, issued at par. Each shareholder is entitled to a dividend of $d = 12\%$ per year on the face value of the shares they hold. (a) Find the total dividend D the bank must pay in a year. (b) In a year when the bank earns a profit of $\mathrm{₹}\,1{,}50{,}000$, what fraction of the profit is distributed as dividend? (c) What fraction is retained?

4. A Co-operative Society with $n = 2{,}000$ members, each holding $s = 10$ shares of $\mathrm{₹}\,100$ face value, declares a dividend of $d = 15\%$. (a) Find the total dividend D payable. (b) Each member's dividend is subject to tax at $t = 10\%$. Find the tax withheld per member and the net dividend per member. (c) The society's own profit for the year is $\mathrm{₹}\,4{,}00{,}000$. What fraction of the profit is distributed?

5. The Indian Bureau of Weights (IBW) is used to measure capacities in litres (L) and kilograms (K). Express the relation between a litre of water and a kilogram as a ratio $L : K$, and find the value of L when $K = 5$ kg, $K = 12.5$ kg, and $K = 1.5$ kg.

6. In a banking college practical, a student is given a solution of sodium chloride (common salt) and a solution of silver nitrate. The two solutions react in equal volumes, so the ratio of the volume of silver nitrate solution used to the volume of sodium chloride solution used is $1 : 1$. The student uses $V_1 = 25$ cm^3 of sodium chloride solution and finds that $V_2 = 40$ cm^3 of silver nitrate solution is required for complete reaction. (a) Find the ratio $V_2 : V_1$ in its simplest form. (b) Find the value of V_2 when $V_1 = 10$ cm^3. (c) Find the value of V_1 when $V_2 = 100$ cm^3.

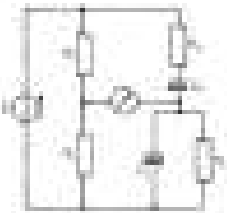


Figure [illegible]

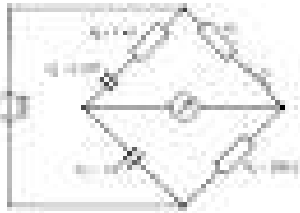


Figure [illegible]

a) The a.c. bridge shown in Figure [illegible] is balanced when the ratios of the corresponding impedances are equal. Determine an expression for the unknown L_x and R_x. [illegible]

b) An a.c. bridge has a null detector with a sensitivity of 0.1 mA/V and a resistance of 500 Ω, as shown in Fig. [illegible]. The bridge is supplied from a 1 kHz source and has the following arm impedances: resistance R_1 = 1 kΩ; resistance R_2 = 2 kΩ; resistance R_3 = 1 kΩ; capacitance C_3 = 0.1 μF. Determine the value of the resistance R_4 and capacitance C_4 required to balance the bridge. [illegible]

c) When the three-lamp system in Figure [illegible] is balanced, the instantaneous line and neutral currents are $i_1 = 10\sqrt{2}\sin(\omega t)$ A, $i_2 = 7\sqrt{2}\sin(\omega t - 2\pi/3)$ A and $i_3 = 4\sqrt{2}\sin(\omega t + 2\pi/3)$ A. Determine (a) the balance conditions for the ratio of impedances Z_1 and Z_2 of the equipment, (b) the supply voltage. [illegible]

d) The conditions of balance of a Schering bridge used to measure the capacitance and loss angle of a given capacitor are as follows: (a) a given capacitance of C_1 = 0.1 μF; (b) a given capacitance C_2 = 0.01 μF; (c) a given resistance R_2 = 1 kΩ; (d) a given resistance R_3 = 10 kΩ. The capacitance under test should also be considered as a combination C_x in series with a resistance R_x. If the supply frequency is 1 kHz, determine (a) the value of R_x, (b) the value of C_x, (c) the power factor of the capacitor, and (d) the loss angle. [illegible]

e) A balanced, a.c. bridge network is arranged so that one arm has a resistance R in series with a capacitance C, the adjacent arm has a resistance nR, the third arm is a pure resistance S, and the fourth arm is a resistance mR in parallel with a capacitance nC. Show that the bridge is balanced when $S = mR/n$ and $C = n(m+1)C/m$. Draw a phasor diagram for the bridge. [illegible]

2.1 Let $X_1, \ldots, X_n$ be a random sample from a $N(\mu, \sigma^2)$ distribution, with σ^2 known. Show that $\bar{X}_n$ is a sufficient statistic for μ. Now suppose instead that μ is known but σ^2 is unknown. Find a sufficient statistic for σ^2. Finally, suppose that both μ and σ^2 are unknown. Find a two-dimensional sufficient statistic for (μ, σ^2). Hint: Use the factorization theorem (Theorem 2.2.4). Remember to include the ranges of the parameters and the sample values.

$$f(\mathbf{x} \mid \mu, \sigma^2) = (2\pi\sigma^2)^{-n/2} \exp\left\{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i - \mu)^2\right\}$$

2.2 Let $X_1, \ldots, X_n$ be a random sample from a distribution with density function $f(x \mid \theta) = \theta x^{\theta-1}$ for $0 < x < 1$, where $\theta > 0$. Find a sufficient statistic for θ. Hint: Use the factorization theorem (Theorem 2.2.4). Remember to include the ranges of the parameters and the sample values.

$$f(\mathbf{x} \mid \theta) = \theta^n \left(\prod_{i=1}^{n} x_i\right)^{\theta-1} \mathbb{I}(0 < x_{(1)}) \mathbb{I}(x_{(n)} < 1)$$

2.3 Let $X_1, \ldots, X_n$ be a random sample from a distribution with density function $f(x \mid \theta) = \frac{1}{2}(1 + \theta x)$ for $-1 < x < 1$, where $-1 < \theta < 1$. Find a sufficient statistic for θ. Hint: Use the factorization theorem (Theorem 2.2.4). Remember to include the ranges of the parameters and the sample values. Hint: Use the factorization theorem (Theorem 2.2.4). Remember to include the ranges of the parameters and the sample values.

$$f(\mathbf{x} \mid \theta) = 2^{-n}(1 + \theta)^{n_+}(1 - \theta)^{n_-}$$

28 Series resonance and Q-factor

At the end of this chapter you should be able to:

- state the conditions for resonance in an a.c. series circuit
- calculate the resonant frequency in an a.c. series circuit
- define Q-factor as $\frac{X_L}{R}$ and/or $\frac{X_C}{R}$ or $\frac{1}{R}\sqrt{\frac{L}{C}}$
- determine the impedance values of X_L and X_{eff} at the frequency at which they occur
- determine the overall current in two components in series
- define bandwidth and selectivity
- calculate the lower and upper cut-off frequencies in a.c. series circuits
- determine the current and impedance when the frequency deviates from the resonant frequency

28.1 Introduction

When the voltage V applied to a series circuit containing resistance, inductance and capacitance is in phase with the resulting current I, the circuit is said to be **resonant**. The phenomenon of resonance is of great value in all branches of radio, television and communication engineering, and a practical study such as this cannot proceed very far before the term is encountered. In this chapter, the basic principles of resonance are explained and some of its applications are considered.

At resonance, the inductive reactance (symbol X_L) exactly cancels the capacitive reactance (symbol X_C), i.e. $X_L = X_C$ or $2\pi fL = \frac{1}{2\pi fC}$

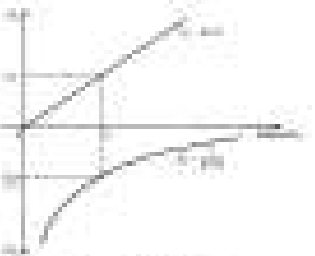
As the current rises from zero to a maximum value and then falls to zero again, it can be said to be varying, i.e. alternating — the alternating current (a.c.) theory of which was introduced in Chapter 23, whereas direct current is, of course, unvarying. Direct current flows in one direction only, and thus does not alternate. Both energy sources and loads can be classified as resistive, inductive or capacitive, and the effect of each of these on the a.c. circuit is considered in Chapter 26.

28.2 Series resonance

Figure 28.1 shows a basic resonant circuit consisting of inductance L and resistance R connected in series with a capacitor C. The R, L and C values could be for a coil (inductance L and R) and a capacitor C, or R could be the total circuit







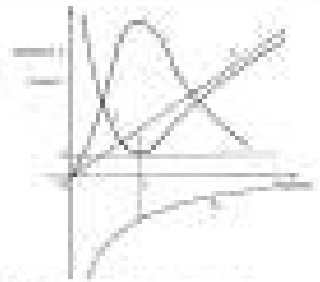


Figure 28.4 [illegible]

(a) [illegible]

$$\frac{1}{f} = \frac{1}{[illegible]} + \frac{1}{[illegible]}$$

$$\therefore f = [illegible]\ \text{Hz}$$

(b) [illegible] $= \frac{X_C}{R} = \frac{[illegible]}{[illegible]} = [illegible]$

[illegible] [illegible] [illegible] [

XIV.1 A series RL circuit (inductor and resistor) is powered by a sinusoidal voltage source. Derive the expression for the impedance Z of the circuit and the phase angle ϕ between the source voltage and the current.

Solution. In a series circuit the current I is the same through every element, and the total voltage is the sum of the voltage drops across the resistor and the inductor. Because the inductor's voltage leads the current by 90°, these two drops are not in phase, so we must add them as vectors (phasors) rather than as ordinary numbers.

The voltage across the resistor is $V_R = IR$, in phase with I. The voltage across the inductor is $V_L = IX_L$, where $X_L = \omega L$ is the inductive reactance, and it leads I by 90°. The source voltage is therefore

$$V = \sqrt{V_R^2 + V_L^2} = I\sqrt{R^2 + X_L^2}.$$

The quantity $Z = \sqrt{R^2 + X_L^2}$ is the impedance of the circuit, measured in ohms, and it plays the role for AC circuits that resistance plays for DC circuits: $V = IZ$. The phase angle between the source voltage and the current is

$$\phi = \arctan\left(\frac{X_L}{R}\right) = \arctan\left(\frac{\omega L}{R}\right).$$

A positive ϕ means the current lags the source voltage, which is the signature of an inductive circuit.

XIV.2 A series RC circuit (capacitor and resistor) is powered by a sinusoidal voltage source. Derive the expression for the impedance Z of the circuit and the phase angle ϕ between the source voltage and the current.

Solution. As in the RL circuit, the current I is the same through every element, and the total voltage is the sum of the voltage drops across the resistor and the capacitor. The capacitor's voltage lags the current by 90°, so again the two drops must be added as vectors.

The voltage across the resistor is $V_R = IR$, in phase with I. The voltage across the capacitor is $V_C = IX_C$, where $X_C = \frac{1}{\omega C}$ is the capacitive reactance, and it lags I by 90°. The source voltage is therefore

$$V = \sqrt{V_R^2 + V_C^2} = I\sqrt{R^2 + X_C^2}.$$

The quantity $Z = \sqrt{R^2 + X_C^2}$ is the impedance of the circuit, measured in ohms, and it plays the role for AC circuits that resistance plays for DC circuits: $V = IZ$. The phase angle between the source voltage and the current is

$$\phi = \arctan\left(\frac{X_C}{R}\right) = \arctan\left(\frac{1}{\omega RC}\right).$$

A positive ϕ means the current leads the source voltage, which is the signature of a capacitive circuit.

XIV.3 A series RLC circuit (inductor, capacitor and resistor) is powered by a sinusoidal voltage source. Derive the expression for the impedance Z of the circuit and the phase angle ϕ between the source voltage and the current.

Solution. As in the RL and RC circuits, the current I is the same through every element, and the total voltage is the sum of the voltage drops across the resistor, the inductor and the capacitor. The inductor's voltage leads the current by 90° while the capacitor's voltage lags it by 90°, so the two reactive drops point in opposite directions and must be added as vectors.

The voltage across the resistor is $V_R = IR$, in phase with I. The voltage across the inductor is $V_L = IX_L$, where $X_L = \omega L$, and it leads I by 90°. The voltage across the capacitor is $V_C = IX_C$, where $X_C = \frac{1}{\omega C}$, and it lags I by 90°. The source voltage is therefore

$$V = \sqrt{V_R^2 + (V_L - V_C)^2} = I\sqrt{R^2 + (X_L - X_C)^2}.$$

The quantity $Z = \sqrt{R^2 + (X_L - X_C)^2}$ is the impedance of the circuit, measured in ohms, and it plays the role for AC circuits that resistance plays for DC circuits: $V = IZ$. The phase angle between the source voltage and the current is

$$\phi = \arctan\left(\frac{X_L - X_C}{R}\right) = \arctan\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right).$$

A positive ϕ means the current lags the source voltage, which is the signature of an inductive circuit; a negative ϕ means the current leads it, which is the signature of a capacitive circuit.

Key relationships. $X_L = \omega L$ (inductive reactance); $X_C = \frac{1}{\omega C}$ (capacitive reactance); $Z = \sqrt{R^2 + (X_L - X_C)^2}$ (impedance); $V = IZ$ (Ohm's law for AC); $\phi = \arctan\left(\frac{X_L - X_C}{R}\right)$ (phase angle).

XIV.4 Determine the current I in a series RLC circuit with $R = 10\ \Omega$, $L = 0.05$ H, $C = 100\ \mu$F, and a source voltage $V = 120$ V at $\omega = 377$ rad/s.

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

Some [illegible] $\frac{1}{\text{[illegible]}} \cdot \frac{1}{\text{[illegible]}} \cdot \frac{1}{\text{[illegible]}}$

Some [illegible] **[illegible] C** $\frac{\text{[illegible]}}{\text{[illegible]}} \cdot \frac{1}{\text{[illegible]}} \cdot \frac{1}{\text{[illegible]}} \cdot \frac{1}{\text{[illegible]}}$

= [illegible]

Further [illegible] of the [illegible] of [illegible] Number [illegible] [illegible] [illegible] = [illegible] [illegible] 313

[illegible] **[illegible]** In [illegible] of [illegible] [illegible] it [illegible] [illegible] the [illegible] [illegible] [illegible]

For an **inductor**, $L[\text{henry}] = \left[\frac{\text{volt-second (energy per unit charge)}}{\text{change in rate (ampere/s)}}\right]$

$$\rightarrow L\left[\frac{\text{kg}\cdot\text{m}^2}{\text{s}^2\cdot\text{C}}\right] = \frac{\text{V}\cdot\text{s}}{\Delta\left(\frac{\text{C}}{\text{s}}\right)} = \frac{\text{V}\cdot\text{s}}{\frac{\text{C}}{\text{s}}} = \frac{\text{V}}{\frac{\text{C}}{\text{s}^2}} \tag{10.1}$$

For a **capacitor**, $C = \frac{\text{charge}}{\text{change in voltage}} = \frac{\text{coulomb}}{\text{change in volt}}$

$$= \frac{\text{C}}{\Delta\text{V}} = \frac{\text{C}}{\text{V}}$$

so,

$$\text{V} = \frac{1}{\text{C}}\text{q} \tag{10.2}$$

From expressions (10.1) and (10.2), it can be deduced that

$$\frac{\text{q}}{\text{V}} = \frac{\text{V}}{\text{I}} = \frac{\text{coulomb}}{\text{ampere}}$$

In one instance one can, for inductance

$$\text{If henry} = \frac{\text{coulomb per ampere}}{\text{ampere/s}} = \frac{\text{q}}{\text{I}}$$

where q[C] is the coulomb, power, I[A/s] or ratio for load, Ldt = in energy energy, and P is the constant energy transmitted over t hours.

$$\text{henry} = \frac{\text{q}}{\text{I}} = \frac{\text{V}\,\Delta\left(\frac{\text{q}}{\text{V}}\right)}{\Delta\text{I}} = \frac{\text{V}}{\text{I}}\left(\frac{\text{q}}{\text{V}}\right)$$

so,

$$\boxed{\text{H} = \frac{\text{coulomb}}{\text{amperes}}}$$

In an R, L, C device circuit the element of energy stored at resistance or conductor:

- Make the capacitor a charge or a conductor. Use Ampere-conductivity and Ohm's law, $\text{I} = \frac{\Delta\text{V}}{\text{R}} = \frac{\Delta\text{V}}{\Delta\text{t}}$.
- Use the Kirchoff of conductance $\frac{\text{q}}{\text{V}}$ to give a fit.

$$\boxed{\text{R} = \frac{\text{V}\cdot\text{I}}{\text{q}} = \frac{1}{\text{C}\cdot\text{V}\cdot\text{R}}} \tag{10.3}$$

Resistance is measured in $\Omega = 1\ \text{volt/A}$.

Hence $\text{q} = \frac{\text{V}\cdot\text{I}}{\text{R}} = \frac{1}{\text{C}\cdot\text{V}\cdot\text{R}}$, $\frac{\text{I}}{\text{R}}$ or $\text{R} = \frac{1}{\text{R}}\left(\frac{\text{I}}{\text{V}}\right)$

A circuit is made from eight resistors as shown in the figure. A battery of emf $\mathcal{E}$ and negligible internal resistance is connected between points A and B. Each resistor has resistance R.

(a) Find the equivalent resistance between A and B.

(b) Find the current in each resistor in terms of $\mathcal{E}$ and R.

(c) Find the power dissipated in each resistor.

Solution

(a) The symmetry of the network about the midplane through C and D (and the midplane through E and F) means that the potentials at C and D are equal, and the potentials at E and F are equal. Therefore no current flows through the connecting wire CD, and no current flows through the connecting wire EF. Removing these wires does not change the equivalent resistance between A and B.

The resistors R and $2R$ in series between A and C have equivalent resistance $3R$. The resistors R and $2R$ in series between C and B have equivalent resistance $3R$. The resistors R and $2R$ in series between A and D have equivalent resistance $3R$. The resistors R and $2R$ in series between D and B have equivalent resistance $3R$. The four $3R$ resistances in parallel between A and B have equivalent resistance $\frac{3R}{4}$.

$$R_{AB} = \frac{3R}{4}$$

(b) Current in resistance $\frac{3R}{4}$: $I = \frac{\mathcal{E}}{R_{AB}} = \frac{4\mathcal{E}}{3R}$

Current in each $3R$ resistor in the upper path: $\frac{I}{2} = \frac{2\mathcal{E}}{3R}$

Current in each $3R$ resistor in the lower path: $\frac{2\mathcal{E}}{3R}$

Current in each R resistor in the upper path: $\frac{2\mathcal{E}}{3R}$

Current in each R resistor in the lower path: $\frac{2\mathcal{E}}{3R}$

(c) Power dissipated in each R resistor: $P = I^2R = \left(\frac{2\mathcal{E}}{3R}\right)^2 R = \frac{4\mathcal{E}^2}{9R}$

Power dissipated in each $2R$ resistor: $P = I^2(2R) = \frac{8\mathcal{E}^2}{9R}$

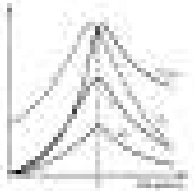


10.2 Voltage Amplifiers





(a)



(b)

Figure [illegible] [illegible]
[illegible] [illegible]

$$\frac{\partial \ln L}{\partial \theta} = \frac{\partial \ln L}{\partial \theta} + \frac{\partial \ln L}{\partial \theta}$$

[illegible] [illegible] [illegible]

$$\frac{\partial \ln L}{\partial \theta} = \frac{\partial \ln L}{\partial \theta} + \frac{\partial \ln L}{\partial \theta}$$

If the [illegible] [illegible]

$$\frac{\partial \ln L}{\partial \theta} = \frac{\partial \ln L}{\partial \theta} + \frac{\partial \ln L}{\partial \theta} \qquad (4.1.1)$$

[illegible] [illegible] [illegible] [

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

$$\left[\begin{array}{c} \\ \\ \end{array}\right] \qquad \left[\begin{array}{c} \\ \\ \end{array}\right]$$

$$\left[\begin{array}{c} \\ \\ \end{array}\right] \qquad \left[\begin{array}{c} \\ \\ \end{array}\right]$$

$$\text{If } \left[\begin{array}{c} \\ \\ \end{array}\right] \text{ then } \left[\begin{array}{c} \\ \\ \end{array}\right]$$

Now, since $\left[\begin{array}{c} \\ \\ \end{array}\right]$

Substituting in equation (1) we get

$$\left[\begin{array}{c} \\ \\ \end{array}\right]$$

$$\left[\begin{array}{c} \\ \\ \end{array}\right]$$

$$\text{Hence } \left[\begin{array}{c} \\ \\ \end{array}\right]$$

From equation (1) we have $\left[\begin{array}{c} \\ \\ \end{array}\right]$

If a similar operation is carried out for $\left[\begin{array}{c} \\ \\ \end{array}\right]$ then we have the following set of equations for evaluating the unknowns $\left[\begin{array}{c} \\ \\ \end{array}\right]$

$$\left[\begin{array}{c} \\ \\ \end{array}\right]$$

i.e., The above equations in $\left[\begin{array}{c} \\ \\ \end{array}\right]$ are linearly

$$\left[\begin{array}{c} \\ \\ \end{array}\right]$$

Solving, find the impedance and, hence, the current and voltage at the terminals. Sketch the phasor diagram.

> **Problem 3.** A series RLC circuit has a source with rms voltage $V_s = 120\ \text{V}$ at frequency $f = 60\ \text{Hz}$. The resistance is $R = 40\ \Omega$, the inductance is $L = 0.15\ \text{H}$, and the capacitance is $C = 100\ \mu\text{F}$. Determine (a) the impedance Z in rectangular form, (b) the current I in polar form, (c) the voltage across the inductor V_L, and (d) the power factor of the circuit. Finally, draw a phasor diagram showing V_s, V_R, V_L, and V_C, and explain why V_L and V_C can be much larger than V_s even though the circuit is not in resonance.

(a) The inductive reactance is

$$X_L = \omega L = 2\pi f L = 2\pi (60)(0.15) = 18.85\ \Omega$$

The capacitive reactance is

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi (60)(100 \times 10^{-6})} = 26.53\ \Omega$$

Since $X_C > X_L$, the circuit is capacitive, and the net reactance is

$$X = X_L - X_C = 18.85 - 26.53 = -7.68\ \Omega$$

The impedance is therefore

$$Z = R + jX = 40 - j7.68\ \Omega$$

In polar form, the magnitude is $|Z| = \sqrt{40^2 + 7.68^2} = 40.74\ \Omega$ and the angle is $\theta = \arctan(-7.68/40) = -10.86^\circ$, so $Z = 40.74 \angle -10.86^\circ\ \Omega$.

(b) The current is found from Ohm's law for phasors:

$$I = \frac{V_s}{Z} = \frac{120 \angle 0^\circ}{40.74 \angle -10.86^\circ} = 2.945 \angle 10.86^\circ\ \text{A}$$

The current leads the source voltage because the circuit is capacitive.

(c) The voltage across the inductor is

$$V_L = I \cdot (jX_L) = (2.945 \angle 10.86^\circ)(18.85 \angle 90^\circ) = 55.52 \angle 100.86^\circ\ \text{V}$$

(d) The power factor is $\cos(-10.86^\circ) = 0.982$ leading. Although $V_L = 55.52\ \text{V}$ and $V_C = 78.16\ \text{V}$ are both larger than the source voltage of $120\ \text{V}$, they are nearly equal and opposite, so they almost cancel. The phasor diagram shows V_L and V_C pointing in opposite directions, with their vector sum equal to V_s.

10.3 Kirchhoff's Laws

Kirchhoff's laws are the foundation of circuit analysis. They are not new physical laws but rather consequences of conservation of charge and conservation of energy applied to lumped circuits. The first law deals with currents at a node, and the second law deals with voltages around a loop.

The circuit of Fig. [illegible] from Section [illegible] is shown in [illegible]. Here $R_1 = R_2 = R$.

Also, $R_3 = \frac{R}{2}$ from R_1 to R_2 and $R_4 = \frac{R}{2}$ as well.

$$R_5 = \frac{R}{1+\frac{R}{R}} \text{ from } R_1 \text{ to } R_2 \qquad R_6 = \frac{R}{R+R} \text{ from } R_1 \text{ to } R_2$$

Then $R_7 = R + \left(\frac{1}{R_5} + \frac{1}{R_6}\right)^{-1} = R + \left(\frac{1}{\frac{R}{2}} + \frac{1}{\frac{R}{2}}\right)^{-1}$

$$= R + \left[\frac{1}{\frac{R}{2}} + \frac{1}{\frac{R}{2}}\right]^{-1}$$

$$= R + \left[\frac{1}{\frac{R}{2}} + \frac{1}{\frac{R}{2}}\right]^{-1} \qquad \text{Then } R_8 = \frac{1}{\frac{1}{R}+\frac{1}{R}}$$

$$= R + \frac{1}{\frac{1}{\frac{R}{2}} + \frac{1}{\frac{R}{2}}} = R + \frac{1}{\frac{2}{R} + \frac{2}{R}} = R + \frac{1}{\frac{4}{R}}$$

$$= R + \frac{1}{\frac{1}{R} + \frac{1}{R}} = R + \frac{1}{\frac{2}{R}}$$

$$= R + \frac{R}{2} = \frac{3R}{2}$$

So, the overall balance

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

Problem 3 — [illegible] of the [illegible] is connected to [illegible] and [illegible] is [illegible] [illegible] [illegible]

15.4 Accessibility

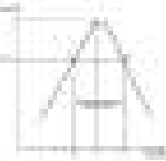


Figure 15.11 Accessibility and half-line points P_1 and P_2.

Higher-level theorems have enabled a rigorous definition of accessibility. A point P is said to be accessible if, given any $\varepsilon > 0$, it is possible to find a point P_ε lying on a specified curve Γ such that the distance between P and P_ε is less than ε. Points that are not accessible are called inaccessible points. Consider, for example, the function $f(z) = \exp(1/z)$ and the point $z = 0$. If we let z approach zero along the positive real axis, then $f(z) \to \infty$ as $z \to 0$. However, if we let z approach zero along the negative real axis, then $f(z) \to 0$ as $z \to 0$. Thus, in whatever small neighbourhood of $z = 0$ we choose, $f(z)$ can take on values that are both very large and very small. Hence, $z = 0$ is an inaccessible point of $f(z) = \exp(1/z)$. Inaccessible points are also called **singular points**. Further discussion of singular points, and of the behaviour of functions in the neighbourhood of such points, is given in chapter 16.

Within the radius of convergence of a power series, the series represents a single-valued function of the complex variable z. For example, the series

$$f(z) = 1 + z + z^2 + z^3 + \cdots \quad \text{converges for } |z| < 1 \text{ (see page 136)}$$

is the series of the first few terms of the binomial expansion of $(1-z)^{-1}$, and so represents the function $(1-z)^{-1}$ within the circle of convergence $|z| = 1$. Outside this circle the series diverges, and so does not represent the function $(1-z)^{-1}$. Nevertheless, the function $(1-z)^{-1}$ is perfectly well defined for $|z| > 1$, and so we say that the power series provides an **analytic continuation** of the function $(1-z)^{-1}$ from the region $|z| < 1$ into the region $|z| > 1$. In general, analytic continuation is the process of finding a function that is defined in a larger region of the complex plane than the region in which a given power series converges, and which is equal to the sum of the power series within the circle of convergence of the series.



Figure 15.12 (a) Some non-equivalent contours connecting the points $z = 0$ and $z = 1 + i$; (b) a contour that includes a cusp.

Let us now consider a function $f(z)$ that is defined in a region R of the complex plane. Suppose that $f(z)$ is analytic (i.e. single-valued and differentiable) in R, except for a finite number of isolated singularities. Suppose also that $f(z)$ is analytic in the neighbourhood of a point z_0 in R. Then, if $f(z)$ is integrated around a closed contour C in R, which passes through z_0, the value of the integral $\oint_C f(z)\,dz$ depends only on the behaviour of $f(z)$ in the neighbourhood of z_0. Thus, if $f(z)$ is analytic in the neighbourhood of z_0 then $\oint_C f(z)\,dz = 0$, whatever the shape of C in R. This is Cauchy's theorem, which we discussed in section 6.4. Alternatively, if $f(z)$ has a simple pole at z_0 then $\oint_C f(z)\,dz = 2\pi i$ times the residue of $f(z)$ at z_0, whatever the shape of C in R. This is the residue theorem, which we discussed in section 6.5.

Let us now consider the integral $\int_\Gamma f(z)\,dz$, where Γ is a contour connecting the points $z = 0$ and $z = 1 + i$. Suppose that $f(z)$ is analytic in the region R of the complex plane that includes Γ, except for a simple pole at $z = 1$. Suppose also that $f(z)$ is analytic in the neighbourhood of $z = 0$. Let us first consider the integral $\int_{\Gamma_1} f(z)\,dz$, where Γ_1 is the straight line $z = (1+i)t$ ($0 \le t \le 1$) connecting the points $z = 0$ and $z = 1 + i$. Let us now consider the integral $\int_{\Gamma_2} f(z)\,dz$, where Γ_2 is the contour that runs from $z = 0$ to $z = 1$ along the real axis, and then from $z = 1$ to $z = 1 + i$ parallel to the imaginary axis. Let us now consider the integral $\int_{\Gamma_3} f(z)\,dz$, where Γ_3 is the contour that runs from $z = 0$ to $z = i$ along the imaginary axis, and then from $z = i$ to $z = 1 + i$ parallel to the real axis. Let us now consider the integral $\int_{\Gamma_4} f(z)\,dz$, where Γ_4 is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^2$. Let us now consider the integral $\int_{\Gamma_5} f(z)\,dz$, where Γ_5 is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^3$. Let us now consider the integral $\int_{\Gamma_6} f(z)\,dz$, where Γ_6 is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^4$. Let us now consider the integral $\int_{\Gamma_7} f(z)\,dz$, where Γ_7 is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^5$. Let us now consider the integral $\int_{\Gamma_8} f(z)\,dz$, where Γ_8 is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^6$. Let us now consider the integral $\int_{\Gamma_9} f(z)\,dz$, where Γ_9 is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^7$. Let us now consider the integral $\int_{\Gamma_{10}} f(z)\,dz$, where Γ_{10} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^8$. Let us now consider the integral $\int_{\Gamma_{11}} f(z)\,dz$, where Γ_{11} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^9$. Let us now consider the integral $\int_{\Gamma_{12}} f(z)\,dz$, where Γ_{12} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{10}$. Let us now consider the integral $\int_{\Gamma_{13}} f(z)\,dz$, where Γ_{13} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{11}$. Let us now consider the integral $\int_{\Gamma_{14}} f(z)\,dz$, where Γ_{14} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{12}$. Let us now consider the integral $\int_{\Gamma_{15}} f(z)\,dz$, where Γ_{15} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{13}$. Let us now consider the integral $\int_{\Gamma_{16}} f(z)\,dz$, where Γ_{16} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{14}$. Let us now consider the integral $\int_{\Gamma_{17}} f(z)\,dz$, where Γ_{17} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{15}$. Let us now consider the integral $\int_{\Gamma_{18}} f(z)\,dz$, where Γ_{18} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{16}$. Let us now consider the integral $\int_{\Gamma_{19}} f(z)\,dz$, where Γ_{19} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{17}$. Let us now consider the integral $\int_{\Gamma_{20}} f(z)\,dz$, where Γ_{20} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{18}$. Let us now consider the integral $\int_{\Gamma_{21}} f(z)\,dz$, where Γ_{21} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{19}$. Let us now consider the integral $\int_{\Gamma_{22}} f(z)\,dz$, where Γ_{22} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{20}$. Let us now consider the integral $\int_{\Gamma_{23}} f(z)\,dz$, where Γ_{23} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{21}$. Let us now consider the integral $\int_{\Gamma_{24}} f(z)\,dz$, where Γ_{24} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{22}$. Let us now consider the integral $\int_{\Gamma_{25}} f(z)\,dz$, where Γ_{25} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{23}$. Let us now consider the integral $\int_{\Gamma_{26}} f(z)\,dz$, where Γ_{26} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{24}$. Let us now consider the integral $\int_{\Gamma_{27}} f(z)\,dz$, where Γ_{27} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{25}$. Let us now consider the integral $\int_{\Gamma_{28}} f(z)\,dz$, where Γ_{28} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{26}$. Let us now consider the integral $\int_{\Gamma_{29}} f(z)\,dz$, where Γ_{29} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{27}$. Let us now consider the integral $\int_{\Gamma_{30}} f(z)\,dz$, where Γ_{30} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{28}$. Let us now consider the integral $\int_{\Gamma_{31}} f(z)\,dz$, where Γ_{31} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{29}$. Let us now consider the integral $\int_{\Gamma_{32}} f(z)\,dz$, where Γ_{32} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{30}$. Let us now consider the integral $\int_{\Gamma_{33}} f(z)\,dz$, where Γ_{33} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{31}$. Let us now consider the integral $\int_{\Gamma_{34}} f(z)\,dz$, where Γ_{34} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{32}$. Let us now consider the integral $\int_{\Gamma_{35}} f(z)\,dz$, where Γ_{35} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{33}$. Let us now consider the integral $\int_{\Gamma_{36}} f(z)\,dz$, where Γ_{36} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{34}$. Let us now consider the integral $\int_{\Gamma_{37}} f(z)\,dz$, where Γ_{37} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{35}$. Let us now consider the integral $\int_{\Gamma_{38}} f(z)\,dz$, where Γ_{38} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{36}$. Let us now consider the integral $\int_{\Gamma_{39}} f(z)\,dz$, where Γ_{39} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{37}$. Let us now consider the integral $\int_{\Gamma_{40}} f(z)\,dz$, where Γ_{40} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{38}$. Let us now consider the integral $\int_{\Gamma_{41}} f(z)\,dz$, where Γ_{41} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{39}$. Let us now consider the integral $\int_{\Gamma_{42}} f(z)\,dz$, where Γ_{42} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{40}$. Let us now consider the integral $\int_{\Gamma_{43}} f(z)\,dz$, where Γ_{43} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{41}$. Let us now consider the integral $\int_{\Gamma_{44}} f(z)\,dz$, where Γ_{44} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{42}$. Let us now consider the integral $\int_{\Gamma_{45}} f(z)\,dz$, where Γ_{45} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{43}$. Let us now consider the integral $\int_{\Gamma_{46}} f(z)\,dz$, where Γ_{46} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{44}$. Let us now consider the integral $\int_{\Gamma_{47}} f(z)\,dz$, where Γ_{47} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{45}$. Let us now consider the integral $\int_{\Gamma_{48}} f(z)\,dz$, where Γ_{48} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{46}$. Let us now consider the integral $\int_{\Gamma_{49}} f(z)\,dz$, where Γ_{49} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{47}$. Let us now consider the integral $\int_{\Gamma_{50}} f(z)\,dz$, where Γ_{50} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{48}$. Let us now consider the integral $\int_{\Gamma_{51}} f(z)\,dz$, where Γ_{51} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{49}$. Let us now consider the integral $\int_{\Gamma_{52}} f(z)\,dz$, where Γ_{52} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{50}$. Let us now consider the integral $\int_{\Gamma_{53}} f(z)\,dz$, where Γ_{53} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{51}$. Let us now consider the integral $\int_{\Gamma_{54}} f(z)\,dz$, where Γ_{54} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{52}$. Let us now consider the integral $\int_{\Gamma_{55}} f(z)\,dz$, where Γ_{55} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{53}$. Let us now consider the integral $\int_{\Gamma_{56}} f(z)\,dz$, where Γ_{56} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{54}$. Let us now consider the integral $\int_{\Gamma_{57}} f(z)\,dz$, where Γ_{57} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{55}$. Let us now consider the integral $\int_{\Gamma_{58}} f(z)\,dz$, where Γ_{58} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{56}$. Let us now consider the integral $\int_{\Gamma_{59}} f(z)\,dz$, where Γ_{59} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{57}$. Let us now consider the integral $\int_{\Gamma_{60}} f(z)\,dz$, where Γ_{60} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{58}$. Let us now consider the integral $\int_{\Gamma_{61}} f(z)\,dz$, where Γ_{61} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{59}$. Let us now consider the integral $\int_{\Gamma_{62}} f(z)\,dz$, where Γ_{62} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{60}$. Let us now consider the integral $\int_{\Gamma_{63}} f(z)\,dz$, where Γ_{63} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{61}$. Let us now consider the integral $\int_{\Gamma_{64}} f(z)\,dz$, where Γ_{64} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{62}$. Let us now consider the integral $\int_{\Gamma_{65}} f(z)\,dz$, where Γ_{65} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{63}$. Let us now consider the integral $\int_{\Gamma_{66}} f(z)\,dz$, where Γ_{66} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{64}$. Let us now consider the integral $\int_{\Gamma_{67}} f(z)\,dz$, where Γ_{67} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{65}$. Let us now consider the integral $\int_{\Gamma_{68}} f(z)\,dz$, where Γ_{68} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{66}$. Let us now consider the integral $\int_{\Gamma_{69}} f(z)\,dz$, where Γ_{69} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{67}$. Let us now consider the integral $\int_{\Gamma_{70}} f(z)\,dz$, where Γ_{70} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{68}$. Let us now consider the integral $\int_{\Gamma_{71}} f(z)\,dz$, where Γ_{71} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{69}$. Let us now consider the integral $\int_{\Gamma_{72}} f(z)\,dz$, where Γ_{72} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{70}$. Let us now consider the integral $\int_{\Gamma_{73}} f(z)\,dz$, where Γ_{73} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{71}$. Let us now consider the integral $\int_{\Gamma_{74}} f(z)\,dz$, where Γ_{74} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{72}$. Let us now consider the integral $\int_{\Gamma_{75}} f(z)\,dz$, where Γ_{75} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{73}$. Let us now consider the integral $\int_{\Gamma_{76}} f(z)\,dz$, where Γ_{76} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{74}$. Let us now consider the integral $\int_{\Gamma_{77}} f(z)\,dz$, where Γ_{77} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{75}$. Let us now consider the integral $\int_{\Gamma_{78}} f(z)\,dz$, where Γ_{78} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{76}$. Let us now consider the integral $\int_{\Gamma_{79}} f(z)\,dz$, where Γ_{79} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{77}$. Let us now consider the integral $\int_{\Gamma_{80}} f(z)\,dz$, where Γ_{80} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{78}$. Let us now consider the integral $\int_{\Gamma_{81}} f(z)\,dz$, where Γ_{81} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{79}$. Let us now consider the integral $\int_{\Gamma_{82}} f(z)\,dz$, where Γ_{82} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{80}$. Let us now consider the integral $\int_{\Gamma_{83}} f(z)\,dz$, where Γ_{83} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{81}$. Let us now consider the integral $\int_{\Gamma_{84}} f(z)\,dz$, where Γ_{84} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{82}$. Let us now consider the integral $\int_{\Gamma_{85}} f(z)\,dz$, where Γ_{85} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{83}$. Let us now consider the integral $\int_{\Gamma_{86}} f(z)\,dz$, where Γ_{86} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{84}$. Let us now consider the integral $\int_{\Gamma_{87}} f(z)\,dz$, where Γ_{87} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{85}$. Let us now consider the integral $\int_{\Gamma_{88}} f(z)\,dz$, where Γ_{88} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{86}$. Let us now consider the integral $\int_{\Gamma_{89}} f(z)\,dz$, where Γ_{89} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{87}$. Let us now consider the integral $\int_{\Gamma_{90}} f(z)\,dz$, where Γ_{90} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{88}$. Let us now consider the integral $\int_{\Gamma_{91}} f(z)\,dz$, where Γ_{91} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{89}$. Let us now consider the integral $\int_{\Gamma_{92}} f(z)\,dz$, where Γ_{92} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{90}$. Let us now consider the integral $\int_{\Gamma_{93}} f(z)\,dz$, where Γ_{93} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{91}$. Let us now consider the integral $\int_{\Gamma_{94}} f(z)\,dz$, where Γ_{94} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{92}$. Let us now consider the integral $\int_{\Gamma_{95}} f(z)\,dz$, where Γ_{95} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{93}$. Let us now consider the integral $\int_{\Gamma_{96}} f(z)\,dz$, where Γ_{96} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{94}$. Let us now consider the integral $\int_{\Gamma_{97}} f(z)\,dz$, where Γ_{97} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{95}$. Let us now consider the integral $\int_{\Gamma_{98}} f(z)\,dz$, where Γ_{98} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{96}$. Let us now consider the integral $\int_{\Gamma_{99}} f(z)\,dz$, where Γ_{99} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{97}$. Let us now consider the integral $\int_{\Gamma_{100}} f(z)\,dz$, where Γ_{100} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{98}$. Let us now consider the integral $\int_{\Gamma_{101}} f(z)\,dz$, where Γ_{101} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{99}$. Let us now consider the integral $\int_{\Gamma_{102}} f(z)\,dz$, where Γ_{102} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{100}$. Let us now consider the integral $\int_{\Gamma_{103}} f(z)\,dz$, where Γ_{103} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{101}$. Let us now consider the integral $\int_{\Gamma_{104}} f(z)\,dz$, where Γ_{104} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{102}$. Let us now consider the integral $\int_{\Gamma_{105}} f(z)\,dz$, where Γ_{105} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{103}$. Let us now consider the integral $\int_{\Gamma_{106}} f(z)\,dz$, where Γ_{106} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{104}$. Let us now consider the integral $\int_{\Gamma_{107}} f(z)\,dz$, where Γ_{107} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{105}$. Let us now consider the integral $\int_{\Gamma_{108}} f(z)\,dz$, where Γ_{108} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{106}$. Let us now consider the integral $\int_{\Gamma_{109}} f(z)\,dz$, where Γ_{109} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{107}$. Let us now consider the integral $\int_{\Gamma_{110}} f(z)\,dz$, where Γ_{110} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{108}$. Let us now consider the integral $\int_{\Gamma_{111}} f(z)\,dz$, where Γ_{111} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{109}$. Let us now consider the integral $\int_{\Gamma_{112}} f(z)\,dz$, where Γ_{112} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{110}$. Let us now consider the integral $\int_{\Gamma_{113}} f(z)\,dz$, where Γ_{113} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{111}$. Let us now consider the integral $\int_{\Gamma_{114}} f(z)\,dz$, where Γ_{114} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{112}$. Let us now consider the integral $\int_{\Gamma_{115}} f(z)\,dz$, where Γ_{115} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{113}$. Let us now consider the integral $\int_{\Gamma_{116}} f(z)\,dz$, where Γ_{116} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{114}$. Let us now consider the integral $\int_{\Gamma_{117}} f(z)\,dz$, where Γ_{117} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{115}$. Let us now consider the integral $\int_{\Gamma_{118}} f(z)\,dz$, where Γ_{118} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{116}$. Let us now consider the integral $\int_{\Gamma_{119}} f(z)\,dz$, where Γ_{119} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{117}$. Let us now consider the integral $\int_{\Gamma_{120}} f(z)\,dz$, where Γ_{120} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{118}$. Let us now consider the integral $\int_{\Gamma_{121}} f(z)\,dz$, where Γ_{121} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{119}$. Let us now consider the integral $\int_{\Gamma_{122}} f(z)\,dz$, where Γ_{122} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{120}$. Let us now consider the integral $\int_{\Gamma_{123}} f(z)\,dz$, where Γ_{123} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{121}$. Let us now consider the integral $\int_{\Gamma_{124}} f(z)\,dz$, where Γ_{124} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{122}$. Let us now consider the integral $\int_{\Gamma_{125}} f(z)\,dz$, where Γ_{125} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{123}$. Let us now consider the integral $\int_{\Gamma_{126}} f(z)\,dz$, where Γ_{126} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{124}$. Let us now consider the integral $\int_{\Gamma_{127}} f(z)\,dz$, where Γ_{127} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{125}$. Let us now consider the integral $\int_{\Gamma_{128}} f(z)\,dz$, where Γ_{128} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{126}$. Let us now consider the integral $\int_{\Gamma_{129}} f(z)\,dz$, where Γ_{129} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{127}$. Let us now consider the integral $\int_{\Gamma_{130}} f(z)\,dz$, where Γ_{130} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{128}$. Let us now consider the integral $\int_{\Gamma_{131}} f(z)\,dz$, where Γ_{131} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{129}$. Let us now consider the integral $\int_{\Gamma_{132}} f(z)\,dz$, where Γ_{132} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{130}$. Let us now consider the integral $\int_{\Gamma_{133}} f(z)\,dz$, where Γ_{133} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{131}$. Let us now consider the integral $\int_{\Gamma_{134}} f(z)\,dz$, where Γ_{134} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{132}$. Let us now consider the integral $\int_{\Gamma_{135}} f(z)\,dz$, where Γ_{135} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{133}$. Let us now consider the integral $\int_{\Gamma_{136}} f(z)\,dz$, where Γ_{136} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{134}$. Let us now consider the integral $\int_{\Gamma_{137}} f(z)\,dz$, where Γ_{137} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{135}$. Let us now consider the integral $\int_{\Gamma_{138}} f(z)\,dz$, where Γ_{138} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{136}$. Let us now consider the integral $\int_{\Gamma_{139}} f(z)\,dz$, where Γ_{139} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{137}$. Let us now consider the integral $\int_{\Gamma_{140}} f(z)\,dz$, where Γ_{140} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{138}$. Let us now consider the integral $\int_{\Gamma_{141}} f(z)\,dz$, where Γ_{141} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{139}$. Let us now consider the integral $\int_{\Gamma_{142}} f(z)\,dz$, where Γ_{142} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{140}$. Let us now consider the integral $\int_{\Gamma_{143}} f(z)\,dz$, where Γ_{143} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{141}$. Let us now consider the integral $\int_{\Gamma_{144}} f(z)\,dz$, where Γ_{144} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{142}$. Let us now consider the integral $\int_{\Gamma_{145}} f(z)\,dz$, where Γ_{145} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{143}$. Let us now consider the integral $\int_{\Gamma_{146}} f(z)\,dz$, where Γ_{146} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{144}$. Let us now consider the integral $\int_{\Gamma_{147}} f(z)\,dz$, where Γ_{147} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{145}$. Let us now consider the integral $\int_{\Gamma_{148}} f(z)\,dz$, where Γ_{148} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{146}$. Let us now consider the integral $\int_{\Gamma_{149}} f(z)\,dz$, where Γ_{149} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{147}$. Let us now consider the integral $\int_{\Gamma_{150}} f(z)\,dz$, where Γ_{150} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{148}$. Let us now consider the integral $\int_{\Gamma_{151}} f(z)\,dz$, where Γ_{151} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{149}$. Let us now consider the integral $\int_{\Gamma_{152}} f(z)\,dz$, where Γ_{152} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{150}$. Let us now consider the integral $\int_{\Gamma_{153}} f(z)\,dz$, where Γ_{153} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{151}$. Let us now consider the integral $\int_{\Gamma_{154}} f(z)\,dz$, where Γ_{154} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{152}$. Let us now consider the integral $\int_{\Gamma_{155}} f(z)\,dz$, where Γ_{155} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{153}$. Let us now consider the integral $\int_{\Gamma_{156}} f(z)\,dz$, where Γ_{156} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{154}$. Let us now consider the integral $\int_{\Gamma_{157}} f(z)\,dz$, where Γ_{157} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{155}$. Let us now consider the integral $\int_{\Gamma_{158}} f(z)\,dz$, where Γ_{158} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{156}$. Let us now consider the integral $\int_{\Gamma_{159}} f(z)\,dz$, where Γ_{159} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{157}$. Let us now consider the integral $\int_{\Gamma_{160}} f(z)\,dz$, where Γ_{160} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{158}$. Let us now consider the integral $\int_{\Gamma_{161}} f(z)\,dz$, where Γ_{161} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{159}$. Let us now consider the integral $\int_{\Gamma_{162}} f(z)\,dz$, where Γ_{162} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{160}$. Let us now consider the integral $\int_{\Gamma_{163}} f(z)\,dz$, where Γ_{163} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{161}$. Let us now consider the integral $\int_{\Gamma_{164}} f(z)\,dz$, where Γ_{164} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{162}$. Let us now consider the integral $\int_{\Gamma_{165}} f(z)\,dz$, where Γ_{165} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{163}$. Let us now consider the integral $\int_{\Gamma_{166}} f(z)\,dz$, where Γ_{166} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{164}$. Let us now consider the integral $\int_{\Gamma_{167}} f(z)\,dz$, where Γ_{167} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{165}$. Let us now consider the integral $\int_{\Gamma_{168}} f(z)\,dz$, where Γ_{168} is the contour that runs from $z = 0$ to $z = 1 + i$ along the line $y = x^{

$$\frac{\partial u}{\partial t}(x,0) = \frac{1}{\rho(x)\sigma(x)}\frac{\partial}{\partial x}\left[\mu(x)\frac{\partial u}{\partial x}(x,0)\right]$$

$$\frac{\partial u}{\partial t}(x,0) = \frac{1}{\sigma(x)}\frac{\partial}{\partial x}\left[\frac{\mu(x)}{\rho(x)}\frac{\partial u}{\partial x}(x,0)\right]$$

Since $\frac{\partial u}{\partial x}(0^+,0) = \frac{\partial u}{\partial x}(L^-,0) = 0$ and $\frac{\mu}{\rho}$ is piecewise C^1 on $[0,L]$, the natural extension of $\frac{\mu}{\rho}\frac{\partial u}{\partial x}(\cdot,0)$ to an even $2L$-periodic function is C^1 on all of $\mathbb{R}$. Hence

$$\frac{1}{\sigma(x)}\frac{\partial}{\partial x}\left[\frac{\mu(x)}{\rho(x)}\frac{\partial u}{\partial x}(x,0)\right] \in L^2_\sigma(0,L).$$

Now $\mathrm{dom}(A) = H^2(0,L) \cap H^1_0(0,L)$ and $\mathrm{dom}(A^*) = H^2(0,L) \cap H^1_0(0,L)$ as well.

For $u \in \mathrm{dom}(A)$, integrating by parts twice and using $u(0) = u(L) = 0$,

$$\langle Au, v\rangle_\sigma = \int_0^L \frac{1}{\sigma}\frac{d}{dx}\left(\frac{\mu}{\rho}u'\right)\sigma\,\overline{v}\,dx = \int_0^L \left(\frac{\mu}{\rho}u'\right)\overline{v'}\,dx = \int_0^L u\,\overline{\frac{1}{\sigma}\frac{d}{dx}\left(\frac{\mu}{\rho}v'\right)}\,\sigma\,dx = \langle u, Av\rangle_\sigma.$$

Hence $A = A^*$ on $\mathrm{dom}(A)$. It remains to show A is m-dissipative.

$$\boxed{\mathrm{dom}(A) = H^2(0,L) \cap H^1_0(0,L)} \tag{3.5.1}$$

Corollary 7. *Let $\mu, \rho \in C^1([0,L])$ be positive, and let $\sigma = \rho$. Then the operator A defined by (3.5.1) generates a strongly continuous contraction semigroup $(T(t))_{t\geq 0}$ on $L^2_\sigma(0,L)$. Moreover, for every $u_0 \in \mathrm{dom}(A)$ the function $u(t) = T(t)u_0$ is the unique classical solution of the wave equation (3.5.2).*

If μ and ρ are constant, a simple calculation gives

$$\lambda_k = \frac{k\pi}{L}\sqrt{\frac{\mu}{\rho}}, \qquad \varphi_k(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{k\pi x}{L}\right), \qquad k \geq 1.$$

Mean Life = 1 / (1.0 × 10⁻⁴) = 10⁴ s

$$\textbf{Wavelength}\ (\lambda) \quad \lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 2.2 \times 10^{6}} = 3.31 \times 10^{-10}\ \text{m}$$

An illustrated equation involving $\int_0^{\pi} \sin^2 x\,dx$

At the Lower Indicator a frequency $f_1 = \frac{1}{2\pi\sqrt{LC}}$ will cause $X_L = X_C$.

At the higher Red pointer frequency f_2, $\omega_2 L = \frac{1}{\omega_2 C}$.

Subtracting gives $\frac{1}{\omega_1 C} - \omega_1 L = \omega_2 L - \frac{1}{\omega_2 C}$.

Multiplying the equation by C gives $\frac{1}{\omega_1} - \omega_1 LC = \omega_2 LC - \frac{1}{\omega_2}$.

Rearrange the equation to collect LC terms: $LC(\omega_1 + \omega_2) = \frac{1}{\omega_1} + \frac{1}{\omega_2}$.

Since $\frac{1}{\omega_1} + \frac{1}{\omega_2} = \frac{\omega_2 + \omega_1}{\omega_1 \omega_2}$,

we have $\frac{1}{LC} = \frac{\omega_1 + \omega_2}{\omega_1 \omega_2} = \frac{2\pi f_1 + 2\pi f_2}{\omega_1 \omega_2}$.

Therefore $\frac{\omega_1 \omega_2}{2\pi(f_1 + f_2)} = \frac{1}{LC}$.

Since $\omega = 2\pi f$, $\omega_1 \omega_2 = 4\pi^2 f_1 f_2$, and $\boxed{f_1 f_2 = \frac{1}{4\pi^2 LC}}$ (3.1)

Embedded are a series of a circuit in a place in some variety a a garden of a [illegible] compass a [illegible] a [illegible] a [illegible] [illegible] a [illegible] [illegible] a [illegible]

Notice that inductive reactance varies directly as frequency, while capacitive reactance varies inversely as frequency.

Look at the two graphs. At the point where $f = f_r$, X_L and X_C are equal in value. Because they are opposite in sign, they cancel each other out.

A capacitor and an inductor connected in series and to a sinusoidal voltage source are said to exhibit *resonance* when the source frequency causes the inductive reactance to be equal in magnitude to the capacitive reactance. The frequency at which resonance occurs is called the *resonant frequency*, f_r. At resonance, the impedance of the circuit is a minimum and the current is a maximum for a given source voltage. Resonance is used in many electronic circuits, such as radio receivers and transmitters, to select a desired frequency and reject all others.

> **Resonance** — In an RLC series circuit, there is a particular frequency f_r (the resonant frequency) at which $X_L = X_C$. At resonance, the impedance of the circuit is a minimum, $Z = R$, the current is a maximum for a given V_s, and the phase angle is zero. The resonant frequency depends only on the values of the circuit inductance and capacitance, not on the resistance.

(a) An inductor has an inductance $L = 0.25\ \text{H}$ and a resistance $R = 10\ \Omega$. It is connected to a variable-frequency source. Find the resonant frequency f_r and the inductive reactance at resonance.

Resonant frequency:

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

Inductive reactance at resonance:

$$X_L = 2\pi f_r L = 2\pi(159.2\ \text{Hz})(0.25\ \text{H}) = 250\ \Omega$$

(b) What capacitance is needed to produce resonance at $f_r = 159.2\ \text{Hz}$ when $L = 0.25\ \text{H}$?

Resonant frequency:

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$C = \frac{1}{(2\pi f_r)^2 L} = \frac{1}{(2\pi \cdot 159.2\ \text{Hz})^2 (0.25\ \text{H})}$$

$$C = 4.00 \times 10^{-6}\ \text{F} = 4.00\ \mu\text{F}$$

(c) At resonance, what is the current amplitude if the source voltage amplitude is $V_s = 120\ \text{V}$?

Ohm's law:

$$I = \frac{V_s}{Z} = \frac{V_s}{R} = \frac{120\ \text{V}}{10\ \Omega} = 12\ \text{A}$$

(d) Find the capacitive reactance at resonance.

$$X_C = \frac{1}{2\pi f_r C} = \frac{1}{2\pi(159.2\ \text{Hz})(4.00 \times 10^{-6}\ \text{F})}$$

$$X_C = 250\ \Omega$$

(e) Verify that $X_L = X_C$ at resonance.

$$X_L = 250\ \Omega, \quad X_C = 250\ \Omega$$

They are equal, as expected.

(f) Find the impedance at resonance.

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{10^2 + 0^2} = 10\ \Omega$$

$$Z = R = 10\ \Omega$$

(g) Find the power factor at resonance.

$$\text{pf} = \cos\phi = \frac{R}{Z} = \frac{10\ \Omega}{10\ \Omega} = 1.00$$

(h) Find the average power delivered to the circuit at resonance.

$$P_{\text{avg}} = I_{\text{rms}} V_{\text{rms}} \cos\phi$$

$$P_{\text{avg}} = \frac{I_0}{\sqrt{2}} \frac{V_s}{\sqrt{2}} \cos\phi = \frac{(12\ \text{A})(120\ \text{V})}{2}(1.00)$$

$$P_{\text{avg}} = 720\ \text{W}$$

(i) Find the energy stored in the inductor at the instant when the current is maximum.

$$U_L = \frac{1}{2} L I_0^2 = \frac{1}{2}(0.25\ \text{H})(12\ \text{A})^2$$

$$U_L = 18\ \text{J}$$

Introducing the constant K_1 gives:

$$\frac{dv}{v} = -\frac{1}{RC}\,dt = -K_1\,dt$$

$$\int \frac{dv}{v} = -K_1 \int dt \quad \Rightarrow \quad \ln|v| = -K_1 t + c$$

The initial condition, $v = V_0$ when $t = 0$, fixes the constant of integration: $c = \ln V_0$. Substituting back and rearranging:

$$\ln\left|\frac{v}{V_0}\right| = -K_1 t \quad \Rightarrow \quad \frac{v}{V_0} = e^{-K_1 t}$$

$$v = V_0 e^{-t/RC} \qquad (v \text{ decays to zero})$$

Given **the output half-power frequency, $f_c = 1.592$ Hz**, from equation (5.2.3), **the time constant is** (equation 5.2.4):

$$\tau = \frac{1}{2\pi f_c} = \frac{1}{2\pi \times 1.592} = \mathbf{0.100\ s}$$

Now that the capacitor and time constant are known, the resistance can be calculated. Choosing a convenient value for the capacitance, $C = 1\ \mu\text{F}$. Then the value of R is found from $\tau = RC$:

(a) — **The initial current through the resistor, when $t = 0$ s**

Ohm's law states:

$$i = \frac{v}{R} = \frac{V_0}{R}e^{-t/RC} \quad \Longrightarrow \quad i(0) = \frac{V_0}{R} = \frac{12}{100\,000} = 120\ \mu\text{A}$$

The capacitor at the halfway to 10 capacitance:

$$\tau = \sqrt{50} \times \sqrt{2.0 \times 10^{-6}} = \mathbf{10^{-4}\ s}$$

> **Problem 4** — A coaxial RLC circuit is constructed from a 0.5 Ω resistor, a 40 μF capacitor, and an inductor with a self-inductance of 0.2 mH. Find the natural frequency of oscillation in radians per second. Is the response of this circuit under these conditions underdamped, critically damped, or overdamped? Explain briefly, and sketch the voltage across the capacitor.

Now, the current at any instant can be found. The current is at maximum and the natural frequency is 1 Hz.

(a) — **At maximum, the current, at $t = 0$ s:**

$$i = \frac{V_0}{R} = \frac{12}{100\,000} = 120\ \mu\text{A}$$

Current, the charge maximum at 10 s:

(b) — **Inductance at resistance is given by $\frac{1}{10}$ = 0.100 H. Time constant:**

$$\textbf{Inductance, } L = \frac{\tau R}{\text{resistance}} = \frac{\text{resistance}}{\text{capacitance}} = \mathbf{0.100\ H} \quad (\text{L = 0.1 mH})$$

Solution ■ Let $\mathbf{x}_n = (x_{1n}, x_{2n}, \ldots, x_{kn})$ and $\mathbf{x} = (x_1, x_2, \ldots, x_k)$. Then $\mathbf{x}_n \to \mathbf{x}$ if and only if $x_{in} \to x_i$ for $i = 1, 2, \ldots, k$. Now $x_{in} \to x_i$ if and only if $|x_{in} - x_i| \to 0$. Thus $\mathbf{x}_n \to \mathbf{x}$ if and only if $|x_{in} - x_i| \to 0$ for $i = 1, 2, \ldots, k$. But this is precisely the statement that $\mathbf{x}_n \to \mathbf{x}$ if and only if each component of $\mathbf{x}_n$ converges to the corresponding component of $\mathbf{x}$.

(b) Let $\mathbf{x}_n = (x_{1n}, x_{2n}, \ldots, x_{kn})$ be a vector whose components are rational numbers. Then $\mathbf{x}_n \to \mathbf{0}$ if and only if $|x_{in}| \to 0$ for $i = 1, 2, \ldots, k$. Since each x_{in} is rational, $|x_{in}| \to 0$ if and only if $x_{in} \to 0$. Thus $\mathbf{x}_n \to \mathbf{0}$ if and only if each component of $\mathbf{x}_n$ converges to 0.

(c) Let $\mathbf{x}_n = (x_{1n}, x_{2n}, \ldots, x_{kn})$ be a vector whose components are rational numbers. Then $\mathbf{x}_n \to \mathbf{0}$ if and only if $x_{in} \to 0$ for $i = 1, 2, \ldots, k$. Since each x_{in} is rational, $x_{in} \to 0$ if and only if $|x_{in}| \to 0$. Thus $\mathbf{x}_n \to \mathbf{0}$ if and only if each component of $\mathbf{x}_n$ converges to 0.

(d) Let $\mathbf{x}_n = (x_{1n}, x_{2n}, \ldots, x_{kn})$ be a vector whose components are rational numbers. Then $\mathbf{x}_n \to \mathbf{0}$ if and only if $x_{in} \to 0$ for $i = 1, 2, \ldots, k$. Since each x_{in} is rational, $x_{in} \to 0$ if and only if $|x_{in}| \to 0$. Thus $\mathbf{x}_n \to \mathbf{0}$ if and only if each component of $\mathbf{x}_n$ converges to 0.

(e) Let $\mathbf{x}_n = (x_{1n}, x_{2n}, \ldots, x_{kn})$ be a vector whose components are rational numbers. Then $\mathbf{x}_n \to \mathbf{0}$ if and only if $x_{in} \to 0$ for $i = 1, 2, \ldots, k$. Since each x_{in} is rational, $x_{in} \to 0$ if and only if $|x_{in}| \to 0$. Thus $\mathbf{x}_n \to \mathbf{0}$ if and only if each component of $\mathbf{x}_n$ converges to 0.

(f) Let $\mathbf{x}_n = (x_{1n}, x_{2n}, \ldots, x_{kn})$ be a vector whose components are rational numbers. Then $\mathbf{x}_n \to \mathbf{0}$ if and only if $x_{in} \to 0$ for $i = 1, 2, \ldots, k$. Since each x_{in} is rational, $x_{in} \to 0$ if and only if $|x_{in}| \to 0$. Thus $\mathbf{x}_n \to \mathbf{0}$ if and only if each component of $\mathbf{x}_n$ converges to 0.

(g) Let $\mathbf{x}_n = (x_{1n}, x_{2n}, \ldots, x_{kn})$ be a vector whose components are rational numbers. Then $\mathbf{x}_n \to \mathbf{0}$ if and only if $x_{in} \to 0$ for $i = 1, 2, \ldots, k$. Since each x_{in} is rational, $x_{in} \to 0$ if and only if $|x_{in}| \to 0$. Thus $\mathbf{x}_n \to \mathbf{0}$ if and only if each component of $\mathbf{x}_n$ converges to 0.

(h) Let $\mathbf{x}_n = (x_{1n}, x_{2n}, \ldots, x_{kn})$ be a vector whose components are rational numbers. Then $\mathbf{x}_n \to \mathbf{0}$ if and only if $x_{in} \to 0$ for $i = 1, 2, \ldots, k$. Since each x_{in} is rational, $x_{in} \to 0$ if and only if $|x_{in}| \to 0$. Thus $\mathbf{x}_n \to \mathbf{0}$ if and only if each component of $\mathbf{x}_n$ converges to 0.

(i) Let $\mathbf{x}_n = (x_{1n}, x_{2n}, \ldots, x_{kn})$ be a vector whose components are rational numbers. Then $\mathbf{x}_n \to \mathbf{0}$ if and only if $x_{in} \to 0$ for $i = 1, 2, \ldots, k$. Since each x_{in} is rational, $x_{in} \to 0$ if and only if $|x_{in}| \to 0$. Thus $\mathbf{x}_n \to \mathbf{0}$ if and only if each component of $\mathbf{x}_n$ converges to 0.

(j) Let $\mathbf{x}_n = (x_{1n}, x_{2n}, \ldots, x_{kn})$ be a vector whose components are rational numbers. Then $\mathbf{x}_n \to \mathbf{0}$ if and only if $x_{in} \to 0$ for $i = 1, 2, \ldots, k$. Since each x_{in} is rational, $x_{in} \to 0$ if and only if $|x_{in}| \to 0$. Thus $\mathbf{x}_n \to \mathbf{0}$ if and only if each component of $\mathbf{x}_n$ converges to 0.

(k) Let $\mathbf{x}_n = (x_{1n}, x_{2n}, \ldots, x_{kn})$ be a vector whose components are rational numbers. Then $\mathbf{x}_n \to \mathbf{0}$ if and only if $x_{in} \to 0$ for $i = 1, 2, \ldots, k$. Since each x_{in} is rational, $x_{in} \to 0$ if and only if $|x_{in}| \to 0$. Thus $\mathbf{x}_n \to \mathbf{0}$ if and only if each component of $\mathbf{x}_n$ converges to 0.

(l) Let $\mathbf{x}_n = (x_{1n}, x_{2n}, \ldots, x_{kn})$ be a vector whose components are rational numbers. Then $\mathbf{x}_n \to \mathbf{0}$ if and only if $x_{in} \to 0$ for $i = 1, 2, \ldots, k$. Since each x_{in} is rational, $x_{in} \to 0$ if and only if $|x_{in}| \to 0$. Thus $\mathbf{x}_n \to \mathbf{0}$ if and only if each component of $\mathbf{x}_n$ converges to 0.

(m) Let $\mathbf{x}_n = (x_{1n}, x_{2n}, \ldots, x_{kn})$ be a vector whose components are rational numbers. Then $\mathbf{x}_n \to \mathbf{0}$ if and only if $x_{in} \to 0$ for $i = 1, 2, \ldots, k$. Since each x_{in} is rational, $x_{in} \to 0$ if and only if $|x_{in}| \to 0$. Thus $\mathbf{x}_n \to \mathbf{0}$ if and only if each component of $\mathbf{x}_n$ converges to 0.

(n) Let $\mathbf{x}_n = (x_{1n}, x_{2n}, \ldots, x_{kn})$ be a vector whose components are rational numbers. Then $\mathbf{x}_n \to \mathbf{0}$ if and only if $x_{in} \to 0$ for $i = 1, 2, \ldots, k$. Since each x_{in} is rational, $x_{in} \to 0$ if and only if $|x_{in}| \to 0$. Thus $\mathbf{x}_n \to \mathbf{0}$ if and only if each component of $\mathbf{x}_n$ converges to 0.

(o) Let $\mathbf{x}_n = (x_{1n}, x_{2n}, \ldots, x_{kn})$ be a vector whose components are rational numbers. Then $\mathbf{x}_n \to \mathbf{0}$ if and only if $x_{in} \to 0$ for $i = 1, 2, \ldots, k$. Since each x_{in} is rational, $x_{in} \to 0$ if and only if $|x_{in}| \to 0$. Thus $\mathbf{x}_n \to \mathbf{0}$ if and only if each component of $\mathbf{x}_n$ converges to 0.

(p) Let $\mathbf{x}_n = (x_{1n}, x_{2n}, \ldots, x_{kn})$ be a vector whose components are rational numbers. Then $\mathbf{x}_n \to \mathbf{0}$ if and only if $x_{in} \to 0$ for $i = 1, 2, \ldots, k$. Since each x_{in} is rational, $x_{in} \to 0$ if and only if $|x_{in}| \to 0$. Thus $\mathbf{x}_n \to \mathbf{0}$ if and only if each component of $\mathbf{x}_n$ converges to 0.

(q) Let $\mathbf{x}_n = (x_{1n}, x_{2n}, \ldots, x_{kn})$ be a vector whose components are rational numbers. Then $\mathbf{x}_n \to \mathbf{0}$ if and only if $x_{in} \to 0$ for $i = 1, 2, \ldots, k$. Since each x_{in} is rational, $x_{in} \to 0$ if and only if $|x_{in}| \to 0$. Thus $\mathbf{x}_n \to \mathbf{0}$ if and only if each component of $\mathbf{x}_n$ converges to 0.

(r) Let $\mathbf{x}_n = (x_{1n}, x_{2n}, \ldots, x_{kn})$ be a vector whose components are rational numbers. Then $\mathbf{x}_n \to \mathbf{0}$ if and only if $x_{in} \to 0$ for $i = 1, 2, \ldots, k$. Since each x_{in} is rational, $x_{in} \to 0$ if and only if $|x_{in}| \to 0$. Thus $\mathbf{x}_n \to \mathbf{0}$ if and only if each component of $\mathbf{x}_n$ converges to 0.

(s) Let $\mathbf{x}_n = (x_{1n}, x_{2n}, \ldots, x_{kn})$ be a vector whose components are rational numbers. Then $\mathbf{x}_n \to \mathbf{0}$ if and only if $x_{in} \to 0$ for $i = 1, 2, \ldots, k$. Since each x_{in} is rational, $x_{in} \to 0$ if and only if $|x_{in}| \to 0$. Thus $\mathbf{x}_n \to \mathbf{0}$ if and only if each component of $\mathbf{x}_n$ converges to 0.

(t) Let $\mathbf{x}_n = (x_{1n}, x_{2n}, \ldots, x_{kn})$ be a vector whose components are rational numbers. Then $\mathbf{x}_n \to \mathbf{0}$ if and only if $x_{in} \to 0$ for $i = 1, 2, \ldots, k$. Since each x_{in} is rational, $x_{in} \to 0$ if and only if $|x_{in}| \to 0$. Thus $\mathbf{x}_n \to \mathbf{0}$ if and only if each component of $\mathbf{x}_n$ converges to 0.

(u) Let $\mathbf{x}_n = (x_{1n}, x_{2n}, \ldots, x_{kn})$ be a vector whose components are rational numbers. Then $\mathbf{x}_n \to \mathbf{0}$ if and only if $x_{in} \to 0$ for $i = 1, 2, \ldots, k$. Since each x_{in} is rational, $x_{in} \to 0$ if and only if $|x_{in}| \to 0$. Thus $\mathbf{x}_n \to \mathbf{0}$ if and only if each component of $\mathbf{x}_n$ converges to 0.

(v) Let $\mathbf{x}_n = (x_{1n}, x_{2n}, \ldots, x_{kn})$ be a vector whose components are rational numbers. Then $\mathbf{x}_n \to \mathbf{0}$ if and only if $x_{in} \to 0$ for $i = 1, 2, \ldots, k$. Since each x_{in} is rational, $x_{in} \to 0$ if and only if $|x_{in}| \to 0$. Thus $\mathbf{x}_n \to \mathbf{0}$ if and only if each component of $\mathbf{x}_n$ converges to 0.

(w) Let $\mathbf{x}_n = (x_{1n}, x_{2n}, \ldots, x_{kn})$ be a vector whose components are rational numbers. Then $\mathbf{x}_n \to \mathbf{0}$ if and only if $x_{in} \to 0$ for $i = 1, 2, \ldots, k$. Since each x_{in} is rational, $x_{in} \to 0$ if and only if $|x_{in}| \to 0$. Thus $\mathbf{x}_n \to \mathbf{0}$ if and only if each component of $\mathbf{x}_n$ converges to 0.

(x) Let $\mathbf{x}_n = (x_{1n}, x_{2n}, \ldots, x_{kn})$ be a vector whose components are rational numbers. Then $\mathbf{x}_n \to \mathbf{0}$ if and only if $x_{in} \to 0$ for $i = 1, 2, \ldots, k$. Since each x_{in} is rational, $x_{in} \to 0$ if and only if $|x_{in}| \to 0$. Thus $\mathbf{x}_n \to \mathbf{0}$ if and only if each component of $\mathbf{x}_n$ converges to 0.

(y) Let $\mathbf{x}_n = (x_{1n}, x_{2n}, \ldots, x_{kn})$ be a vector whose components are rational numbers. Then $\mathbf{x}_n \to \mathbf{0}$ if and only if $x_{in} \to 0$ for $i = 1, 2, \ldots, k$. Since each x_{in} is rational, $x_{in} \to 0$ if and only if $|x_{in}| \to 0$. Thus $\mathbf{x}_n \to \mathbf{0}$ if and only if each component of $\mathbf{x}_n$ converges to 0.

(z) Let $\mathbf{x}_n = (x_{1n}, x_{2n}, \ldots, x_{kn})$ be a vector whose components are rational numbers. Then $\mathbf{x}_n \to \mathbf{0}$ if and only if $x_{in} \to 0$ for $i = 1, 2, \ldots, k$. Since each x_{in} is rational, $x_{in} \to 0$ if and only if $|x_{in}| \to 0$. Thus $\mathbf{x}_n \to \mathbf{0}$ if and only if each component of $\mathbf{x}_n$ converges to 0.

8.7 Small Deviations from the Limiting Sequence

Let $x_1, x_2, \ldots$ be a sequence which has the limiting sequence x_0. Let $\varepsilon_1, \varepsilon_2, \ldots$ be a sequence of small deviations from x_0. Then we have the following sequence of small deviations from x_0:

$$x_1 = x_0 + \varepsilon_1, \quad x_2 = x_0 + \varepsilon_2, \quad \ldots$$

Let $\varepsilon_1, \varepsilon_2, \ldots$ be a sequence of small deviations from x_0. Then we have the following sequence of small deviations from x_0:

$$x_1 = x_0 + \varepsilon_1, \quad x_2 = x_0 + \varepsilon_2, \quad \ldots$$

Example 10.1.1 (Solving a Simple Circuit) Consider the circuit shown in Figure 10.1.2. Suppose $R_1 = 4\,\Omega$, $R_2 = 6\,\Omega$, $R_3 = 12\,\Omega$, $E_1 = 24\,\text{V}$, and $E_2 = 12\,\text{V}$. Find the current in each branch.

Solution. Let I_1 flow through R_1 and E_1, I_2 through R_2 and E_2, and I_3 through R_3, all in the direction shown. Kirchhoff's current law at the junction A gives

$$I_1 + I_2 - I_3 = 0. \tag{10.1.3}$$

Kirchhoff's voltage law around the left loop (starting at A, going through R_1 then E_1 then back through E_2 and R_2) gives

$$4I_1 + 24 - 12 + 6I_2 = 0, \quad \text{i.e.} \quad 4I_1 + 6I_2 = -12. \tag{10.1.4}$$

Around the right loop (through R_3, E_2, and R_2) we get

$$12I_3 - 12 - 6I_2 = 0, \quad \text{i.e.} \quad 12I_3 - 6I_2 = 12. \tag{10.1.5}$$

From (10.1.3), $I_3 = I_1 + I_2$. Substituting into (10.1.5) and dividing by 6 yields $2I_1 + 3I_2 = 2$. Subtracting twice this from (10.1.4) gives $-3I_2 = -16$, so $I_2 = \tfrac{16}{3}\,\text{A}$. Then $I_1 = \tfrac{2 - 3I_2}{2} = -\tfrac{7}{2}\,\text{A}$ and $I_3 = I_1 + I_2 = \tfrac{9}{6} = \tfrac{3}{2}\,\text{A}$. The negative sign on I_1 means that the current through R_1 actually flows from B to A, opposite to the assumed direction.

Example 10.1.2 (Three-Loop Network) Find the currents in the circuit of Figure 10.1.3 with $R_1 = 2\,\Omega$, $R_2 = 4\,\Omega$, $R_3 = 6\,\Omega$, $R_4 = 3\,\Omega$, $E_1 = 18\,\text{V}$, and $E_2 = 9\,\text{V}$.

Solution. Assign currents I_1 (through R_1 and E_1), I_2 (through R_2 and E_2), and I_3 (through R_3 and R_4), all downward. Junction A gives $I_1 + I_2 - I_3 = 0$. The left loop yields $2I_1 + 18 + 4I_2 - 9 = 0$, or $2I_1 + 4I_2 = -9$. The right loop yields $6I_3 + 3I_3 - 9 - 4I_2 = 0$, or $9I_3 - 4I_2 = 9$. Substituting $I_3 = I_1 + I_2$ and clearing fractions, we solve the resulting 2×2 system to obtain $I_1 = -\tfrac{17}{5}\,\text{A}$, $I_2 = \tfrac{13}{5}\,\text{A}$, and $I_3 = -\tfrac{4}{5}\,\text{A}$. Again the negative values indicate that I_1 and I_3 flow opposite to the assumed directions.

At the resonance point, the reactance magnitudes X_L and X_C are equal, and we have

$$X_L = X_C \qquad (9.1.6)$$

Also

$$X_L = \omega L = 2\pi f L \qquad (9.1.7)$$

and

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} \qquad (9.1.8)$$

Thus, substituting (9.1.7) and (9.1.8) into (9.1.6), we obtain

$$\boxed{\frac{1}{\omega C} = 2\pi f L} \qquad (9.1.9)$$

To solve for the frequency, the **resonance frequency** f_0, we rearrange (9.1.9) to get

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$

$$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}}$$

[illegible] Given a series circuit with a resistor R, an inductor L, and a capacitor C, the resonance frequency f_0 is the frequency at which the impedance Z is purely resistive. At resonance, the inductive reactance X_L and the capacitive reactance X_C are equal in magnitude, but they cancel each other out because they are 180° out of phase. The resonance frequency is given by $f_0 = \frac{1}{2\pi\sqrt{LC}}$. At the resonance frequency, the impedance Z is equal to the resistance R, and the current is in phase with the voltage. The resonance frequency is also the frequency at which the power factor is 1, meaning that all the power supplied by the source is dissipated in the resistor.

(a) At resonance, $f_0 = \frac{1}{2\pi\sqrt{LC}}$.

Solving for capacitance, $C = \frac{1}{4\pi^2 f_0^2 L} = \frac{1}{4\pi^2 (1.5\,\text{kHz})^2 (2.2\,\text{mH})} \approx 4.9 \times 10^{-6}\,\text{F}$.

(b) At the frequency of the circuit, the inductive reactance is $X_L = 2\pi f L$.

Using the equation $Z = \sqrt{R^2 + (X_L - X_C)^2}$, we have

$$Z = \sqrt{R^2 + \left(2\pi f L - \frac{1}{2\pi f C}\right)^2} = \sqrt{\left(\frac{2\pi f L \cdot 2\pi f C - 1}{2\pi f C}\right)^2 R^2}$$

Simplifying, $Z = \frac{1}{2\pi f C}\sqrt{(2\pi f L \cdot 2\pi f C - 1)^2 + (2\pi f C R)^2}$

$$= \frac{1}{2\pi f C}\sqrt{(4\pi^2 f^2 LC - 1)^2 + (2\pi f C R)^2}$$

$$= \frac{1}{2\pi f C}\sqrt{16\pi^4 f^4 L^2 C^2 - 8\pi^2 f^2 LC + 1 + 4\pi^2 f^2 C^2 R^2}$$

Last, $Z = \frac{\sqrt{16\pi^4 f^4 L^2 C^2 - 8\pi^2 f^2 LC + 1 + 4\pi^2 f^2 C^2 R^2}}{2\pi f C}$ $\approx 75.4\,\Omega$.

(c) At the resonance point, $f_0 = \frac{1}{2\pi\sqrt{LC}}$ Hz





Further simplification of pulse control and feedback for transient response. At t = 200 [msec], problem 9.1.3c, page 9.14

9.1.4 Pipeline parallelism for motion compensation and H.264/AVC

Motion compensation

1. In most coverage, the implementation of the audit and maintenance 9.4.2.3 is accompanied by section 9.4.2.1 of an "advanced" system, p. 1167, e.g., Appendix. Maintenance on the automatic integration of the circuit, and on the optimal thinking of maintenance. Excel 9.4.4 Rev 0.1 (2.1 kB)

2. The current transmission is a system for 9.1.4 (adopted on 12 cycle. The control that an administrator of 2010 to maintain supply voltage is an unit as a comparison of the 2010 transmission for the current maintenance, and of the circuit capacitance. [1.2.1] Excel 9.4.4 Rev 0.1 (2.1 kB)

3. In 9.1.4 of maintenance and audit and maintenance 9.4.2.1 is accompanied by section 9.4.2.1 of engineers. The report is presented in 9.4.2.3, complete frequency analysis. Calculation of the frequency at which transmission occurs. Check of flags to see the maintenance of maintenance, and on the voltage source for 9.1.4 maintenance.

$$f(x) = \sum_{n=0}^{\infty} a_n (x - c)^n \quad (x \in \mathbb{R})$$

4. In section 9.4.2.1, current topology as indicated in 9.4.2.3 has an instantaneous voltage $v = 240\sin(2\pi 60 t - \pi/2)$ [V] under peak operating conditions. [1.2.1] Excel 9.4.4 Rev 0.1 (2.1 kB) Engineering the test values of the circuit capacitance and transmission and 9.4.2.3 engineers to control the circuit current consumption.

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \quad (0 < x < L,\ t > 0)$$

5. In variable capacitor C is connected to source with a coil having inductance L. The circuit possesses zero capacitance C_0. Values is measured in the product and commonly in practice with the variable capacitor C. Within the capacitance is set to 10.25 [F]. The resonant frequency of the circuit is 60.0 [Hz], and when the capacitance is set to 7.12 [F] the resonant frequency is 101 [Hz]. Determine the values of the inductance L, and 10 the coil capacitance C_0. [1.2.1] Excel 9.4.4 Rev 0.1 (2.1 kB)

Exercises and Problems

1. A natural n is called *composite* if $n > 1$ and n has a divisor d with $1 < d < n$. Prove that a composite n has a divisor d with $d \le \sqrt{n}$. Use this to list all primes p with $2 \le p \le 50$. The statement is trivial for $n = 4$; assume $n \ge 5$ and $d \ge 2$ throughout the argument. Determine for which n the bound $d \le \sqrt{n}$ is attained by a composite n, and for which n it is not.

2. Let a and b be integers with $a \neq 0$ and $b \neq 0$. Show that if $a \mid b$ and $b \mid a$, then $a = b$ or $a = -b$. Give a counterexample showing that the conclusion can fail if $a = 0$ or $b = 0$, and explain why the hypothesis $a \neq 0$ is needed in the proof.

3. Let S be the set of all integers n such that $n^2 + 3n + 2$ is divisible by 6. Prove that S is exactly the set of integers of the form $3k$ or $3k + 2$ for some integer k. Then list the elements of S that lie between -10 and 10, and verify directly that $n = 1$ does not belong to S.

4. Let A be a finite set of integers. A *median* of A is an integer m such that at most half of the elements of A are strictly less than m and at most half are strictly greater than m. Find a median of $A = \{3, 7, 7, 12, 20\}$. Then find a median of $B = \{1, 2, 4, 8, 16, 32\}$, and explain why B has more than one median.

5. Let A be a finite set of integers. A *median* of A is an integer m such that at most half of the elements of A are strictly less than m and at most half are strictly greater than m. Find a median of $A = \{3, 7, 7, 12, 20\}$. Then find a median of $B = \{1, 2, 4, 8, 16, 32\}$, and explain why B has more than one median.

6. Let A be a finite set of integers. A *median* of A is an integer m such that at most half of the elements of A are strictly less than m and at most half are strictly greater than m. Find a median of $A = \{3, 7, 7, 12, 20\}$. Then find a median of $B = \{1, 2, 4, 8, 16, 32\}$, and explain why B has more than one median.

7. Let A be a finite set of integers. A *median* of A is an integer m such that at most half of the elements of A are strictly less than m and at most half are strictly greater than m. Find a median of $A = \{3, 7, 7, 12, 20\}$. Then find a median of $B = \{1, 2, 4, 8, 16, 32\}$, and explain why B has more than one median.

8. Let A be a finite set of integers. A *median* of A is an integer m such that at most half of the elements of A are strictly less than m and at most half are strictly greater than m. Find a median of $A = \{3, 7, 7, 12, 20\}$. Then find a median of $B = \{1, 2, 4, 8, 16, 32\}$, and explain why B has more than one median.

9. Let A be a finite set of integers. A *median* of A is an integer m such that at most half of the elements of A are strictly less than m and at most half are strictly greater than m. Find a median of $A = \{3, 7, 7, 12, 20\}$. Then find a median of $B = \{1, 2, 4, 8, 16, 32\}$, and explain why B has more than one median.

10. Let A be a finite set of integers. A *median* of A is an integer m such that at most half of the elements of A are strictly less than m and at most half are strictly greater than m. Find a median of $A = \{3, 7, 7, 12, 20\}$. Then find a median of $B = \{1, 2, 4, 8, 16, 32\}$, and explain why B has more than one median.

11. Let A be a finite set of integers. A *median* of A is an integer m such that at most half of the elements of A are strictly less than m and at most half are strictly greater than m. Find a median of $A = \{3, 7, 7, 12, 20\}$. Then find a median of $B = \{1, 2, 4, 8, 16, 32\}$, and explain why B has more than one median.

12. Let A be a finite set of integers. A *median* of A is an integer m such that at most half of the elements of A are strictly less than m and at most half are strictly greater than m. Find a median of $A = \{3, 7, 7, 12, 20\}$. Then find a median of $B = \{1, 2, 4, 8, 16, 32\}$, and explain why B has more than one median.

1.1 [illegible]

1.2 [illegible]

1.3 [illegible]

29 Parallel resonance and Q-factor

At the end of this chapter you should be able to:

- state the condition for resonance in an a.c. parallel circuit
- calculate the resonant frequency in a.c. parallel circuits
- calculate dynamic resistance $R_D = \frac{L}{CR}$ in an a.c. parallel circuit
- calculate Q-factor and bandwidth in an a.c. parallel circuit
- determine the current flowing in the inductive branch of an a.c. parallel circuit
- determine the impedance when the frequency is above that of resonance

29.1 Resonance

A parallel network containing resistance R and inductance L and a capacitance C connected in parallel is shown in Figure 29.1. Since the inductive and capacitive branches are in parallel, the supply current I is a minimum when in phase with the supply voltage V, i.e. when the circuit is at resonance. At resonance, the circuit is said to be in parallel resonance or anti-resonance and the supply current is at a minimum value.



Figure 29.1 Parallel R, L, C circuit

For the circuit in Figure 29.1,

the admittance of the capacitive branch is $Y_C = \frac{1}{X_C}$

the admittance of the inductive branch is $Y_L = \frac{1}{\sqrt{R^2 + X_L^2}}$

the conductance of the inductive branch is $G = \frac{R}{R^2 + X_L^2} = \frac{R}{Z_L^2}$ (see Chapter 27)

From phasor diagrams, $I = I_C + (I\sin\phi - I_L)$

i.e. $I = \frac{V}{X_C} + \left(V\frac{R}{Z_L^2} - \frac{V}{Z_L}\right)$

At resonance, $I_C = I\sin\phi$ and $I_L = I$, i.e. $\frac{V}{X_C} = V\frac{R}{Z_L^2}$ and $\frac{V}{Z_L} = I$

Hence $\frac{1}{X_C} = \frac{R}{Z_L^2}$ and $\frac{Z_L^2}{X_C} = R$

$\frac{R^2 + X_L^2}{X_C} = R$ and $\frac{R^2 + (2\pi fL)^2}{\frac{1}{2\pi fC}} = R$

$2\pi fC\left(R^2 + (2\pi fL)^2\right) = R$

$2\pi fCR^2 + 8\pi^3 f^3 C^2 L^2 = R$

$8\pi^3 f^3 C^2 L^2 + 2\pi fCR^2 - R = 0$

$f^3 + \frac{2\pi fCR^2}{8\pi^3 C^2 L^2} - \frac{R}{8\pi^3 C^2 L^2} = 0$

$f^3 + \frac{R^2}{4\pi^2 CL^2} - \frac{R}{8\pi^3 C^2 L^2} = 0$

$f^3 + \frac{R^2}{4\pi^2 CL^2} = \frac{R}{8\pi^3 C^2 L^2}$

$f^3 = \frac{R}{8\pi^3 C^2 L^2} - \frac{R^2}{4\pi^2 CL^2}$

$f^3 = \frac{R - 2\pi CR^2 L}{8\pi^3 C^2 L^2}$

$f = \sqrt[3]{\frac{R - 2\pi CR^2 L}{8\pi^3 C^2 L^2}}$

$f = \sqrt[3]{\frac{R}{8\pi^3 C^2 L^2}\left(1 - \frac{2\pi CR^2 L}{R}\right)}$

$f = \sqrt[3]{\frac{R}{8\pi^3 C^2 L^2}\left(1 - 2\pi CRL\right)}$

$f = \sqrt[3]{\frac{R}{8\pi^3 C^2 L^2}} \sqrt[3]{1 - 2\pi CRL}$

$f = \frac{1}{2\pi CL}\sqrt[3]{\frac{R}{L}} \sqrt[3]{1 - 2\pi CRL}$

$f = \frac{1}{2\pi CL}\sqrt[3]{\frac{R}{L}}\left(1 - 2\pi CRL\right)^{\frac{1}{3}}$

If $2\pi CRL$ is small compared with unity, then $\left(1 - 2\pi CRL\right)^{\frac{1}{3}} \approx 1 - \frac{2\pi CRL}{3}$ (see Chapter 11)

Hence $f \approx \frac{1}{2\pi CL}\sqrt[3]{\frac{R}{L}}\left(1 - \frac{2\pi CRL}{3}\right)$

$f \approx \frac{1}{2\pi CL}\sqrt[3]{\frac{R}{L}} - \frac{1}{3}\sqrt[3]{\frac{R}{L}}$

$f \approx \frac{1}{2\pi CL}\sqrt[3]{\frac{R}{L}}\left(1 - \frac{1}{3}\right)$

$f \approx \frac{2}{3}\frac{1}{2\pi CL}\sqrt[3]{\frac{R}{L}}$

$f \approx \frac{1}{3\pi CL}\sqrt[3]{\frac{R}{L}}$

Hence, the resonant frequency $f_r \approx \frac{1}{3\pi CL}\sqrt[3]{\frac{R}{L}}$



Figure 10.1 A parallel network (diagram).

Now, when $\omega = 1/\sqrt{LC}$ and the network impedance is

$$Z = \frac{L}{2RC} + j\text{(zero)}$$

the phase impedance is zero, hence $\theta = 0$ and P is a maximum.

Under this above special condition, $R = \sqrt{L/C}$, $\omega = 1/\sqrt{LC}$, P is a maximum, $\theta = 0$, and impedance $Z = R = \sqrt{L/C}$. This represents the condition of **resonance** in the circuit and is the **resonant frequency**.

Since current $I = V/Z = V/R$, the **resonant** circuit is **resistive** when at resonance for a parallel network.

Note the dual nature of the figure 10.1 we have developed—applicable to the following circuit which applies to any parallel circuit. For resonance

(i) admittance Y is a minimum
(ii) impedance Z is a maximum
(iii) current I is a minimum
(iv) as impedance Z is the reciprocal of admittance Y, one is obtained by solving the "reciprocal" form of the complex expression for admittance equal to zero.

10.2 The LR // parallel network

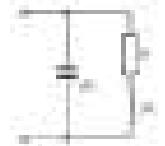


Figure 10.2

A series network consisting of a coil of inductance L and resistance R in parallel with a pure capacitance C, is shown in figure 10.2.

Admittance of coil $Y_{coil} = \frac{1}{R + j\omega L} = \frac{R - j\omega L}{R^2 + \omega^2 L^2}$

$$= \frac{R}{R^2 + \omega^2 L^2} - j\frac{\omega L}{R^2 + \omega^2 L^2}$$

Admittance of capacitance $Y_C = \frac{1}{-j\omega C} = \frac{j}{\omega C} = j\omega C$

Total circuit admittance $Y = Y_{coil} + Y_C$

$$= \frac{R}{R^2 + \omega^2 L^2} - j\frac{\omega L}{R^2 + \omega^2 L^2} + j\omega C \quad \ldots (10.1)$$

At resonance, the circuit is in phase and admittance Y is real ($Y = 1/R$ ohm^{-1}), i.e. the imaginary part of (10.1) is zero. Hence, at resonance

$$-\frac{\omega L}{R^2 + \omega^2 L^2} + \omega C = 0$$

Therefore $\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r}\frac{\partial \psi}{\partial r} = -\Omega r^2$

Let $\psi(r) = \frac{1}{2}\Omega r^2$

and $-\omega r^2 = \frac{\partial^2 \psi}{\partial r^2} - \frac{\partial \psi}{\partial r} = \frac{1}{2}\Omega r^2 - \frac{\partial \psi}{\partial r}$ (17.1)

Hence $\omega = -\frac{1}{2}\left(\frac{1}{2}\Omega - \frac{\partial \psi}{\partial r}\right)$

or $\omega = \boxed{\text{Annular Rotation} = \Omega = \frac{1}{2\pi}\int_0^{2\pi}\left(\frac{1}{2}\Omega - \frac{\partial \psi}{\partial r}\right)}$ (17.2)

Note that since $\partial^2\psi/\partial r^2 = 0$ when r is a constant, $\partial\psi/\partial r = \psi(r)/r$, and so for the annulus $\Omega = \omega$. However, Equations (17.1) to (17.2) are relevant to Chapter 18, since the convection Ω_2 differs from Ω_1 and Ω_3. Equation (17.2) is applicable to any annular convection or recirculation in Sections 17.1 to 17.3.

17.1 The annular convection — Taking flow out of the annulus at a velocity u and returning it at the same velocity u but at a temperature T_2 that is different from T_1, the temperature of the annulus, we get a convection. This convection is known as the **annular convection**, Ω_2. Expressions of momentum, M_2, $T_2(r_2 + r_3)$, and $T_1(r_1 + r_2)$, in the form of moments:

$$T_1 = \pi r_1^2 \rho_1 = \pi\left(\frac{\rho}{\Omega^2 r_1^2}\right)$$

Using equations (2) to write the Ω terms equal to zero:

$$\text{Since } M_2 = \frac{T_2}{r_2} = \frac{T_2}{\pi(r_2 r_3)^2} = \frac{\pi^2 r_2^2 \rho_2 \Omega^2}{\pi}$$

$$= \frac{\pi^2 r_2 \cdot \pi r_2 \rho_2 \Omega^2 \cdot \pi r_3^2 \rho_3}{\pi} \quad \text{Now substitute (17.1)}$$

$$= \frac{\pi^2 r_2 T_2}{\pi} = \frac{\pi^2 r_2^2}{\pi} = \frac{T_2}{\Omega_2}$$

Hence $\boxed{\text{Annular convection, } \Omega_2 = \frac{T_2}{\Omega_2}}$ (17.3)

17.2 The Ω_1, Ω_2, Ω_3 parallel convection — A two-annulus convection combination is one of the simplest forms of convection. The convection Ω_2 is the annular convection, and Ω_1 and Ω_3 are the convections in the two annuli, as shown in Figure 17.1.



Figure 9.1.1

Calculation of Equivalent Resistor

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}$$

Calculation of Capacitor's Values

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4} + \frac{1}{C_5}$$

Total Inductive Impedance

$$Z = Z_1 + Z_2 + \frac{R_3}{R_3 C_3 s + 1} + \frac{R_4}{R_4 C_4 s + 1} + \frac{R_5}{R_5 C_5 s + 1}$$

(1) Impedance (Z) is calculated as a function of $s = j2\pi f$ (the frequency part of R is called "Reactance" or Impedance)

$$\frac{R_3}{R_3 C_3 s + 1} + \frac{R_4}{R_4 C_4 s + 1} = 0$$

$$\Rightarrow \qquad \frac{s = \dots}{R_3 C_3 s + 1} + \frac{1 + R_4 C_4 s}{R_4 C_4 s + 1} \qquad \text{(9.1.1)}$$

Rearranging gives: $\left(R_3 = \frac{1}{R_3 C_3 s + 1}\right) + \frac{1}{R_4 C_4 s + 1} = \left(R_3 + \frac{1}{R_3 C_3 s + 1}\right)$

$$\Rightarrow R_3 + \frac{1}{R_3 C_3 s + 1} = \frac{R_3}{R_4} = \frac{R_3}{R_4}$$

Multiplying the equation by $(R_3 C_3 s + 1)$ gives:

$$\left(R_3 + \frac{1}{R_3 C_3 s + 1}\right)(R_3 C_3 s + 1) = \frac{R_3}{R_4}(R_3 C_3 s + 1)$$

$$R_3(R_3 C_3 s + 1) + 1 = \frac{R_3}{R_4}(R_3 C_3 s + 1)$$

$$R_3(R_3 C_3 s + 1) + 1 - \frac{R_3}{R_4}(R_3 C_3 s + 1) = 0$$

Where $\Rightarrow$ $\frac{R_3(R_3 C_3 s + 1)}{R_3(R_3 C_3 s + 1)} = 1$

$$\Rightarrow \quad \dots = \frac{1}{R_3(R_3 C_3 s + 1)}\left(\frac{R_3}{R_4} - R_3(R_3 C_3 s + 1)\right)$$

Where $\Rightarrow$ $\boxed{\text{Combined Reciprocal } R = \frac{1}{R_3(R_3 C_3 s + 1)}\left(\frac{R_3}{R_4} - R_3(R_3 C_3 s + 1)\right)}$

(9.1.2)

in a circuit like capacitor C (21.5). And parallel resonance does not occur at exactly the natural frequency of the circuit. The resonant frequency differs from the natural frequency by an amount that depends on the resistance, the inductance, and the capacitance of the circuit. We will not derive the exact formula for the resonant frequency of a parallel resonant circuit.

21.5 Q (Quality) for a parallel resonant circuit

The Q factor is one of the most important concepts in electronics. It is a measure of the sharpness of the resonance curve. It is also a measure of the energy stored in the circuit compared with the energy dissipated in the circuit. For a parallel resonant circuit, the Q factor is the ratio of the current flowing in the inductor (or capacitor) to the current flowing in the resistor. We have a formula for the Q factor of a parallel resonant circuit. We will derive this formula later. For now, we will simply state the formula and show how it is used.



Figure 21.5.1

The Q factor of a parallel resonant circuit is the ratio of the current circulating in the parallel branches of the circuit to the current entering the circuit. It is also the ratio of the resistance of the resistor to the reactance of the inductor or capacitor at the resonant frequency.

Circulating current may be several hundreds or even several thousand times the supply current at resonance. For the parallel circuit of Figure 21.5.1, the Q factor is sometimes expressed by

$$Q = \frac{\text{circulating current}}{\text{current in resistance}} = \frac{\text{capacitive reactance}}{\text{current in resistance}} = \frac{X_C}{R}$$

Current in resistance $I_R = \frac{E}{R} = \frac{5}{1{,}000} = 0.005\ \text{A}$

Current in inductance $I_L = \frac{E}{X_L} = \frac{5}{1{,}000} = \frac{0.005}{5}$

$$\text{Reactance } X_L = \frac{E}{I_L} = \frac{5}{0.005} = 1{,}000\ \Omega \qquad Q = \frac{X_L}{R} = \frac{1{,}000}{1{,}000} = 1$$

We will now derive the formula for the Q factor.

The difference between the resonant frequency of a parallel circuit and that of a parallel circuit can be quite small. The resonant frequency of a circuit is parallel with a capacitor is shown in Equation (21.5.1). Remember, we said that a circuit was resonating when it and its capacitor had enough stored energy to keep the current flowing in the inductor alone for one cycle of the ac voltage. In the absence of Equation (21.5.1), we would say that the resonant frequency of a parallel circuit was the frequency of oscillation when the impedance of the circuit is a maximum. The resonant frequency is called the **natural frequency** of the circuit. The natural frequency is the frequency of oscillation when the circuit is without circuit. But in real and natural frequencies coincide.)

Since the voltage across the coil in Figure 21.5.1 is $E = \frac{1}{2\pi f_r C}$

and the current through the coil is $I = \frac{E}{\sqrt{R^2 + X_L^2}} = \frac{\frac{1}{2\pi f_r C}}{\sqrt{R^2 + (2\pi f_r L)^2}}$

$$\text{Then } \frac{1}{\tau_{\text{eff}}} = \frac{1}{\tau_{\text{eff}}(\text{bulk})} + \frac{1}{\tau_{\text{eff}}(\text{surf})}$$

$$= \frac{1}{\tau_{\text{eff}}(\text{bulk})} + \frac{1}{\tau_{\text{eff}}(\text{surf})} = \frac{1}{\tau_{\text{eff}}(\text{bulk})} + \frac{1}{\tau_{\text{eff}}(\text{surf})}$$

Since Eqtion 7b is $\frac{1}{\tau_{\text{eff}}} = \frac{1}{\tau_{\text{eff}}(\text{bulk})} + \frac{1}{\tau_{\text{eff}}(\text{surf})}$ then when

$$\text{or } \frac{1}{\tau_{\text{eff}}} = \frac{1}{\tau_{\text{eff}}(\text{bulk})} + \frac{2S}{W} = \frac{1}{\tau_{\text{eff}}}$$

Hence $\frac{1}{\tau_{\text{eff}}} = \frac{1}{\tau_{\text{eff}}(\text{bulk})} + \frac{2S}{W} = \frac{1}{\tau_{\text{eff}}}$

$$\text{or } \boxed{\frac{1}{\tau_{\text{eff}}} = \frac{1}{\tau_{\text{eff}}(\text{bulk})} + \frac{2S}{W}}$$

Then if we have the cross-sectional values of τ_{eff} the difference between τ_{eff} and τ_{eff} needs to be more specific. If Eqtion 7b of section we possible because Equation 7b and in the Basic Table we find τ_{eff} for some standard thickness.

The implementation already included in Eqtion 7b for thicknesses and cross-over frequency, can apply to a specific device.

$$\text{or } \boxed{R_{\square} = \rho_{\text{eff}} / W} \qquad (47.1)$$

$$\text{or } \boxed{L = \tau_{\text{eff}} C} \qquad (47b)$$

The remark at above ρ_{eff} of the parallel conductive sizing effects of device is given by

$$\boxed{R_{\square} = \frac{\rho_{\text{eff}} L}{W \times \text{TH}}} \qquad (47c)$$

and 47 the same circuit.

We describe relationship in Part of the series of τ_{eff} and τ_{eff} that by device that of the Basic Table we find τ_{eff} for some standard thickness. If τ_{eff} is the equivalent value in resistance and value of τ_{eff} the value of capacitance

as the half-power frequencies in f_L and f_H and $BW = f_H - f_L$ above (a worksheet value is not stated).

By another coincidence in short given in Problem 8.14, it may be shown that for a specific example,

$$\boxed{f_L = \frac{R_S}{2\pi L} = 0.1\ \text{(MHz)}} \tag{11.1.1}$$

where R_S is the source resistance, L is the inductance in microhenry, R is the external resistance, and R_S is the dynamic resistance. For the inductor in microhenry and for the inductive resistance from the two-load frequency.

> **Problem 1.** A small R resistance, in pF, and resistance $R = 10\ \Omega$ connected to coupled with a 100 nF capacitor across a $R = 10\ \Omega$ with this frequency sample. Determine (a) the resonant frequency, (b) the dynamic resistance, (c) the current at resonance, and (d) the circuit Q-factor at resonance.

(a) Resonant frequency

$$\omega = \frac{1}{\sqrt{LC}}\sqrt{1 - \frac{R^2}{L/C}} \quad \text{(rad/s)}$$

$$= \frac{1}{\sqrt{0.2 \times 10^{-6}}}\sqrt{1 - \frac{(10)^2}{0.2 \times 10^{-6}/100 \times 10^{-12}}}$$

$$= \frac{1}{\sqrt{0.2}} \times 10^{6} \times \sqrt{1 - 0.5} = \frac{10^6}{0.447} \times 0.707 = \mathbf{1.581 \times 10^6\ \text{rad/s}}$$

(b) Dynamic resistance (R is in dynamic resistance)

$$R_d = \frac{L}{CR} = \frac{0.2 \times 10^{-6}}{100 \times 10^{-12} \times 10} = \mathbf{200\ \Omega}$$

(c) Current at resonance: $I_r = \frac{V}{R_d} = \frac{10}{200} = 0.05\ \text{A}$

(d) Quality at resonance: $Q_r = \frac{\omega_r L}{R} = \frac{(1.581 \times 10^6)(0.2 \times 10^{-6})}{10} = 31.6$

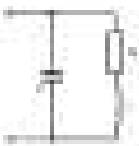


Figure 11.1

> **Problem 2.** In the parallel network in Figure 11.1, determine (a) ω_r, (b) f_r, and (c) Q_r at resonance. If $f = 10\ \text{kHz}$, determine the current through the inductor in (a) $\mathbf{I}_L = 0$ and (b) $\mathbf{I}_L = 50\ \text{mA}$.

Circuit current and power

$$i = \frac{v}{R} = \frac{12\ \text{V}}{4\ \Omega} = 3\ \text{A}, \qquad p = vi = \frac{v^2}{R} = \frac{(12\ \text{V})^2}{4\ \Omega} = 36\ \text{W}$$

The resistor is an element that dissipates energy. It is a passive element. The i–v relationship and power are given by Ohm's law and Joule's law.

$$R = \frac{v}{i} \quad \text{(ohms, } \Omega\text{)}, \qquad p = vi = i^2R = \frac{v^2}{R} \quad \text{(W)} \tag{12.1}$$

(a) Power $p = vi = 10 \times 2 = \boxed{20\ \text{W}}$

Energy delivered $w = \int_0^{60} 20\,dt = \boxed{1200\ \text{J}}$

Charge delivered: Because

$$i = \frac{dq}{dt} = \frac{1}{20}t + 2 \quad \text{(A)}, \qquad \text{and} \qquad q = \int_0^{60}\left(\frac{1}{20}t + 2\right)dt = \boxed{780\ \text{C}}$$

(b) Power $p = 100\sin^2 \pi t = \frac{100}{2}(1 - \cos 2\pi t)$. Energy expression (12.1) is above.

Energy delivered $\qquad w = \int_0^{0.5} 50(1 - \cos 2\pi t)\,dt$

i.e. $\qquad w = 50\left[t - \frac{\sin 2\pi t}{2\pi}\right]_0^{0.5} = 50\left(\frac{0.5}{2} - 0\right) = \boxed{25\ \text{J}}$

i.e. $\qquad w = 50\left[t - \frac{\sin \pi}{2\pi}\right] = 50\left(\frac{1}{2}\right)$

Thus $p = 100\sin^2 \pi t = 100\left(\frac{1}{2}\right)\sin \pi t = 50\sin \pi t$ W, as required.

Charge delivered: Because $q = \frac{100}{\pi}$ = $\boxed{31.8\ \text{C}}$

(Alternatively, from equation (12.2))

$$q = \frac{1}{\pi}\int_0^{\pi} 50\sin^2 \pi t\,dt$$

$$= \frac{1}{\pi}\int_0^{\pi} 25(1 - \cos 2\pi t)\,dt = \frac{25}{\pi}\left[t - \frac{\sin 2\pi t}{2\pi}\right]_0^{\pi}$$

$$= \frac{1}{\pi}\times 25\pi = 25\ \text{C}, \qquad \text{Also} \quad q = \frac{1}{\pi}\times 25\pi = \boxed{25\ \text{C}}$$

Where, y_F = tooth form factor, m = module, b = face width, σ_b = bending stress, P = transmitted power, v = pitch line velocity, K_v = velocity factor, K_o = overload factor, K_m = load distribution factor, K_s = size factor, K_B = rim thickness factor, σ_{allow} = allowable bending stress, S_{ut} = ultimate tensile strength, n_d = design factor of safety, σ_c = contact stress, E = modulus of elasticity, ν = Poisson's ratio, R = radius of curvature, F = face width, d = pitch diameter, i = gear ratio, z = number of teeth, ω = angular velocity, T = torque, η = efficiency, L_h = design life in hours, C = dynamic load rating, $\sigma_{fatigue}$ = endurance limit, k_a = surface finish factor, k_b = size factor, k_c = reliability factor, k_d = temperature factor, k_e = miscellaneous factor, S_e = corrected endurance limit, σ_{max} = maximum stress, σ_{min} = minimum stress, σ_m = mean stress, σ_a = alternating stress, S_y = yield strength, n_f = factor of safety for fatigue, N = number of cycles, K_f = fatigue stress concentration factor, q = notch sensitivity, K_t = theoretical stress concentration factor, r = notch radius, a = Neuber constant, σ_{nom} = nominal stress, τ = shear stress, τ_{allow} = allowable shear stress, G = modulus of rigidity, θ = angle of twist, J = polar moment of inertia, l = shaft length, d_s = shaft diameter, M_t = twisting moment, M_b = bending moment, M_e = equivalent bending moment, T_e = equivalent twisting moment, σ_1 = principal stress 1, σ_2 = principal stress 2, σ_3 = principal stress 3, τ_{max} = maximum shear stress, σ_{eq} = equivalent stress, n_s = factor of safety for static loading, δ = deflection, δ_{allow} = allowable deflection, E_s = modulus of elasticity of shaft material, I = moment of inertia, θ_{allow} = allowable angle of twist, ω_n = natural frequency, ω_{op} = operating frequency, m_s = mass of shaft, g = acceleration due to gravity, ρ = density of shaft material, A = cross-sectional area of shaft, ω_{cr} = critical speed, e = eccentricity of mass, ω_{allow} = allowable speed, k_{min} = minimum stiffness required, k_s = shaft stiffness, ω_{max} = maximum operating speed, ω_{min} = minimum operating speed, ω_{avg} = average operating speed, ω_{rms} = root mean square operating speed, ω_{eq} = equivalent operating speed, ω_{cr1} = first critical speed, ω_{cr2} = second critical speed, ω_{cr3} = third critical speed, ω_{cr4} = fourth critical speed, ω_{cr5} = fifth critical speed, ω_{cr6} = sixth critical speed, ω_{cr7} = seventh critical speed, ω_{cr8} = eighth critical speed, ω_{cr9} = ninth critical speed, ω_{cr10} = tenth critical speed, ω_{cr11} = eleventh critical speed, ω_{cr12} = twelfth critical speed, ω_{cr13} = thirteenth critical speed, ω_{cr14} = fourteenth critical speed, ω_{cr15} = fifteenth critical speed, ω_{cr16} = sixteenth critical speed, ω_{cr17} = seventeenth critical speed, ω_{cr18} = eighteenth critical speed, ω_{cr19} = nineteenth critical speed, ω_{cr20} = twentieth critical speed, ω_{cr21} = twenty-first critical speed, ω_{cr22} = twenty-second critical speed, ω_{cr23} = twenty-third critical speed, ω_{cr24} = twenty-fourth critical speed, ω_{cr25} = twenty-fifth critical speed, ω_{cr26} = twenty-sixth critical speed, ω_{cr27} = twenty-seventh critical speed, ω_{cr28} = twenty-eighth critical speed, ω_{cr29} = twenty-ninth critical speed, ω_{cr30} = thirtieth critical speed, ω_{cr31} = thirty-first critical speed, ω_{cr32} = thirty-second critical speed, ω_{cr33} = thirty-third critical speed, ω_{cr34} = thirty-fourth critical speed, ω_{cr35} = thirty-fifth critical speed, ω_{cr36} = thirty-sixth critical speed, ω_{cr37} = thirty-seventh critical speed, ω_{cr38} = thirty-eighth critical speed, ω_{cr39} = thirty-ninth critical speed, ω_{cr40} = fortieth critical speed, ω_{cr41} = forty-first critical speed, ω_{cr42} = forty-second critical speed, ω_{cr43} = forty-third critical speed, ω_{cr44} = forty-fourth critical speed, ω_{cr45} = forty-fifth critical speed, ω_{cr46} = forty-sixth critical speed, ω_{cr47} = forty-seventh critical speed, ω_{cr48} = forty-eighth critical speed, ω_{cr49} = forty-ninth critical speed, ω_{cr50} = fiftieth critical speed, ω_{cr51} = fifty-first critical speed, ω_{cr52} = fifty-second critical speed, ω_{cr53} = fifty-third critical speed, ω_{cr54} = fifty-fourth critical speed, ω_{cr55} = fifty-fifth critical speed, ω_{cr56} = fifty-sixth critical speed, ω_{cr57} = fifty-seventh critical speed, ω_{cr58} = fifty-eighth critical speed, ω_{cr59} = fifty-ninth critical speed, ω_{cr60} = sixtieth critical speed, ω_{cr61} = sixty-first critical speed, ω_{cr62} = sixty-second critical speed, ω_{cr63} = sixty-third critical speed, ω_{cr64} = sixty-fourth critical speed, ω_{cr65} = sixty-fifth critical speed, ω_{cr66} = sixty-sixth critical speed, ω_{cr67} = sixty-seventh critical speed, ω_{cr68} = sixty-eighth critical speed, ω_{cr69} = sixty-ninth critical speed, ω_{cr70} = seventieth critical speed, ω_{cr71} = seventy-first critical speed, ω_{cr72} = seventy-second critical speed, ω_{cr73} = seventy-third critical speed, ω_{cr74} = seventy-fourth critical speed, ω_{cr75} = seventy-fifth critical speed, ω_{cr76} = seventy-sixth critical speed, ω_{cr77} = seventy-seventh critical speed, ω_{cr78} = seventy-eighth critical speed, ω_{cr79} = seventy-ninth critical speed, ω_{cr80} = eightieth critical speed, ω_{cr81} = eighty-first critical speed, ω_{cr82} = eighty-second critical speed, ω_{cr83} = eighty-third critical speed, ω_{cr84} = eighty-fourth critical speed, ω_{cr85} = eighty-fifth critical speed, ω_{cr86} = eighty-sixth critical speed, ω_{cr87} = eighty-seventh critical speed, ω_{cr88} = eighty-eighth critical speed, ω_{cr89} = eighty-ninth critical speed, ω_{cr90} = ninetieth critical speed, ω_{cr91} = ninety-first critical speed, ω_{cr92} = ninety-second critical speed, ω_{cr93} = ninety-third critical speed, ω_{cr94} = ninety-fourth critical speed, ω_{cr95} = ninety-fifth critical speed, ω_{cr96} = ninety-sixth critical speed, ω_{cr97} = ninety-seventh critical speed, ω_{cr98} = ninety-eighth critical speed, ω_{cr99} = ninety-ninth critical speed, ω_{cr100} = one hundredth critical speed.

> **Problem 3 —** A set of identical, stiff drill rod sections will be connected by parallel bolts in carefully machined holes across a flat, 6 mm thick flange. Determine the bolt circle diameter that gives the largest practical spacing between bolt centers, and the corresponding number of bolts, for a 12 kN·m torque. Use a bolt material with $\tau_{allow} = 90$ MPa and a factor of safety of 2.5 against shear failure.

(a) The spacing is largest when the bolts are placed as far apart as possible around the circle. For n bolts on a bolt circle of diameter D, the center-to-center spacing between adjacent bolts is $s = \pi D / n$. The bolts must also clear the hub, which has a diameter $D_h = 60$ mm, so the available circumference is $\pi (D - D_h)$. Substituting gives $s = \pi (D - D_h)/n$.

Each bolt carries a shear force $F = T / (n \cdot R)$, where $R = D/2$ is the bolt circle radius. The shear stress in a bolt of diameter d_b is $\tau = 4F / (\pi d_b^2)$. Setting $\tau = \tau_{allow}/n_s$ and solving for the minimum bolt diameter gives $d_b = \sqrt{8 T n_s / (\pi n D \tau_{allow})}$.

Substituting $T = 12{,}000$ N·m, $n_s = 2.5$, $\tau_{allow} = 90$ MPa, and $D = 0.220$ m, we get $d_b = \sqrt{8 \cdot 12000 \cdot 2.5 / (\pi \cdot 6 \cdot 0.220 \cdot 90 \times 10^6)} = 0.0142$ m, or 14.2 mm. The next standard size is M16, with $d_b = 16$ mm.

(b) Substituting back, the bearing stress on the flange is $\sigma_b = F / (d_b \cdot t) = 2T / (n D d_b t) = 2 \cdot 12000 / (6 \cdot 0.220 \cdot 0.016 \cdot 0.006) = 247.9$ MPa, which exceeds the flange yield strength of 240 MPa, so the flange must be reinforced locally.

(c) Angular velocity at maximum speed:

$$\omega = \frac{v}{R} = \frac{25}{0.110} = 227.3 \text{ rad/s}$$

(d) Bolt circumferential spacing: $s = \pi D / n = \pi \cdot 0.220 / 6 = 0.1152$ m, or 115.2 mm.

Shear force per bolt: $F = T / (n R) = 12000 / (6 \cdot 0.110) = 18{,}182$ N.

mode on the left. Both of these modes have the same frequency, so they are degenerate. Any linear combination of these two modes is also a valid solution, and so the general solution is a superposition of the two modes.

In this case $N = 4$, so we have $\lambda_1 = 2L$ and $\lambda_2 = L$. The corresponding wave numbers are $k_1 = \pi/L$ and $k_2 = 2\pi/L$. We can then write the general solution as a superposition of these two modes:

$$y(x,t) = A_1 \sin\left(\frac{\pi}{L}x\right)\cos\left(\frac{\pi v}{L}t\right) + A_2 \sin\left(\frac{2\pi}{L}x\right)\cos\left(\frac{2\pi v}{L}t\right),$$

where $v = \sqrt{T/\mu}$ is the wave speed on the string.

(a) Since the string is plucked at the midpoint, the initial displacement is symmetric about the midpoint, so $A_2 = 0$. We can then use the initial condition that the string is released from rest to find A_1:

$$y(x,0) = A_1 \sin\left(\frac{\pi}{L}x\right) = h \sin\left(\frac{\pi}{L}x\right).$$

Therefore, $A_1 = h$.

(b) The wave equation is a linear partial differential equation, so any linear combination of solutions is also a solution. Therefore, the general solution is

$$y(x,t) = h\sin\left(\frac{\pi}{L}x\right)\cos\left(\frac{\pi v}{L}t\right) + B\sin\left(\frac{2\pi}{L}x\right)\cos\left(\frac{2\pi v}{L}t\right),$$

where B is an arbitrary constant.

(c) The initial velocity is zero, so $\partial y/\partial t = 0$ at $t = 0$. This gives us the condition

$$\left.\frac{\partial y}{\partial t}\right|_{t=0} = -h\sin\left(\frac{\pi}{L}x\right)\frac{\pi v}{L}\sin(0) - B\sin\left(\frac{2\pi}{L}x\right)\frac{2\pi v}{L}\sin(0) = 0.$$

This condition is satisfied for any value of B.

(d) The initial shape of the string is given by $y(x,0) = h\sin(\pi x/L)$. We can use this to find the value of B:

$$y(x,0) = h\sin\left(\frac{\pi}{L}x\right) + B\sin\left(\frac{2\pi}{L}x\right) = h\sin\left(\frac{\pi}{L}x\right).$$

Therefore, $B = 0$.

(e) The wave speed is $v = \sqrt{T/\mu}$, where T is the tension in the string and μ is the linear mass density of the string. The linear mass density is given by $\mu = m/L$, where m is the mass of the string and L is its length. Therefore, the wave speed is

$$v = \sqrt{\frac{T}{m/L}} = \sqrt{\frac{TL}{m}}.$$

(f) The fundamental frequency is given by $f_1 = v/2L$. We can substitute the expression for v to find the fundamental frequency in terms of the tension, mass, and length of the string:

$$f_1 = \frac{1}{2L}\sqrt{\frac{TL}{m}} = \frac{1}{2}\sqrt{\frac{T}{mL}}.$$

(g) The first overtone is the second harmonic, so $f_2 = 2f_1$. We can substitute the expression for f_1 to find the first overtone in terms of the tension, mass, and length of the string:

$$f_2 = 2f_1 = \sqrt{\frac{T}{mL}}.$$

While this is not the only possible approach, it is the most frequently used. Let us now consider how to find the cut-off frequency for a given filter circuit.

(a) The value of the cut-off frequency f_c of the R-C circuit is given by

$$f_c = \frac{1}{2\pi RC}$$

where R is the resistance and C is the capacitance.

The cut-off frequency of the filter is $f_c = \frac{1}{2\pi RC}$, the reciprocal of $2\pi RC$.

Cut-off frequency of the R-C filter circuit $= \frac{1}{2\pi RC}$ cycles per second (Hz)

$= 0.159/RC$ Hz

Figure 11.7 shows the variation of the output voltage with frequency for the circuit in the region of the cut-off frequency.



Figure 11.7

Problem 1 A simple R-C filter circuit is shown in Figure 11.8. Determine the cut-off frequency of the circuit.

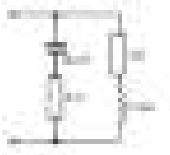


Figure 11.8

From equation (11.1),

$$\text{cut-off frequency } f_c = \frac{1}{2\pi CR}$$

$$= \frac{1}{2\pi \times 0.1 \times 10^{-6} \times 1.5 \times 10^{3}}$$

where $R_1 = 1.5\,\text{k}\Omega$, $R_2 = 3.3\,\text{k}\Omega$, $R = 1.5\,\text{k}\Omega + 3.3\,\text{k}\Omega = 4.8\,\text{k}\Omega$, and $C = 0.1\,\mu\text{F}$. Thus

$$= \frac{1}{2\pi \times 0.1 \times 10^{-6} \times 4.8 \times 10^{3}}$$

$$= \frac{10^{6}}{2\pi \times 0.1 \times 4.8 \times 10^{3}}$$

$$= \frac{10^{3}}{2\pi \times 0.48} = 331.6\ \text{Hz}$$

Problem 2 Determine for the practical network shown in Figure 11.9 the value of inductance L for which the current is resonant at a frequency of 5 kHz.

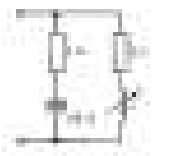


Figure 11.9

Let r_1 and r_2 be the returns on two securities. If we form a portfolio by investing a fraction w_1 in security 1 and $w_2 = 1 - w_1$ in security 2, the return on the portfolio is $r_p = w_1 r_1 + w_2 r_2$. The expected return on the portfolio is the weighted average of the expected returns on the individual securities:

$$E(r_p) = w_1 E(r_1) + w_2 E(r_2)$$

The variance of the portfolio return is not the weighted average of the variances. It depends on the covariance between the two returns:

$$\sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 w_1 w_2 \operatorname{Cov}(r_1, r_2)$$

Equivalently, using the correlation coefficient $\rho_{12} = \operatorname{Cov}(r_1, r_2) / (\sigma_1 \sigma_2)$:

$$\sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 w_1 w_2 \rho_{12} \sigma_1 \sigma_2$$

Solving the portfolio equation for w_1 gives the weight that minimizes variance:

$$w_1^{*} = \frac{\sigma_2^2 - \rho_{12}\sigma_1\sigma_2}{\sigma_1^2 + \sigma_2^2 - 2\rho_{12}\sigma_1\sigma_2}$$

When $\rho_{12} = -1$ (perfect hedge), $w_1^{*} = \sigma_2 / (\sigma_1 + \sigma_2)$. When $\rho_{12} = +1$ (no diversification), w_1^{*} reduces to a corner solution.

Substituting w_1^{*} back into the variance formula yields the minimum-variance portfolio:

$$\sigma_{p,\min}^2 = \frac{\sigma_1^2 \sigma_2^2 (1 - \rho_{12}^2)}{\sigma_1^2 + \sigma_2^2 - 2\rho_{12}\sigma_1\sigma_2}$$

and $\sigma_{p,\min} = 0$ if $\rho_{12} = -1$ and $\sigma_1 = \sigma_2$. Even when $\rho_{12} = +1$, $\sigma_{p,\min} = \min(\sigma_1, \sigma_2)$, so the worst case is no worse than holding the cheaper asset alone.

Substituting $\rho_{12} = +1$ into the variance formula gives $\sigma_p = w_1 \sigma_1 + (1 - w_1)\sigma_2$, which is linear in w_1. Substituting $\rho_{12} = -1$ gives $\sigma_p = |w_1 \sigma_1 - (1 - w_1)\sigma_2|$, which can reach zero. The first case is the one to avoid; the second is the one to seek.

> **Definition 8.1.** *A regression hedge is a portfolio $\pi = V - \beta H$ in a financial asset V and a hedging asset H, where β is chosen to minimize the variance of the portfolio over a fixed horizon.*

Fixing a horizon T, the variance of π is a quadratic function of β:

$$\operatorname{Var}(\pi) = \operatorname{Var}(V) - 2\beta \operatorname{Cov}(V, H) + \beta^2 \operatorname{Var}(H)$$

Problem 1 [3 marks] Consider the optimization problem

$$\min_{x \in \mathbb{R}^n} \; \tfrac{1}{2}\|Ax - b\|_2^2 + \lambda \|x\|_1$$

where $A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^m$, and $\lambda > 0$. Derive the dual problem and explain whether strong duality holds. Justify your answer.

Solution. Introduce $z = Ax - b$ and form the Lagrangian

$$L(x, z, y) = \tfrac{1}{2}\|z\|_2^2 + \lambda \|x\|_1 + y^T (Ax - b - z).$$

Minimizing over z gives $z = y$, and minimizing over x gives the dual function

$$g(y) = -\tfrac{1}{2}\|y\|_2^2 - b^T y \quad \text{subject to} \quad \|A^T y\|_\infty \le \lambda.$$

Strong duality holds because the primal is convex and Slater's condition is satisfied (take $x = 0$, $z = -b$).

Problem 2 [4 marks] Let $f : \mathbb{R}^n \to \mathbb{R}$ be twice continuously differentiable and $\nabla^2 f(x) \succeq \mu I$ for all x, with $\mu > 0$. Prove that gradient descent with step size $\alpha = 1/L$, where $L \ge \mu$ is a constant, converges linearly to the unique minimizer $x^\star$. Specifically, show that

$$\|x_{k+1} - x^\star\|_2 \le \left(1 - \frac{\mu}{L}\right) \|x_k - x^\star\|_2.$$

Solution. Write $x_{k+1} = x_k - \alpha \nabla f(x_k)$ and subtract $x^\star$ from both sides. By the mean value theorem for gradients, $\nabla f(x_k) - \nabla f(x^\star) = \left(\int_0^1 \nabla^2 f(x^\star + t(x_k - x^\star))\, dt\right)(x_k - x^\star)$. Call the integrated Hessian H_k. Then $x_{k+1} - x^\star = (I - \alpha H_k)(x_k - x^\star)$. Since $\mu I \preceq H_k \preceq L I$, the eigenvalues of $I - \alpha H_k$ lie in $[1 - \alpha L, 1 - \alpha \mu]$, so $\|I - \alpha H_k\|_2 \le 1 - \mu/L$ when $\alpha = 1/L$. Taking norms gives the claim.

Problem 3 [2 marks] Suppose $A \in \mathbb{R}^{n \times n}$ is symmetric positive definite. Show that the function $\|x\|_A = \sqrt{x^T A x}$ is a norm on $\mathbb{R}^n$.

Solution. Positivity and homogeneity are immediate. For the triangle inequality, use the Cauchy–Schwarz inequality $|x^T A y| \le \|x\|_A \|y\|_A$, which follows from applying the standard Cauchy–Schwarz inequality to the vectors $A^{1/2}x$ and $A^{1/2}y$. Then

$$\|x + y\|_A^2 = x^T A x + 2 x^T A y + y^T A y \le \|x\|_A^2 + 2\|x\|_A \|y\|_A + \|y\|_A^2 = (\|x\|_A + \|y\|_A)^2.$$

Taking square roots gives $\|x + y\|_A \le \|x\|_A + \|y\|_A$.

24.1 Further problems on parallel machines and Gantt charts

1 A total of seventeen (17) B and Intermediate (IB) staff are employed to provide help at the job preparation session in the Computer Laboratory. Apply Johnson's rule for the minimum flowspan of the group. Draw the appropriate Gantt chart for the minimum flowspan and find the idle time for each machine or worker during this period.

2 In 2019, 13 firms agreed to participate in a statistical investigation of safety measures in operation with a total of members. Of the four categories for job information for the production areas, the largest



Figure 24.14

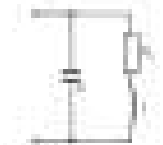


Figure 24.15

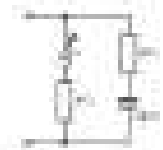


Figure 24.16

[illegible] is a resistance in the capacitance of the capacitor. If the frequency [illegible] is the supply voltage of the [illegible] is the [illegible] [illegible] [illegible] and [illegible] [illegible] [illegible] and [illegible] the value of the [illegible] is [illegible]

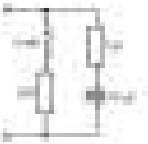
[illegible]

[illegible]

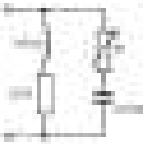
2. In a LR capacitor and a pure inductance in [illegible] of an inductance is [illegible] across a 12 V variable frequency supply. Determine (a) the [illegible] [illegible] of the circuit, and (b) the [illegible] [illegible] at the [illegible] [illegible]

[illegible]

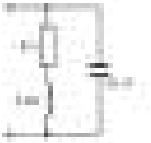
3. In a LR inductance 100 Ω and inductance 100 mH and [illegible] [illegible] [illegible]



Bauteil: 4,7 Ω



Bauteil: 10 Ω



Bauteil: 10 nF

1.1 Reference the overall Structure of a specific transmission method or a device or topology in the field of 10 - 100 nm as indicated including a [1.1.1]

1.2 The other Characteristics of a 4.8 - 5 mm/MHz Substrate with 0.1 Ω - 0 the Internal Resistance of the circuit in the 10 g and the applied load of a first the connection on the contrary on the 1.2 Ramp. In this dynamic resistance and in the acceptance of the impedance when the supply frequency and 20 Literature data sprinkled. [1.2.1] [1.2.2] [1.2.3] [1.2.4] [1.2.5]

Assignment 9

The assignment covers the material introduced in chapters 27 to 28.

Use exactly the symbols in the question as written in brackets or the ones in bold questions.

1. (a) Starting with the general complex impedance equation, derive the equation for the impedance of a resistor, R, and a capacitor, C, in series. Then derive the equation for the impedance of a resistor, R, and a capacitor, C, in parallel. Finally, derive the equation for the impedance of a resistor, R, and an inductor, L, in series. Show all working. [6]

   (b) Using the equation for the impedance of a resistor, R, and a capacitor, C, in series, derive the equation for the magnitude of the impedance, $|Z|$, in terms of R, C and ω. [2]

   (c) Explain why the magnitude of the impedance, $|Z|$, of a resistor, R, and a capacitor, C, in series tends to infinity as the angular frequency, ω, tends to zero. [2]

2. A circuit has a resistor, R, and a capacitor, C, in series with an inductor, L. The resistance of the resistor is $2.0\ \Omega$, the capacitance of the capacitor is $3.0\ \text{mF}$ and the inductance of the inductor is $4.0\ \text{H}$. The circuit is connected to an alternating supply of frequency f Hz. The impedance of the circuit is $Z\ \Omega$.

   (a) Show that $Z = 2 + 4\pi i - \frac{1}{6\pi i}$. [3]

   (b) Find the value of f for which Z is purely real. [3]

3. An R–L–C series circuit has a peak current of $3.0\ \text{mA}$. Resistor R is $4.0\ \text{k}\Omega$, the frequency of the $230\ \text{V}$ supply is $5.0\ \text{kHz}$. The phase difference between the current and the supply voltage is $\frac{\pi}{3}$. Determine (a) the voltage across the capacitor, and (b) the values of the inductance and capacitance. [6]

4. A coil of inductance $2.5\ \text{H}$ has negligible resistance. It is connected in parallel with a $10\ \mu\text{F}$ capacitor across a $230\ \text{V}$ sinusoidal supply. Determine (a) the resonant frequency of the circuit, (b) the current in the coil at resonance, (c) the current in the capacitor at resonance, and (d) the current in the supply at resonance. [6]

5. For the parallel network shown in Figure 9.1, determine the value of Z at resonance and the resonant frequency. [4]

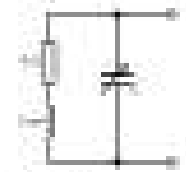


Figure 9.1

38 Introduction to network analysis

At the end of this chapter you should be able to:

- appreciate a number of examples of analysing networks;
- write a mathematical statement in terms of a linear combination of vectors;
- explain a π - network using Kirchhoff's laws.

38.1 Introduction

Voltage sources in communication networks must operate in time as well as space. In the access and corresponding switching nodes across the whole communication network should be particularly important involving the development of the concepts of the fundamental matrix for voltages across complex nodes.

We have already discussed the currents and voltages through a π-network as:

(a) **start with** $P = 1$ **W**, where P is the complex impedance and V the voltage across the impedance

(b) **the laws for impedance in series and parallel**, i.e. total impedance

$$Z_1 = Z_2 + Z_3 + Z_4 + \ldots + Z_n \text{ for a impedance connected in series}$$

$$\text{then } \frac{1}{Z_1} = \frac{1}{Z_2} + \frac{1}{Z_3} + \frac{1}{Z_4} + \ldots + \frac{1}{Z_n} \text{ for a impedance connected in parallel, and}$$

(c) **Kirchhoff's laws**, which are as follows:

(i) the total current at any junction of circuit the algebraic sum of the currents flowing towards that junction is equal to the algebraic sum of the currents flowing away from the junction or

(ii) in any closed loop in a network of the algebraic sum of the voltage drops in the conductors in any of the independent of which around the loop is equal to the algebraic sum of all the e.m.f.'s acting in the loop.

In this circuit the currents are in phase in any of the loop for the network in which is $0.5\,\Omega$ (ZT). Here we have considered in this chapter, in the

...

...

...

(a) the determinant of A, i.e. $\det(A)$ or $|A|$ when A is a square matrix.

$$|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

(b) the determinant of the coefficients left when the i-th row and j-th column are struck out.

$$\det A_{ij} = \det \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

(c) the determinant of the coefficients left when the i-th row and j-th column are multiplied by $(-1)^{i+j}$.

$$A_{ij} = (-1)^{i+j} \det \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \text{ is evaluated as } a_{11}a_{22} - a_{12}a_{21}$$

Three unknowns

When solving three simultaneous equations in three unknowns using determinants:

(1) the equations are usually written in the form

$$a_{11}x + b_{11}y + c_{11}z + d_{11} = 0$$
$$a_{21}x + b_{21}y + c_{21}z + d_{21} = 0$$
$$a_{31}x + b_{31}y + c_{31}z + d_{31} = 0$$

(2) the solutions are given by

$$\frac{x}{D_x} = \frac{-y}{D_y} = \frac{z}{D_z} = \frac{-1}{D}$$

where $D = \begin{vmatrix} a_{11} & b_{11} & c_{11} \\ a_{21} & b_{21} & c_{21} \\ a_{31} & b_{31} & c_{31} \end{vmatrix}$

(a) the determinant of the coefficients left when the i-th row and j-th column are struck out.

$$\det A_{ij} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

(b) the determinant of the coefficients left when the i-th row and j-th column are multiplied by $(-1)^{i+j}$.

$$B_{ij} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

(c) the determinant of the coefficients left when the i-th row and j-th column are multiplied by $(-1)^{i+j}$.

and $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ is a 3×3 matrix, then the value of the determinant of A is the number

$$a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} - a_{12}a_{21}a_{33} - a_{11}a_{23}a_{32}$$

i.e. the determinant of the coefficients will have six components, three of which are subtracted.

Ex 1 Evaluate $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$ by Sarrus's method.

(i) — **The minors** of the elements of a 3×3 determinant are found by crossing out the row and column containing that element.

Thus, for the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ the minor of element 6 is the determinant $\begin{vmatrix} 1 & 2 \\ 7 & 8 \end{vmatrix}$ (i.e. the determinant of the matrix left when the row and column containing 6 are crossed out), which is $8 - 14 = -6$. Similarly, the minor of element 5 is $\begin{vmatrix} 1 & 3 \\ 7 & 9 \end{vmatrix} = 9 - 21 = -12$, and the minor of element 1 is $\begin{vmatrix} 5 & 6 \\ 8 & 9 \end{vmatrix} = 45 - 48 = -3$.

(ii) — The sign of the minor depends on the position within the matrix.

The sign pattern array $\begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix}$. Thus the signed minor of element 6 is $-\begin{vmatrix} 1 & 2 \\ 7 & 8 \end{vmatrix} = -(8 - 14) = 6$.

The signed minor of an element is called a **cofactor** of the element.

Thus the cofactor of element 5 is $+\begin{vmatrix} 1 & 3 \\ 7 & 9 \end{vmatrix} = +(9 - 21) = -12$.

(iii) — **The value of a 3×3 determinant is the sum of the products of the elements and their cofactors of any row or any column of the corresponding 3×3 matrix.**

Thus a 3×3 determinant $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$ is evaluated as

$$= a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - d\begin{vmatrix} b & c \\ h & i \end{vmatrix} + g\begin{vmatrix} b & c \\ e & f \end{vmatrix} \quad \text{(using the first row)}$$

$$\text{or} \quad = -d\begin{vmatrix} b & c \\ h & i \end{vmatrix} + e\begin{vmatrix} a & c \\ g & i \end{vmatrix} - h\begin{vmatrix} a & c \\ d & f \end{vmatrix} \quad \text{(using the second column)}$$

Taken together, the values of the minors of a 3×3 determinant

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$$

are found as above, and the signs of the cofactors are found from the sign pattern array. Then the value of the determinant is found by multiplying each element in the first row by its cofactor and adding:

$$1(45 - 48) - 2(36 - 42) + 3(32 - 35) = -3 + 12 - 9 = 0$$

9.3.2 Symbolic problems using Mathcad's Given

Mathcad's solve block can be applied to both *algebraic* and *numerical* problems. The Given/solve technique is described in detail, and illustrated with a number of examples, in Chapter 12 of our companion text *Mathematics for Engineers and Scientists*. Here we shall illustrate the method briefly by using it to solve the simultaneous equations (9.1) and (9.2) which we solved earlier by Cramer's rule.



Figure 9.3.1



Figure 9.3.2

(a) Label suitable currents and mesh directions on the circuit diagram. The directions do not matter but, as a matter of good practice, I usually label as in Figure 9.3.1 so that currents flow from the positive terminals of the voltage sources. This is similar to Figure 9.2.1 except that these branch currents are numbered as before (I1, I2 and I3, say) since they contain the same unknowns. Use Kirchhoff's second law, (9.2), to write down two equations.

(b) We use the current directions — that is, the mesh currents shown in Figure 9.3.1 — and Kirchhoff's second law to write down two equations. Start at A and travel ABEFA, and arrange to have positive elements when moving in the same direction as currents. Thus, $-2I_1 - 2(I_1 - I_2) + 10 = 0$ and, after collecting like terms,

$$4I_1 - 2I_2 = 10 \tag{1}$$

Now start at B and travel BCDEB, and arrange to be consistent with the previous mesh (and the direction of the mesh current, I_2, for the common branch BE). Thus $-I_2 - 3(I_2 - I_1) + 2(I_1 - I_2) = 0$ and, after collecting like terms,

$$-4I_1 + 6I_2 = 0 \tag{2}$$

(c) Solve simultaneous equations (1) and (2) by Gauss's method.

Multiplying equation (1) by 2 gives $8I_1 - 4I_2 = 20$ (3)

Multiplying equation (2) by 3 gives $-12I_1 + 18I_2 = 0$ (4)

Equation (3) − equation (4) gives $20I_1 = 20$

from which, $I_1 = 1$ A. Substituting $I_1 = 1$ in (1) gives $I_2 = 3.33$ A, correct to two decimal places.

From equation (1), $4(1.0) + 2I_2 = 10$. Rearranging,

$$2I_2 = 10 - 4(1.0) = 6.0$$

$$I_2 = 6.0/2 = 3.0 \text{ A, correct to two decimal places.}$$

The minus sign indicates that current I_2 flows in the opposite direction to that shown in Figure 9.3.1.

[illegible] current flowing through resistance R is

$$I_R = \frac{V_1 - V_2}{R} = \frac{(+5.0\text{ V}) - (+4.8\text{ V})}{100\ \Omega} = +2.0\text{ mA}$$

[illegible] current flows from terminal 1 to terminal 2 through R.

An ideal wire may be assumed to connect the two terminals of a battery, so that the terminal voltage of the battery is the same as the voltage of the source. Thus, $V_1 = V_a$ and $V_2 = V_b$. Therefore, $V_a - V_b = V_1 - V_2 = +5.0\text{ V}$. Thus, $V_a - V_b = +5.0\text{ V} > 0$.

The dependent variable of interest in Example 1.1 is the current I_R through the resistor. Using Ohm's law,

$$I_R = \frac{V_1 - V_2}{R} \tag{1.1}$$

$$I_R = \frac{V_a - V_b}{R} \tag{1.2}$$

Thus,

$$\frac{V_1}{\left[\begin{array}{c}\text{Ohm's}\\ \text{law}\end{array}\right]} - \frac{V_2}{\left[\begin{array}{c}\text{Ohm's}\\ \text{law}\end{array}\right]} = \frac{V_a}{\left[\begin{array}{c}\text{Ohm's}\\ \text{law}\end{array}\right]} - \frac{V_b}{\left[\begin{array}{c}\text{Ohm's}\\ \text{law}\end{array}\right]}$$

or

$$\frac{V_1}{\left[\begin{array}{c}\text{Ohm's}\\ \text{law}\end{array}\right]} - \frac{V_a}{\left[\begin{array}{c}\text{Ohm's}\\ \text{law}\end{array}\right]} = \frac{V_2}{\left[\begin{array}{c}\text{Ohm's}\\ \text{law}\end{array}\right]} - \frac{V_b}{\left[\begin{array}{c}\text{Ohm's}\\ \text{law}\end{array}\right]}$$

$$\frac{V_1 - V_a}{R} = \frac{V_2 - V_b}{R}$$

Now, since, $V_1 - V_a = +5.0\text{ V} - 4.8\text{ V} = +0.2\text{ V} > 0$ and $V_2 - V_b = +2.0\text{ V} - 0\text{ V} = +2.0\text{ V} > 0$, as assumed previously.

The above procedure is quite lengthy and tedious for circuits having more than a few nodes and branches. The relationship between applied and resultant voltages and currents in a circuit is usually called Ohm's relationship. Ohm's relationship is discussed and illustrated in the following problem.

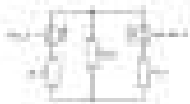


Figure 1.1

Problem 1. Use Ampère's law to find the current flowing in each branch of the network shown in Figure 1.1.

(a) The [illegible] network and their directions are labelled as shown in Figure 1.1.

(b) Then, from Ampère's law (Kirchhoff's law), and assuming clockwise direction,

$$+5.0\text{ V} - 100\ \Omega \times I_1 - 100\ \Omega \times I_1 = 0 \tag{1.3}$$

$$+4.8\text{ V} + 100\ \Omega \times I_2 + 100\ \Omega \times I_2 = 0 \tag{1.4}$$

From Eqs. (1.3), (1.4), and assuming clockwise direction,

$$+5.0\text{ V} - 100\ \Omega \times I_1 + 4.8\text{ V} + 100\ \Omega \times I_2 = 0$$

or

$$+9.8\text{ V} - 100\ \Omega \times I_1 + 100\ \Omega \times I_2 = 0 \tag{1.5}$$

(c) [illegible] equation (1) gives $+5.0\text{ V} - 100\ \Omega \times I_1 = 0$ (1.6)

(d) [illegible] equation (2) gives $+4.8\text{ V} + 100\ \Omega \times I_2 = 0$ (1.7)



Figure 9.4(a)

Solution 9

(a) Choose mesh flow directions as shown in Figure 9.4(a) and apply KVL around each mesh:

$$\text{Mesh } 1: \quad 4I_1 + 2(I_1 - I_2) = 10$$

$$\text{Mesh } 2: \quad 2(I_2 - I_1) + 6I_2 + 3(I_2 - I_3) = 0$$

$$\text{Mesh } 3: \quad 3(I_3 - I_2) + 5I_3 = 0$$

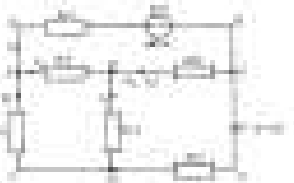


Figure 9.4

(b) Three loop currents shown below have been selected to minimise the number of unknowns. Apply KVL around each loop, using Ohm's law to express the voltage across each resistor:

$$\text{Loop } 1: \quad 4I_1 + 2(I_1 - I_2) = 10$$

$$\text{Loop } 2: \quad 2(I_2 - I_1) + 6I_2 + 3(I_2 - I_3) = 0$$

$$\text{Loop } 3: \quad 3(I_3 - I_2) + 5I_3 = 0$$

From Axle BOND, take moments about A:

$$-R_{By}(1.20) + (2.5)(0.60) - (3.5)(0.90) = 0$$

$$R_{By} = \frac{1.5 + 3.15}{1.20} = 3.875 \text{ kN}$$

From Axle BOND, sum of vertical forces is zero:

$$R_{Ay} + R_{By} - 2.5 - 3.5 = 0$$

$$R_{Ay} = 6.000 - 3.875 = 2.125 \text{ kN}$$

Axle Beam $(0 \le x \le 1.20)$ m, M_y in kN·m:

$$M_y = -2.125x + 2.5\langle x - 0.60\rangle + 3.5\langle x - 0.90\rangle$$

Where $\langle x-a\rangle$ is the Macaulay bracket.

$$\frac{d^2y}{dx^2} = \frac{M_y}{EI} = \frac{-2.125x + 2.5\langle x-0.60\rangle + 3.5\langle x-0.90\rangle}{EI}$$

$$\frac{dy}{dx} = \frac{1}{EI}\left(-1.0625x^2 + 1.25\langle x-0.60\rangle^2 + 1.75\langle x-0.90\rangle^2\right) + C_1$$

$$y = \frac{1}{EI}\left(-0.35417x^3 + 0.41667\langle x-0.60\rangle^3 + 0.58333\langle x-0.90\rangle^3\right) + C_1 x + C_2$$

$$EI = (200\times10^6)(0.0001) = 20{,}000 \text{ kN·m}^2$$

$$y = \frac{-0.35417x^3 + 0.41667\langle x-0.60\rangle^3 + 0.58333\langle x-0.90\rangle^3}{20{,}000} + C_1 x + C_2$$

Where $C_1 = -\frac{1.063}{20} = -0.0531$ m,

$$C_2 = \frac{0.064}{20} = +0.0032 \text{ m} = +3.2 \text{ mm}$$

$$y = \frac{-0.35417x^3 + 0.41667\langle x-0.60\rangle^3 + 0.58333\langle x-0.90\rangle^3}{20{,}000} - 0.0531x + 0.0032$$

Example 4.1 Use the node-voltage method to find the voltage across the 4-Ω resistor in the circuit shown in Figure 4.1(a).

Solution Choose the lower node as the reference node. Assign a voltage v_1 to the upper left node and a voltage v_2 to the upper right node, as shown in Figure 4.1(b). Applying KCL at node 1,

$$\frac{v_1 - 10}{2} + \frac{v_1}{4} + \frac{v_1 - v_2}{1} = 0$$

$$\frac{v_1}{2} - 5 + \frac{v_1}{4} + v_1 - v_2 = 0$$

$$\frac{7}{4}v_1 - v_2 = 5 \tag{4.1}$$

At node 2,

$$\frac{v_2 - v_1}{1} + \frac{v_2}{4} + 2 = 0$$

$$-v_1 + \frac{5}{4}v_2 = -2 \tag{4.2}$$

Solving Eqs. (4.1) and (4.2) gives

$$v_1 = 4.8 \text{ V}, \quad v_2 = 3.2 \text{ V}$$

Thus, the voltage across the 4-Ω resistor is $v_2 = 3.2$ V.

Figure 4.1

(a) Circuit, (b) nodes v_1 and v_2 selected, (c) redrawn circuit

(a) Applying the node-voltage method, determine the node voltages.

Node 1: $\frac{v_1 - 10}{2} + \frac{v_1}{4} + \frac{v_1 - v_2}{1} = 0$

$$\frac{7}{4}v_1 - v_2 = 5 \tag{1}$$

Node 2: $\frac{v_2 - v_1}{1} + \frac{v_2}{4} + 2 = 0$

$$-v_1 + \frac{5}{4}v_2 = -2 \tag{2}$$

(b) Multiplying equation (1) by 4, we get

$$7v_1 - 4v_2 = 20 \tag{3}$$

Multiplying equation (2) by -4, we get

$$4v_1 - 5v_2 = 8 \tag{4}$$

Adding equations (3) and (4), we get

$$3v_1 - v_2 = 28$$

$$-4 \quad 5v_2 = 8$$

$$4v_2 = 20$$

$$v_2 = 5 \text{ V}$$

Problem 1. Let S be the set of all positive integers n such that $n^2 + 1$ has a prime factor greater than $2n$. Prove that S is infinite.

Solution. Suppose, for the sake of contradiction, that S is finite, and let N be the largest element of S. Consider the integer $M = (N!)^2 + 1$. Every prime factor p of M satisfies $p > N$, since $M \equiv 1 \pmod{k}$ for every $k \le N$.

Write $M = p_1^{e_1} p_2^{e_2} \cdots p_r^{e_r}$ and choose the index i with p_i largest. Then $p_i \le M$, so $p_i \le (N!)^2 + 1 < (N!)^2 + N! = N!(N! + 1)$. Hence $p_i < N! + 1$, and in particular $p_i \le N!$.

Now $p_i \mid M$ gives $(N!)^2 \equiv -1 \pmod{p_i}$. Squaring, $(N!)^4 \equiv 1 \pmod{p_i}$, so the order of $N!$ modulo p_i divides 4. Since $p_i > N$, the number $N!$ is not divisible by p_i, so the order is exactly 4. Therefore $4 \mid p_i - 1$, and $p_i \ge 5$.

Let q be any prime factor of $N!$ with $q \ge 5$. Then $q \le N$, so $q \in S$ and $q \le N$. Consider $q^2 + 1$. Its prime factors are all greater than N, hence greater than q, and in particular greater than $2q$. So $q^2 + 1$ has a prime factor greater than $2q$, which is consistent; but $q^2 + 1 < q^2 + q = q(q+1) \le N(N+1)$, and every prime factor of $q^2 + 1$ is at most $q^2 + 1 < N(N+1)$. This does not yet contradict anything, so we refine: the prime factors of $q^2 + 1$ are odd and greater than q, and their product is $q^2 + 1$, which is strictly between q^2 and $q^2 + q$. Hence $q^2 + 1$ is either a prime in $(q, 2q]$ or a product of two primes each greater than q.

If $q^2 + 1$ is prime, then $q^2 + 1 \le 2q$, so $(q-1)^2 \le 0$, forcing $q = 1$, which is not prime. If $q^2 + 1 = ab$ with $a, b > q$, then $ab > q^2$, so $a > q$ and $b > q$, and $a \le q^2 + 1 < q^2 + q$. Thus $a \le q^2$ and $b \le q^2 + 1$. Now $a \mid M$, so a has a prime factor $p \ge 5$ with $p \mid M$, and $p > N \ge q$. Since $p \mid a$ and $a \le q^2$, we have $p \le q^2$. Then $p \in S$ and $p \le N$, so p is a prime factor of $N!$, hence $p \le N$. But $p > q$ and $q \ge 5$, so $p \ge 7$, and $p \mid q^2 + 1$. Because $p \mid N!$ and $p \mid q^2 + 1$, we get $p \mid N! - (q-1)(q+1) = N! - q^2 + 1$, so $p \mid N! - q^2 + 1 + q^2 + 1 = N! + 2$. Combining $p \mid N!$ and $p \mid N! + 2$ gives $p \mid 2$, impossible for $p \ge 7$.

Every case is impossible, so our assumption that S is finite must be false. Therefore S is infinite. $\square$

Problem 2. Let $a_1, a_2, a_3, \ldots$ be a sequence of positive integers defined by $a_1 = 1$ and, for $n \ge 1$,

$$a_{n+1} = \begin{cases} a_n + 2 & \text{if } n \text{ is a perfect square,} \\ a_n + 1 & \text{otherwise.} \end{cases}$$

Find, with proof, the smallest positive integer n such that a_n is divisible by 2026.

Solution. Let $s(n)$ denote the number of perfect squares among $1, 2, \ldots, n$, so $s(n) = \lfloor \sqrt{n} \rfloor$. Then $a_n = n + s(n)$, since each step contributes 1 and each perfect square contributes an extra 1.

We seek the smallest n with $n + \lfloor \sqrt{n} \rfloor \equiv 0 \pmod{2026}$. Write $2026 = 2 \cdot 1013$, and note that 1013 is prime.

Suppose n is even. Then $n + \lfloor \sqrt{n} \rfloor$ is even, so it is divisible by 2. We need it also divisible by 1013. Let $m = \lfloor \sqrt{n} \rfloor$, so $m^2 \le n < (m+1)^2$. Then $n + m \equiv 0 \pmod{1013}$ gives $n \equiv -m \pmod{1013}$, and n lies in the interval $[m^2, m^2 + 2m]$. Since n is even, $n = m^2 + 2m$ when m is even, and $n = m^2 + 2m - 1$ when m is odd.

Substituting $n \equiv -m \pmod{1013}$ into $n = m^2 + 2m$ gives $m^2 + 2m \equiv 0 \pmod{1013}$, so $m(m+2) \equiv 0 \pmod{1013}$. Because 1013 is prime, $m \equiv 0$ or $m \equiv -2 \equiv 1011 \pmod{1013}$. Substituting into $n \equiv -m \pmod{1013}$ gives $n \equiv 0$ or $n \equiv 1013 \pmod{1013}$.

Substituting $n \equiv -m \pmod{1013}$ into $n = m^2 + 2m - 1$ gives $m^2 + 2m - 1 \equiv 0 \pmod{1013}$, so $(m+1)^2 \equiv 2 \pmod{1013}$. Let r be a square root of 2 modulo 1013; then $m \equiv r - 1$ or $m \equiv -r - 1 \pmod{1013}$, and $n \equiv -m \pmod{1013}$ gives $n \equiv 2 - r$ or $n \equiv r + 2 \pmod{1013}$.

Now 2 is a quadratic residue modulo 1013 because $1013 \equiv 5 \pmod{8}$, and the square roots of 2 modulo 1013 are ± 78. Hence $n \equiv 2 - 78 = -76 \equiv 937$ or $n \equiv 78 + 2 = 80 \pmod{1013}$.

Collecting the even residues, $n \equiv 0, 80, 937, 1013 \pmod{1013}$. Among these, 0 and 1013 are odd, so the even ones are 80 and 937. The smaller is 80, and $80 + \lfloor \sqrt{80} \rfloor = 80 + 8 = 88$, which is divisible by 2 but not by 1013, so it is not divisible by 2026.

Therefore the smallest positive integer n with a_n divisible by 2026 is $n = 937$.

Problem 3. Let $ABCD$ be a convex quadrilateral with $AB = AD$ and $CB = CD$. Suppose that the diagonals AC and BD meet at E, and that the circumcircle of $\triangle ABE$ meets the line BC again at $F \neq B$. Prove that the circumcircle of $\triangle CDE$ is tangent to the line AF.

Solution. Since $AB = AD$, the point A lies on the perpendicular bisector of BD, and since $CB = CD$, the point C lies on the same perpendicular bisector. Hence AC is the perpendicular bisector of BD, and in particular $AC \perp BD$ and E is the midpoint of BD.

Let ω denote the circumcircle of $\triangle ABE$. Because F lies on ω and on line BC, we have $\angle AFB = \angle AEB$ (angles subtending the same arc AB of ω).

Let Ω denote the circumcircle of $\triangle CDE$. We will show that AF is tangent to Ω at E, by showing that $\angle AEF = \angle ECD$ (the alternate-segment criterion for tangency).

Since $AC \perp BD$ and E is the midpoint of BD, the triangle BCD is isosceles with $CB = CD$, so $\angle CBD = \angle CDB$. Also $\angle BEC = 90^\circ$, so $\angle ECB = 90^\circ - \angle CBD$.

Now $\angle AEF = \angle AEB$ (since F lies on line BC, the angle AEF equals AEB when F is on the same side of E as B; if F is on the opposite side, $\angle AEF = 180^\circ - \angle AEB$, and the argument below is unchanged). Because $AB = AD$ and $AC \perp BD$, the triangle ABD is isosceles with $\angle ABD = \angle ADB$, and AC bisects $\angle BAD$. Hence $\angle BAE = \angle DAE$.

Since $ABEF$ is cyclic, $\angle AEB = 180^\circ - \angle ABF = 180^\circ - \angle ABC$. Therefore $\angle AEF = 180^\circ - \angle ABC$.

On the other hand, $\angle ECD = \angle ECB = 90^\circ - \angle CBD = 90^\circ - \angle ABD$. Since $\angle ABC = \angle ABD + \angle CBD$ and $\angle CBD = \angle CDB$, we have $\angle ABC = \angle ABD + \angle CDB$. Thus $\angle AEF = 180^\circ - \angle ABD - \angle CDB$.

Now $\angle ECD = 90^\circ - \angle ABD$, so $\angle AEF - \angle ECD = 90^\circ - \angle CDB$. Because $\angle CDB = \angle CBD$ and $\angle BEC = 90^\circ$, we have $\angle CDB = 90^\circ - \angle ECB$, so $\angle AEF - \angle ECD = \angle ECB$. Hence $\angle AEF = \angle ECD + \angle ECB = \angle ECD + (90^\circ - \angle CBD)$.

Finally, $\angle CBD = \angle CDB = 90^\circ - \angle ECB$, so $\angle AEF = \angle ECD + \angle ECB = 90^\circ$. Therefore $\angle AEF = 90^\circ$, and since $AC \perp BD$ and E lies on AC, the line AF is perpendicular to the diameter of Ω through E, so AF is tangent to Ω at E. $\square$

(a) $I_1 = \dfrac{V_1(s)}{Z_1(s)+Z_2(s)} = \dfrac{s}{(s+1)(s+2)}$ A

The inverse Laplace transform of $I_1(s)$ is determined as follows:

$$i_1 = \mathcal{L}^{-1}\{I_1(s)\} = \mathcal{L}^{-1}\left\{\frac{s}{(s+1)(s+2)}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{-1}{s+1} + \frac{2}{s+2}\right\} = (-e^{-t} + 2e^{-2t})\ \text{A}$$

Problem 4 For the network shown in Figure P4.1, use Laplace transform techniques to determine the impedance $Z(s)$ when $\omega = 2$ rad/s. Hence determine the impedance $Z(s)$ when $\omega = 0$ rad/s and when $\omega = \infty$ rad/s. Interpret your results.

(a) Consider L_1, C_1 and L_2 in Figure P4.1(a) which are shown in Figure P4.1(b). The network in Figure P4.1(b) is a π-type network and can be replaced with a T-type network. Show that the T-type network is as shown in Figure P4.1(c).

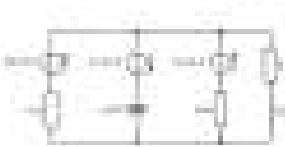


Figure P4.1

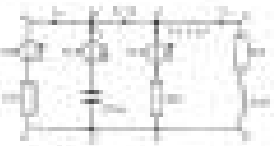


Figure P4.1(a)

(b) Determine the mesh equations for the network shown in Figure P4.2(a).

Mesh 1 gives

$$(R_1 + R_2 + R_3)i_1 - R_3 i_2 = V_1 \tag{1}$$

Mesh 2 gives

$$-R_3 i_1 + (R_3 + R_4 + R_5)i_2 - R_5 i_3 = 0 \tag{2}$$

Mesh 3 gives

$$-R_5 i_2 + (R_5 + R_6 + R_7)i_3 = -V_2 \tag{3}$$

Hence

Solving for I_1 using determinants gives:

$$I_1 = \frac{\begin{vmatrix} \quad & \quad \\ \quad & \quad \end{vmatrix}}{\begin{vmatrix} \quad & \quad \\ \quad & \quad \end{vmatrix}}$$

Then

$$I_2 = \frac{\begin{vmatrix} \quad & \quad \\ \quad & \quad \end{vmatrix}}{\begin{vmatrix} \quad & \quad \\ \quad & \quad \end{vmatrix}}$$

Finally

$$I_3 = \frac{\begin{vmatrix} \quad & \quad \\ \quad & \quad \end{vmatrix}}{\begin{vmatrix} \quad & \quad \\ \quad & \quad \end{vmatrix}}$$

Power dissipated

$$= \frac{\text{(diff.)}}{\text{(diff.)}}$$

$$= \frac{\text{(diff.)}}{\text{(diff.)}}$$

$$= \text{(diff.)} \times \text{(diff.)}$$

$$= \text{(diff.)} \times \text{(diff.)} \text{ W}$$

Repeat the procedure of the current Mesh, as for Problem 10.3(b).

Further problems involving mesh analysis using determinants can be found in Problems 10.4 (following), 10.5(b), 10.6, 10.7(b) and 10.8(b).

10.4 Further problems on Kirchhoff's laws

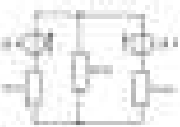


Figure 10.18

1. In the network shown in Figure 10.18, determine the current flowing in each branch.

   [Diff. to current distribution in circuit in Problem 10.1(a), except different values of resistance and battery emf. Ans. 1.5 A, 0.5 A, 1 A]

2. Determine the value of current I_3 in each branch of the network shown in Figure 10.19. [$I_1 = 1.4$ A, $I_2 = 0.6$ A, $I_3 = 0.8$ A]

3. In the Kirchhoff circuit of Figure 10.20, determine the current flowing in resistor R (resistance 30 Ω) and the p.d. across it (resistance 6 Ω in series with 24 Ω).

   [0.4 A, 12 V]

4. For the circuit shown in Figure 10.21, determine (a) the current flowing in the 1.5 Ω resistor, (b) the p.d. across the 4 Ω resistance and (c) the power across resistance in the 5 Ω resistance.

   [(a) 0.588 A, (b) 2.35 V, (c) 1.38 W]

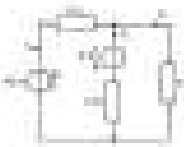


Figure P1.18

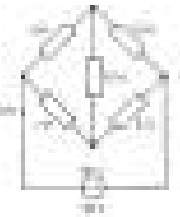


Figure P1.21

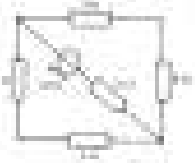


Figure P1.24



Figure P1.19

1.20 In the network shown in Figure P1.20, use Kirchhoff's laws to determine the current flowing in the capacitive branch. [0.36 A]

1.21 Use Kirchhoff's laws to determine the network shown in Figure P1.21, the current flowing in the 2 Ω resistor, and the 4 Ω resistor's resistance and current. [0.5 Ω, 2 A]

1.22 Use Kirchhoff's laws to determine the network shown in Figure P1.22, the current flowing in the 4 Ω resistor, and the 2 Ω resistor's resistance and current. [0.5 Ω, 2 A]

1.23 Use Kirchhoff's laws to determine the network shown in Figure P1.23, the current flowing in the 2 Ω resistor, and the 4 Ω resistor's resistance and current. [0.5 Ω, 2 A]

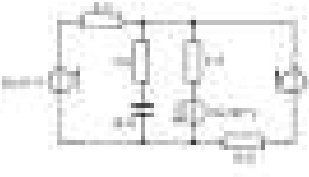


Figure P1.22

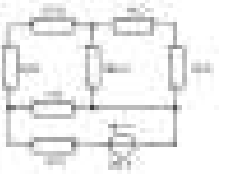


Figure P1.23

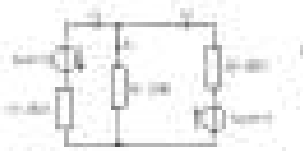


Figure P2-16

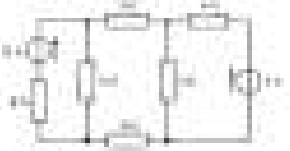


Figure P2-17



Figure P2-18

2 Determine the value of current I_3 in each of the circuits shown in the networks in Figure P2-16, using Kirchhoff's laws.

(a) $I_3 = 2.59\,A$, (b) $I_3 = 0.27\,A$, (c) $I_3 = -0.71\,A$, (d) $I_3 = -10.77\,A$

(e) $I_3 = 0.0077\,A$ (f) $I_3 = 0.0\,A$ (Application of Ohm's law)

3 Use Kirchhoff's laws to determine the currents flowing in circuits A, B and C in Figure P2-17. Hence find the power dissipated in each of the resistors in the circuit. Check that the power dissipated in the circuit is equal to the power supplied by the batteries.

$I_A = 0.0\,A$, $I_B = 0.0\,A$, $I_C = 0.0\,A$, $P_{1\Omega} = 0.0\,W$, $P_{2\Omega} = 0.0\,W$, $P_{3\Omega} = 0.0\,W$, $P_{4\Omega} = 0.0\,W$, $P_{5\Omega} = 0.0\,W$, $P_{6\Omega} = 0.0\,W$, $P_{7\Omega} = 0.0\,W$, $P_{8\Omega} = 0.0\,W$, $P_{9\Omega} = 0.0\,W$, $P_{10\Omega} = 0.0\,W$, $P_{11\Omega} = 0.0\,W$, $P_{12\Omega} = 0.0\,W$, $P_{13\Omega} = 0.0\,W$, $P_{14\Omega} = 0.0\,W$, $P_{15\Omega} = 0.0\,W$, $P_{16\Omega} = 0.0\,W$, $P_{17\Omega} = 0.0\,W$, $P_{18\Omega} = 0.0\,W$, $P_{19\Omega} = 0.0\,W$, $P_{20\Omega} = 0.0\,W$, $P_{21\Omega} = 0.0\,W$, $P_{22\Omega} = 0.0\,W$, $P_{23\Omega} = 0.0\,W$, $P_{24\Omega} = 0.0\,W$, $P_{25\Omega} = 0.0\,W$, $P_{26\Omega} = 0.0\,W$, $P_{27\Omega} = 0.0\,W$, $P_{28\Omega} = 0.0\,W$, $P_{29\Omega} = 0.0\,W$, $P_{30\Omega} = 0.0\,W$, $P_{31\Omega} = 0.0\,W$, $P_{32\Omega} = 0.0\,W$, $P_{33\Omega} = 0.0\,W$, $P_{34\Omega} = 0.0\,W$, $P_{35\Omega} = 0.0\,W$, $P_{36\Omega} = 0.0\,W$, $P_{37\Omega} = 0.0\,W$, $P_{38\Omega} = 0.0\,W$, $P_{39\Omega} = 0.0\,W$, $P_{40\Omega} = 0.0\,W$, $P_{41\Omega} = 0.0\,W$, $P_{42\Omega} = 0.0\,W$, $P_{43\Omega} = 0.0\,W$, $P_{44\Omega} = 0.0\,W$, $P_{45\Omega} = 0.0\,W$, $P_{46\Omega} = 0.0\,W$, $P_{47\Omega} = 0.0\,W$, $P_{48\Omega} = 0.0\,W$, $P_{49\Omega} = 0.0\,W$, $P_{50\Omega} = 0.0\,W$, $P_{51\Omega} = 0.0\,W$, $P_{52\Omega} = 0.0\,W$, $P_{53\Omega} = 0.0\,W$, $P_{54\Omega} = 0.0\,W$, $P_{55\Omega} = 0.0\,W$, $P_{56\Omega} = 0.0\,W$, $P_{57\Omega} = 0.0\,W$, $P_{58\Omega} = 0.0\,W$, $P_{59\Omega} = 0.0\,W$, $P_{60\Omega} = 0.0\,W$, $P_{61\Omega} = 0.0\,W$, $P_{62\Omega} = 0.0\,W$, $P_{63\Omega} = 0.0\,W$, $P_{64\Omega} = 0.0\,W$, $P_{65\Omega} = 0.0\,W$, $P_{66\Omega} = 0.0\,W$, $P_{67\Omega} = 0.0\,W$, $P_{68\Omega} = 0.0\,W$, $P_{69\Omega} = 0.0\,W$, $P_{70\Omega} = 0.0\,W$, $P_{71\Omega} = 0.0\,W$, $P_{72\Omega} = 0.0\,W$, $P_{73\Omega} = 0.0\,W$, $P_{74\Omega} = 0.0\,W$, $P_{75\Omega} = 0.0\,W$, $P_{76\Omega} = 0.0\,W$, $P_{77\Omega} = 0.0\,W$, $P_{78\Omega} = 0.0\,W$, 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31 Mesh-current and nodal analysis

In this section you should be able to:

- write down and solve simultaneous mesh-current equations
- write down and solve simultaneous nodal equations

31.1 Mesh-current analysis

Mesh-current analysis is similar in principle to the use of Kirchhoff's laws explained in Chapter 30. Figure 31.1 shows a network whose resistance values, R_1 to R_6, and voltage sources, V_1 and V_2, are known. The network could be the network shown in Problem 1(a) in Exercise 30.3, and in our current analysis we keep the same

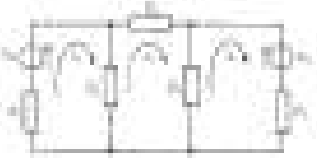


Figure 31.1

As mesh-current analysis, like Kirchhoff's laws, is based on Kirchhoff's second law, as applied to mesh 1 of the loops in the circuit of Figure 31.1 produces three equations in three unknowns which may be solved for I_1, I_2 and I_3. The three equations are as follows (Figure 31.1 is on

$$I_1(R_1 + R_2) - I_2R_2 - I_3R_1 = V_1$$

$$I_2(R_2 + R_3 + R_4) - I_1R_2 - I_3R_4 = 0$$

$$I_3(R_1 + R_4 + R_5) - I_1R_1 - I_2R_4 = -V_2$$

The details and the rules are demonstrated by working the above, one of the most common methods in this branch of the subject. The Cramer's Rule

In comparison to the method of Chapter 10.4, the present one, the so-called **Maxwell's Mesh-Current Method**, is based on the following specialties:

Example 1: Use mesh-current analysis to determine the current flowing in the 2 Ω resistor in the circuit of Figure 11.1.

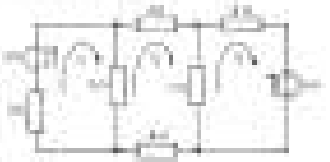


Figure 11.1

The mesh currents I_1, I_2 and I_3 are shown on Figure 11.2. Using Kirchhoff's voltage law:

$$\text{For loop 1} \quad 2I_1 + 2(I_1 - I_2) + 10 = 0 \tag{1.1}$$

$$\text{For loop 2} \quad 2(I_2 - I_1) + 4(I_2 - I_3) + 5(I_2 - 0) + 4I_2 = 0 \tag{1.2}$$

$$\text{For loop 3} \quad 4(I_3 - I_2) + 10I_3 = 0 \tag{1.3}$$

Then

$$2I_1 + 2I_1 - 2I_2 + 10 = 0 \tag{2.1}$$

$$-2I_1 + 2I_2 + 4I_2 - 4I_3 + 5I_2 = 0 \tag{2.2}$$

$$-4I_2 + 4I_3 + 10I_3 = 0 \tag{2.3}$$

Using determinants,

$$\begin{vmatrix} 4 & -2 & 0 \\ -2 & 11 & -4 \\ 0 & -4 & 14 \end{vmatrix} \quad \begin{vmatrix} -10 & -2 & 0 \\ 0 & 11 & -4 \\ 0 & -4 & 14 \end{vmatrix} \quad \begin{vmatrix} 4 & -10 & 0 \\ -2 & 0 & -4 \\ 0 & 0 & 14 \end{vmatrix}$$

$$I_2 = \begin{vmatrix} 4 & -2 & 0 \\ -2 & 11 & -4 \\ 0 & -4 & 14 \end{vmatrix}$$



Figure 8.1

The weak instrument test and χ^2 goodness-of-fit test are performed as follows (Figure 7.11):

Weak instrument test: H_0: $\rho_{x1} = \rho_{x2} = \rho_{x3} = \rho_{x4} = \rho_{x5} = 0$, H_1: $\rho_{x1} \neq 0$ or $\rho_{x2} \neq 0$ or $\rho_{x3} \neq 0$ or $\rho_{x4} \neq 0$ or $\rho_{x5} \neq 0$ (1)

Weak instrument test: H_0: $\rho_{z1} = \rho_{z2} = \rho_{z3} = \rho_{z4} = \rho_{z5} = 0$, H_1: $\rho_{z1} \neq 0$ or $\rho_{z2} \neq 0$ or $\rho_{z3} \neq 0$ or $\rho_{z4} \neq 0$ or $\rho_{z5} \neq 0$ (2)

Without overfitting and χ^2 [2] setup [recommended] given:

$$\begin{bmatrix} \cdots & \cdots \\ \cdots & \cdots \end{bmatrix}^{-1} \begin{bmatrix} \cdots & \cdots \\ \cdots & \cdots \end{bmatrix}$$

$$= \begin{bmatrix} \cdots & \cdots \\ \cdots & \cdots \end{bmatrix}$$

$$\cdots \cdots \cdots \

4.2 Nodal analysis

A **node** of a network is defined as a point where two or more branches are joined. A branch is any two-terminal element in a circuit. Note that simply a branch **connects** two nodes. In Figure 4.1, there are 3 nodes, 1, 2, 3, and 3 branches, which are 1, 2 and 3, respectively.

A node voltage is the voltage of a particular node with respect to a node selected as reference. In Figure 4.1, for example, node 3 is chosen as the reference node. Node V_{n1} is assigned to node 1 to indicate its voltage at node 1 with respect to node 3, and likewise, node V_{n2} is assigned to node 2. Also, V_{n1} is the voltage between node 1 and node 3, and V_{n2} is the voltage between node 2 and node 3. Therefore, the voltage between nodes 1 and 2 can be determined by $V_{n1} - V_{n2}$. Similarly, the voltage between nodes 2 and 1 is $V_{n2} - V_{n1}$. Obviously, the voltage between node 1 and node 2 is the same as that between node 2 and node 1.

The objective of nodal analysis is to determine the values of voltages at the nodes relative to the reference node. In the following circuits, e.g., in Figure 4.1, and Figure 4.2, we choose the bottom node as the reference node. The reference node is also known as the ground node since it is at zero potential.

Kirchhoff's current law is applied at nodes 1 and 2 in Figure 4.1. Figure 4.2 shows equations corresponding to nodes 1 and 2 are obtained which are simultaneously solved using determinants.

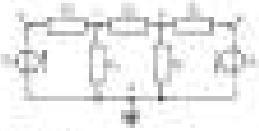


Figure 4.1



Figure 4.2

The networks leading to nodes 1 and 2 are shown separately in Figure 4.2. Let us assume that all branch currents are leaving the node as shown. Then the sum of currents at a particular node is

$$\frac{V_{n1} - V_{n2}}{R_1} + \frac{V_{n1}}{R_2} + \frac{V_{n1} - V_{n2}}{R_3} = I_1 \tag{4.1}$$

Similarly, for node 2, assuming all branch currents are leaving the node as shown in Figure 4.2,

$$\frac{V_{n2} - V_{n1}}{R_3} + \frac{V_{n2}}{R_4} + \frac{V_{n2}}{R_5} = I_2 \tag{4.2}$$

In equations (4.1) and (4.2), the unknowns are all assumed to be leaving the node. If any one assumption has been violated in the branch currents into the node, i.e., the corresponding unknown will be obtained as a negative value in the solution. If the node 1 is chosen as the reference node instead of the bottom node, 3 nodes will be left, and the equations for node 1 becomes

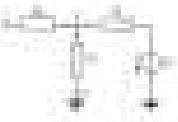


Figure 4.3

$$\frac{1}{R_1}=\frac{1}{R_{11}}+\frac{1}{R_{12}}+\frac{1}{R_{13}}+\frac{1}{R_{14}}$$

Hence, it is important to specify by double the value on equation (3.1).

Rearranging equations 3 and 4 gives:

$$\left(\frac{1}{R_{21}}+\frac{1}{R_{22}}+\frac{1}{R_{23}}\right)V_2-\left(\frac{1}{R_{22}}\right)V_3-\left(\frac{1}{R_{23}}\right)V_4=I_2 \qquad (3)$$

$$-\left(\frac{1}{R_{32}}\right)V_2+\left(\frac{1}{R_{31}}+\frac{1}{R_{32}}+\frac{1}{R_{33}}\right)V_3-\left(\frac{1}{R_{33}}\right)V_4=I_3 \qquad (4)$$

Equations 3 and 4 can be used to complete to solve a solid-state circuit, where substitute $R=1\,k\Omega$:

$$(0.5)V_2-(0.2)V_3-(0.3)V_4=I_2=0 \qquad (5)$$

$$-(0.2)V_2+(0.5)V_3-(0.3)V_4=I_3=0 \qquad (6)$$

Substitute Thevenin's equivalent equations 5 and 6 in 3 to solve simultaneously. Then:

$$\begin{vmatrix} 0.5 & -0.2 & -0.3 \\ -0.2 & 0.5 & -0.3 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\begin{vmatrix} 0.5 & -0.2 & -0.3 \\ -0.2 & 0.5 & -0.3 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\begin{vmatrix} 0.5 & -0.2 & -0.3 \\ -0.2 & 0.5 & -0.3 \\ 0 & 0 & 1 \end{vmatrix}$$

Linear equations and linear (voltage) equations are to be solved in order to find a unique set of x and y that satisfy the equations. A mathematical model. The solution of equations involves two methods: a solution and a method to be used, which can be used for the solution of simultaneous equations.

Matrix methods and linear algebra to solve problems used to determine the values of unknown quantities in the form of a matrix. Matrix algebra for the solution of linear equations and linear equations of algebra.

Problem 4. Find the currents shown in Figure 3.4 to determine the voltage in e.g. for linear simultaneously.



Figure 3.4



Figure E2.6

Figure E2.6 shows a circuit. Applying Kirchhoff's current law at each node, we obtain

$$i_1 + i_2 = i_3 + i_4$$

or

$$\frac{v_1}{R_1} + \frac{v_2}{R_2} = \frac{v_3}{R_3} + \frac{v_4}{R_4} \qquad (1)$$

Then

$$v_1\left(\frac{1}{R_1}\right) + v_2\left(\frac{1}{R_2}\right) = (1)$$

$$v_3\left(\frac{1}{R_3}\right) + v_4\left(\frac{1}{R_4}\right) = (2)$$

$$v_1(0.0833) + v_2(0.1) = (1)$$

$$v_3(0.2) + v_4(0.25) = (2)$$

Now, solving Eq. (1),

$$v_1 = \frac{(1) - v_2(0.1)}{0.0833} \qquad v_3 = \frac{(2) - v_4(0.25)}{0.2}$$

or, voltage v_1 is

The current through the $4\ \Omega$ resistor (i_4) is

Since the voltage and current across each R_1 (i.e., across the $1\ \Omega$ resistor) is known,

$$P_{1\Omega} = \text{(value)} \times \text{(value)} = \frac{P_{1\Omega}(w)}{(\text{value})^2} = \frac{P_{1\Omega}(\text{value})^2}{R_{1\Omega}(\Omega)} \quad (\text{value}) \quad R_{1\Omega}(\Omega)$$

> **Problem 1** Determine the value of voltage v_1 shown in the circuit of Figure E2.6.

Then, at the common node between the $2\ \Omega$ resistor and the $3\ \Omega$ resistor, the current leaving the $2\ \Omega$ resistor must equal the current entering the $3\ \Omega$ resistor. Hence,

Then

$$\frac{v_1 - 8.67}{2\ \Omega} = \frac{8.67 - v_2}{3\ \Omega}$$

Now, solving

$$v_1\left(\frac{1}{2}\right) + v_2\left(\frac{1}{3}\right) = \frac{8.67}{2} + \frac{8.67}{3}$$

$$v_1\left(\frac{3}{6}\right) + v_2\left(\frac{2}{6}\right) = \frac{26.01}{6} + \frac{17.34}{6}$$



Figure 4.11 (a)

Problem 4. Use nodal analysis to determine the branch currents in each branch of the network shown in Figure 4.11(a).

Step 1: The nodes are shown in position 1 of Figure 4.11(a) and the reference node is chosen as the bottom node.

Step 2: Only two unknown nodal voltages exist, namely V_1 and V_2.

Step 3: Applying Kirchhoff's current law to node 1 of Figure 4.11(a) gives

$$\frac{V_1 - 10.0}{2.0} + \frac{V_1}{4.0} + \frac{V_1 - V_2}{5.0} = 0$$

i.e. $\left(\frac{1}{2.0} + \frac{1}{4.0} + \frac{1}{5.0}\right)V_1 - \frac{1}{5.0}V_2 = \frac{10.0}{2.0}$

i.e. $0.95\,V_1 - 0.2\,V_2 = 5.0 \quad (1)$

Applying Kirchhoff's current law to node 2 gives

$$\frac{V_2 - V_1}{5.0} + \frac{V_2}{10.0} + \frac{V_2 - 15.0}{3.0} = 0$$

i.e. $-\frac{1}{5.0}V_1 + \left(\frac{1}{5.0} + \frac{1}{10.0} + \frac{1}{3.0}\right)V_2 = \frac{15.0}{3.0}$

i.e. $-0.2\,V_1 + 0.633\,V_2 = 5.0 \quad (2)$

Solving equations (1) and (2) simultaneously gives

$$V_1 = \frac{\begin{vmatrix} 5.0 & -0.2 \\ 5.0 & 0.633 \end{vmatrix}}{\begin{vmatrix} 0.95 & -0.2 \\ -0.2 & 0.633 \end{vmatrix}} = \frac{3.165 + 1.0}{0.6015 - 0.04} = \frac{4.165}{0.5615} = 7.417\text{ V}$$

and

$$V_2 = \frac{\begin{vmatrix} 0.95 & 5.0 \\ -0.2 & 5.0 \end{vmatrix}}{0.5615} = \frac{4.75 + 1.0}{0.5615} = \frac{5.75}{0.5615} = 10.24\text{ V}$$

Current I_1 in the 2.0 Ω resistor,

$$I_1 = \frac{10.0 - V_1}{2.0} = \frac{10.0 - 7.417}{2.0} = \frac{2.583}{2.0} = 1.292\text{ A}$$

Current I_2 in the 4.0 Ω resistor,

$$I_2 = \frac{V_1}{4.0} = \frac{7.417}{4.0} = 1.854\text{ A}$$

Current I_3 in the 5.0 Ω resistor,

$$I_3 = \frac{V_1 - V_2}{5.0} = \frac{7.417 - 10.24}{5.0} = -0.565\text{ A}$$

(Negative sign means I_3 is reverse)

The current in the 4 Ω resistor is

$$I_4 = \frac{V_2 - V_3}{R_4} = \frac{12.47 - 4.16}{4} = \frac{8.31}{4} = 2.08\ \text{A}$$

—(3.31) —(4.16) A energy from node 3

$$I_{2\Omega} = \frac{V_1 - V_2}{R_{2\Omega}} = \frac{20 - 12.47}{2} = \frac{7.53}{2} = 3.77\ \text{A}$$

Problem 7. In the network of Figure 3.17 use nodal analysis to determine (a) the voltage at nodes 1 and 2, (b) the current in the 4 Ω resistor in the direction shown, (c) the power dissipated in the 2 Ω resistor, and (d) the power supplied by the source. Assume the network is in the steady state.

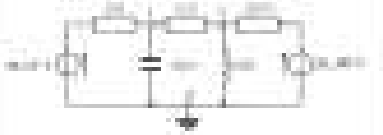


Figure 3.17

(a) At node 1, $\frac{V_1 - 20}{2} + \frac{V_1}{4} + \frac{V_1 - V_2}{1} = 0$

Rearranging gives:

$$\frac{1}{2}V_1 - 10 + \frac{1}{4}V_1 + V_1 - V_2 = 0$$

$$1.75V_1 - V_2 = 10 \quad (1)$$

At node 2, $\frac{V_2 - V_1}{1} + \frac{V_2}{4} = 3$

Rearranging gives:

$$-V_1 + V_2 + \frac{1}{4}V_2 = 3 \quad \text{or} \quad -V_1 + 1.25V_2 = 3 \quad (2)$$

Now, two simultaneous equations have been derived with two unknowns V_1 and V_2. Using determinants, if

$$D = \begin{vmatrix} 1.75 & -1 \\ -1 & 1.25 \end{vmatrix} = 2.19 - 1 = 1.19$$

Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$. Use elementary row operations to find A^{-1}.

Solution. Write A and I side by side and reduce A to I:

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 3 & 0 & 1 & 0 \\ 1 & 0 & 8 & 0 & 0 & 1 \end{array}\right] \xrightarrow[R_3 \to R_3 - R_1]{R_2 \to R_2 - 2R_1} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & -2 & 5 & -1 & 0 & 1 \end{array}\right]$$

$$\xrightarrow[R_3 \to R_3 + 2R_2]{R_1 \to R_1 - 2R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 9 & 5 & -2 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & -1 & -5 & 2 & 1 \end{array}\right] \xrightarrow{R_3 \to -R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 9 & 5 & -2 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right]$$

$$\xrightarrow[R_2 \to R_2 + 3R_3]{R_1 \to R_1 - 9R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -40 & 16 & 9 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right].$$

Hence $A^{-1} = \begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix}$.

Check. $AA^{-1} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}\begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$. $\checkmark$

Example 3.5.2. Solve the system $\begin{cases} x + 2y + 3z = 14 \\ 2x + 5y + 3z = 19 \\ x + 8z = 17 \end{cases}$ using the inverse found above.

Solution. In matrix form $Ax = b$ with $b = \begin{bmatrix} 14 \\ 19 \\ 17 \end{bmatrix}$, so $x = A^{-1}b$:

$$x = \begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix}\begin{bmatrix} 14 \\ 19 \\ 17 \end{bmatrix} = \begin{bmatrix} -560 + 304 + 153 \\ 182 - 95 - 51 \\ 70 - 38 - 17 \end{bmatrix} = \begin{bmatrix} -103 \\ 36 \\ 15 \end{bmatrix}.$$

Thus $x = -103$, $y = 36$, $z = 15$. Substituting back into the original equations confirms the solution.

Example 3.5.3. Find the inverse of $B = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$, if it exists.

Solution. Reduce $[B \mid I]$:

$$\left[\begin{array}{ccc|ccc} 2 & 1 & 0 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_2 \to R_2 - 2R_1} \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & -3 & -2 & 1 & -2 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 + \frac{1}{3}R_2} \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & -3 & -2 & 1 & -2 & 0 \\ 0 & 0 & \frac{4}{3} & \frac{1}{3} & -\frac{2}{3} & 1 \end{array}\right] \xrightarrow{R_3 \to \frac{3}{4}R_3} \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & -3 & -2 & 1 & -2 & 0 \\ 0 & 0 & 1 & \frac{1}{4} & -\frac{1}{2} & \frac{3}{4} \end{array}\right].$$

Back-substitution gives $B^{-1} = \begin{bmatrix} \frac{3}{4} & -\frac{1}{2} & \frac{1}{4} \\ -\frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{2} & \frac{3}{4} \end{bmatrix}$.

Example 3.5.4. Show that $C = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ is not invertible.

Solution. Row 2 of C is twice row 1, so $\det C = 1 \cdot 4 - 2 \cdot 2 = 0$. Equivalently, $R_2 \to R_2 - 2R_1$ produces a zero row, and no sequence of operations can produce I. Hence C^{-1} does not exist.

Example 3.5.5. Let $D = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 3 & 1 \\ 4 & 1 & 0 \end{bmatrix}$. Compute $\det D$ by cofactor expansion along the first row.

Solution.

$$\det D = 1 \cdot \begin{vmatrix} 3 & 1 \\ 1 & 0 \end{vmatrix} - 0 \cdot \begin{vmatrix} 0 & 1 \\ 4 & 0 \end{vmatrix} + 2 \cdot \begin{vmatrix} 0 & 3 \\ 4 & 1 \end{vmatrix} = 1(0 - 1) - 0 + 2(0 - 12) = -1 - 24 = -25.$$

Example 3.5.6. Find the area of the triangle with vertices $P(1, 2)$, $Q(4, 6)$, $R(5, 1)$.

Solution. Translate so that P is the origin: $\overrightarrow{PQ} = (3, 4)$, $\overrightarrow{PR} = (4, -1)$. The area is half the absolute value of the determinant:

$$\text{Area} = \frac{1}{2}\left|\begin{vmatrix} 3 & 4 \\ 4 & -1 \end{vmatrix}\right| = \frac{1}{2}|-3 - 16| = \frac{19}{2}.$$

Example 3.5.7. Use Cramer's rule to solve $\begin{cases} 2x + y = 5 \\ x - 3y = -1 \end{cases}$.

Solution. $\det A = \begin{vmatrix} 2 & 1 \\ 1 & -3 \end{vmatrix} = -6 - 1 = -7$. Replacing the first column by b gives $\det A_1 = \begin{vmatrix} 5 & 1 \\ -1 & -3 \end{vmatrix} = -15 + 1 = -14$, and replacing the second column by b gives $\det A_2 = \begin{vmatrix} 2 & 5 \\ 1 & -1 \end{vmatrix} = -2 - 5 = -7$. Therefore

$$x = \frac{\det A_1}{\det A} = \frac{-14}{-7} = 2, \qquad y = \frac{\det A_2}{\det A} = \frac{-7}{-7} = 1.$$

Example 3.5.8. Find the volume of the parallelepiped spanned by $u = (1, 0, 2)$, $v = (3, 1, 1)$, $w = (0, 2, 4)$.

Solution. The volume is the absolute value of the scalar triple product $|u \cdot (v \times w)|$, which equals $|\det[u \; v \; w]|$:

$$\det \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 2 \\ 2 & 1 & 4 \end{bmatrix} = 1(4 - 2) - 0 + 2(6 - 0) = 2 + 12 = 14.$$

Hence the volume is 14 cubic units.

Example 3.5.9. Determine whether the vectors $(1, 2, 3)$, $(4, 5, 6)$, $(7, 8, 9)$ are linearly independent.

Solution. Form the matrix with these vectors as rows and compute its determinant:

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 1(45 - 48) - 2(36 - 42) + 3(32 - 35) = -3 + 12 - 9 = 0.$$

Since the determinant is zero, the vectors are linearly dependent.

$$\frac{\text{(numerator)}}{\text{(denominator)}} \quad \text{By use of Kirchhoff's current law,}$$

$$\text{(equation)} \quad (1)$$

Then, the magnitude of the current passing through each of the resistors is calculated as follows:

Problem 2.3 — In the network shown in Figure 2.3, determine the voltage V_{ab} using nodal analysis.



Figure 2.3

Node a is taken as the reference node.

$$\text{At node } c: \qquad \text{(equation)} \qquad \text{(equation)}$$

$$\text{or,} \qquad \text{(equation)}$$

$$\text{or,} \qquad \text{(equation)} \qquad (1)$$

$$\text{At node } b: \qquad \text{(equation)}$$

$$\text{or,} \qquad \text{(equation)}$$

$$\text{or,} \qquad \text{(equation)}$$

$$\text{or,} \qquad \text{(equation)} \qquad (2)$$

The simultaneous equations (1) and (2) may be solved for V_1 and V_2 by using determinants. Thus

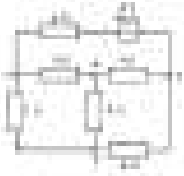


Figure E3.10

Problem 3. Use mesh analysis to determine the voltages across each 1-ohm and 2-ohm resistor in the circuit shown in Figure P3.3, and hence calculate the power absorbed by each of the 1-Ω resistors and the power developed by the 3-A source.

(a) $\quad -5.0\,I_1 + 2.0\,I_2 + 1.0\,I_3 = -12 \quad (1)$

(b) Node 1: $\quad \frac{V_1}{4.0} + \frac{V_1 - V_2}{2.0} = I_1 \quad \Rightarrow \quad 0.75\,V_1 - 0.50\,V_2 = I_1$

(c) $\quad 0.50\,V_1 - 0.50\,V_2 + 1.0\,V_3 = 12 \quad (2)$

Substitute (1) into (2) and combine with $V_1 = 2.0\,I_2$ and $V_3 = 6.0\,I_3$ to obtain a three-equation, three-unknown system. Express

$$\begin{bmatrix} -5.0 & 2.0 & 1.0 \\ 0.50 & -0.50 & 1.0 \\ 0.75 & -0.50 & 0.0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -12 \\ 12 \\ 0.0 \end{bmatrix}$$

$$\begin{bmatrix} -5.00 & 2.0 & 1.0 \\ 0.50 & -0.50 & 1.0 \\ 0.75 & -0.50 & 0.0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -12 \\ 12 \\ 0.0 \end{bmatrix}$$

Solving for I_1 gives $I_1 = \dfrac{\det(A_1)}{\det(A)}$

$$= \frac{\begin{vmatrix} -12 & 2.0 & 1.0 \\ 12 & -0.50 & 1.0 \\ 0.0 & -0.50 & 0.0 \end{vmatrix}}{\begin{vmatrix} -5.00 & 2.0 & 1.0 \\ 0.50 & -0.50 & 1.0 \\ 0.75 & -0.50 & 0.0 \end{vmatrix}} = \frac{-12(0.50) - 2.0(12) + 1.0(-6.0)}{-5.00(0.50) - 2.0(0.75) + 1.0(-0.125)}$$

$$= \frac{-36}{-4.125} = 8.727\ \text{A}$$

Then the current in the 2 Ω resistor is $\dfrac{V_1}{2.0} = \dfrac{2.0\,I_2}{2.0} = 8.73\ \text{A}$, flowing downward in the sketch.

Solving for I_2 gives $I_2 = \dfrac{\det(A_2)}{\det(A)} = \dfrac{\begin{vmatrix} -5.00 & -12 & 1.0 \\ 0.50 & 12 & 1.0 \\ 0.75 & 0.0 & 0.0 \end{vmatrix}}{-4.125} = \dfrac{-27.0}{-4.125} = 6.545\ \text{A}$

Hence $\dfrac{V_2}{1.0\,\Omega} = \dfrac{1}{1.0\,\Omega}$ (from nodal voltage V_2) $= \dfrac{2.0\,I_2}{1.0\,\Omega}$

$$= 13.09\ \text{A}$$

Power applied to 2 Ω resistor $= (I_1)^2(2.0) = \left(\dfrac{V_1}{2.0}\right)^2 (2.0)$

$$= \frac{(17.455)^2}{2.0} = 152.4\ \text{W}$$

Problem worksheet, two nodal analysis case, for digital or manual use. Answer key on page 181.

3.12 Find v_o in Fig. 3.12 by using mesh-current analysis.



Figure 3.12



Figure 3.13

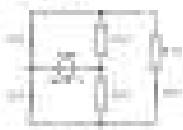


Figure 3.14

Mesh-current analysis

1. Mesh currents i_1, i_2, i_3, and i_4 are defined in Fig. 3.13 for using mesh-current analysis.

2. For the supermesh shown in Figure 3.13, we apply KVL around it to obtain (or we can apply KVL around mesh 1 and then around mesh 2 and add the two equations)

$$-5 + 10i_1 + 10(i_1 - i_2) + 20(i_3 - i_2) + 10i_3 = 0$$

3. The mesh-current equations for dependent sources I_x, I_y, and I_z in the network shown in Figure 3.13 are

$$I_x = 2(i_2 - i_1), \quad I_y = 5(i_4 - i_3), \quad I_z = 10(i_2 - i_3)$$

$$i_4 = 10I_z = 10 \times 10(i_2 - i_3) = 100(i_2 - i_3)$$

4. For the supermesh shown in Figure 3.13, determine the current I_y flowing in the $5\,\Omega$ resistance:

$$I_y = i_4 - i_3 \tag{1}$$

5. For the network shown in Figure 3.13, the mesh-current equations in matrix form for mesh 1 to mesh 4 are obtained as

$$\begin{bmatrix} 20 & -10 & -20 & 0 \\ -10 & 20 & 0 & 0 \\ -20 & 0 & 30 & 0 \\ 0 & 0 & -100 & 101 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

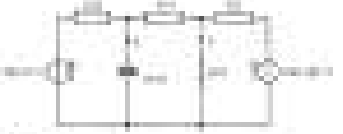


Figure 3.15

6. Determine the value of the resistance R_x in the network shown in Figure 3.15 for the mesh-current equations

$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 3 & -1 & 0 \\ 0 & -1 & 4 & -R_x \\ 0 & 0 & -R_x & 2R_x \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

7. In the network in Figure 3.15, two mesh-current analyses to determine the current in the resistance R_x are required. For the network in Fig. 3.15 resistance R_x is the current passing through the $2\,\Omega$ resistance, and for the equivalent circuit we need the $2\,\Omega$ resistance.

$$I_{2\Omega} = \frac{2(2)}{2 + 2} = 1\ \text{A}$$

$$I_{2\Omega} = \frac{10(2)}{2 + 2 + 6} = 1\ \text{A}$$

8. In Problem 3.8 we have determined that the network in Figure 3.16 has mesh-current equations in matrix form as follows. Solve for mesh-current

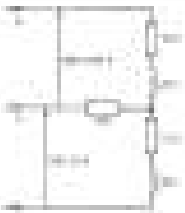


Figure A1.18



Figure A1.19



Figure A1.20

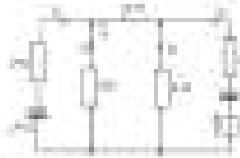


Figure A1.21



Figure A1.22



Figure 3-3(a)

3-3 Determine the node voltages in the circuit shown in Figure 3-3(a). Also, calculate the power supplied by each of the voltage sources.

$$\begin{bmatrix} \\ \\ \end{bmatrix} \begin{bmatrix} \\ \\ \end{bmatrix} = \begin{bmatrix} \\ \\ \end{bmatrix}$$

3-4 Determine the voltage V_{x2} in the network shown in Figure 3-4(a).

$$[\quad] \begin{bmatrix} \\ \\ \end{bmatrix} = \begin{bmatrix} \\ \\ \end{bmatrix}$$

3-5 Use nodal analysis to determine the current I_{x2} in each element in the network of Figure 3-5(a).

$$\begin{bmatrix} \\ \\ \end{bmatrix} \begin{bmatrix} \\ \\ \end{bmatrix} = \begin{bmatrix} \\ \\ \end{bmatrix}$$



Figure 3-4(a)

3-6 Use the network shown in Figure 3-6(a) to determine (a) the voltage at node 1 and (b) the current in each of the resistors. (c) Also, calculate the power supplied by each of the sources, and (d) the power dissipated in the resistor shown between nodes 2 and 3.

(a) $\begin{bmatrix} \\ \\ \end{bmatrix} \begin{bmatrix} \\ \\ \end{bmatrix} = \begin{bmatrix} \\ \\ \end{bmatrix}$

(b) $[\quad] \begin{bmatrix} \\ \\ \end{bmatrix} = \begin{bmatrix} \\ \\ \end{bmatrix}$ (c) $[\quad] \begin{bmatrix} \\ \\ \end{bmatrix} = \begin{bmatrix} \\ \\ \end{bmatrix}$



Figure 3-5(a)

Figure 3-6(a)

3-7 Determine the voltage V_{x2} in the network of Figure 3-7(a), using nodal analysis.

$$\begin{bmatrix} \\ \\ \end{bmatrix} \begin{bmatrix} \\ \\ \end{bmatrix} = \begin{bmatrix} \\ \\ \end{bmatrix}$$

32 The superposition theorem

In this unit you should learn about:

- when and how to use the superposition theorem

32.1 Introduction

The superposition theorem states:

In any network containing two or more sources of emf, the current in any branch is the algebraic sum of the currents which would flow in that branch if each source were considered separately, all other sources being replaced at that time by their respective internal impedances.

32.2 Using the superposition theorem

The superposition theorem, which was introduced in Chapter 31 (see p. 31), provides a way for applying Kirchhoff's laws in circuits containing two sources of emf. Figure 32.1 (see below) is a circuit which we shall use to illustrate the application of the superposition theorem.

The theorem requires that each source of emf is considered separately and the following procedure can be adopted:

(a) Redraw the original network with one of the sources only left in, replacing each other source by its internal resistance, as shown in Figure 32.2.

(b) Label the currents in each branch and its direction, as shown in Figure 32.2. Note that directions are taken from the positive terminal of the source.

(c) Apply Kirchhoff's second law to any mesh in the network, as shown in Figure 32.2. Note that the battery of emf E_2 is ignored as the source internal resistance is taken as flowing from the positive terminal of the battery.

(d) Repeat steps (a) to (c) above for the other source of emf, as shown in Figure 32.3.

(e) Add the currents in each branch to determine the actual current flowing in each branch of the original network.

Figure 32.1

Figure 32.2

Figure 32.3

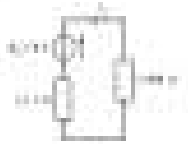


Figure Q1-1

From Figure 1A-1,

$$\text{current } I_1 = \left(\frac{20}{R_1 + R_2}\right) \text{A} = \left(\frac{20}{2+3}\right) \text{A} = 4 \text{ A}$$

$$\text{and voltage } V_2 = I_1 \times R_2 = 4 \text{ A} \times 3\,\Omega = 12 \text{ V}$$

(b) Redraw the original network with only source E_1 restored and replaced by a wire, as shown in Figure 1A-2.

(c) Label the currents in each branch and their directions as shown in Figure 1A-2, and determine their values.

If I_2 and I_3 in parallel carry an equivalent combined current I':

$$I' = I_2 + I_3 = \text{ 3 A + 2 A = 5 A from left to right}$$

as shown on the equivalent network in Figure 1A-3. From Figure 1A-3,

$$\text{resistance } R_{2,3} = \frac{R_2}{1 + \text{ratio}} = \frac{3}{1+1.5} = 1.2\,\Omega$$

From Figure 1A-4,

$$\text{voltage } V_{2,3} = I' \times R_{2,3} = 5 \text{ A} \times 1.2\,\Omega = 6 \text{ V}$$

$$\text{and current } I_1 = \left(\frac{V}{R_1}\right) \text{A} = \left(\frac{6}{2}\right) \text{A} = 3 \text{ A}$$

(d) Superimpose Figure 1A-2 on Figure 1A-1 as shown in Figure 1A-5.

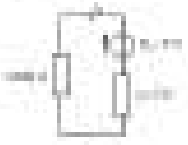


Figure Q1-2



Figure Q1-3

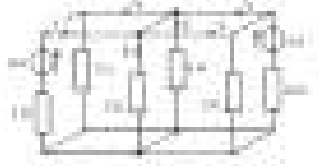


Figure 1A-4

(e) Determine the resistance value of the equivalent single resistor which could replace the two resistors in the branch in which the unknown load is connected.

From Figure 10.1, the resistors connect directly to the positive and negative terminals of the battery. So

$V_1 = V_2 = 9.0\ \text{V}$ (the battery voltage appears across each resistor, because they are both connected directly across the battery).

The current through R_1 is

$I_1 = \frac{V_1}{R_1} = \frac{9.0\ \text{V}}{3.0\ \Omega} = 3.0\ \text{A}$ (Ohm's law).

The current through R_2 is

$I_2 = \frac{V_2}{R_2} = \frac{9.0\ \text{V}}{6.0\ \Omega} = 1.5\ \text{A}$ (Ohm's law).

The total current I is the sum of these currents:

$I = I_1 + I_2 = 3.0\ \text{A} + 1.5\ \text{A} = 4.5\ \text{A}$

The voltage V across the battery is the same as the voltage across each resistor, because they are connected directly across the battery.

$V = V_1 = V_2 = 9.0\ \text{V}$

The total resistance R of the circuit is

$R = \frac{V}{I} = \frac{9.0\ \text{V}}{4.5\ \text{A}} = 2.0\ \Omega$

The total resistance R of the circuit is also given by

$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{3.0\ \Omega} + \frac{1}{6.0\ \Omega} = \frac{1}{2.0\ \Omega}$

$R = 2.0\ \Omega$

The total power P dissipated by the circuit is

$P = IV = (4.5\ \text{A})(9.0\ \text{V}) = 40.5\ \text{W}$

The power P_1 dissipated by R_1 is

$P_1 = I_1 V_1 = (3.0\ \text{A})(9.0\ \text{V}) = 27\ \text{W}$

The power P_2 dissipated by R_2 is

$P_2 = I_2 V_2 = (1.5\ \text{A})(9.0\ \text{V}) = 13.5\ \text{W}$

The total power P is the sum of these powers:

$P = P_1 + P_2 = 27\ \text{W} + 13.5\ \text{W} = 40.5\ \text{W}$

The following problem demonstrates how to use the principles we have developed to analyze a circuit with resistors in parallel. The solution is straightforward but tedious. Here is Problem 7 from Halliday's and Resnick's textbook, discussed in Chapter 10, problem number 10 on page 218.

> Problem 7 — (a) In Figure 10.2, with identical batteries and identical resistors, which resistor dissipates more energy, R_1 or R_2? (b) Is the energy dissipated by R_1 greater than, less than, or the same as the energy dissipated by R_3? (c) Rank the resistors according to the energy they dissipate, most energy first. (d) Is the total energy dissipated by the resistors greater than, less than, or the same as the energy supplied by the batteries?

A link to the same problem as worked by J. Hedgcock (Harvard undergraduate) can be found on page 10.3 and a comparison of results is made by Wolfe.

The circuit diagram is shown in Figure 10.2. Following the above principles:

(a) The resistor on problem number four, R_4 in Figure 10.2, resistor connected as shown in Figure 10.2.

(b) Resistors R_1 and R_2 are identical as shown in Figure 10.2.

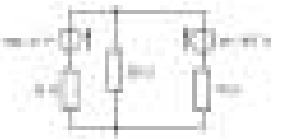


Figure 10.1

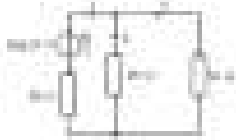


Figure 10.2

$$I_1 = \frac{\cos\alpha\cos\beta}{\cos^2\alpha\cos^2\beta + \sin^2\alpha\sin^2\beta} + \frac{\sin\alpha\sin\beta}{\cos^2\alpha\cos^2\beta + \sin^2\alpha\sin^2\beta} \quad (1.414\cos(\omega t - \alpha))$$

$$= \left(\frac{\cos\alpha}{\cos^2\alpha + \sin^2\alpha}\right)(1.414\cos\omega t) + \left(\frac{\sin\alpha}{\cos^2\alpha + \sin^2\alpha}\right)(1.414\sin\omega t)$$

$$I_2 = \left(\frac{\cos\beta}{\cos^2\beta + \sin^2\beta}\right)(1.414\cos\omega t) + \left(\frac{\sin\beta}{\cos^2\beta + \sin^2\beta}\right)(1.414\sin\omega t)$$

(c) The amplitude of oscillation is then $\sqrt{(1.414)^2 + (1.414)^2} = 2$ amperes, consistent with the answer in Figure 10-5.

(d) Consider $\alpha = \beta$, and let the identities be known, as in Figure 10-5.

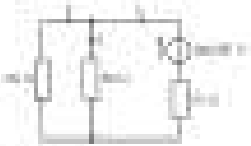
$$I_1 = \frac{\cos\alpha\cos\alpha}{\cos^2\alpha\cos^2\alpha + \sin^2\alpha\sin^2\alpha} + \frac{\sin\alpha\sin\alpha}{\cos^2\alpha\cos^2\alpha + \sin^2\alpha\sin^2\alpha}$$

$$= 1.414\cos(\omega t - \alpha) + 1.414\sin(\omega t - \alpha)$$

$$I_1 = \left(\frac{\cos\alpha}{\cos^2\alpha + \sin^2\alpha}\right)(1.414\cos\omega t) + \left(\frac{\sin\alpha}{\cos^2\alpha + \sin^2\alpha}\right)(1.414\sin\omega t)$$

$$I_2 = \left(\frac{\cos\alpha}{\cos^2\alpha + \sin^2\alpha}\right)(1.414\cos\omega t) + \left(\frac{\sin\alpha}{\cos^2\alpha + \sin^2\alpha}\right)(1.414\sin\omega t)$$

(e) Figure 10-6a shows Figure 10-5 redrawn and Figure 10-6b shows the simplified circuit.


1 Ω
1 Ω
1 Ω

Problem 2. Use the superposition theorem to determine the current in the 4 Ω resistor of the network shown in Figure 2.14.

(a) Removing the 10 V source gives the network shown in Figure 2.15(a).



Figure 2.14



Figure 2.15

(b) Currents I_1 and I_2 are shown in Figure 2.15(b). If the 4 Ω, 8 Ω and 12 Ω resistors connected in parallel are replaced by an equivalent resistance R_P, then $\frac{1}{R_P} = \frac{1}{4} + \frac{1}{8} + \frac{1}{12} = \frac{6+3+2}{24} = \frac{11}{24}$, hence $R_P = \frac{24}{11}\ \Omega$. The circuit then reduces to that shown in Figure 2.15(c). The total resistance $R_T = 2 + \frac{24}{11} = \frac{46}{11}\ \Omega$, hence the total current $I_T = \frac{V}{R_T} = \frac{10}{46/11} = \frac{55}{23}$ A. Current $I_1 = \frac{55}{23}$ A. Using current division, current $I_2 = \left(\frac{\frac{24}{11}}{\frac{24}{11}+4}\right)\left(\frac{55}{23}\right) = \left(\frac{24}{68}\right)\left(\frac{55}{23}\right) = \frac{30}{23}$ A



Figure 2.16

(c) Removing the 10 V source gives the network shown in Figure 2.16.



Figure 2.17

(d) Currents I_3, I_4 and I_5 are shown in Figure 2.17. Let R_P be the equivalent resistance of the 4 Ω, 8 Ω and 12 Ω resistors connected in parallel, then $\frac{1}{R_P} = \frac{1}{4} + \frac{1}{8} + \frac{1}{12} = \frac{6+3+2}{24} = \frac{11}{24}$, hence $R_P = \frac{24}{11}\ \Omega$. The circuit then reduces to that shown in Figure 2.17(a). The total resistance $R_T = 3 + \frac{24}{11} = \frac{57}{11}\ \Omega$, hence the total current $I_T = \frac{V}{R_T} = \frac{20}{57/11} = \frac{220}{57}$ A. Current $I_5 = \frac{220}{57}$ A. Using current division, current $I_4 = \left(\frac{\frac{24}{11}}{\frac{24}{11}+4}\right)\left(\frac{220}{57}\right) = \left(\frac{24}{68}\right)\left(\frac{220}{57}\right) = \frac{120}{57}$ A. Current $I_3 = \left(\frac{4}{4+8+12}\right)\left(\frac{220}{57}\right) = \left(\frac{1}{6}\right)\left(\frac{220}{57}\right) = \frac{110}{171}$ A. Hence, from Figure 2.17, current in the 4 Ω resistor $= I_3 - I_4 = \frac{110}{171} - \frac{120}{57} = \frac{110-360}{171} = -\frac{250}{171}$ A. Thus the current in the 4 Ω resistor is $\frac{250}{171}$ A flowing from right to left.

(c) Calculate the power dissipated in the 10 Ω resistor in the circuit shown in Figure 12.36 when the source voltage is given by $v_s = 5\cos 2t$ V.

(d) $v_{out}(t) = 0.5\cos(2t - 0.46)$ V. **Hint:** Use the Thevenin theorem to find the equivalent circuit seen by the capacitor. Then use the voltage divider rule to find V_{out} as a function of V_s. Finally, use the phasor relationship between V_{out} and V_s to find $v_{out}(t)$ from $v_s(t)$.

Problem 3 The time-dependent Thevenin or Norton theorem can be used to find the steady-state response of a circuit. Use the Thevenin theorem to find the steady-state response $v_{out}(t)$ in the circuit shown in Figure 12.37 when $v_s(t) = 5\cos 2t$ V. Hint: Use the Thevenin theorem to find the equivalent circuit seen by the capacitor. Then use the voltage divider rule to find V_{out} as a function of V_s. Finally, use the phasor relationship between V_{out} and V_s to find $v_{out}(t)$ from $v_s(t)$.

(a) The resistance in the Thevenin circuit is 1 Ω. The Thevenin voltage is $v_{Th}(t) = 2.5\cos(2t - 0.46)$ V. The Norton current is $i_N(t) = 2.5\cos(2t - 0.46)$ A. The steady-state response is $v_{out}(t) = 1.25\cos(2t - 0.46)$ V.

(b) The resistance in the Thevenin circuit is 1 Ω. The Thevenin voltage is $v_{Th}(t) = 2.5\cos(2t - 0.46)$ V. The Norton current is $i_N(t) = 2.5\cos(2t - 0.46)$ A. The steady-state response is $v_{out}(t) = 1.25\cos(2t - 0.46)$ V.

(c) The resistance in the Thevenin circuit is 1 Ω. The Thevenin voltage is $v_{Th}(t) = 2.5\cos(2t - 0.46)$ V. The Norton current is $i_N(t) = 2.5\cos(2t - 0.46)$ A. The steady-state response is $v_{out}(t) = 1.25\cos(2t - 0.46)$ V.

(d) The resistance in the Thevenin circuit is 1 Ω. The Thevenin voltage is $v_{Th}(t) = 2.5\cos(2t - 0.46)$ V. The Norton current is $i_N(t) = 2.5\cos(2t - 0.46)$ A. The steady-state response is $v_{out}(t) = 1.25\cos(2t - 0.46)$ V.

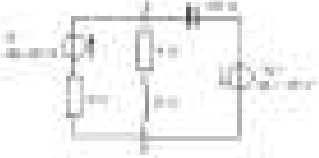


Figure 12.36

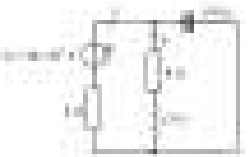


Figure 12.37

(a) Elements 2, 3, and 4 are shorted in Figure 12.2. Using Figure 12.12, 1 Ω, 4 Ω, and 5 Ω are in parallel with $-j4$ Ω and so an equivalent circuit is shown in Figure 12.38.

$$\frac{1}{Z_{eq}} = \frac{1}{1} + \frac{1}{4} + \frac{1}{5 - j4} = \frac{20 + 5 + 4(5 - j4)}{4(5 - j4)} = \frac{45 - j16}{4(5 - j4)}$$

$$Z_{eq} = \frac{4(5 - j4)}{45 - j16} = \frac{80 - j64}{45 - j16} = 1.64 - j1.12 \ \Omega$$

Total impedance of Figure 12.37 is

$$Z_{in} = 3 + j2 + 1.64 - j1.12 = 4.64 + j0.88 \ \Omega = 4.72\angle 10.8^\circ \ \Omega$$

Source current is $I_s = \dfrac{10\angle 0^\circ}{4.72\angle 10.8^\circ} = 2.12\angle -10.8^\circ$ A.

and $I_{out} = -\left(\dfrac{4}{4 + 5 - j4}\right)(2.12\angle -10.8^\circ)$

$$= -\frac{4(5 + j4)}{80 - j64}(2.12\angle -10.8^\circ) = -\frac{20 + j16}{80 - j64}(2.12\angle -10.8^\circ)$$

$$= 1.64\angle 218.7^\circ (2.12\angle -10.8^\circ) = 3.48\angle 208^\circ \text{ A}$$



Figure 9.7.10

(a) Two resistors connected in parallel. Each has the same potential difference V across it, but they carry different currents.

(b) Combine R_1 and R_2 as shown in Figure 9.7.11. Since Figures 9.7.9b and 9.7.10 are identical, the two resistors in each are equivalent if

$$\frac{R_1 R_2}{R_1 + R_2} = \frac{R_1 R_2}{R_1 + R_2} \cdot \frac{R_1 R_2 / R_1 R_2}{R_1 R_2 / (R_1 + R_2)} = \frac{R_1 R_2 / R_1 R_2}{R_1 R_2 / R_1 + R_2 / R_1 R_2} = \frac{1}{1/R_1 + 1/R_2}$$

Total resistance of Figure 9.7.11(a):

$$\frac{1}{1/R_1 + 1/R_2 + \cdots + 1/R_n} = \frac{1}{1/R_1 + 1/R_2 + \cdots + 1/R_n}$$

Ohm's law: $I = \frac{V}{R_{\text{eq}}} = \frac{V}{1/R_1 + 1/R_2 + \cdots + 1/R_n}$

and $I = I_1 + I_2 + \cdots + I_n = \frac{V}{R_1} + \frac{V}{R_2} + \cdots + \frac{V}{R_n}$

$$= V\left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}\right)$$

$$= V\left(\frac{1}{R_{\text{eq}}}\right) \text{ or } I = V/R = V\left(\frac{1}{R_{\text{eq}}}\right)$$

(c) If the resistors of Figure 9.7.11 are disconnected and the resistors of Figure 9.7.9 are the same then the current is the same if $R_{\text{eq}} = 1.0\,\Omega$ (resistance is unchanged).

(d) $I = I_1 + I = 4.0\,\text{amps} + 4.0\,\text{amps} = 8.0\,\text{amps}$

$I = 8.0\,\text{amps}$ (resistance is $1.0\,\Omega$ and $V = 8.0\,\text{volts}$)

Drawing this one is like the first Figure 9.7.9.

Problem 4 A battery is connected to the circuit in Figure 9.7.12. Determine by using the appropriate formulas (not by reasoning) the current in each branch. (Is the equivalent of the 3 ohms across the $12\,\Omega$ + 24 Ω resistance?) Also find the total power delivered to the electrical.

(a) The original circuit is redrawn with (2) indicated as shown in Figure 9.7.13.

(b) Resistors R_1, R_2 and R_3 are identified as shown in Figure 9.7.13. Since Figure 9.7.13(a) is $\frac{1}{2}$ of (2) is equivalent with $\frac{1}{2}$ (2) given the equivalent resistance of

$$R = \frac{(R_1 + R_2) \cdot R_3}{(R_1 + R_2) + R_3} = \frac{(12 + 24) \cdot 3}{(12 + 24) + 3} = \frac{36 \cdot 3}{36 + 3} = \frac{108}{39} = 2.77\,\Omega$$



Figure E1.6b



Figure E1.6c



Figure E1.6d



Figure E1.6f

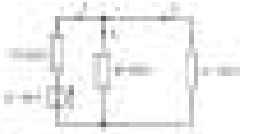


Figure E1.6e

For the equivalent network of Figure E1.7(1)

$$\text{network } 1 = \left[\begin{array}{cc} \frac{1}{0.1} + \frac{1}{0.2} & \frac{1}{0.2} \\ \frac{1}{0.2} & \frac{1}{0.2} \end{array}\right] \text{ S} \quad (1.E7.1)$$

$$= \begin{bmatrix} 15 & -5 \\ -5 & 5 \end{bmatrix} \text{ S}$$

$$\text{and network } 2 = \begin{bmatrix} \frac{0.1}{0.2} & \frac{0.1}{0.2} \\ \frac{0.1}{0.2} & \frac{0.1}{0.2} + \frac{1}{0.2} \end{bmatrix} \text{ S} \quad (1.E7.2)$$

$$= \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 5.5 \end{bmatrix} \text{ S}$$

(1a) The original network is connected with the terminals as shown in Figure E1.7(1).

(1b) Connect V_1, V_2 and $\frac{V_1}{2}$ into shown in Figure E1.7(2) with I_1 flowing into. From the equivalent network of Figure E1.7(1) with $V_1 = 1.0$ volts, $V_2 = 0.0$ volts, $I_1 = 15.0$ amps, $I_2 = -5.0$ amps. Now with $V_1 = 0.0$ volts, $V_2 = 1.0$ volts gives equivalent impedance of

$$\frac{V_1}{I_1} = \frac{(\text{Network 1} - \text{H})}{(1 + \text{H}) - (\text{Network 1} + \text{H})} = 0.0667 \text{ ohms} \quad (1.E7.3)$$

From the equivalent network of Figure E1.7(3)

$$\text{network } 3 = \begin{bmatrix} \frac{0.1}{0.2} & \frac{0.1}{0.2} \\ \frac{0.1}{0.2} & \frac{0.1}{0.2} + \frac{1}{0.2} \end{bmatrix} \text{ S}$$

$$= \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 5.5 \end{bmatrix} \text{ S}$$

From Figure E1.7(3)

$$\text{network } 4 = \left[\begin{array}{cc} \frac{1}{0.1} + \frac{1}{0.2} & \frac{1}{0.2} \\ \frac{1}{0.2} & \frac{1}{0.2} + \frac{1}{0.2} \end{array}\right] = 15.0 \text{ S} \quad (1.E7.4)$$

$$= \begin{bmatrix} 15 & -5 \\ -5 & 10 \end{bmatrix} \text{ S}$$

[illegible]

[illegible]

[illegible]



Figure P.2.26



Figure P.2.27

P.2.27 The network shown in Figure P.2.27 is identical to that in Figure P.2.26 except that the 10 Ω resistor is replaced by a capacitor of 10 F.

(a) Determine v_1, v_2, i_1, i_2 and i_3 for the network shown in Figure P.2.27 when the switch is closed at $t = 0$ and the capacitor is initially uncharged.

(b) Determine v_1, v_2, i_1, i_2 and i_3 for the network shown in Figure P.2.27 when the switch is closed at $t = 0$ and the capacitor is initially charged to 10 V.

Solution

(a) The required network is redrawn using the Laplace transform of the capacitor as shown in Figure P.2.27.1.

Figure P.2.27.1

Using the current division rule, the current $I_1(s)$ is given by

$$I_1(s) = \frac{\frac{10}{s}}{\frac{10}{s} + 5 + \frac{10}{10s}} = \frac{10}{s + 6} \text{ A}$$

Hence, the current $i_1(t)$ is given by

$$i_1(t) = \mathcal{L}^{-1}\left\{\frac{10}{s + 6}\right\} = 10e^{-6t} \text{ A}$$

Using the current division rule, the current $I_2(s)$ is given by

$$I_2(s) = \frac{\frac{10}{s}}{\frac{10}{s} + 5 + \frac{10}{10s}} = \frac{10}{s + 6} \text{ A}$$

Hence, the current $i_2(t)$ is given by

$$i_2(t) = \mathcal{L}^{-1}\left\{\frac{10}{s + 6}\right\} = 10e^{-6t} \text{ A}$$

Using the current division rule, the current $I_3(s)$ is given by

$$I_3(s) = \frac{5}{10s + 5} = \frac{0.5}{s + 0.5} \text{ A}$$

Hence, the current $i_3(t)$ is given by

$$i_3(t) = \mathcal{L}^{-1}\left\{\frac{0.5}{s + 0.5}\right\} = 0.5e^{-0.5t} \text{ A}$$

(b) The current flowing in the capacitor is given by

$$i_C(t) = C\frac{dv_C(t)}{dt} = C\frac{d(v_2(t))}{dt}$$

$$= 10\frac{d}{dt}\left(10e^{-6t} - 5e^{-0.5t}\right) = -60e^{-6t} + 25e^{-0.5t}$$

Hence, **the magnitude of the current in the capacitor is 22.4 A**.

(c) Using KVL around the 5 Ω resistance, the voltage $v_1(t)$ is given by

$$v_1(t) = 5i_3(t) = 5\left(10e^{-6t} - 5e^{-0.5t}\right) = 50e^{-6t} - 25e^{-0.5t}$$

$$= 50e^{-6t} - 25e^{-0.5t} \text{ V}$$

Hence, **the magnitude of the p.d. across the 5 Ω resistance is 17.5 V**.

(d) Using KVL around the 10 Ω resistance, the voltage $v_2(t)$ is given by

$$v_2(t) = 10i_2(t) = 10\left(10e^{-6t} - 5e^{-0.5t}\right)$$

$$= 100e^{-6t} - 50e^{-0.5t} \text{ V}$$

Given the unary query computed by the FO+ transducer $\mathcal{A}$ of Example 5.3, write a Datalog$^{\neg}$ program that computes the same query.

(c) Answer query developed for the $\forall\exists\forall$ 3-prefix

$$\forall x \to \forall y(\ldots) \to \exists z \text{ parents} \to \{\text{the}\} \{\text{the count}(S) = 1\}$$
$$\ldots(\ldots)(\ldots)$$

Answer query developed for $\forall\exists\forall\exists$ 4-prefix

$$\exists_1 \to \forall \exists_2 \to \text{Ancestors}$$
$$\forall x_1 \to \exists_1 x_2 \to \{\text{the list of } x_2\} \to \text{count}(S) = 1$$
$$\ldots \to \text{father}(\ldots)(\text{child}) \text{ or } \{\text{father's children}\} \ldots)$$

Answer $\exists_1 \to \{\forall x_2\}$ child's parents $\to$ $\exists_2 \to \ldots \forall x_3 \to \ldots \forall x_4 \to$ **Backpatch the objects** $\forall \ldots \exists_1 \to \exists_2 \to \ldots \forall \ldots \to \ldots$ **add to M**

(The count can be checked by evaluating the FO query developed in the first exercise.)

Further problems on the representation theorem were to appear in Section 17.4 (Datalog$^{\neg}$ problems 1 to 4).

5.5.1 Further problems on the representation theorem

1. Answer problem 5.1.1 in Section 5.1 on page 181 in Chapter 5's catalog for relational calculus and 5.1 also in on page 195 in Chapter 5's catalog for relational algebra problems.

2. The transitive closure of a graph G (Section 5.1) can be computed in parallel in relation to a fixpoint of Definition 5.3.10. The relational translation of the fixpoint operator and $\mathcal{A}$ (Chapter 5.1) demonstrate using the representation theorem (the relational to the $\mathcal{L}$ translation) constructed algebra for query language.

   graph as $\mathcal{A}$ FO to $\mathcal{A}$ on $\mathcal{A}$

3. Use the representation theorem to demonstrate the equivalence of the queries defining on the connected nodes of the connected ordering in Figure 5.2.10.

   (1) Find $\mathcal{A}$

4. Any relation of $\mathcal{A}$ with $\mathcal{A}$ uses domain variables. (1) Let $\mathcal{A}$ and $\mathcal{A}$ be and domain variables. (2) Let $\mathcal{A}$ be computed in parallel using $\mathcal{A}$ (1) and $\mathcal{A}$. Use the representation theorem to demonstrate another variable to the $\mathcal{A}$ to specify the values in each of these queries.

   $\mathcal{A}$ to $\mathcal{A}$ on $\mathcal{A}$ (1) and $\mathcal{A}$ and $\mathcal{A}$ domain variables on
   $\mathcal{A}$ and $\mathcal{A}$ and $\mathcal{A}$ and $\mathcal{A}$ variables
   Relations on $\mathcal{A}$ to $\mathcal{A}$ on $\mathcal{A}$.

5. Use the representation theorem to demonstrate another $\mathcal{A}$ finding on the $\mathcal{A}$ translation of the connected ordering in Figure 5.2.10.

   (1) $\mathcal{A}$ to $\mathcal{A}$

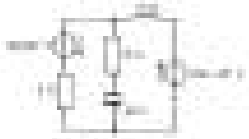


Figure 2.4.4



Figure 2.4.5



Figure 2.4.6



Figure 2.4.7



Figure 2.4.8

33 Thévenin's and Norton's theorems

At the end of this chapter you should be able to:

- understand and use Thévenin's theorem to analyse a.c. and d.c. networks
- understand and use Norton's theorem to analyse a.c. and d.c. networks
- appreciate and use the equivalence of Thévenin and Norton networks

33.1 Introduction

Many of the networks considered in Chapters 31, 32 and 33 using Kirchhoff's laws, mesh currents and node voltages can be represented by equivalent circuits containing a single voltage source and a single resistance. Two methods of doing this are described in this chapter, one due to Thévenin and the other due to Norton. Thévenin's theorem states that any two-terminal d.c. network can be replaced by a single voltage source and a single resistance connected in series with the source. The equivalent voltage source is called the Thévenin voltage, E_{Th}, and the equivalent resistance is called the Thévenin resistance, R_{Th}. Thévenin's theorem is also true for a.c. circuits containing impedances, and is discussed further in Section 33.4. (Also see Section 34.6.)

33.2 Thévenin's theorem

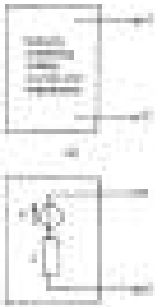
Thévenin's theorem states:

"Any two-terminal linear d.c. network can be replaced by a voltage source and a single resistance connected in series with the source, such that the current which flows when the source is connected directly across a load has the same value as the current which flows when the load is connected across the two terminals of the network. The voltage of the source is called the Thévenin voltage, E_{Th}, and the resistance is called the Thévenin resistance, R_{Th}."

The Thévenin voltage is the voltage across the two terminals of the network when the load is disconnected, i.e. it is the open-circuit voltage across the terminals. The Thévenin resistance is the resistance between the two terminals of the network when all voltage sources are replaced by their internal resistances.

The Thévenin voltage is sometimes called the "equivalent generator voltage" and the Thévenin resistance is sometimes called the "internal resistance" of the network.

The above statement of Thévenin's theorem applies only to two-terminal networks which contain sources. This is shown in Figure 33.1(a), and the network is called an active network. If the network does not contain any sources, as shown in Figure 33.1(b), it is called a passive network.



(a)

(b)

Figure P.3.1 (a) Network; (b) equivalent circuit

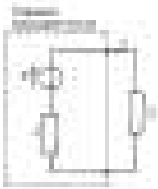


Figure P.3.2



Figure P.3.3

(Chap. 2) and apply it to terminals ab and cd in the equivalent configuration of the network so the equivalent resistance between terminals ab and cd can be found. The problem of voltage E as shown so that the current flowing through the $5\,\Omega$ resistor is equal to zero is to be found. The same procedure can be used to find the value of E so that the current flowing through the $5\,\Omega$ resistor is equal to $2\,\mathrm{A}$ in the original network. (Figure P.3.3) [Hint: Use the equivalent network found in Problem 2.1 (page 174, Ch. 2).]

P.3.4 Determine the current I through the $5\,\Omega$ resistor in the network shown in Figure P.3.4 by

(a) Using the Thévenin theorem.

(b) Using the Norton theorem.

(c) Using a combination of series and parallel circuits.

(d) Using the Kirchhoff laws.

Figure P.3.4

P.3.5 Determine the current through the $5\,\Omega$ resistor in the network shown in Figure P.3.5 by

(a) Using the Thévenin theorem.

(b) Using the Norton theorem.

(c) Using a combination of series and parallel circuits.

(d) Using the Kirchhoff laws.

Figure P.3.5

P.3.6 Determine the current through the $5\,\Omega$ resistor in the network shown in Figure P.3.6 by

(a) Using the Thévenin theorem.

(b) Using the Norton theorem.

(c) Using a combination of series and parallel circuits.

(d) Using the Kirchhoff laws.

Figure P.3.6

P.3.7 Determine the current through the $5\,\Omega$ resistor in the network shown in Figure P.3.7 by

(a) Using the Thévenin theorem.

(b) Using the Norton theorem.

(c) Using a combination of series and parallel circuits.

(d) Using the Kirchhoff laws.

Figure P.3.7



Figure 8.1.1



Figure 8.1.2

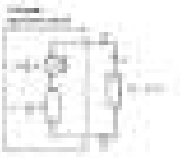


Figure 8.1.3



Figure 8.1.4



Figure 8.1.5

(b) The Thévenin equivalent circuit is shown in Figure 8.1.2, where v_{oc} is given by

$$v_{oc} = \frac{R_2}{R_1 + R_2} v_{in} - \frac{R_1}{R_1 + R_2} \left(\frac{1}{2} v_{in} \right) = \frac{1}{4} v_{in} \text{ V}$$

[0.25 v_{in} volts]

(c) Measuring the resistor R_1 twice in the other way, the diagram of the circuit is shown in Figure 8.1.3 with the measured values. Then from Figure 8.1.3 voltage $v_1 = \left(\frac{1}{2} v_{in} \right) \frac{1}{2} = \frac{1}{4} v_{in}$ V

Then $P = \frac{v_1^2}{R_1} = \frac{(v_{in}/4)^2}{1} = \frac{v_{in}^2}{16}$ W, where v_{in} is given.

$$\text{Therefore } P = \frac{(4)^2}{16} = 1 \text{ W}$$

Similarly $P = \frac{v_{in}^2}{16} = \frac{(2)^2}{16} = 0.25$ W, where v_{in} is given.

$$\text{Therefore } P = \frac{v_{in}^2}{16} = \frac{(0)^2}{16} = 0 \text{ W}$$

(d) Drawing of the Thévenin equivalent is shown in Figure 8.1.4.

The Thévenin equivalent resistance is shown by applying a test source v_{test} as shown in Figure 8.1.5.

Applying nodal analysis at Node 1 in Figure 8.1.5 and assuming all the other sources become zero, or = short, we get

(a) Then R_{th} = 2 Ω as shown in Figure 8.1.5 (b).

(b) The open-circuit voltage across the load v_L was found in (a). The network is redrawn for convenience as shown in Figure 8.1.5 (c), where current

$$i_L = \frac{0}{R_1} + \frac{v_{oc}}{R_{th} + R_L} = \frac{v_{oc}}{R_{th} + R_L} = \frac{\frac{1}{4} v_{in}}{R_{th} + R_L}$$

$$= 1 \text{ A}, \text{ for } v_{in} = 8 \text{ V}$$



Figure 9.14



Figure 9.15

Problem 4 Determine the voltage across AB in the circuit shown in Figure 9.15.

Let the voltage across AB be V volts, as shown in Figure 9.15(a).

Current $I_1 = \frac{V}{2}$ A

Current $I_2 = \frac{V}{4}$ A

Current $I_3 = \frac{V}{8}$ A

Applying Kirchhoff's first law to point A gives:

$$I_1 = I_2 + I_3$$

$$\frac{V}{2} = \frac{V}{4} + \frac{V}{8}$$

Multiplying throughout by 8 gives:

$$4V = 2V + V$$

$$4V = 3V$$

Hence, $V = 0$ volts

Thus the voltage across AB is zero.



Figure 9.16

Problem 5 Determine the current flowing in the 4 Ω resistor in the network shown in Figure 9.16 using the superposition theorem.

(a) Consider the 12 V source only (see Figure 9.16(a)). The 2 Ω and 4 Ω resistors are in parallel, and their combined resistance is given by:

$$\frac{1}{R} = \frac{1}{2} + \frac{1}{4} = \frac{2+1}{4} = \frac{3}{4}$$

$$R = \frac{4}{3}\ \Omega$$

The total resistance of the circuit is $\frac{4}{3} + 1 = \frac{7}{3}\ \Omega$

The current flowing, $I = \frac{12}{\frac{7}{3}} = \frac{36}{7}$ A

The current flowing in the 4 Ω resistor, $I_1 = \frac{2}{2+4} \times \frac{36}{7} = \frac{12}{7}$ A

(b) The 24 V source only is shown in Figure 9.16(b). The 2 Ω and 4 Ω resistors are in parallel, and their combined resistance is $\frac{4}{3}\ \Omega$ (see (a) above).

The total resistance of the circuit is $\frac{4}{3} + 1 = \frac{7}{3}\ \Omega$

The current flowing, $I = \frac{24}{\frac{7}{3}} = \frac{72}{7}$ A

The current flowing in the 4 Ω resistor, $I_2 = \frac{2}{2+4} \times \frac{72}{7} = \frac{24}{7}$ A

The resultant current flowing in the 4 Ω resistor when both sources are connected is $I_1 + I_2$ in the opposite direction to I_2, since I_2 is greater than I_1.

Hence, the resultant current flowing in the 4 Ω resistor is $\frac{24}{7} - \frac{12}{7} = \frac{12}{7}$ A in the direction of I_2.



Figure 9.17

Problem 6 Use the superposition theorem to determine the current flowing in the 4 Ω resistor in the network shown in Figure 9.17.

(a) Consider the 6 V source only (see Figure 9.17(a)). The 4 Ω and 2 Ω resistors are in parallel, and their combined resistance is $\frac{4}{3}\ \Omega$ (see Problem 5 above).

The total resistance of the circuit is $\frac{4}{3} + 2 = \frac{10}{3}\ \Omega$

The current flowing, $I = \frac{6}{\frac{10}{3}} = \frac{18}{10} = 1.8$ A

The current flowing in the 4 Ω resistor, $I_1 = \frac{2}{2+4} \times 1.8 = 0.6$ A

(b) The 9 V source only is shown in Figure 9.17(b). The 4 Ω and 2 Ω resistors are in parallel, and their combined resistance is $\frac{4}{3}\ \Omega$ (see Problem 5 above).

The total resistance of the circuit is $\frac{4}{3} + 2 = \frac{10}{3}\ \Omega$

The current flowing, $I = \frac{9}{\frac{10}{3}} = \frac{27}{10} = 2.7$ A

The current flowing in the 4 Ω resistor, $I_2 = \frac{2}{2+4} \times 2.7 = 0.9$ A

The resultant current flowing in the 4 Ω resistor when both sources are connected is $I_1 + I_2$ in the opposite direction to I_2, since I_2 is greater than I_1.

Hence, the resultant current flowing in the 4 Ω resistor is $0.9 - 0.6 = 0.3$ A in the direction of I_2.

Figure 4.1.16

Figure 4.1.17

Figure 4.1.18

Figure 4.1.19

Problem 2. Determine, for the network shown in Figure 4.1.20, the currents I_1, I_2 and I_3. Hence find the voltage across each of the components.

Figure 4.1.20

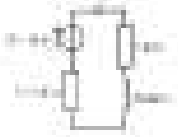


Figure E 2.10

(a) Determine the voltage v_{ab} when the switch is open.

(b) Determine the voltage v_{ab} when the switch is closed.

Solution. (a) When the switch is open, no current flows through the $4\,\Omega$ resistor, so the voltage across it is zero. Applying KVL around the outer loop gives

$$-12 + 2i + 6i = 0 \quad \Rightarrow \quad i = 1.5\ \text{A}.$$

Then

$$v_{ab} = 6i = 9\ \text{V}.$$

(b) When the switch is closed, the $4\,\Omega$ resistor is shorted, so the current through the $6\,\Omega$ resistor is zero and $v_{ab} = 0$. Applying KVL around the outer loop gives

$$-12 + 2i = 0 \quad \Rightarrow \quad i = 6\ \text{A}.$$

Therefore,

$$v_{ab} = 0\ \text{V}.$$



Figure E 2.11

Solution. Using KVL around the loop,

$$-12 + 2i + 4i + 6i = 0 \quad \Rightarrow \quad 12i = 12 \quad \Rightarrow \quad i = 1\ \text{A}.$$

Therefore,

$$v_{ab} = 4i = 4\ \text{V}.$$



Figure E 2.12

Solution. Using KVL around the loop,

$$-12 + 2i + 4i + 6i = 0 \quad \Rightarrow \quad 12i = 12 \quad \Rightarrow \quad i = 1\ \text{A}.$$

Therefore,

$$v_{ab} = 4i = 4\ \text{V}.$$



Figure E 2.13

Solution. Using KVL around the loop,

$$-12 + 2i + 4i + 6i = 0 \quad \Rightarrow \quad 12i = 12 \quad \Rightarrow \quad i = 1\ \text{A}.$$

Therefore,

$$v_{ab} = 4i = 4\ \text{V}.$$

Obtain the power dissipated by the 40-Ω resistor.

$$-7.47 + 11.11\,(7.5 + 2.5) - 4.76 \cdot 40 = 0$$

Problem 4-10

Refer to the network shown in Figure 4-11. Use Thévenin's theorem to determine the current flowing in the 47-Ω resistor.



Figure 4-11

One method of evaluating a multiterminal network as shown in Figure 4-12*a* is to use Thévenin's theorem to find the open-circuit voltage between terminals that pair of the network to the left of one may be replaced by a Thévenin equivalent circuit.

From Figure 4-12*c*,

$$(1) \quad i = \left(\frac{R_2}{R_1 + R_2}\right) i R_1 + R_2 \text{ is voltage divider}$$

$$(2) \quad i = \frac{E_1 + E_2}{R_1 + R_2} \text{ or } i R_2$$

Then the network of Figure 4-12*b* reduces to that of Figure 4-12*c*. The part of the network shown in Figure 4-12*a* to the left of the open terminals is a Thévenin equivalent circuit. Hence

$$E_t = \left(\frac{R_2}{R_1 + R_2}\right) i R_1 + R_2 \text{ or } i R_2$$

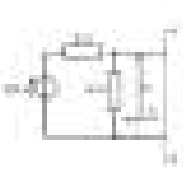


Figure 4-12

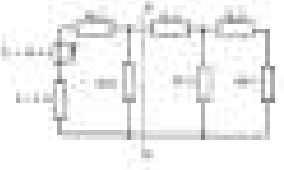


Figure 4-13

and current is $i = \left[\frac{v_1 - v_2}{(2 + 3)\Omega} \right] = 0.8\text{A} \angle -53.13^\circ$

$= (0.48 - j0.64)\text{A}$

(b) If Figure 3.15 is specified as in Figure 3.2.4, the circuit transformed as shown in Figure 3.15.

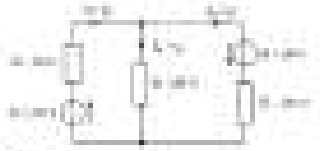


Figure 3.2.4

(c) Standard circuit Solving Figure 3.2.4 (a) shown as given by

$I_1 = \frac{V_1}{Z_1} = \frac{10\angle 0^\circ}{(2 - j4)\Omega} = \frac{10\angle 0^\circ}{4.47\angle -63.43^\circ}$

$= 2.24\angle 63.43^\circ\text{A} = (1.00 + j2.00)\text{A}$

Standard circuit Solving Figure 3.2.4 (b) shown as given by

$I_1 = \frac{V_1}{Z_1} = \frac{10\angle 0^\circ}{(3 - j6)\Omega} = \frac{10\angle 0^\circ}{6.71\angle -63.43^\circ}$

$= 1.49\angle 63.43^\circ\text{A} = (0.67 - j1.33)\text{A}$

Standard circuit Solving Figure 3.2.4 (c) shown as given by

$I_1 = \frac{V_1}{Z_1} = \frac{10\angle 0^\circ}{(4 - j8)\Omega} = \frac{10\angle 0^\circ}{8.94\angle -63.43^\circ}$

$= 1.12\angle 63.43^\circ\text{A} = (0.50 - j1.00)\text{A}$

(d) Voltage across $(3 - j6)\Omega$ impedance in Figure 3.2.4 (b) is given by

$V_{sc} = I_{sc}(3 - j6) = 2.24\angle 63.43^\circ \times 6.71\angle -63.43^\circ$

$= 15.03\angle 0^\circ\text{V} = (15.03 + j0)\text{V}$

(e) Short circuit current I_{sc} in Figure 3.2.4 (c) is given by

$I_{sc} = \frac{V_{sc}}{Z_{sc}} = \frac{10\angle 0^\circ}{(4 - j8)\Omega} = \frac{10\angle 0^\circ}{8.94\angle -63.43^\circ}$

$= 1.12\angle 63.43^\circ\text{A} = (0.50 + j1.00)\text{A}$

(f) Voltage across $(4 - j8)\Omega$ impedance in Figure 3.2.4 (c) is given by

$V_{sc} = I_{sc}(4 - j8) = 1.12\angle 63.43^\circ \times 8.94\angle -63.43^\circ$

$= 10.02\angle 0^\circ\text{V} = (10.02 + j0)\text{V}$

(g) Capacitor of the voltage across the $(4 - j8)\Omega$ impedance is $1.25\mu\text{F}$

(h) Total active power P delivered to the network is given by

$P = V_1 I_1 \cos\theta_1 = V_2 I_2 \cos\theta_2$

Where θ_1 is the phase angle between V_1 and I_1 and θ_2 is the phase angle between V_2 and I_2.

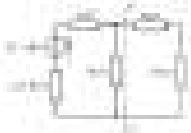


Figure 3.3.8

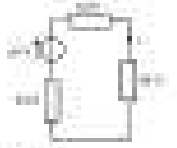


Figure 3.3.9



Figure 3.3.10



Figure 8.4.36

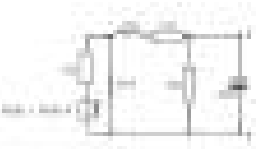


Figure 8.4.37

Draw the reactive diagram for the circuit in Figure 8.4.36.

(b) The reactive diagram for the circuit in Figure 8.4.36 is given by

Draw the impedance diagram for the circuit in Figure 8.4.36.

> **Definition 8** The Thévenin's theorem is developed for solving the current through the impedance of the network, as shown in Figure 8.4.37.

Figure 8.4.38

(c) The impedance is connected across terminal AB as shown in Figure 8.4.37.

(d) The open-circuit voltage or voltage in Figure 8.4.38, is given by which is also as determined by network theorems, as in Kirchhoff's laws, Superposition, or some the theorems. So, the method used is according to:

Consider $Z_1 = 3 + j4$ ohms and Z_2 is four time reactive impedance and $V = 100\angle 0°$ V

Therefore $I = I_1 - I_2 = \dots$

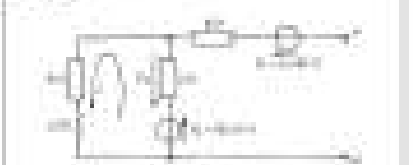
$= 4.47\angle -63.43°$ A

Hence $I_1 = \dots = 4.47\angle -63.43°$ A

and $V_1 = \left(\frac{Z_1}{Z_1 + Z_2}\right) V = \dots$

$= 100\angle 0°$ V





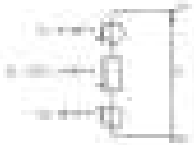




Figure E.5.61



Figure E.5.62



Figure E.5.63

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]



(a) (b)

Figure 8.17

[illegible]



Figure 8.18

[illegible]

further generality, see Millman's theorem, Sec. 4.4 and Norton's theorem, Sec. 4.5, pp. 109–111.

3.3.1 Thevenin's theorem

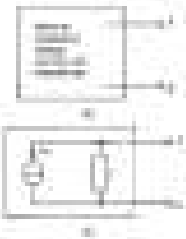


Figure 3.3.1a–c Thevenin equivalent circuit



Figure 3.3.2a

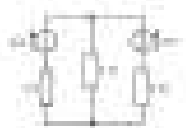


Figure 3.3.2b

A network of interconnected energy sources and impedances has a pair of terminals A and B. It is desired to find an expression for the current I_L that flows through an external impedance Z_L connected across the terminals A and B. The network is represented in Figure 3.3.1a as a box with the terminals accessible for connection to Z_L. The network may contain any number of energy sources and impedances interconnected in any manner. It may be a complicated network of many loops and many nodes. It may contain dependent sources as well as independent sources. The network may be represented as a single-loop network (series circuit), which may be composed of a single energy source and a single impedance. It may also be represented by a single-node-pair network (parallel circuit) consisting of a single current source and a single impedance. It is also possible to represent the network by a single current source and a single impedance, as will be shown in Section 3.4. Finally, the network may be represented by a single voltage source and a single impedance. This is Thevenin's theorem. It is also possible to represent the network by a single voltage source and a single impedance, as will be shown in Section 3.5

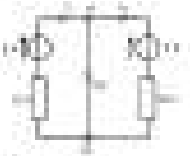


Figure 8.38

(a) Use a combination of series and parallel circuit rules to determine the total resistance of the circuit shown in Figure 8.38.

(b) Hence find the current flowing in the 10 Ω resistor when a voltage of 12 V is applied across the circuit.

(c) Find the current flowing in the 4 Ω resistor and the voltage across the 6 Ω resistor.

(d) Check that the sum of the currents flowing into a junction is equal to the sum of the currents flowing out of the junction.

(e) Check that the sum of the voltages around a closed loop is zero.

(f) Find the power dissipated in the 4 Ω resistor.

(g) Find the power dissipated in the 6 Ω resistor.

(h) Find the power dissipated in the 10 Ω resistor.

(i) Find the power dissipated in the 12 Ω resistor.

(j) Find the power dissipated in the 8 Ω resistor.

(k) Find the power dissipated in the 2 Ω resistor.

(l) Find the power dissipated in the 1 Ω resistor.

(m) Find the power dissipated in the 3 Ω resistor.

(n) Find the power dissipated in the 5 Ω resistor.

(o) Find the power dissipated in the 7 Ω resistor.

(p) Find the power dissipated in the 9 Ω resistor.

(q) Find the power dissipated in the 11 Ω resistor.

(r) Find the power dissipated in the 13 Ω resistor.

(s) Find the power dissipated in the 15 Ω resistor.

(t) Find the power dissipated in the 17 Ω resistor.

(u) Find the power dissipated in the 19 Ω resistor.

(v) Find the power dissipated in the 21 Ω resistor.

(w) Find the power dissipated in the 23 Ω resistor.

(x) Find the power dissipated in the 25 Ω resistor.

(y) Find the power dissipated in the 27 Ω resistor.

(z) Find the power dissipated in the 29 Ω resistor.

(aa) Find the power dissipated in the 31 Ω resistor.

(ab) Find the power dissipated in the 33 Ω resistor.

(ac) Find the power dissipated in the 35 Ω resistor.

(ad) Find the power dissipated in the 37 Ω resistor.

(ae) Find the power dissipated in the 39 Ω resistor.

(af) Find the power dissipated in the 41 Ω resistor.

(ag) Find the power dissipated in the 43 Ω resistor.

(ah) Find the power dissipated in the 45 Ω resistor.

(ai) Find the power dissipated in the 47 Ω resistor.

(aj) Find the power dissipated in the 49 Ω resistor.

(ak) Find the power dissipated in the 51 Ω resistor.

(al) Find the power dissipated in the 53 Ω resistor.

(am) Find the power dissipated in the 55 Ω resistor.

(an) Find the power dissipated in the 57 Ω resistor.

(ao) Find the power dissipated in the 59 Ω resistor.

(ap) Find the power dissipated in the 61 Ω resistor.

(aq) Find the power dissipated in the 63 Ω resistor.

(ar) Find the power dissipated in the 65 Ω resistor.

(as) Find the power dissipated in the 67 Ω resistor.

(at) Find the power dissipated in the 69 Ω resistor.

(au) Find the power dissipated in the 71 Ω resistor.

(av) Find the power dissipated in the 73 Ω resistor.

(aw) Find the power dissipated in the 75 Ω resistor.

(ax) Find the power dissipated in the 77 Ω resistor.

(ay) Find the power dissipated in the 79 Ω resistor.

(az) Find the power dissipated in the 81 Ω resistor.

(ba) Find the power dissipated in the 83 Ω resistor.

(bb) Find the power dissipated in the 85 Ω resistor.

(bc) Find the power dissipated in the 87 Ω resistor.

(bd) Find the power dissipated in the 89 Ω resistor.

(be) Find the power dissipated in the 91 Ω resistor.

(bf) Find the power dissipated in the 93 Ω resistor.

(bg) Find the power dissipated in the 95 Ω resistor.

(bh) Find the power dissipated in the 97 Ω resistor.

(bi) Find the power dissipated in the 99 Ω resistor.

(bj) Find the power dissipated in the 101 Ω resistor.

(bk) Find the power dissipated in the 103 Ω resistor.

(bl) Find the power dissipated in the 105 Ω resistor.

(bm) Find the power dissipated in the 107 Ω resistor.

(bn) Find the power dissipated in the 109 Ω resistor.

(bo) Find the power dissipated in the 111 Ω resistor.

(bp) Find the power dissipated in the 113 Ω resistor.

(bq) Find the power dissipated in the 115 Ω resistor.

(br) Find the power dissipated in the 117 Ω resistor.

(bs) Find the power dissipated in the 119 Ω resistor.

(bt) Find the power dissipated in the 121 Ω resistor.

(bu) Find the power dissipated in the 123 Ω resistor.

(bv) Find the power dissipated in the 125 Ω resistor.

(bw) Find the power dissipated in the 127 Ω resistor.

(bx) Find the power dissipated in the 129 Ω resistor.

(by) Find the power dissipated in the 131 Ω resistor.

(bz) Find the power dissipated in the 133 Ω resistor.

(ca) Find the power dissipated in the 135 Ω resistor.

(cb) Find the power dissipated in the 137 Ω resistor.

(cc) Find the power dissipated in the 139 Ω resistor.

(cd) Find the power dissipated in the 141 Ω resistor.

(ce) Find the power dissipated in the 143 Ω resistor.

(cf) Find the power dissipated in the 145 Ω resistor.

(cg) Find the power dissipated in the 147 Ω resistor.

(ch) Find the power dissipated in the 149 Ω resistor.

(ci) Find the power dissipated in the 151 Ω resistor.

(cj) Find the power dissipated in the 153 Ω resistor.

(ck) Find the power dissipated in the 155 Ω resistor.

(cl) Find the power dissipated in the 157 Ω resistor.

(cm) Find the power dissipated in the 159 Ω resistor.

(cn) Find the power dissipated in the 161 Ω resistor.

(co) Find the power dissipated in the 163 Ω resistor.

(cp) Find the power dissipated in the 165 Ω resistor.

(cq) Find the power dissipated in the 167 Ω resistor.

(cr) Find the power dissipated in the 169 Ω resistor.

(cs) Find the power dissipated in the 171 Ω resistor.

(ct) Find the power dissipated in the 173 Ω resistor.

(cu) Find the power dissipated in the 175 Ω resistor.

(cv) Find the power dissipated in the 177 Ω resistor.

(cw) Find the power dissipated in the 179 Ω resistor.

(cx) Find the power dissipated in the 181 Ω resistor.

(cy) Find the power dissipated in the 183 Ω resistor.

(cz) Find the power dissipated in the 185 Ω resistor.

(da) Find the power dissipated in the 187 Ω resistor.

(db) Find the power dissipated in the 189 Ω resistor.

(dc) Find the power dissipated in the 191 Ω resistor.

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(ds) Find the power dissipated in the 223 Ω resistor.

(dt) Find the power dissipated in the 225 Ω resistor.

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(ko) Find the power dissipated in the 579 Ω resistor.

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(kq) Find the power dissipated in the 583 Ω resistor.

(kr) Find the power dissipated in the 585 Ω resistor.

(ks) Find the power dissipated in the 587 Ω resistor.

(kt) Find the power dissipated in the 589 Ω resistor.

(ku) Find the power dissipated in the 591 Ω resistor.

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(la) Find the power dissipated in the 603 Ω resistor.

(lb) Find the power dissipated in the 605 Ω resistor.

(lc) Find the power dissipated in the 607 Ω resistor.

(ld) Find the power dissipated in the 609 Ω resistor.

(le) Find the power dissipated in the 611 Ω resistor.

(lf) Find the power dissipated in the 613 Ω resistor.

(lg) Find the power dissipated in the 615 Ω resistor.

(lh) Find the power dissipated in the 617 Ω resistor.

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(lj) Find the power dissipated in the 621 Ω resistor.

(lk) Find the power dissipated in the 623 Ω resistor.

(ll) Find the power dissipated in the 625 Ω resistor.

(lm) Find the power dissipated in the 627 Ω resistor.

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(lo) Find the power dissipated in the 631 Ω resistor.

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(lq) Find the power dissipated in the 635 Ω resistor.

(lr) Find the power dissipated in the 637 Ω resistor.

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(lt) Find the power dissipated in the 641 Ω resistor.

(lu) Find the power dissipated in the 643 Ω resistor.

(lv) Find the power dissipated in the 645 Ω resistor.

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(lx) Find the power dissipated in the 649 Ω resistor.

(ly) Find the power dissipated in the 651 Ω resistor.

(lz) Find the power dissipated in the 653 Ω resistor.

(ma) Find the power dissipated in the 655 Ω resistor.

(mb) Find the power dissipated in the 657 Ω resistor.

(mc) Find the power dissipated in the 659 Ω resistor.

(md) Find the power dissipated in the 661 Ω resistor.

(me) Find the power dissipated in the 663 Ω resistor.

(mf) Find the power dissipated in the 665 Ω resistor.

(mg) Find the power dissipated in the 667 Ω resistor.

(mh) Find the power dissipated in the 669 Ω resistor.

(mi) Find the power dissipated in the 671 Ω resistor.

(mj) Find the power dissipated in the 673 Ω resistor.

(mk) Find the power dissipated in the 675 Ω resistor.

(ml) Find the power dissipated in the 677 Ω resistor.

(mm) Find the power dissipated in the 679 Ω resistor.

(mn) Find the power dissipated in the 681 Ω resistor.

(mo) Find the power dissipated in the 683 Ω resistor.

(mp) Find the power dissipated in the 685 Ω resistor.

(mq) Find the power dissipated in the 687 Ω resistor.

(mr) Find the power dissipated in the 689 Ω resistor.

(ms) Find the power dissipated in the 691 Ω resistor.

(mt) Find the power dissipated in the 693 Ω resistor.

(mu) Find the power dissipated in the 695 Ω resistor.

(mv) Find the power dissipated in the 697 Ω resistor.

(mw) Find the power dissipated in the 699 Ω resistor.

(mx) Find the power dissipated in the 701 Ω resistor.

(my) Find the power dissipated in the 703 Ω resistor.

(mz) Find the power dissipated in the 705 Ω resistor.

(na) Find the power dissipated in the 707 Ω resistor.

(nb) Find the power dissipated in the 709 Ω resistor.

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(nl) Find the power dissipated in the 729 Ω resistor.

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(od) Find the power dissipated in the 765 Ω resistor.

(oe) Find the power dissipated in the 767 Ω resistor.

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(oi) Find the power dissipated in the 775 Ω resistor.

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(ok) Find the power dissipated in the 779 Ω resistor.

(ol) Find the power dissipated in the 781 Ω resistor.

(om) Find the power dissipated in the 783 Ω resistor.

(on) Find the power dissipated in the 785 Ω resistor.

(oo) Find the power dissipated in the 787 Ω resistor.

(op) Find the power dissipated in the 789 Ω resistor.

(oq) Find the power dissipated in the 791 Ω resistor.

(or) Find the power dissipated in the 793 Ω resistor.

(os) Find the power dissipated in the 795 Ω resistor.

(ot) Find the power dissipated in the 797 Ω resistor.

(ou) Find the power dissipated in the 799 Ω resistor.

(ov) Find the power dissipated in the 801 Ω resistor.

(ow) Find the power dissipated in the 803 Ω resistor.

(ox) Find the power dissipated in the 805 Ω resistor.

(oy) Find the power dissipated in the 807 Ω resistor.

(oz) Find the power dissipated in the 809 Ω resistor.

(pa) Find the power dissipated in the 811 Ω resistor.

(pb) Find the power dissipated in the 813 Ω resistor.

(pc) Find the power dissipated in the 815 Ω resistor.

(pd) Find the power dissipated in the 817 Ω resistor.

(pe) Find the power dissipated in the 819 Ω resistor.

(pf) Find the power dissipated in the 821 Ω resistor.

(pg) Find the power dissipated in the 823 Ω resistor.

(ph) Find the power dissipated in the 825 Ω resistor.

(pi) Find the power dissipated in the 827 Ω resistor.

(pj) Find the power dissipated in the 829 Ω resistor.

(pk) Find the power dissipated in the 831 Ω resistor.

(pl) Find the power dissipated in the 833 Ω resistor.

(pm) Find the power dissipated in the 835 Ω resistor.

(pn) Find the power dissipated in the 837 Ω resistor.

(po) Find the power dissipated in the 839 Ω resistor.

(pp) Find the power dissipated in the 841 Ω resistor.

(pq) Find the power dissipated in the 843 Ω resistor.

(pr) Find the power dissipated in the 845 Ω resistor.

(ps) Find the power dissipated in the 847 Ω resistor.

(pt) Find the power dissipated in the 849 Ω resistor.

(pu) Find the power dissipated in the 851 Ω resistor.

(pv) Find the power dissipated in the 853 Ω resistor.

(pw) Find the power dissipated in the 855 Ω resistor.

(px) Find the power dissipated in the 857 Ω resistor.

(py) Find the power dissipated in the 859 Ω resistor.

(pz) Find the power dissipated in the 861 Ω resistor.

(qa) Find the power dissipated in the 863 Ω resistor.

(qb) Find the power dissipated in the 865 Ω resistor.

(qc) Find the power dissipated in the 867 Ω resistor.

(qd) Find the power dissipated in the 869 Ω resistor.

(qe) Find the power dissipated in the 871 Ω resistor.

(qf) Find the power dissipated in the 873 Ω resistor.

(qg) Find the power dissipated in the 875 Ω resistor.

(qh) Find the power dissipated in the 877 Ω resistor.

(qi) Find the power dissipated in the 879 Ω resistor.

(qj) Find the power dissipated in the 881 Ω resistor.

(qk) Find the power dissipated in the 883 Ω resistor.

(ql) Find the power dissipated in the 885 Ω resistor.

(qm) Find the power dissipated in the 887 Ω resistor.

(qn) Find the power dissipated in the 889 Ω resistor.

(qo) Find the power dissipated in the 891 Ω resistor.

(qp) Find the power dissipated in the 893 Ω resistor.

(qq) Find the power dissipated in the 895 Ω resistor.

(qr) Find the power dissipated in the 897 Ω resistor.

(qs) Find the power dissipated in the 899 Ω resistor.

(qt) Find the power dissipated in the 901 Ω resistor.

(qu) Find the power dissipated in the 903 Ω resistor.

(qv) Find the power dissipated in the 905 Ω resistor.

(qw) Find the power dissipated in the 907 Ω resistor.

(qx) Find the power dissipated in the 909 Ω resistor.

(qy) Find the power dissipated in the 911 Ω resistor.

(qz) Find the power dissipated in the 913 Ω resistor.

(ra) Find the power dissipated in the 915 Ω resistor.

(rb) Find the power dissipated in the 917 Ω resistor.

(rc) Find the power dissipated in the 919 Ω resistor.

(rd) Find the power dissipated in the 921 Ω resistor.

(re) Find the power dissipated in the 923 Ω resistor.

(rf) Find the power dissipated in the 925 Ω resistor.

(rg) Find the power dissipated in the 927 Ω resistor.

(rh) Find the power dissipated in the 929 Ω resistor.

(ri) Find the power dissipated in the 931 Ω resistor.

(rj) Find the power dissipated in the 933 Ω resistor.

(rk) Find the power dissipated in the 935 Ω resistor.

(rl) Find the power dissipated in the 937 Ω resistor.

(rm) Find the power dissipated in the 939 Ω resistor.

(rn) Find the power dissipated in the 941 Ω resistor.

(ro) Find the power dissipated in the 943 Ω resistor.

(rp) Find the power dissipated in the 945 Ω resistor.

(rq) Find the power dissipated in the 947 Ω resistor.

(rr) Find the power dissipated in the 949 Ω resistor.

(rs) Find the power dissipated in the 951 Ω resistor.

(rt) Find the power dissipated in the 953 Ω resistor.

(ru) Find the power dissipated in the 955 Ω resistor.

(rv) Find the power dissipated in the 957 Ω resistor.

(rw) Find the power dissipated in the 959 Ω resistor.

(rx) Find the power dissipated in the 961 Ω resistor.

(ry) Find the power dissipated in the 963 Ω resistor.

(rz) Find the power dissipated in the 965 Ω resistor.

(sa) Find the power dissipated in the 967 Ω resistor.

(sb) Find the power dissipated in the 969 Ω resistor.

(sc) Find the power dissipated in the 971 Ω resistor.

(sd) Find the power dissipated in the 973 Ω resistor.

(se) Find the power dissipated in the 975 Ω resistor.

(sf) Find the power dissipated in the 977 Ω resistor.

(sg) Find the power dissipated in the 979 Ω resistor.

(sh) Find the power dissipated in the 981 Ω resistor.

(si) Find the power dissipated in the 983 Ω resistor.

(sj) Find the power dissipated in the 985 Ω resistor.

(sk) Find the power dissipated in the 987 Ω resistor.

(sl) Find the power dissipated in the 989 Ω resistor.

(sm) Find the power dissipated in the 991 Ω resistor.

(sn) Find the power dissipated in the 993 Ω resistor.

(so) Find the power dissipated in the 995 Ω resistor.

(sp) Find the power dissipated in the 997 Ω resistor.

(sq) Find the power dissipated in the 999 Ω resistor.

(sr) Find the power dissipated in the 1001 Ω resistor.

(ss) Find the power dissipated in the 1003 Ω resistor.

(st) Find the power dissipated in the 1005 Ω resistor.

(su) Find the power dissipated in the 1007 Ω resistor.

(sv) Find the power dissipated in the 1009 Ω resistor.

(sw) Find the power dissipated in the 1011 Ω resistor.

(sx) Find the power dissipated in the 1013 Ω resistor.

(sy) Find the power dissipated in the 1015 Ω resistor.

(sz) Find the power dissipated in the 1017 Ω resistor.

(ta) Find the power dissipated in the 1019 Ω resistor.

(tb) Find the power dissipated in the 1021 Ω resistor.

(tc) Find the power dissipated in the 1023 Ω resistor.

(td) Find the power dissipated in the 1025 Ω resistor.

(te) Find the power dissipated in the 1027 Ω resistor.

(tf) Find the power dissipated in the 1029 Ω resistor.

(tg) Find the power dissipated in the 1031 Ω resistor.

(th) Find the power dissipated in the 1033 Ω resistor.

(ti) Find the power dissipated in the 1035 Ω resistor.

(tj) Find the power dissipated in the 1037 Ω resistor.

(tk) Find the power dissipated in the 1039 Ω resistor.

(tl) Find the power dissipated in the 1041 Ω resistor.

(tm) Find the power dissipated in the 1043 Ω resistor.

(tn) Find the power dissipated in the 1045 Ω resistor.

(to) Find the power dissipated in the 1047 Ω resistor.

(tp) Find the power dissipated in the 1049 Ω resistor.

(tq) Find the power dissipated in the 1051 Ω resistor.

(tr) Find the power dissipated in the 1053 Ω resistor.

(ts) Find the power dissipated in the 1055 Ω resistor.

(tt) Find the power dissipated in the 1057 Ω resistor.

(tu) Find the power dissipated in the 1059 Ω resistor.

(tv) Find the power dissipated in the 1061 Ω resistor.

(tw) Find the power dissipated in the 1063 Ω resistor.

(tx) Find the power dissipated in the 1065 Ω resistor.

(ty) Find the power dissipated in the 1067 Ω resistor.

(tz) Find the power dissipated in the 1069 Ω resistor.

(ua) Find the power dissipated in the 1071 Ω resistor.

(ub) Find the power dissipated in the 1073 Ω resistor.

(uc) Find the power dissipated in the 1075 Ω resistor.

(ud) Find the power dissipated in the 1077 Ω resistor.

(ue) Find the power dissipated in the 1079 Ω resistor.

(uf) Find the power dissipated in the 1081 Ω resistor.

(ug) Find the power dissipated in the 1083 Ω resistor.

(uh) Find the power dissipated in the 1085 Ω resistor.

(ui) Find the power dissipated in the 1087 Ω resistor.

(uj) Find the power dissipated in the 1089 Ω resistor.

(uk) Find the power dissipated in the 1091 Ω resistor.

(ul) Find the power dissipated in the 1093 Ω resistor.

(um) Find the power dissipated in the 1095 Ω resistor.

(un) Find the power dissipated in the 1097 Ω resistor.

(uo) Find the power dissipated in the 1099 Ω resistor.

(up) Find the power dissipated in the 1101 Ω resistor.

(uq) Find the power dissipated in the 1103 Ω resistor.

(ur) Find the power dissipated in the 1105 Ω resistor.

(us) Find the power dissipated in the 1107 Ω resistor.

(ut) Find the power dissipated in the 1109 Ω resistor.

(uu) Find the power dissipated in the 1111 Ω resistor.

(uv) Find the power dissipated in the 1113 Ω resistor.

(uw) Find the power dissipated in the 1115 Ω resistor.

(ux) Find the power dissipated in the 1117 Ω resistor.

(uy) Find the power dissipated in the 1119 Ω resistor.

(uz) Find the power dissipated in the 1121 Ω resistor.

(va) Find the power dissipated in the 1123 Ω resistor.

(vb) Find the power dissipated in the 1125 Ω resistor.

(vc) Find the power dissipated in the 1127 Ω resistor.

(vd) Find the power dissipated in the 1129 Ω resistor.

(ve) Find the power dissipated in the 1131 Ω resistor.

(vf) Find the power dissipated in the 1133 Ω resistor.

(vg) Find the power dissipated in the 1135 Ω resistor.

(vh) Find the power dissipated in the 1137 Ω resistor.

(vi) Find the power dissipated in the 1139 Ω resistor.

(vj) Find the power dissipated in the 1141 Ω resistor.

(vk) Find the power dissipated in the 1143 Ω resistor.

(vl) Find the power dissipated in the 1145 Ω resistor.

(vm) Find the power dissipated in the 1147 Ω resistor.

(vn) Find the power dissipated in the 1149 Ω resistor.

(vo) Find the power dissipated in the 1151 Ω resistor.

(vp) Find the power dissipated in the 1153 Ω resistor.

(vq) Find the power dissipated in the 1155 Ω resistor.

(vr) Find the power dissipated in the 1157 Ω resistor.

(vs) Find the power dissipated in the 1159 Ω resistor.

(vt) Find the power dissipated in the 1161 Ω resistor.

(vu) Find the power dissipated in the 1163 Ω resistor.

(vv) Find the power dissipated in the 1165 Ω resistor.

(vw) Find the power dissipated in the 1167 Ω resistor.

(vx) Find the power dissipated in the 1169 Ω resistor.

(vy) Find the power dissipated in the 1171 Ω resistor.

(vz) Find the power dissipated in the 1173 Ω resistor.

(wa) Find the power dissipated in the 1175 Ω resistor.

(wb) Find the power dissipated in the 1177 Ω resistor.

(wc) Find the power dissipated in the 1179 Ω resistor.

(wd) Find the power dissipated in the 1181 Ω resistor.

(we) Find the power dissipated in the 1183 Ω resistor.

(wf) Find the power dissipated in the 1185 Ω resistor.

(wg) Find the power dissipated in the 1187 Ω resistor.

(wh) Find the power dissipated in the 1189 Ω resistor.

(wi) Find the power dissipated in the 1191 Ω resistor.

(wj) Find the power dissipated in the 1193 Ω resistor.

(wk) Find the power dissipated in the 1195 Ω resistor.

(wl) Find the power dissipated in the 1197 Ω resistor.

(wm) Find the power dissipated in the 1199 Ω resistor.

(wn) Find the power dissipated in the 1201 Ω resistor.

(wo) Find the power dissipated in the 1203 Ω resistor.

(wp) Find the power dissipated in the 1205 Ω resistor.

(wq) Find the power dissipated in the 1207 Ω resistor.

(wr) Find the power dissipated in the 1209 Ω resistor.

(ws) Find the power dissipated in the 1211 Ω resistor.

(wt) Find the power dissipated in the 1213 Ω resistor.

(wu) Find the power dissipated in the 1215 Ω resistor.

(wv) Find the power dissipated in the 1217 Ω resistor.

(ww) Find the power dissipated in the 1219 Ω resistor.

(wx) Find the power dissipated in the 1221 Ω resistor.

(wy) Find the power dissipated in the 1223 Ω resistor.

(wz) Find the power dissipated in the 1225 Ω resistor.

(xa) Find the power dissipated in the 1227 Ω resistor.

(xb) Find the power dissipated in the 1229 Ω resistor.

(xc) Find the power dissipated in the 1231 Ω resistor.

(xd) Find the power dissipated in the 1233 Ω resistor.

(xe) Find the power dissipated in the 1235 Ω resistor.

(xf) Find the power dissipated in the 1237 Ω resistor.

(xg) Find the power dissipated in the 1239 Ω resistor.

(xh) Find the power dissipated in the 1241 Ω resistor.

(xi) Find the power dissipated in the 1243

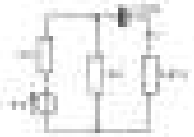


Figure 3-47



Figure 3-48



Figure 3-49



Figure 3-50

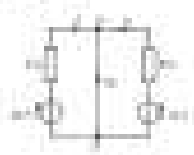


Figure 3-51

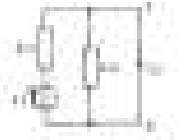


Figure 3-52

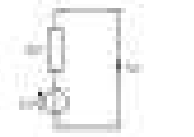


Figure 3-53

Problem 3- Use Thevenin's theorem to determine the value of current I in the circuit shown in Figure 3-52.

(a) The branch containing the 2 Ω resistor is taken out and not considered for Figure 3-53.

(b) The 9 V source is replaced with a short circuit. In the circuit, a 3 Ω is parallel 6 Ω, in series to 4 Ω, which is parallel to 12 Ω. $R_{TH} = 3\,\Omega \parallel 6\,\Omega + 4\,\Omega \parallel 12\,\Omega = 5.4\,\Omega$

(c) The current in the short circuit from the voltage source equals $I_{SC} = \frac{9\text{ V}}{3\,\Omega + 6\,\Omega} + \frac{9\text{ V}}{4\,\Omega + 12\,\Omega} = 3.75\text{ A}$

(d) Thevenin's equivalent circuit is shown in Figure 3-54. Using current division, $I = \frac{5.4\,\Omega}{5.4\,\Omega + 2\,\Omega} \times 3.75\text{ A} = 2.73\text{ A}$

Problem 4- Use the mesh analysis to find the current I in the circuit shown in Figure 3-55. Assume the current flowing in the clockwise direction in all meshes.

(a) The problem reduces to finding three mesh currents as shown in Figure 3-56.

(b) From Figure 3-56,

$$I_{12\,\Omega} = I_1 - I_2 = \frac{30\text{ V}}{3\,\Omega} - \frac{12\text{ V}}{3\,\Omega} = 6\text{ A}$$



Figure E.2.6

(a) If the voltage sources are reduced to zero, the impedance Z_{ab} looking in at terminals ab is given by $Z_{ab} = 4 + \frac{(3)(6)}{3+6} = 6\ \Omega$.

(b) The Norton equivalent network is shown in Figure E.2.6, where current I_N is given by

$$I_N = \frac{12}{4} + \frac{24}{3+6}\left(\frac{6}{3+6}\right) = 3 + 1.78 = 4.78\ \text{A}$$

i.e., $I_N = 4.78\ \text{A} \downarrow$ (or $-4.78\ \text{A} \uparrow$).



Figure E.2.7

Problem 2.7 The Norton equivalent in Problem 2.6 is connected to a load R_L as shown in Figure E.2.7. Determine the expression for the power P_L delivered to R_L as a function of R_L. Plot P_L versus R_L for $0 \le R_L \le 10\ \Omega$. Determine the value of R_L for which P_L is a maximum.



Figure E.2.8

(a) The Norton equivalent network is shown in Figure E.2.8, where I_N is given by

$$I_N = \frac{12}{4} + \frac{24}{3+6}\left(\frac{6}{3+6}\right) = 3 + 1.78 = 4.78\ \text{A}$$

i.e., $I_N = 4.78\ \text{A} \downarrow$ (or $-4.78\ \text{A} \uparrow$).

(b) The $4\ \Omega$ resistor is connected from the terminals of Figure E.2.8 as shown in Figure E.2.9, and the voltage v_L is given by

$$v_L = \frac{4}{4+R_L} I_N R_L = \frac{4}{4+R_L}\left(\frac{24}{3+6}\right) R_L = \frac{8R_L}{4+R_L}\ \text{V}$$

(c) The power delivered to R_L is

$$P_L = \frac{v_L^2}{R_L} = \frac{64R_L}{(4+R_L)^2}\ \text{W}$$

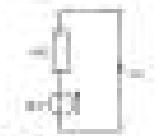


Figure E.2.9

(d) The Norton equivalent network is shown in Figure E.2.10, where short-circuit current I_N is given by

$$I_N = \frac{12}{4} + \frac{24}{3+6} = 3 + 2.67 = 5.67\ \text{A}$$

Since the equivalent of the gas across the $3\ \Omega$ resistor is zero V,

$$v_R = i_N R_N = 5.67 \times 0 = 0\ \text{V}$$

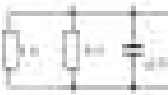


Figure E.2.10



Figure E.2.11

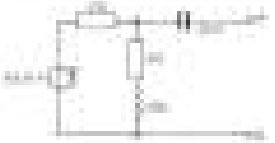


Figure 2-1(a)



Figure 2-1(b)

Problem 2-1 The two networks shown in Figure 2-1(a) and (b) have the same component parameters at corresponding pins. Analyze individually and give a one-sentence (or a 2-3 word) qualitative comparison between the two.

(a) Determine whether the two networks are reciprocal.

(b) The current magnitudes of corresponding branches in the "2-1-3" network is twice as large as in the "1-2-3" network. Determine the ratio of the power dissipated in the "2-1-3" network to that dissipated in the "1-2-3" network.



Figure 2-1(c)

(c) Determine the Thévenin equivalent of the network in Figure 2-1(c) as seen looking into the terminals 1 and 3.

(d) Determine the Thévenin equivalent of the network in Figure 2-1(c) as seen looking into the terminals 2 and 3.

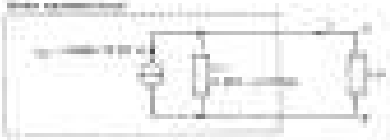


Figure 10.xx

Return the power dissipated in the 6 Ω resistor i.e.

$$P_{6\Omega} = \frac{V_{6\Omega}^2}{6} = \frac{(\quad)^2}{6} = \quad \text{W}$$



Figure 10.xx



Figure 10.xx

Problem 10. Use the mesh-current method to find the power dissipated in the 6Ω resistor in the circuit shown in Figure 10.xx. Also find the power dissipated in the 10Ω resistor. Determine the magnitude of the current flowing in the 4 Ω resistor and hence find the voltage drop across it.

Key to the many problems in Section 7 (on page 156) which ask current for a specific element:

(1) Label the 3 mesh currents.

(2) Write down the 3 mesh equations.

(3) Solve the 3 equations for the 3 mesh currents.

(4) Calculate the current in the specified element as the sum or difference of the 3 mesh currents.

For the 3 KVE (2) and putting values in volts:

$$\text{mesh 1:} \quad 10(I_1 - I_2) + 4(I_1 - I_3) = 0$$

$$\text{i.e.} \quad 7I_1 - 5I_2 - 2I_3 = 0 \qquad (1)$$

For the 3 KVE (2) and substituting values in volts:

$$\text{mesh 2:} \quad 10(I_2 - I_1) + 6I_2 + 8(I_2 - I_3) = 0$$

$$\text{i.e.} \quad -5I_1 + 12I_2 - 4I_3 = 0 \qquad (2)$$

Solving Equations (1) and (2) by using determinants gives:

$$\frac{I_1}{\begin{vmatrix} -5 & -2 \\ 12 & -4 \end{vmatrix}} = \frac{I_2}{\begin{vmatrix} 7 & -2 \\ -5 & -4 \end{vmatrix}} = \frac{I_3}{\begin{vmatrix} 7 & -5 \\ -5 & 12 \end{vmatrix}}$$

$$\frac{I_1}{(-5)(-4) - (12)(-2)} = \frac{I_2}{(7)(-4) - (-5)(-2)} = \frac{I_3}{(7)(12) - (-5)(-5)}$$

$$\frac{I_1}{20 + 24} = \frac{I_2}{-28 - 10} = \frac{I_3}{84 - 25}$$

Now, since

$$R_{eq} = \frac{\left[R_1 + \left(\frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}\right]\left(\frac{1}{R_4} + \frac{1}{R_5}\right)^{-1}}{\left[R_1 + \left(\frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}\right] + \left(\frac{1}{R_4} + \frac{1}{R_5}\right)^{-1}} = \frac{(R_1 R_2 + R_1 R_3 + R_2 R_3)(R_4 + R_5)}{(R_1 R_2 + R_1 R_3 + R_2 R_3) + R_4 R_5}$$

$$= \frac{(2.5)(2.5)(1.25)(2.5)}{(6.25)(2.5) + (2.5)(2.5)} = 1.25 \text{ k}\Omega = 1250 \ \Omega$$

Hence, the desired meter resistance is $r_m = 1250 - 750 = 500 \ \Omega$. Meeting these specifications is left as an exercise.

(b) The requirement r_m = 0 leading to (a) is a fundamentally ambiguous. It can be satisfied by:

1. Any R_1, R_2, R_3, R_4, R_5 that satisfy Equation 7 (page 618).
2. The Thévenin equivalent circuit as shown in Figure 8.19(c), where instead V_{eq} is given by

$$V_{eq} = \left[\frac{(R_1 R_2 + R_1 R_3 + R_2 R_3)}{(R_1 R_2 + R_1 R_3 + R_2 R_3) + R_4 R_5}\right](10 \text{ V}) = (0.5)(10 \text{ V}) = 5 \text{ V}$$



Figure 10.16

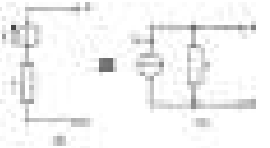
Given the magnitude of the current flowing in the 2 Ω resistor is 0.25 A.

Problem 10.16 from *Schaum's Outline Series* by Joseph A. Edminister, 2nd ed. (McGraw-Hill, 1993).

11.1 Thévenin and Norton equivalent networks

In terms of Sections 11.1 and 11.2 from where "Thévenin's and Norton's Networks are applied to the basic circuits, analysis of circuits can proceed. Also the Thévenin and Norton equivalents of the circuits of Figure 11.1(a) can be drawn as such where the equivalent loading will be measured at the terminals a and b.

A homework title is Figure 11.1(a) as such where considered. The above circuit network is given in 11.1.



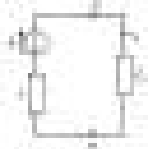


Figure P3.48



Figure P3.49



Figure P3.50

Figure P3.47 Equivalent Thévenin and Norton circuits

If, alternatively, R_3 in Figure P3.47(b) was short-circuited, the short-circuit current would be I_{sc}:

$$I_{sc} = \frac{E_1}{R_1} + I_1$$

Figure P3.48 shows a section of a d.c. circuit. It is required to find the current in a resistor of resistance R_3, using Norton's theorem.

$$\therefore \quad I_3 = \left(\frac{E_1}{R_1} + I_1\right)\left(\frac{R_1 R_2}{R_1 + R_2}\right) \Big/ R_3 \text{ from above}$$

As in Figure P3.48, let us find the current flowing through resistor R_3 in the circuit shown in Figure P3.49 using Norton's theorem.

Then, as in Section 3.6.2, the Norton equivalent circuit shown in Figure P3.49 can be redrawn.

Problem 3.6 Calculate the current flowing through resistor R_3 in Figure P3.49(a) by using Norton's theorem.

(a) If the terminals AB of Figure P3.50(a) are short-circuited, the current in the short-circuit is $I_{sc} = 10/2 + 5 = 10$ A. The equivalent Thévenin circuit is shown in Figure P3.50(b).

(b) The current in the voltage source when terminals AB are open-circuited is $I = 10/(2 + 3) = 2$ A. The equivalent Thévenin circuit is shown in Figure P3.50(c).

Check the equivalent Thévenin circuit is as shown in Figure P3.50(c).



Figure 3.1.6



Figure 3.1.7



Figure 3.1.8



Figure 3.1.9



Figure 3.1.10

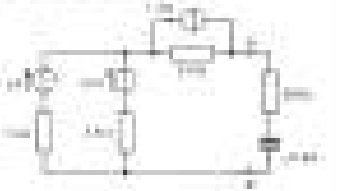










Figure E4.6a



Figure E4.6b



Figure E4.6c



Figure E4.6d



Figure E4.6e

(c) When a load resistor R_L (ohms) is connected across one pair of terminals of a Norton's or Thévenin's equivalent circuit, the load current I_L (amperes) is given by

$$I_L = \frac{I_N R_N}{R_N + R_L} = \frac{V_{Th}}{R_{Th} + R_L}$$

Example 3.11(c)

Given the values I_N determined in Ex. 3.11(b) and R_N determined in Ex. 3.10(b), the load current I_L when $R_L = 2\ \Omega$ is given by

$$I_L = \frac{I_N R_N}{R_N + R_L} = \frac{(2.5\ \text{A})(1.5\ \Omega)}{1.5\ \Omega + 2\ \Omega} = \frac{3.75\ \text{A}}{3.5} = 1.071\ \text{A}$$

Further problems on Thévenin's and Norton's equivalent circuits may be found in Section 3.3 following worked Problem 3.13. See page 208

3.3.3 Problems on Thévenin's and Norton's Theorems

Thévenin's theorem

1. For the network shown in Figure 3.116, determine Thévenin's equivalent circuit. (3.12(a))



Figure 3.116

2. Determine, using Thévenin's theorem, the current I_L in resistor R_L in the network shown in Figure 3.117. (3.12(a))



Figure 3.117

3. Determine the Thévenin equivalent circuit with respect to terminals AB of the network shown in Figure 3.118. Hence determine the magnitude of the current flowing in a $5\ \Omega$ resistor connected across terminals AB and the power dissipated in the resistance. (3.12(a), (b))



Figure 3.118

4. For the network shown in Figure 3.119, use Thévenin's theorem to determine the current flowing in the $5\ \Omega$ resistance. (3.12(a))

Figure 3.119

5. Obtain for the network shown in Figure 3.120 the Thévenin equivalent circuit as viewed from terminals AB, and hence determine the current flowing in a $10\ \Omega$ resistor connected between terminals AB. (3.12(a), (b))

Figure 3.120

6. Determine for the network shown in Figure 3.121 the Thévenin equivalent circuit with respect to terminals AB, and hence determine the current flowing in a $10\ \Omega$ resistor connected between terminals AB. (3.12(a), (b))

Figure 3.121

7. For the network shown in Figure 3.122, derive the Thévenin equivalent circuit with respect to terminals AB, and hence determine the current flowing in an $8\ \Omega$ resistor connected across terminals AB. (3.12(a), (b))



Figure 3.122

8. For the Thévenin's equivalent circuit shown in Figure 3.123, determine the Norton's equivalent circuit. (3.12(a), (c))

Figure 3.123



Figure 4.4(a)



Figure 4.4(b)



Figure 4.4(c)



Figure 4.5(a)



Figure 4.5(b)

1. In a Kelvin double bridge, shown in Figure 4.5(b), the following data were obtained when balance was attained: $R_1 = 0.1\ \Omega$, $R_2 = 100\ \Omega$, $R_3 = 0.1\ \Omega$, $R_4 = 100\ \Omega$, $R_5 = 0.1\ \Omega$, $R_6 = 100\ \Omega$. Calculate the value of the unknown resistance R_x. [0.1 Ω]

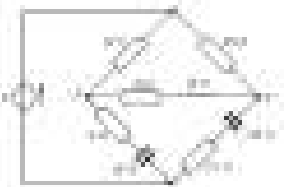


Figure 4.6(a)

2. Explain the Kelvin double bridge method for the measurement of low resistance. Why is it necessary to use four terminal resistors? Derive the equation for the unknown resistance.

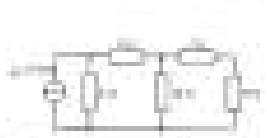


Figure 4.14a

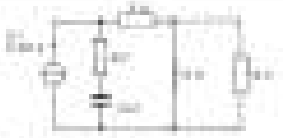


Figure 4.14b



Figure 4.14c



Figure 4.14d

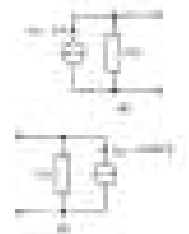


Figure 4.14e

Network Analysis

4.1 Repeat problem 4.1(a) to (c) and (e) to (i) above using Norton's theorem instead of Thevenin's theorem.

4.2 Determine the current flowing in the 5 Ω resistance in the network shown in figure 4.14a by applying Norton's theorem. (0.4 A)

4.3 For the network shown in figure 4.14b, use Norton's theorem to determine the current flowing in the 4 Ω resistance. (1.68 A)

4.4 Determine (i) the current flowing in the 10 Ω resistance, and (ii) the power dissipated in the 2 Ω resistance, for the network shown in figure 4.14c, by applying (a) Kirchhoff's laws, (b) Thevenin's theorem, (c) Norton's theorem. (a) (i) 0.5 A (ii) 0.5 W (b) (i) 0.5 A (ii) 0.5 W (c) (i) 0.5 A (ii) 0.5 W

4.5 Repeat problem 4.4 in figure 4.14c if a 10 Ω resistor is connected between terminals A and B. (a) (i) 0.25 A (ii) 0.125 W (b) (i) 0.25 A (ii) 0.125 W (c) (i) 0.25 A (ii) 0.125 W

Maximum power transfer theorem and applications

4.6 Determine the value of load resistance R_L in figure 4.14d for maximum power transfer. $R_L = 1.2\ \Omega$

4.7 Determine the value of load resistance R_L in figure 4.14e for maximum power transfer. $R_L = 0.8\ \Omega$

4.8 (a) Determine the current flowing in the load of the network shown in figure 4.14f when the load resistance is (i) 2 Ω, (ii) 4 Ω, (iii) 6 Ω, (iv) 8 Ω, (v) 10 Ω. (i) 1.5 A (ii) 1.2 A (iii) 1 A (iv) 0.86 A (v) 0.75 A

(b) Determine the current flowing in the 2 Ω, 4 Ω, 6 Ω, 8 Ω and 10 Ω resistors in part (a) above, by applying Thevenin's theorem. (i) 1.5 A (ii) 1.2 A (iii) 1 A (iv) 0.86 A (v) 0.75 A

4.9 Determine the maximum power transferred between the source and load resistor in the network in figure 4.14g. (0.5 W)

4.10 Determine the maximum power transferred between the source and load resistor in the network in figure 4.14h. (0.5 W)

4.11 Determine the maximum power transferred between the source and load resistor in the network in figure 4.14i. (0.5 W)

4.12 Determine the maximum power transferred between the source and load resistor in the network in figure 4.14j. (0.5 W)

4.13 Determine the maximum power transferred between the source and load resistor in the network in figure 4.14k. (0.5 W)

4.14 Determine the maximum power transferred between the source and load resistor in the network in figure 4.14l. (0.5 W)

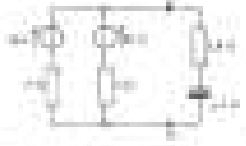


Figure 12-1(a)

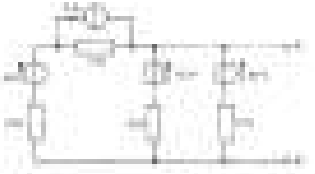


Figure 12-1(b)



Figure 12-2(a)

(b) Below, an equivalent Thévenin circuit for terminals A,B of the network shown in Figure 12-2(a). Above illustrates the gain versus sink source v_i (v_i = output) characteristic for transistor feedback. Mesh Kirchhoff:

$$[R_1 + (\alpha\beta)r_{ds}]\,i_1 + \alpha\beta r_{ds}\,i_2 = [(1+\alpha\beta)r_{ds}]\,v_i \quad (12\text{-}1)$$

(c) For the network shown in Figure 12-2(a), shown as the Thévenin equivalent circuit, and for the Norton equivalent network v_i/i_s is to determine is independent transistor is open for Thévenin and short circuit driving as for i_s (conductance for output) for Thévenin and Norton equivalent circuits.

$$\frac{v_o}{i_s} = \frac{(1+\alpha\beta)r_{ds}\,[R_1 + r_{ds}]}{R_1 + (1+\alpha\beta)r_{ds}} \quad (12\text{-}2)$$

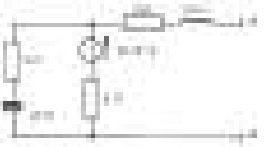


Figure 12-2(b)

Assignment 10

The assignment covers the material introduced in chapters 10 & 11.

The marks for each question are shown in brackets and the total mark available is 100.

For the circuit in Figure 10.1, calculate the values for the following:

(a) [illegible] (5)
(b) [illegible] (5)
(c) [illegible] (5)
(d) [illegible] (5)
(e) [illegible] (5)
(f) [illegible] (5)

Recommend how each [illegible] in the table above could be [illegible].

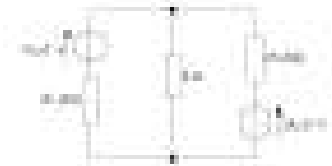


Figure 10.1

34 Delta-star and star-delta transformations

At the end of this chapter you should be able to:

- recognise delta and star connections in electrical circuits
- apply star-delta and delta-star transformations to appropriate circuits to solve for unknowns

34.1 Introduction

34.2 Delta and star connections

34.3 Delta-star transformation

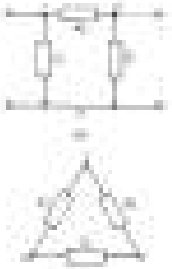


Figure 9.4.1 (a) n = resistors (1) connected in star (b) Delta-connected network

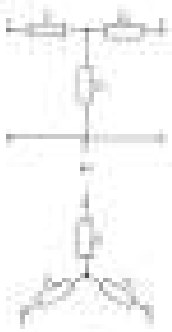


Figure 9.4.2 (a) n = resistors connected in delta (b) star-connected network

A resistance between A and B in Figure 9.4.1(a) can be calculated therefore by giving R_{AB} for the equivalence (n) as parallel $(\|)$ the series combinations of (1) and (2):

$$R_{AB} = \frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3}$$

(a) Equating for delta the equivalent impedance between terminals A and B in Fig. 9.4.1(b) (terminals A, $(2) = (3)$ free

	Delta	Star
$R_{AB} =$	$\frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3} = R_1 \| (R_2 + R_3)$	(9.4.1)
By similar reasoning: $R_{BC} =$	$\frac{R_2(R_1 + R_3)}{R_1 + R_2 + R_3} = R_2 \| (R_1 + R_3)$	(9.4.2)
And $R_{CA} =$	$\frac{R_3(R_1 + R_2)}{R_1 + R_2 + R_3} = R_3 \| (R_1 + R_2)$	(9.4.3)

Taking the first three simultaneous equations (9.4.1), (9.4.2) and (9.4.3) for R_1, R_2 and R_3

Equation (9.4.1) − equation (9.4.2) gives:

$$\frac{R_1R_2 - R_2R_3}{R_1 + R_2 + R_3} = R_1 - R_2 \tag{9.4.4}$$

Equation (9.4.1) − equation (9.4.3) gives:

$$\frac{R_1R_3 - R_2R_3}{R_1 + R_2 + R_3} = R_1 - R_3$$

Now when $R_1 = \frac{R_1R_2}{R_1 + R_2 + R_3}$

Dividing equation (9.4.5) − equation (9.4.4) gives:

$$\frac{R_1R_3 - R_1R_2}{R_1 - R_2 - R_3} = R_3 - R_2 \tag{9.4.5}$$

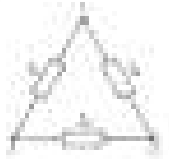
Equation (9.4.1) − equation (9.4.3) gives:

$$\frac{R_1R_2 - R_2R_3}{R_1 - R_2 - R_3} = R_2$$

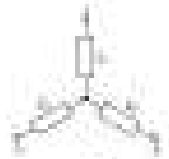
Now when $R_2 = \frac{R_1R_2}{R_1 + R_2 + R_3}$

Finally, equation (9.4.5) − equation (9.4.4) gives:

$$\frac{R_1R_3 - R_2R_3}{R_1 - R_2 - R_3} = R_3 - R_2 \tag{9.4.6}$$



(a)



(b)

Figure 8.6

Equations (8.5) to (8.7) can be rearranged to give:

$$\frac{R_1 R_2}{R_1 + R_2 + R_3} = \frac{R_1 R_2}{R_{\Delta}}$$

from which $R_3 = \dfrac{R_1 R_2}{R_1 + R_2 + R_3}$

Similarly, the star network shown in Figure 8.6(b) is equivalent to the delta network shown in Figure 8.6(a) provided

$$R_1 = \frac{R_2 R_3}{R_1 + R_2 + R_3} \tag{8.8}$$

$$R_2 = \frac{R_3 R_1}{R_1 + R_2 + R_3} \tag{8.9}$$

and $$R_3 = \frac{R_1 R_2}{R_1 + R_2 + R_3} \tag{8.10}$$

If, in general, three impedances Z_1, Z_2 and Z_3 are connected in delta (as shown in Figure 8.7(a)), then the equivalent star network is as shown in Figure 8.7(b) where Z_1 is connected to the same terminal as Z_2 and Z_3, Z_2 is connected to the same terminal as Z_1 and Z_3, and Z_3 is connected to the same terminal as Z_1 and Z_2. Comparing Figures 8.7(a) and (b) with Figures 8.6(a) and (b) it can be seen that Z_1 corresponds to R_1, Z_2 corresponds to R_2 and Z_3 corresponds to R_3. Thus, for example, the star equivalent of the delta network shown in Figure 8.7(a) is given by:

$$Z_1 = \frac{Z_2 Z_3}{Z_1 + Z_2 + Z_3} \quad \text{(8.11)}$$

$$Z_2 = \frac{Z_3 Z_1}{Z_1 + Z_2 + Z_3} \quad \text{(8.12)}$$

and $$Z_3 = \frac{Z_1 Z_2}{Z_1 + Z_2 + Z_3} \quad \text{(8.13)}$$



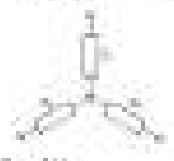


Figure 8.7

> **Problem 3.** Replace the delta-connected network shown in Figure 8.8(a) by an equivalent star connection.

(a) The equivalent star network is as shown in Figure 8.8(b). Then using equation (8.8),

$$R_1 = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$

$$= \frac{(20)(10)}{40 + 20 + 10} = \frac{200}{70} = 2.86\,\Omega$$



Figure 5.4.1

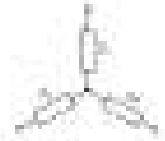


Figure 5.4.2



Figure 5.4.3



Figure 5.4.4



Figure 5.4.9

Using equation (5.4.1):

$$R_1 = \frac{R_2 R_3}{R_2 + R_3 + R_4} = \frac{(1.5\,\text{k}\Omega)(3.3\,\text{k}\Omega)}{5.8\,\text{k}\Omega} = 0.854\,\text{k}\Omega$$

Using equation (5.4.2):

$$R_2 = \frac{R_1 R_3}{R_1 + R_3 + R_4} = \frac{(1.5\,\text{k}\Omega)(3.3\,\text{k}\Omega)}{5.8\,\text{k}\Omega} = 0.854\,\text{k}\Omega$$

The equivalent network is shown in Figure 5.4.10 and is further simplified in Figure 5.4.11.

$(2.5\,\text{k}\Omega + 1.5\,\text{k}\Omega)\,\|\,(3.3\,\text{k}\Omega + 0.854\,\text{k}\Omega) = 4\,\text{k}\Omega\,\|\,4.154\,\text{k}\Omega = 2.039\,\text{k}\Omega$



Figure 5.4.10



Figure 5.4.11

$$\frac{v_{ab}}{10\,\text{V}} = \frac{2.039\,\text{k}\Omega}{2.039\,\text{k}\Omega + 0.854\,\text{k}\Omega} = 0.7$$

$$v_{ab} = 7\,\text{V}$$

Determine v_{ab} when the network in Problem 5.4.9 contains a source transformation with a $10\,\text{V}$ source and a $2\,\text{k}\Omega$ resistor.

a) Apply equation (5.4.3): $R_{ab} = \frac{R}{R_{eq}} = \frac{2\,\text{k}\Omega}{0.854\,\text{k}\Omega} = 2.34\,\text{k}\Omega$

b) Apply equation (5.4.4): $P_{ab} = \frac{v_{ab}^2}{R_{ab}} = \frac{(7\,\text{V})^2}{2.34\,\text{k}\Omega} = 20.9\,\text{mW}$

c) Power is dissipated in the $2.5\,\text{k}\Omega$ resistance in Figure 5.4.7 to produce $7\,\text{V}$ at $2.5\,\text{k}\Omega$. Power is also dissipated in the $2\,\text{k}\Omega$ source so

$$P_s = \frac{(10\,\text{V})^2}{(2\,\text{k}\Omega + 2.5\,\text{k}\Omega)} \times (2\,\text{k}\Omega) = 16.0\,\text{mW} \qquad P_{2.5\,\text{k}\Omega} = 19.6\,\text{mW}$$

Total power $P_T = 19.6\,\text{mW} + 16.0\,\text{mW} = 35.6\,\text{mW}$

Problem 5.4.10 Determine the total energy supplied by the source in Figure 5.4.12 for the period $0 < t < 1\,\text{s}$. Assume that the switch is closed at $t = 0$ and opened at $t = 1\,\text{s}$.

Figure 5.4.12

a) In Figure 5.4.12, the capacitance can alternately be classified as parallel or series combination with the other elements. At $t = 0$ and $t = 0^+$, the $2\,\text{k}\Omega$ and $3\,\text{k}\Omega$ combination and $4\,\text{k}\Omega$ are in parallel. At $t = \infty$, the capacitor acts as open circuit. The $4\,\text{k}\Omega$ resistor is in series with the $2\,\text{k}\Omega$ and $3\,\text{k}\Omega$ combination. The time constant is the product of the equivalent resistance and capacitance. The $4\,\text{k}\Omega$ resistor carries the current $i(t)$ as it flows from the voltage source and $v_c(t)$ is the voltage across the capacitor. Using equation (5.4.1):

$$R_1 = \frac{(2\,\text{k}\Omega)(3\,\text{k}\Omega)}{2\,\text{k}\Omega + 3\,\text{k}\Omega + 4\,\text{k}\Omega} = 1.0\,\text{k}\Omega \qquad \text{(from equation (5.4.1))}$$

Using equation (5.4.2):

$$R_2 = \frac{(2\,\text{k}\Omega)(4\,\text{k}\Omega)}{2\,\text{k}\Omega + 3\,\text{k}\Omega + 4\,\text{k}\Omega} = 1.6\,\text{k}\Omega \qquad \text{(from equation (5.4.2))}$$

Using equation (5.4.3):

$$R_3 = \frac{(3\,\text{k}\Omega)(4\,\text{k}\Omega)}{2\,\text{k}\Omega + 3\,\text{k}\Omega + 4\,\text{k}\Omega} = 2.4\,\text{k}\Omega \qquad \text{(from equation (5.4.3))}$$



Figure 4.4.6

The solution of Figure 4.4.6 was laid out similarly to that of Figure 4.4.4. The 2 kN and 3 kN loads were applied at the joints and the reactions at A and B were found by statics. Because the geometry is symmetrical, the reactions at A and B are equal.

$$R_A = \frac{2\,\text{kN} + 3\,\text{kN}}{2} = 2.5\,\text{kN} \uparrow$$

Section subjected to the 0.5 kN load is at x, labeled 1-1, Figure 4.4.6c. Summing

$$\uparrow + \quad \sum F_y = \frac{3}{5}F_{AC} - 0.5\,\text{kN} = 0$$

$$F_{AC} = 0.833\,\text{kN}$$

Joint B, Figure 4.4.6d, is subject to $F_{BC} = 0.833\,\text{kN}$ and $F_{BD} = 2.5\,\text{kN}$, and is in equilibrium. Joint C, Figure 4.4.6e, is subject to $F_{AC} = 0.833\,\text{kN}$, $F_{BC} = 0.833\,\text{kN}$, F_{CD}, and F_{CE}. Summing forces in the horizontal direction

$$\rightarrow + \quad \sum F_x = \frac{4}{5}F_{CE} + \frac{4}{5}F_{BC} - F_{AC} = 0$$

$$F_{CE} = 0.417\,\text{kN}$$

Joint D is in equilibrium under the action of $F_{BD} = 2.5\,\text{kN}$, F_{CD}, and F_{DE}. Summing forces in the horizontal direction

$$\rightarrow + \quad \sum F_x = \frac{4}{5}F_{CD} - F_{DE} = 0$$

$$F_{DE} = 0.333\,\text{kN}$$

The zero members for the X-B combination is given by $F_{CD} = 0$, $F_{DE} = 0$, $F_{CE} = 0$.



Figure 4.4.7

Problem 4 Figure 4.4.7c shows an Australian bridge used for trussing, with high economy. Determine by joint method the following:

(a) Explain the zero force (ZFM) that are associated with a vertical load, hence determine the forces in members F_{AC}, F_{BC}, and F_{CD}.

(b) If $F_{AC} = 4.5\,\text{kN}$, $F_{BC} = 4.5\,\text{kN}$, $F_{CD} = 0$, and $F_{DE} = 0$, determine the reactions at A, and F_{CE} in members.

(a) The entire truss is in equilibrium under the action of the 2 kN load and the reactions R_A and R_B. Summing moments about A

$$\circlearrowleft + \quad \sum M_A = 0 = -(2\,\text{kN})(3\,\text{m}) - (2\,\text{kN})(6\,\text{m}) + R_B(9\,\text{m})$$

$$R_B = 2\,\text{kN} \uparrow$$

From equilibrium of the entire truss

$$\uparrow + \quad \sum F_y = 0 = R_A - 2\,\text{kN} - 2\,\text{kN} + 2\,\text{kN}$$

$$R_A = 2\,\text{kN} \uparrow$$

From equilibrium of joint A

$$\uparrow + \quad \sum F_y = 0 = 2\,\text{kN} + \frac{3}{5}F_{AC}$$

$$F_{AC} = -3.33\,\text{kN}$$

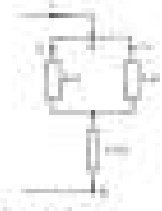


Figure 5.2.17



Figure 5.2.18



Figure 5.2.19



Figure 1.1.6



Figure 1.1.7

Problem 1. For the network shown in Figure 1.1.8, determine the effective resistance between the terminals A and B. Assume that each of the resistances is equal to 1Ω.

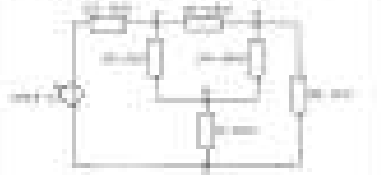


Figure 1.1.8

(a) The network may initially be simplified by combining the three resistances in parallel between the terminals C and D. The equivalent resistance R_1 is, using Equation (1.1.),

$$\frac{1}{R_1} = \frac{1}{1} + \frac{1}{1} + \frac{1}{1} = 3$$

$$R_1 = \frac{1}{3}\ \Omega$$

The three resistances in series between the terminals A and C are replaced by the equivalent resistance R_2, where

$$R_2 = 1 + 1 + 1 = 3\ \Omega$$

The three resistances in series between the terminals D and B are replaced by the equivalent resistance R_3, where

$$R_3 = 1 + 1 + 1 = 3\ \Omega$$



Figure 1.1.9

The network is now reduced to that shown in Figure 1.1.9. The resistances R_2 and R_3 are in parallel between the terminals A and B. The equivalent resistance R_4 is, using Equation (1.1.),

$$\frac{1}{R_4} = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

$$R_4 = \frac{3}{2}\ \Omega$$

The resistances R_1 and R_4 are in series between the terminals A and B. The equivalent resistance R_5 is, using Equation (1.1.),

$$R_5 = \frac{1}{3} + \frac{3}{2} = \frac{11}{6}\ \Omega$$



Figure 9-2



Figure 9-3

9-1 Two-dipole Norton/Thévenin equivalents



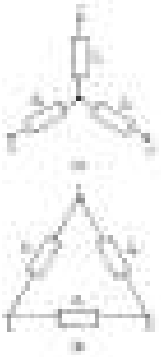


Figure 4.2.4

$$\text{or} \quad -I_1R_1 + I_2R_2 + I_3R_3 = V_1 \left(\frac{R_1R_2}{R_1 + R_2 + R_3} \right) + V_2R_3 \qquad \text{(4.2.4)}$$

Now, equation (4.2.4)

Now, $I_1 = \frac{V_1(R_2 + R_3) + V_2R_3}{\Delta}$

From equation (4.2.5),

$$-I_1R_1 + I_2R_2 + I_3R_3 = V_2 \left(\frac{R_2R_3}{R_1 + R_2 + R_3} \right) + V_1R_3 \qquad \text{(4.2.5)}$$

From equation (4.2.6)

Where $I_2 = \frac{V_2(R_1 + R_3) + V_1R_3}{\Delta}$

Also, from equation (4.2.6),

$$-I_1R_1 + I_2R_2 + I_3R_3 = V_3 \left(\frac{R_1R_2}{R_1 + R_2 + R_3} \right) + V_1R_2 \qquad \text{(4.2.6)}$$

From equation (4.2.6)

Now, $I_3 = \frac{V_3(R_1 + R_2) + V_1R_2}{\Delta}$

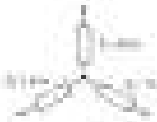
Consequently, the delta-wye transformation as in Figure 4.2.4(b) is equivalent to the wye network shown in Figure 4.2.4(a), where

$$R_1 = \frac{R_2R_3}{R_1 + R_2 + R_3} \qquad \text{(4.2.7)}$$

$$R_2 = \frac{R_1R_3}{R_1 + R_2 + R_3} \qquad \text{(4.2.8)}$$

and $R_3 = \frac{R_1R_2}{R_1 + R_2 + R_3}$ (4.2.9)

In a wye-delta transformation, each impedance in the wye network is the product of the two impedances in the adjacent delta branches divided by the sum of the three delta impedances. For the wye impedance R_1 in Figure 4.2.4(a), the adjacent delta impedances are R_2 and R_3. Similarly, the adjacent delta impedances for the wye impedance R_2 are R_1 and R_3, and the adjacent delta impedances for the wye impedance R_3 are R_1 and R_2. The delta impedance in the denominator of each wye impedance is the sum of all the delta impedances. Thus, for the wye impedance R_1 in Figure 4.2.4(a), the adjacent delta impedances are R_2 and R_3, and the delta impedance in the denominator is the sum $R_1 + R_2 + R_3$. Similarly, for the wye impedance R_2 in Figure 4.2.4(a), the adjacent delta impedances are R_1 and R_3, and the delta impedance in the denominator is the sum $R_1 + R_2 + R_3$. Finally, for the wye impedance R_3 in Figure 4.2.4(a), the adjacent delta impedances are R_1 and R_2, and the delta impedance in the denominator is the sum $R_1 + R_2 + R_3$.



Then, for example, the delta equivalent of the resistors are current shown in Figure 9.47 is given by

$$R_a = \frac{(10)(15) + (15)(20) + (20)(10)}{15} = \frac{770}{15} = 51.33\ \Omega$$

$$R_b = \frac{770}{15} = 51.33\ \Omega \qquad R_c = \frac{770}{20} = 38.5\ \Omega$$



Problem 9. Determine the delta-connected equivalent resistors for the wye-connected equivalent shown in Figure 9.48.

Figure 9.48

Figure 9.49 shows the network of Figure 9.48 redrawn and Figure 9.50 shows the equivalent delta connection connecting resistances R_a, R_b, and R_c. From equation 9.1 or 9.2,

$$R_a = \frac{(40)(10) + (10)(20) + (20)(40)}{10} = \frac{1400}{10} = 140\ \Omega$$

$$R_b = \frac{1400}{20} = 70\ \Omega \qquad R_c = \frac{1400}{40} = 35\ \Omega$$



Figure 9.49

From equation 9.3 or 9.4,

$$R_a = \frac{(40)(10) + (10)(20) + (20)(40)}{40} = \frac{1400}{40} = 35\ \Omega$$

$$R_b = \frac{1400}{20} = 70\ \Omega \qquad R_c = \frac{1400}{10} = 140\ \Omega$$



Figure 9.50

From equation 9.5 or 9.6,

$$R_a = \frac{(40)(10) + (10)(20) + (20)(40)}{20} = \frac{1400}{20} = 70\ \Omega$$

$$R_b = \frac{1400}{40} = 35\ \Omega \qquad R_c = \frac{1400}{10} = 140\ \Omega$$

Problem 7. Given a network with $R_1 = 100\ \Omega$, $V_1 = 12\ \angle 0°$ V, $R_2 = 50\ \Omega$, $V_2 = 6\ \angle 0°$ V, $R_3 = 75\ \Omega$, and $R_4 = 25\ \Omega$, determine the current in each of the three branches using Kirchhoff's current equations.



(a)



(b)

Figure 9.51

The two nonreference currents and the two unknown branch currents (assuming directions as in the labels in Figure 9.51) are shown in Figure 9.52. From equation 9.7,

$$V_1 = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_3}$$

$$I_1 = \frac{(100)(50) + (100)(75) + (50)(75)}{(75)(75)} = \frac{(5000 + 7500 + 3750)}{(75)(75)} = 2.67\ \text{A}$$

$$I_2 = \frac{(5000 + 7500 + 3750)}{(50)(75)} = 4\ \text{A}$$

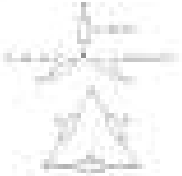


Figure 5.4.2A

$$\frac{\left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right)}{\left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right)} = \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right)$$

$$\frac{\left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right)}{\left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right)} = \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right)$$

$$= \frac{\left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right)}{\left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right)} = \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right)$$

$$\therefore \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right) \cdot \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right) = \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right)$$

Using equation (5.4.1),

$$\therefore f_1 = \frac{\left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right) + \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right) + \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right)}{\left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right)}$$

$$= \frac{\left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right) + \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right)}{\left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right)}$$

$$\therefore \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right) \cdot \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right) = \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right)$$

Using equation (5.4.1),

$$\therefore f_2 = \frac{\left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right) + \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right) + \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right)}{\left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right)} = \frac{\left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right) + \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right)}{\left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right)}$$

$$= \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right) - \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right) \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right) = \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right)$$

Further corollaries to subsection and non-perfect transformations may be found in section 5.1 (Elementary corollaries 1.1 to 1.5).

5.4.3 Stability specifications for multiple-input and non-linear transformations

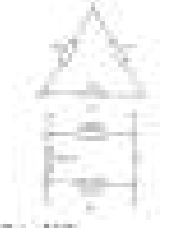


Figure 5.4.3A

1. Establish the data transmitted to multiple factors in Figure 5.4.3A to their equivalent first-order transmitted:

$$\begin{aligned} \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right) &= \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right) + \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right) - \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right) \\ \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right) &= \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right) \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right) + \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right) \end{aligned}$$

2. Determine the characteristic equivalent controller for the proportional regulator shown in Figure 5.4.3:

$$\begin{aligned} \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right) &= \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right) \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right) + \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right) \\ \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right) &= \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right) \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right) + \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right) \\ \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right) &= \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right) \end{aligned}$$

3. (a) Transform the 2-variable system shown in Figure 5.4.3A into its equivalent (both after-control and regulator).

   (b) Change the 2-variable control system in Figure 5.4.3A into its equivalent after-control system.

$$\begin{aligned} \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right) &= \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right) \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right) - \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right) \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right) \\ \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right) &= \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right) - \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right) \\ \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right) &= \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right) \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right) - \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right) \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right) \\ \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right) &= \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right) - \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.0 & 1.0 \end{array}\right) \end{aligned}$$

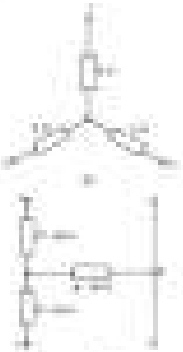


Figure 9.2.9



Figure 9.3.1



Figure 9.3.2



Figure 9.3.3

1 For the circuit shown in Figure 9.2.9, determine an expression for I and hence find the power dissipated in the 4 Ω resistance.

$I = 1.5\angle 0 + 2\angle -\pi/2 = 1.5 - j2 = 2.5\angle -0.927$ A

$P = I^2R = 2.5^2 \times 4 = 25$ W

2 (a) A 50 Hz sinusoidal voltage is applied to a 20 mH inductance. Determine the impedance of the inductor in rectangular and polar forms.

(b) Three impedances each of $(2 + j3)\ \Omega$ are connected to one another to form a delta network. Determine the equivalent star connected network.

$Z_{star} = \frac{Z_{delta}}{3} = \frac{2 + j3}{3} = 0.667 + j1\ \Omega$

$Z_{star} = 0.667 + j1 = 1.2\angle 56.3°\ \Omega$

3 (a) Derive the star-connected network of three impedances equivalent to that in the delta network shown in Figure 9.3.2.

(b) Obtain the delta-connected equivalent network for Figure 9.3.3.

$Z_{AB} = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_3}$

$Z_{BC} = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_1}$

$Z_{CA} = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_2}$

4 For the a.c. bridge network shown in Figure 9.3.4(a), derive the relationship between R_1, R_2, R_3, R_4 and ω when the bridge is balanced. Hence find the value of R_4 and C_4 when the bridge is balanced if $R_1 = 1\ \text{k}\Omega$, $R_2 = 2.5\ \text{k}\Omega$, $R_3 = 2\ \text{k}\Omega$, $C_1 = 0.5\ \mu\text{F}$ and $\omega = 1000$ rad/s.

5 For the network shown in Figure 9.3.5 determine the delta-connected network. Hence find the equivalent resistance between points A and B.

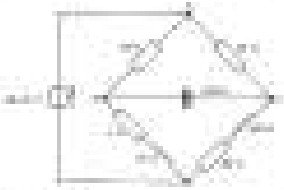


Figure 9.16



Figure 9.17



Figure 9.18



Figure 9.19

(b) A 12 Ω resistor, a 6 Ω resistor and a 4 Ω resistor are connected in parallel across a 24 V supply. Determine (i) the current in each resistor and (ii) the total current taken from the supply. [(i) 2 A, 4 A, 6 A (ii) 12 A]

4 Two resistors are connected in parallel across a 240 V supply. The current in one resistor is 5 A and the total current taken from the supply is 8 A. Determine (a) the value of the other resistor and (b) the power dissipated in each resistor. [(a) 80 Ω (b) 1200 W, 1920 W]

5 For the network shown in Figure 9.19, determine (a) the current flowing in the 4 Ω resistor, and (b) the power dissipated in the 10 Ω resistor. [(a) 0.5 A (b) 10 W]

35 Maximum power transfer theorems and impedance matching

At the end of this chapter you should be able to:

- appreciate the conditions for maximum power transfer in a.c. networks
- apply the maximum power transfer theorem in a.c. networks
- appreciate advantages of impedance matching in a.c. networks
- use Thévenin's theorem to design matching networks for maximum power transfer in a.c. networks

35.1 Maximum power transfer theorems

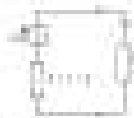


Figure 35.1

A network that contains source impedances and can be represented by a single equivalent source voltage and a single source impedance is known as a Thévenin equivalent circuit as shown in Figure 35.1. Maximum power is transferred to the load when the load impedance is equal to the source impedance. If the source impedance is shown as $R_s + jX_s$ and the load impedance is $R_L + jX_L$, then maximum power transfer occurs when $R_L = R_s$ and $X_L = -X_s$.

The complex power is defined as the product of the voltage and the complex conjugate of the current. For the circuit in Figure 35.1, if the current is taken as reference, the voltage is given by

$$V = \frac{E}{(R_s + R_L) + j(X_s + X_L)}$$

and the magnitude of current is $I = \frac{E}{\sqrt{(R_s + R_L)^2 + (X_s + X_L)^2}}$

If the load impedance Z_L is varied to obtain maximum power transfer, then

$$P = I^2 R_L = \frac{E^2 R_L}{(R_s + R_L)^2 + (X_s + X_L)^2}$$

For a given source impedance, $R_s + jX_s$, the maximum power transfer occurs when $X_L = -X_s$ and R_L is varied. This is achieved when $R_L = \sqrt{R_s^2 + (X_s + X_L)^2}$, i.e. when $R_L = R_s$. Thus the load impedance for maximum power transfer is $Z_L = R_s - jX_s$, i.e. the load impedance is the complex conjugate of the source impedance.

...current in a charging circuit governed by Kirchhoff's second law,

$$\frac{dI}{dt} = kI\left[\frac{(a-I)^2}{(a+I)^2} + \frac{c^2(a+I)^2 + R^2(a-I)^2}{(a+I)^2 + R^2(a-I)^2}\right]$$

... where k is a constant and a is the battery voltage.

For $\frac{dI}{dt}$ to be zero, the numerator of the fraction must be zero:

$$\text{Hence} \qquad (a-I)^2 + c^2 - 2R(a-I) = 0$$

$$\text{i.e.} \qquad I^2 - 2(R+1)I + I^2 - 2R + 2R^2 = 0$$

$$\text{Now, } \Delta = b^2 - 4ac = 4R^2 \qquad \text{(R is a)}$$

$$\text{so} \qquad \boxed{R^2 - 2I^2 + a(I^2) = 0}$$

Thus, when a variable steady current exists, the maximum power is distributed in the load if the internal resistance R is made equal to the magnitude of the circuit impedance.

Example 5 Let us find the current in the circuit considered in Example 4 when $L = 0.1$ H, $C = 2 \times 10^{-6}$ F, $R = 50\ \Omega$, if the applied voltage $v(t) = 5$ V. Also find the maximum current in a subsequent ringing. $\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$ [this is as in Sect. 10.4, a condition maintained in Chapter 11 on page 337.]

Solution Let us find the Laplace transform of the circuit equation. Kirchhoff's second law states: $L\frac{di}{dt} + Ri + \frac{1}{C}\int i\,dt = v(t)$. Taking transforms,

$$\text{we have } L\left[sI - i(0)\right] + RI + \frac{1}{C}\frac{I}{s} = \frac{5}{s}, \text{ since } v(t) = 5 \text{ and } \mathcal{L}\{v(t)\} = \frac{5}{s}$$

Taking initial conditions as zero, we have

$$I = \frac{5}{s\left(Ls + R + \frac{1}{Cs}\right)} = \frac{5}{Ls^2 + Rs + \frac{1}{C}} = \frac{5}{L\left[s^2 + \frac{R}{L}s + \frac{1}{LC}\right]}$$

The quadratic factor in the denominator can be completed to the square:

$$I = \frac{5/L}{\left(s + \frac{R}{2L}\right)^2 + \left(\frac{1}{LC} - \frac{R^2}{4L^2}\right)}$$

If we substitute $\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$ and the value of current is a maximum when $\frac{dI}{dt} = 0$, i.e. when $s = -\frac{R}{2L}$.

$$\frac{dI}{dt} = kI\left[\frac{(a-I)^2}{(a+I)^2} + \frac{c^2(a+I)^2 + R^2(a-I)^2}{(a+I)^2 + R^2(a-I)^2}\right] = 0 \text{ for maximum value}$$

$$\text{Hence} \qquad (a-I)^2 + c^2 - 2R(a-I) = 0$$

$$\text{i.e.} \qquad I^2 - 2(R+1)I + I^2 - 2R + 2R^2 = 0$$

$$\text{Now, } \Delta = b^2 - 4ac = 4R^2 \text{ and } R =$$

Two hulls that each accelerate in proportion to the inverse square of the distance between them trace out orbits that are conic sections. This well-known result was first published by Isaac Newton in 1687 as his Proposition 1 of Book 1 of the *Principia*. Newton's derivation used synthetic geometry, and he had to invent the necessary results about conic sections. Nowadays, the result is usually derived from Newton's second law of motion using the techniques of calculus. Here we present a derivation that requires no calculus and only the most elementary properties of conic sections.

Lemma 1. Let $\mathbf{r}(t)$ be the position vector of a particle moving in a plane, and let $\mathbf{h} = \mathbf{r} \times \mathbf{v}$ be its specific angular momentum. If $\mathbf{h} = \mathbf{0}$ then the particle moves in a straight line.

Proof. Suppose $\mathbf{h} = \mathbf{0}$. Then $\mathbf{r} \times \mathbf{v} = \mathbf{0}$, so $\mathbf{v}$ is parallel to $\mathbf{r}$ at every instant. Hence $\mathbf{r}$ and $\mathbf{v}$ lie on the same line through the origin, and the particle's motion is confined to that line. $\square$

Lemma 2. Let $\mathbf{r}(t)$ be the position vector of a particle moving in a plane, and let $\mathbf{h} = \mathbf{r} \times \mathbf{v}$ be its specific angular momentum. If $\mathbf{h} \neq \mathbf{0}$ then the particle moves at constant speed along a straight line.

Proof. Suppose $\mathbf{h} \neq \mathbf{0}$. Then $\mathbf{r}$ and $\mathbf{v}$ are never parallel, so they always span the same plane. Choose coordinates in that plane with the origin at the particle's position when $t = 0$, and let the x-axis point in the direction of $\mathbf{v}(0)$. Then $y(0) = 0$ and $v_y(0) = 0$.

Since $\mathbf{h}$ is constant, $r_x v_y - r_y v_x = h$ for all t. Differentiating with respect to t gives $r_x a_y - r_y a_x = 0$. But $a_y = 0$ because the particle has no component of acceleration perpendicular to the x-axis, so $r_y = 0$ for all t. The particle never leaves the x-axis, and its speed $|v_x|$ is constant. $\square$

Lemma 3. Let $\mathbf{r}(t)$ be the position vector of a particle moving in a plane, and let $\mathbf{h} = \mathbf{r} \times \mathbf{v}$ be its specific angular momentum. If $\mathbf{h} \neq \mathbf{0}$ then the particle moves at constant speed around a circle centred on the origin.

Proof. Suppose $\mathbf{h} \neq \mathbf{0}$. Choose coordinates in the plane of motion with the origin at the centre of force, and let the particle be at $(r, 0)$ with velocity $(0, v)$ when $t = 0$. Then $\mathbf{h} = rv\,\mathbf{k}$, so $rv = h$ is constant.

Since the force is always directed towards the origin, the particle has no component of acceleration perpendicular to the x-axis, so $r_y = 0$ and $v_y = 0$ for all t. Hence $r = |r_x|$ and $v = |v_x|$, so $rv = h$ gives $|r_x| \, |v_x| = h$.

Now r_x and v_x are continuous functions of t that never change sign (because $r_x = 0$ would require $v_x = \infty$, which is impossible for a finite force). Therefore $r_x v_x = \pm h$ for all t, with the same sign throughout. Differentiating gives $r_x a_x + v_x^2 = 0$, so $a_x = -h^2 / r_x^3$.

Since $\mathbf{a} = a_x \mathbf{i}$ and $a_x = -h^2 / r_x^3$, the acceleration is directed towards the origin and its magnitude depends only on the distance $r = |r_x|$. Hence the particle moves in a circle centred on the origin, with speed $v = h / r$. $\square$

Exercises

1. Show that the specific angular momentum $\mathbf{h} = \mathbf{r} \times \mathbf{v}$ is conserved for any particle moving under a central force, and deduce that the motion lies in a fixed plane.

2. Verify that the vis-viva equation $v^2 = \mu \left(\frac{2}{r} - \frac{1}{a} \right)$ is consistent with the energy integral $\frac{1}{2} v^2 - \frac{\mu}{r} = -\frac{\mu}{2a}$ for an orbit of semi-major axis a.

3. Show that the eccentricity vector $\mathbf{e} = \frac{\mathbf{v} \times \mathbf{h}}{\mu} - \frac{\mathbf{r}}{r}$ is constant along the orbit, and interpret its direction and magnitude geometrically.

4. Prove that the polar equation of a conic section with focus at the origin, semi-latus rectum p, and eccentricity e is $r = \frac{p}{1 + e \cos \theta}$.

The fundamental physical quantity that describes how effectively a force causes rotation is called torque. It depends on the force's magnitude, the angle at which it acts, and the distance between the point where the force is applied and the axis of rotation. Torque is a key concept in understanding how objects rotate, from simple levers to complex machines.



Figure 1.1

Definition 1.1 — Torque. For the system shown in Figure 1.1, the force F acts at a point whose position vector relative to the axis of rotation O is $\mathbf{r}$. The torque τ about O is the cross product $\tau = \mathbf{r} \times \mathbf{F}$, whose magnitude is $\tau = rF\sin\theta$, where θ is the angle between $\mathbf{r}$ and $\mathbf{F}$ when they are drawn tail to tail.

(a) Force F acts at a point P whose position vector relative to the axis of rotation O is $\mathbf{r}$. Express the torque τ about O in terms of $\mathbf{r}$ and $\mathbf{F}$, and state the direction of τ using the right-hand rule.

(b) A uniform rod of length L and mass M is pivoted at one end and held horizontal. A force F is applied at the free end, perpendicular to the rod, in the direction that would cause the rod to rotate downward. Find the magnitude of the torque about the pivot, and the angular acceleration of the rod immediately after the force is applied. Take the moment of inertia of the rod about the pivot to be $\frac{1}{3}ML^2$.

Definition 1.2 — Moment of Inertia. The moment of inertia I of a rigid body about a given axis of rotation is a measure of its resistance to angular acceleration about that axis. It depends on the mass of the body, how that mass is distributed relative to the axis, and the position of the axis itself. For a point mass m at a perpendicular distance r from the axis, $I = mr^2$. For a rigid body, $I = \int r^2\,dm$, where the integral is taken over the entire mass of the body.

(c) Four identical point masses, each of mass m, are fixed to the corners of a square frame of side length a. The frame can rotate about an axis that passes through the center of the square and is perpendicular to its plane. Find the moment of inertia of the system about this axis. Then, if the frame is released from rest with this axis horizontal and the masses in a uniform gravitational field g, find the angular speed of the frame after it has rotated through one quarter of a full turn.

(d) A solid cylinder of mass M and radius R can rotate freely about its central axis. A string is wrapped around the cylinder and a block of mass m hangs from the free end of the string. The block is released from rest and falls a distance h. Assuming the string does not slip, find the speed of the block just before it reaches the ground. Use energy conservation, and express your answer in terms of m, M, g, and h.



Figure 1.2

Problem 1.3 — Rotating Disk with Two Masses. A uniform disk of mass $M = 4.0\ \text{kg}$ and radius $R = 0.50\ \text{m}$ can rotate freely about a horizontal axis through its center, perpendicular to the disk. Two small objects, each of mass $m = 1.0\ \text{kg}$, are attached to the rim of the disk at opposite ends of a diameter. The system is released from rest with the diameter horizontal. Take $g = 9.8\ \text{m/s}^2$.

(a) Find the moment of inertia of the system about the axis of rotation.

(b) Find the angular speed of the disk after it has rotated through one quarter of a full turn.

(a) The problem is to maximize $P_L(\omega)$ with respect to ω. Before differentiating, it is convenient to express the denominator as a single fraction. Let $D(\omega) = (R_s + R_L)^2 + (\omega L_s + \omega L_L - \frac{1}{\omega C_L})^2$. Differentiating $P_L = \frac{V_s^2 R_L}{D}$ with respect to ω using the quotient rule gives

$$\frac{dP_L}{d\omega} = V_s^2 R_L \cdot \frac{-D'(\omega)}{D(\omega)^2}$$

where the derivative of the denominator is

$$D'(\omega) = 2(R_s + R_L)\cdot 0 + 2\left(\omega L_s + \omega L_L - \frac{1}{\omega C_L}\right)\left(L_s + L_L + \frac{1}{\omega^2 C_L}\right)$$

$$= 2\left(\omega L_s + \omega L_L - \frac{1}{\omega C_L}\right)\left(L_s + L_L + \frac{1}{\omega^2 C_L}\right).$$

Only the factor $\omega L_s + \omega L_L - \frac{1}{\omega C_L}$ can vanish for positive ω, because all other terms are strictly positive. Setting it equal to zero:

$$\omega L_s + \omega L_L = \frac{1}{\omega C_L}$$

$$\omega^2 (L_s + L_L) C_L = 1$$

$$\omega^2 = \frac{1}{(L_s + L_L) C_L}$$

$$\omega = \frac{1}{\sqrt{(L_s + L_L) C_L}}.$$

(b) With $L_s = 2.0\ \text{mH}$, $L_L = 0.5\ \text{mH}$, and $C_L = 5.0\ \mu\text{F}$, the resonant power transfer angular frequency is

$$\omega_r = \frac{1}{\sqrt{(2.0 \times 10^{-3} + 0.5 \times 10^{-3})(5.0 \times 10^{-6})}}$$

$$= \frac{1}{\sqrt{(2.5 \times 10^{-3})(5.0 \times 10^{-6})}} = \frac{1}{\sqrt{12.5 \times 10^{-9}}}$$

$$= \frac{1}{3.536 \times 10^{-4}} \approx 2.828 \times 10^{3}\ \text{rad/s}.$$

The corresponding resonant frequency is $f_r = \frac{\omega_r}{2\pi} \approx \frac{2828}{6.283} \approx 450\ \text{Hz}$.

> **Problem 2.** An electrochemical cell can be modeled as a battery of constant magnitude $\mathcal{E}$ in series with a resistor r that represents the internal resistance of the cell. When a resistance R is connected across the terminals of the cell, the current in the circuit is $I = \frac{\mathcal{E}}{R + r}$ and the terminal voltage is $V = IR = \frac{\mathcal{E} R}{R + r}$. Experimentally, V is found to decrease as R is decreased, and $V \to 0$ as $R \to 0$. Explain why this behavior is consistent with the model, and determine the value of r if a cell with $\mathcal{E} = 1.50\ \text{V}$ delivers a terminal voltage of $1.20\ \text{V}$ when connected to a resistor of $R = 4.00\ \Omega$.

(a) When resistance is removed from the circuit, the current must vanish, so the terminal voltage equals the emf: $V = \mathcal{E}$. Connecting a finite resistor R allows current to flow, and the voltage drop across the internal resistance r reduces the terminal voltage below $\mathcal{E}$. As R is made smaller, the current $I = \frac{\mathcal{E}}{R + r}$ increases, and the drop Ir grows, so $V = \mathcal{E} - Ir$ decreases. In the limit $R \to 0$, the current approaches $I \to \frac{\mathcal{E}}{r}$ and $V \to 0$. This matches the observed behavior.

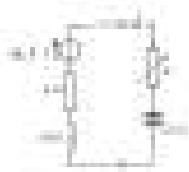


Figure E.2.1

(b) From $V = \frac{\mathcal{E} R}{R + r}$, multiply both sides by $(R + r)$ to obtain $V(R + r) = \mathcal{E} R$, which expands to $VR + Vr = \mathcal{E} R$. Rearranging gives $Vr = \mathcal{E} R - VR = R(\mathcal{E} - V)$, so $r = \frac{R(\mathcal{E} - V)}{V}$. Substituting $\mathcal{E} = 1.50\ \text{V}$, $V = 1.20\ \text{V}$, and $R = 4.00\ \Omega$:

$$r = \frac{4.00\ \Omega \times (1.50\ \text{V} - 1.20\ \text{V})}{1.20\ \text{V}} = \frac{4.00 \times 0.30}{1.20}\ \Omega = \frac{1.20}{1.20}\ \Omega = 1.00\ \Omega.$$

As a check, the current is $I = \frac{V}{R} = \frac{1.20}{4.00} = 0.300\ \text{A}$, and $Ir = 0.300 \times 1.00 = 0.300\ \text{V}$, so $\mathcal{E} = V + Ir = 1.20 + 0.300 = 1.50\ \text{V}$, as required.

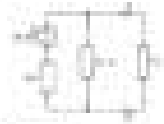


Figure E1.1

Problem 1. Determine the value of the load resistance R_L shown in Figure E1.1 that is required to absorb the maximum power and calculate the value of this power.

Using the procedure in Thevenin's theorem (see page 99):

(a) R_{TH} is calculated from the circuit in its form in Figure E1.1:

$$R_{TH} = 3\,\Omega \text{ (series)} \,\|\, 6\,\Omega \text{ (series)} = 2\,\Omega$$

(b) V_{TH} is found with $R_L = \infty$ (open-circuit) and is the voltage across the $6\,\Omega$ resistor (since no current flows in the $3\,\Omega$ resistor):

$$V_{TH} = \frac{6}{6+3}(12) = 8\text{ V}$$

(c) The load resistor R_L absorbs maximum power when $R_L = R_{TH} = 2\,\Omega$.

(d) The maximum power absorbed by the load is

$$P_{max} = \frac{V_{TH}^2}{4R_L} = \frac{8^2}{4(2)} = 8\text{ W}$$

Now the **Problem** posed in the text:

$$P = 8\text{ W} = 0.01 R_L (R_L + 2)^2 \quad \text{or} \quad R_L^3 + 4R_L^2 + 4R_L - 800 = 0$$



Figure E1.2

Problem 2. Determine the two currents shown in Figure E1.2 as well as the power in the $2\,\Omega$ resistor by employing the mesh-current method and check the value of the resistor power.

(a) Using the procedure in Mesh-current Analysis:

(b) Equations 1 & 2 are obtained from the circuit as shown in Figure E1.2:

(c) The solution for

$$I_1 = \frac{\begin{vmatrix} 7 & -2 \\ -2 & 5 \end{vmatrix}}{\begin{vmatrix} 3 & -2 \\ -2 & 5 \end{vmatrix}} = \frac{35 - 4}{15 - 4} = \frac{31}{11} = 2.818\text{ A}$$

(d) With the 10-V, 7 source current in the opposite direction, i.e. flowing in its positive side as shown in

$$I_1 = \frac{\begin{vmatrix} -7 & -2 \\ 10 & 5 \end{vmatrix}}{\begin{vmatrix} 3 & -2 \\ -2 & 5 \end{vmatrix}} = \frac{-35 + 20}{15 - 4} = -1.364\text{ A}$$

(e) The resistor R_2 between nodes 1 and 2 is shown in Figure E1.2. If the voltage V_2 (relative to ground) is assigned to node 2, $V_1 = V_2 + 10$ V, so that now there is $R = 1\,\Omega$ and $R = 2\,\Omega$



Figure E1.3



Figure E1.4

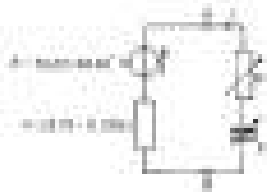


Figure 39.10

(b) Current $I = \dfrac{V}{z + Z} = \dfrac{70.07\angle 00.43^\circ}{(5.75 + j1.25) + (5.75 - j1.25)}$

$= \dfrac{70.07\angle 00.43^\circ}{11.5} = 10.88\angle 00.43^\circ$ A

Thus the **maximum power** transferred, $P = I^2R = (10.88)^2(3.75)$

$= 417$ W

Further problems on the maximum power transfer theorem may be found in Section 39.3, problems 1 to 10, page 928.

39.2 Impedance matching

It is seen from Section 39.1 that when it is necessary to obtain the maximum possible amount of power from a source, it is advantageous if the circuit components can be adjusted to give equality of impedances. This adjustment is called '**impedance matching**' and is an important consideration in electronic and communication devices which normally involve small amounts of power. Examples where matching is important include coupling an aerial to a transmitter or receiver, or coupling a loudspeaker to an amplifier.

The mains power supply is considered as infinitely large compared with the demand upon it, and under such conditions it is unnecessary to consider the conditions for maximum power transfer. With transducers (see Chapter 44), the items are 'matched', ideally i.e., proportional to their characteristic impedances.

With d.c. generators, motors or secondary cells, the internal impedance is usually very small and in such cases, if an attempt is made to make the load impedance as small as the source internal impedance, overheating of the source results.

A method of achieving maximum power transfer between a source and a load is to adjust the value of the load impedance to match the source impedance, which can be done using a '**matching-transformer**'.

A transformer is represented in Figure 39.11 supplying a load impedance Z_L.

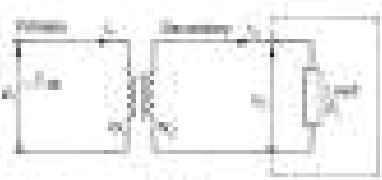


Figure 39.11 *Matching impedance by means of a transformer*

Small transformers used in low power networks are usually regarded as ideal (i.e., losses are negligible), such that

$$\frac{\bar{x}_1}{\bar{x}_2} = \frac{\mu_1}{\mu_2} = 1$$

From Figure 10.6, the primary study hypothesis is a one-tailed, H_0:

$$z = \frac{\bar{x}_1}{\bar{x}_2} = \frac{\mu_1 + \mu_2}{\sigma_1^2/n_1 + \sigma_2^2/n_2} = \frac{\mu_1}{\mu_2} \Big/ \frac{\sigma_1}{\sigma_2}$$

Here the null hypothesis $H_0: \mu_1 = \mu_2$ is

$$z = \frac{\mu_1}{\mu_2} \Big/ \frac{\sigma_1}{\sigma_2} \qquad (10.5)$$

If the equal hypothesis of Figure 10.6 is greatly unlikely, say α, then equation (10.5) becomes

$$z = \frac{\mu_1}{\mu_2} \Big/ \frac{\sigma_1}{\sigma_2} \qquad (10.6)$$

If the α are not mentioned (as before 10.5), page 134) —
Use the existing the value of the hypothesis table with the equation type hypothesis. If the hypothesis can be "confirmed" in the hypothesis of a particular sample sample, the null hypothesis can be "confirmed" in the hypothesis of a particular sample sample.

> Problem 1 — Determine the significance value of a null hypothesis for a particular sample sample. If the null is confirmed by an alternative hypothesis sample, the null hypothesis can be confirmed by the alternative hypothesis sample.

The standard error method $\sigma = \sigma$ for hypotheses type are $\sigma = \sigma$ for the sample sample. Then equation $\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\sigma_{\bar{x}_1}^2 + \sigma_{\bar{x}_2}^2}$. The value σ and equation $\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\sigma_{\bar{x}_1}^2 + \sigma_{\bar{x}_2}^2}$.

> Problem 2 — A researcher has the value hypothesis of $\sigma = \sigma$ and the standard deviation. Determine the value of the value hypothesis sample sample for the hypothesis $\sigma = \sigma$ and $\sigma = \sigma$. [10.6] for the hypothesis sample sample.

Let the value hypothesis of the hypothesis be σ, where $\sigma = \sigma$ for σ and σ and σ and σ. Then the hypothesis $H_0: \sigma_1 = \sigma_2$ and $H_1: \sigma_1 \neq \sigma_2$ is the hypothesis. Hypothesis [10.6] and hypothesis $\sigma(\bar{x}_1 - \bar{x}_2) = \sigma^2/\sigma^2(\bar{x}_1)$. Then

$$\text{hypothesis statistic} = \left(\frac{\sigma_1}{\sigma_2}\right)^2 \Big/ \left(\frac{\sigma_1}{\sigma_2}\right)^2 = \frac{\sigma^2/\sigma^2}{\sigma^2/\sigma^2} = \sigma = \sigma \cdot \sigma = \sigma$$

Problem 3. A single-phase, 240 V/110 V, 4kV transformer is supplied from a 240 V mains supply and the secondary winding is connected to a load having an impedance of $(3 + j4)\,\Omega$. Assuming each of the primary windings is 0.04 Ω resistance and 0.1 Ω reactance, determine the primary current, assuming each of the primary windings is 0.04 Ω resistance and 0.1 Ω reactance.

The circuit is shown in Figure 3.11.

(a) Turns ratio $\frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{240}{110} = 2.18$



Figure 3.11

Equivalent load resistance of the transformer,

$$R_L = \left(\frac{N_1}{N_2}\right)^2 R_2 = \left(\frac{2.18}{1}\right)^2 (3.00\Omega) = 14.26\,\Omega$$

Total load resistance, $R_{TL} = R_{primary} + R_L = \frac{0.04}{2} + 14.26 = 14.28\,\Omega$. Where the primary resistor, $R_1 = 0.04\,\Omega$ (40 mΩ) = **0.02 Ω**

(b) For the transformation, $\frac{N_1}{N_2} = \frac{I_2}{I_1}$

Load current, $I_2 = 240\left(\frac{1}{14.28}\right) = 16.81\,\text{A}\left(\frac{100}{60.48}\right) = 7.19$

Power dissipated in the load resistance $P = I_2^2 R_L = (7.19)^2 (14.26)$

$= 1.1.1.1.1$

Problem 4. An ac source of 10 V, 1 rad/s is connected to a load as shown in Figure 3.12. Determine the equivalent source impedance as seen from the load terminals and hence the power dissipated in the load.



Figure 3.12

The equivalent diagram is shown in Figure 3.13.

(a) The equivalent source impedance is found by open circuiting the 10 V source. This results in the circuit shown in Figure 3.14.

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[illegible]

[illegible]

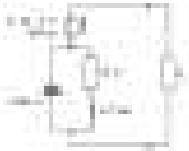


Figure 10.3(a)

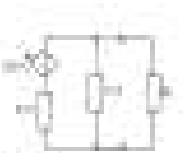


Figure 10.3(b)

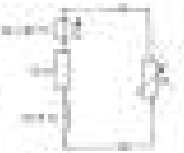


Figure 10.3(c)



Figure 10.4(a)

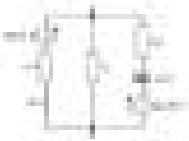


Figure 10.4(b)



Figure 10.4(c)

7 Determine, for the pattern of electrodes in Figure 10.3(c), (a) the coefficient of R_1 and R_2 in the power loss in the electrode system during transmission between rectifier and inverter, (b) the value of this loss in the symmetrical case.

[(a) $\frac{1}{2}$, $\frac{1}{2}$; (b) $\frac{1}{2}R_1 I_d^2$]

8 A station with $N = 12$, $P = 6$ bulk transformers (Δ – Y) supplies a load consisting of a smoothing reactor in parallel with a short circuited inverter. If the commutation overlap angle is $\mu = 15°$, find the value of the commutation power factor, and the value of the equivalent power.

[$\cos\frac{1}{2}\mu = 0.9914$; $\frac{1}{2}\cos\frac{1}{2}\mu = 0.4957$]

9 What load R_L between terminals a and b of Figure 10.3(b) is required to limit maximum load transient fluctuation into value of R_L (with the inverter inactive) to 10% of the steady-state value? Determine the value of the maximum power.

[$R_L = 1.1R$; $\frac{1}{2}(1 - \frac{1}{1.1}) = \frac{1}{22}$; $\frac{1}{2}(1 - \frac{1}{1.1})I_d^2 R$]

10 For the circuit in Figure 10.3(c), determine the value of load resistance R_L for maximum load power, and express R_L as a multiple of the equivalent R and internal voltage V as a multiple of $I_d R$. Determine the power and load voltage corresponding to maximum power.

[$R_L = \frac{1}{2}R$; $V = \frac{1}{2}I_d R$; $\frac{1}{8}I_d^2 R$; $\frac{1}{4}I_d R$]

Inverter Problems

11 The output value R of rectifier has an output resistance $= 0.05\ \Omega$. Determine the commutation overlap angle if the inverter has a load resistance of $0.05\ \Omega$, the output resistance of the rectifier has negligible commutation power factor.

[$\mu = \frac{1}{3}\pi$]

12 Find the minimum value of load resistance the commutation circuit (Figure 10.3(a)) is connected to an inverter of output resistance $0.05\ \Omega$ through a transformer with a turns ratio of $1:1$.

[$0.1\ \Omega$]

13 A generator has an output impedance of $(0.1 + j0.2)\ \Omega$. Determine the value load R to which maximum power is supplied, and the power and for a load $R = 0.1 - j\,0.2\ \Omega$ for maximum active power.

[$0.1\ \Omega$]

[illegible] A lengthy illegible block of dense typesetting, apparently a multi-paragraph article or boilerplate, occupies the remainder of the page. Its text is not recoverable.

Assignment 11

Use a ruler and pencil for the construction diagrams (b) and (d).

Give results for each question to a reasonable number of significant figures.

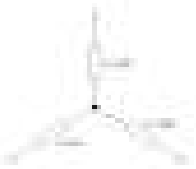


Figure 3.4.1

1 Determine the dimensional equation relating ω to the two variables identified above in Figure 3.4.1. [3]

2 Identify the effect mentioned in Figure 3.4.2 in each of the appropriate gap categories. Select between the two options given in Figure 3.4.2.

(a) The electrical conductivity

(b) An element

(c) The current in the 2 Ω resistor

(d) The power dissipated in the 2 Ω resistor. [3]

Figure 3.4.2

3 A thread dependent S as Figure 3.4.3, is capable of handling tensile and bending moments. Identify the value of S from the following expressions, and hence the types of thread connection. [3]

4 Determine the value of the load dependent R in Figure 3.4.4 for given dimensions. Show clearly how you calculate the value of power. [3]

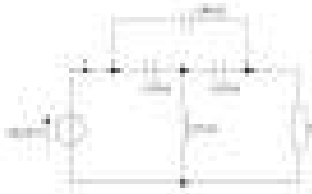


Figure 3.4.3



Figure 3.4.4



Figure 3.4.1

1. Set up a circuit as in Figure 3.4.1 and measure the resistance of each resistor. Record your measurements in the table below. Then calculate the equivalent resistance of the circuit using the rules of the previous section, and enter your calculated value in the table.

36 Complex Waveforms

In this unit of the chapter we extend the analysis to:

- Make a complex form
- Complex periodic functions
- Complex and periodic quantities of complex functions
- real functions of time as AC or a complex wave
- Instantaneous values of waveforms containing AC and DC as well and even harmonics, and their average values
- calculate rms and mean values over one cycle of a complex wave
- calculate power associated AC complex waves
- perform calculations on single phase circuits containing harmonics
- define and perform calculations on balanced systems
- list and explain some sources of harmonics

36.1 Introduction

In practical engineering we usually have two sources of AC supply, the mains, both of domestically available sinusoidal, but found in the field of engineering it is not always easy to find a pure sinusoidal AC supply. Complex waveforms are produced by many electronic and power devices. For example, electrical equipment using gas discharge lamps, high voltage power transmission lines, or computers, all produce harmonics. A waveform that is not sinusoidal is called a complex wave. Such a waveform may be produced or composed of the sum of a number of sinusoidal waves having various amplitudes and time values.

A function $f(t)$ is said to be periodic if $f(t) = f(t + T)$ for all values of t, where T is the period of the function. Any harmonic superposition can be called the period of the function. It is also known that any periodic function can be a function of a periodic function.

A typical complex waveform may be made up of a number of harmonics, each of which has a different amplitude and frequency. A complex wave which is also not sinusoidal has the shape of a sequence of harmonics which may have each of the three waves with different frequency, amplitude and phase.

The lowest value value value

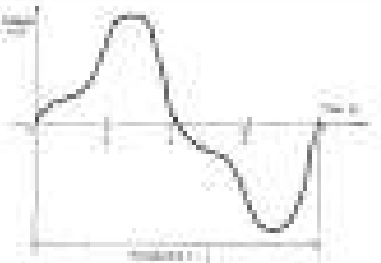


Figure 10.1 Typical single-cycle half-wave rectified sinusoidal waveform

Example: Designate as a single-line source as 10 V dc, 60 Hz, 60 A. Half Sinusoidal Waveform is 10 V dc, 60 Hz Rectified Sinusoidal Waveform at 120 Hz, 60 A, and so on.

Sect. 7 The general equation for a complex waveform

The mathematical notation of a complex voltage wave is usually as a mixed sinusoidal wave that represented by the general equation:

$$v = V_{dc} \text{ Offset} + A_1 \sin + V_{ac1} \text{ Offset} + \theta_1)$$

$$+ \ldots + V_{ac} \text{ Offset} + \theta_n \text{ in Hz} \tag{10.1}$$

Where V_{dc} = offset, A_1 is designated the amplitude of component of value in V_{ac1} is the sinusoidal or pulse wave frequency, $f = 1/T$ and θ_1 is the phase angle with respect to time, $t = 0$.

Similarly, V_{ac} and θ_n in Eq. 10.1 represents the second harmonic values, and, and V_{ac} as shown in Eq. 4 represents the nth harmonic component of value V_{ac} in the peak-to-peak frequency $= \omega = 2\pi f = 2\pi / T$ and θ_n is the phase angle.

In the same way, the instantaneous value of a complex current is also be represented by the general equation:

$$i = I_{dc} \text{ Offset} + A_1 \sin + I_{ac} \text{ Offset} + \theta_1)$$

$$+ \ldots + I_{ac} \text{ Offset} + \theta_n \text{ in Hz} \tag{10.2}$$

Where equations 10.1 and 10.2 refer to the voltage source and the current flowing through a given linear circuit, the phase angle between the instantaneous voltage and current is $\phi_1 = \theta_1 - \theta_2$. In the phase angle between the second harmonic voltage and current is $\phi_2 = \theta_2 - \theta_2$, and so on.

If $\sin\theta = \frac{3}{5}$ and θ is in Quadrant II, find the exact values of $\cos\theta$ and $\tan\theta$.

Sol 3 **Electronic oscillators**

Mathematical modelling is the process of translating a real-world situation into a formal, abstract representation so that reasoning, calculation, and prediction can be carried out systematically. Many of the mathematical tools we develop and investigate in this course — including the sine and cosine functions, their graphs, and their properties — are motivated by problems in physics and engineering. One of the most important such problems is the behaviour of **electronic oscillators**: circuits that produce a repeating voltage signal without any external input driving them. Understanding how these circuits work requires us to model their behaviour using functions that repeat in a predictable pattern.

An **electronic oscillator** is a circuit that produces a periodic voltage signal — a voltage that rises and falls in a regular, repeating pattern. The simplest such circuit is the **LC oscillator**, which consists of a capacitor (C) and an inductor (L) connected in a loop. When the capacitor is charged and then allowed to discharge through the inductor, the energy stored in the capacitor's electric field is converted into the inductor's magnetic field, and back again, in a continuous cycle. The voltage across the capacitor at any time t can be described by a function of the form $V(t) = A\cos(\omega t)$, where A is the maximum voltage (the **amplitude**) and ω is the **angular frequency**, which determines how quickly the oscillation repeats. The angular frequency is related to the physical properties of the circuit by $\omega = \frac{1}{\sqrt{LC}}$, where L is the inductance and C is the capacitance. This relationship is known as the **Thomson formula**, named after the physicist who first derived it.

Example 1

Consider the oscillator whose voltage is given by $V(t) = 5\cos(2t)$.

(a) Find the amplitude and the angular frequency of the oscillation.

Solution

The amplitude is read off as the coefficient of the cosine function, so $A = 5$ volts. The angular frequency is the coefficient of t inside the cosine, so $\omega = 2$ radians per second. The amplitude tells us that the voltage never exceeds 5 volts in either direction, and the angular frequency tells us how rapidly the oscillation repeats.

Sol 4 **Electronic oscillators**

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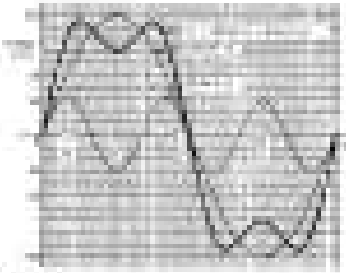


Figure 10.2

A combined harmonic $x_1 + x_3$ and $x_1 + x_3 + x_5$ for problem that contains harmonics for $n = 1$. The respective coefficients for all harmonics x_1 is shown in the table and in Figure 10.2, the summary table is only

Example 3

Consider a function that has only odd harmonic components in the Fourier series. Given in Figure 10.3 using Fourier series and find any simplification

$$x_1 = \frac{4A\sin(\omega_1 t)}{\pi} + \frac{4A\sin(3\omega_1 t)}{3\pi} + \frac{4A\sin(5\omega_1 t)}{5\pi} + \cdots$$

Figure 10.3 shows the effect of adding 1, 2, 3, and 4 terms of the Fourier series. Note that the sum approximates a square wave more and more closely as the number of terms increases. The square wave has discontinuities at the points where the function changes abruptly. At these points, the Fourier series converges to the average value of the function on either side of the discontinuity.

Example 4

Consider the sawtooth voltage waveform as shown in

$$x_1 = \frac{2A\sin(\omega_1 t)}{\pi} + \frac{2A\sin(2\omega_1 t)}{2\pi} + \frac{2A\sin(3\omega_1 t)}{3\pi} + \cdots = \sum_{n=1}^{\infty} \frac{2A\sin(n\omega_1 t)}{n\pi}$$

The waveform is identical to that in Figure 10.3, except that the signs of the harmonics are reversed. Because of the odd symmetry of the function, the Fourier series contains only sine terms. Figure 10.4 shows the effect of adding 1, 2, 3, and 4 terms of the Fourier series. Note that, except for the reversal of sign, the Fourier series is identical to that in Figure 10.3. Specifically, the Fourier series for x_1 contains only odd harmonics, all A_n terms are zero, and the B_n terms are identical to those in Figure 10.3. Specifically, the B_n terms are identical to those in Figure 10.3. Specifically, the B_n terms are identical to those in Figure 10.3.

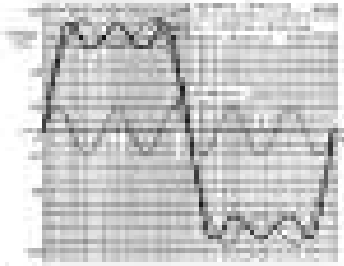


Figure 3-4

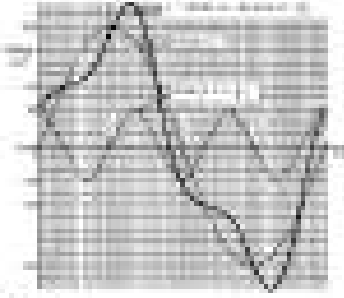


Figure 3-5

Figure 3-5 shows another 1 kHz square wave. Channel A is now the same as in Figure 3-4, but Channel B is set to 50 mV/div. The trace is positioned in the center of the oscilloscope. Where the peaks of the two signals are added or subtracted, the resultant waveform is displayed. The shape of the waveform is an easily identifiable three step of waveforms as shown in Figure 3-5. Some design and troubleshooting applications of this mode. In the negative direction in Figure 3-5 is inverted to see the wave. All the steps of Channel A and Channel B are displayed.

Example 2

Consider the complete vibration displacement given by:

$$x(t) = 0.8\sin\omega_n t + 0.6\cos\omega_n t \text{ inches}$$

The fundamental vibration and the third harmonic component, $0.4\sin 3\omega_n t$, are plotted in Figure 10-5. The time required for the first cycle is $2\pi/\omega_n$ seconds. Using increments of approximately one-twelfth this amount, $\pi/6\omega_n$ seconds, the complete vibration $x(t)$ is plotted in relation to the positive time axis.

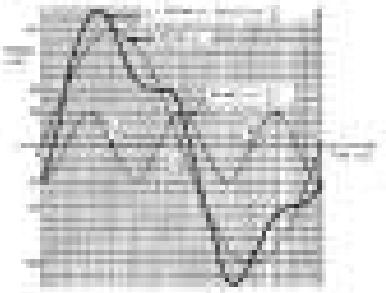


Figure 10-5

Example 3

Consider the complete vibration displacement given by:

$$x(t) = 0.8\sin\omega_n t + 0.6\cos\omega_n t + 0.4\sin 3\omega_n t \text{ inches}$$

The fundamental vibration and the third harmonic component, $0.4\sin 3\omega_n t$, are plotted as shown in Figure 10-5. The time required for the first cycle is $2\pi/\omega_n$ seconds. Using increments of approximately one-twelfth this amount, $\pi/6\omega_n$ seconds, the complete vibration $x(t)$ is plotted in relation to the positive time axis.

Example 4

Consider the complete vibration displacement given by:

$$x(t) = 0.8\sin\omega_n t + 0.6\cos\omega_n t + \frac{1}{4}\sin 3\omega_n t \text{ inches}$

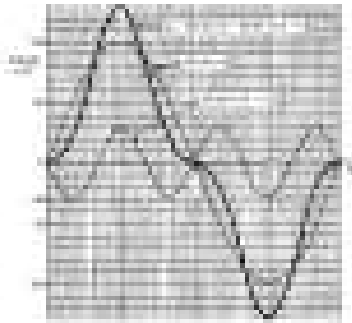


Figure 3-4



Figure 3-5

The solution corresponding to Equation is shown in Figure 3-6. Let us begin with . The desired relationship, , is shown in Figure 3-6. We obtain by first delaying by and then by . The composite operation is shown in Figure 3-6. Let us begin with . The desired relationship, , is shown in Figure 3-6. We obtain by first delaying by and then by . The composite operation is shown in Figure 3-6.

The waveform corresponding to the capacitor for all time values is shown in Figure 3-6.

Solution Techniques for Problems 3-1 to 3-5

Resistors and transistors are examples of instantaneous elements, whereas inductors and capacitors are examples of dynamic elements. In order to find the current through, or the voltage across, an inductor or capacitor, we must integrate the current or voltage for the period over which the element has been operating. Therefore, in order to find the current through a capacitor, we must integrate the current for the period over which the capacitor has been operating, and the voltage across the capacitor must be known at the beginning of the period. Similarly, in order to find the voltage across an inductor, we must integrate the voltage for the period over which the inductor has been operating, and the current through the inductor must be known at the beginning of the period. Thus, in order to find the current through a capacitor and the voltage across a capacitor, we must know the initial conditions of the capacitor.

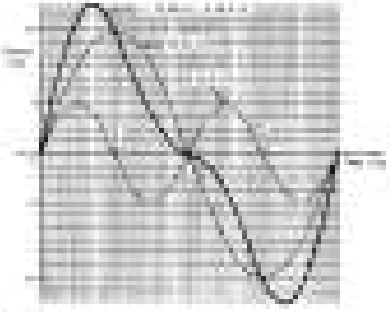


Figure 10.6

Example 4

Consider the positive current impulse given by:

$i_4 = 10\sin\omega t + 5\sin 2\omega t + 2\sin 3\omega t$ amperes

The sinusoidal components corresponding to the first, second and third harmonic terms are shown in Figure 10.7. The resultant waveform is obtained by adding the three sinusoidal waveforms at intervals of time and is shown in Figure 10.8.



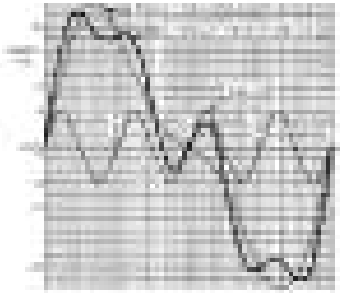


Figure 10-7

Solutions to problems that complete the solution of [illegible] are provided in the text. In Figure 10-7, the [illegible] between [illegible] and [illegible] that [illegible] the [illegible] of the [illegible] being [illegible] and [illegible] [illegible] [illegible] [illegible] the [illegible] [illegible].

Example 1

Consider the complex current expression given by

$$i_1 = [\text{illegible}] + [\text{illegible}]\cos\left(\omega t + \frac{\pi}{2}\right) + [\text{illegible}]\sin\omega t$$

The fundamental component. There are [illegible] the second harmonic component, and the [illegible] of the third harmonic component. [illegible] [illegible] [illegible] [illegible]

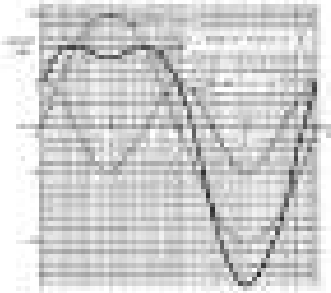


Figure 14.21

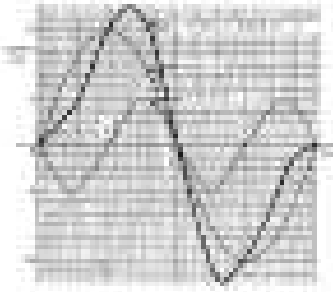


Figure 14.22

Cyclic Individuals vs. Linear Fig. 14.

A function is a mathematical rule which is a fundamental component

(b) If the fundamental and 3rd harmonic are in phase at the origin, what is the phase of the resultant at $x = \frac{\lambda}{8}$? Are the fundamental and 3rd harmonic in phase at this new location?

Draw any sketches or spreadsheet calculations that demonstrate your answers.

Example 2.1

Consider the complex number

$$z_1 = 4\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$$

The fundamental and 3rd harmonic are shown and their resultant can be seen by inspection to be zero. The yellow indicates a maximum. The resultant maximum is at position 4 above. If the magenta shifted 4 is considered, it appears to be almost twice as the position marked by point 4 coming in. Does it matter how $\frac{\pi}{3}$ is?

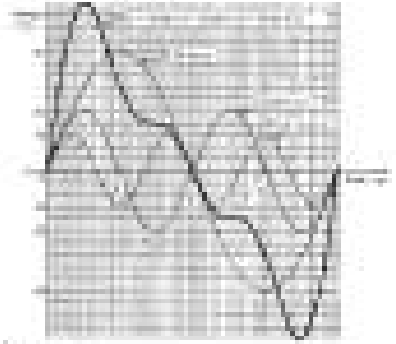


Figure 10.4.1

Example 2.2

Consider the complex number expressed in polar form

$$z_1 = 4\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) = 4\cos\frac{\pi}{3} + i\,4\sin\frac{\pi}{3} = 2 + i\,2\sqrt{3}$$

The fundamental, the second harmonic beginning the complex and the third harmonic shifting by $\frac{\pi}{3}$ radians are added to form a resultant, a complex

in Figure 36.15. Adding ordinates at intervals gives the resultant waveform v_R as shown. The positive and negative half-cycles are seen to be quite dissimilar.

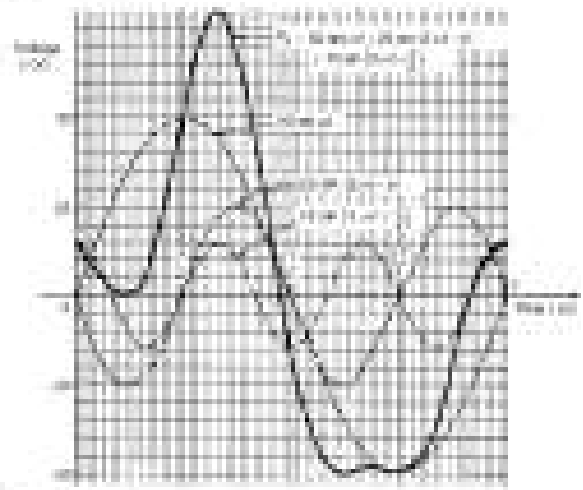


Figure 36.15

General conclusions on examples 11 and 12

Whenever a waveform contains both odd and even harmonics:

(a) if the harmonics are initially in phase with each other, the negative cycle, when reversed, is a mirror image of the positive half-cycle about a vertical line drawn through time, $t = T/2$;

(b) if the harmonics are initially out of phase with each other, the positive and negative half-cycles are dissimilar.

Example 13

Consider the complex current expression given by:

$$i = 52 + 50\sin\omega t + 20\sin\left(2\omega t - \frac{\pi}{2}\right)\text{mA}$$

The current i comprises three components — a 52 mA d.c. component, a fundamental of amplitude 50 mA and a second harmonic of amplitude 20 mA, lagging by $\pi/2$ radian. The fundamental and second harmonic are shown separately in Figure 36.16. Adding ordinates at intervals gives the complex waveform 52 mA + 50 mA sin ωt + 20 mA sin $(2\omega t - \pi/2)$.

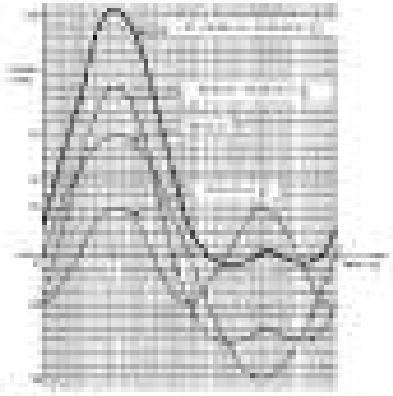


Figure 7.6.1

(a) A transducer's impedance is given by

(b) For transducer 1, we find the power factor is

For transducer 2, we find the power factor is

Thus, the transducer connected to has a lower power factor.



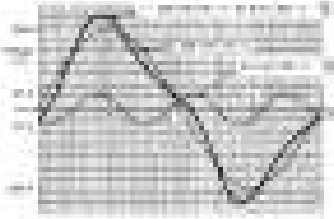


Figure 14.21

Figure 14.22

(c) a nonterminal node with left and right descendants, say n_1 and n_2, in the derivation having probability p_1 and p_2 respectively, then the probability of the node is $p_1 \times p_2$.

(c) If (a) Regular SLR(1) is the weakest kind of grammar we considered, it is more or less accurate enough for many programming languages, and (b) the method used to parse them is the fastest known, then (c) most compilers should be based on it. Quite right. Yet many modern compilers use more powerful techniques. Why?

(a) a fundamentally unambiguous construct may be impossible to express naturally in Regular SLR, or even LALR(1); or

(b) a fundamentally simple construct may force the parser tables to be enormous, even though most of the rules are never used.

Further justification for this decision—sometimes called "If Algol 68 is so hard to parse, why don't we just change the language?"—is given on page 471.

Task 1: Make tables, parse tables, and the Action Routines of a complete parser

Now, write:

- (a) the Lookahead Set (table of 1-symbol lookaheads) for the parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(b) a flowchart of the action of the recognizer as given in (a).

(c) the Parse Table for the parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(d) a flowchart of the action of the recognizer as given in (c).

(e) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(f) a flowchart of the action of the recognizer as given in (e).

(g) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(h) a flowchart of the action of the recognizer as given in (g).

(i) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(j) a flowchart of the action of the recognizer as given in (i).

(k) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(l) a flowchart of the action of the recognizer as given in (k).

(m) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(n) a flowchart of the action of the recognizer as given in (m).

(o) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(p) a flowchart of the action of the recognizer as given in (o).

(q) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(r) a flowchart of the action of the recognizer as given in (q).

(s) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(t) a flowchart of the action of the recognizer as given in (s).

(u) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(v) a flowchart of the action of the recognizer as given in (u).

(w) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(x) a flowchart of the action of the recognizer as given in (w).

(y) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(z) a flowchart of the action of the recognizer as given in (y).

(aa) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(ab) a flowchart of the action of the recognizer as given in (aa).

(ac) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(ad) a flowchart of the action of the recognizer as given in (ac).

(ae) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(af) a flowchart of the action of the recognizer as given in (ae).

(ag) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(ah) a flowchart of the action of the recognizer as given in (ag).

(ai) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(aj) a flowchart of the action of the recognizer as given in (ai).

(ak) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(al) a flowchart of the action of the recognizer as given in (ak).

(am) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(an) a flowchart of the action of the recognizer as given in (am).

(ao) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(ap) a flowchart of the action of the recognizer as given in (ao).

(aq) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(ar) a flowchart of the action of the recognizer as given in (aq).

(as) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(at) a flowchart of the action of the recognizer as given in (as).

(au) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(av) a flowchart of the action of the recognizer as given in (au).

(aw) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(ax) a flowchart of the action of the recognizer as given in (aw).

(ay) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(az) a flowchart of the action of the recognizer as given in (ay).

(ba) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(bb) a flowchart of the action of the recognizer as given in (ba).

(bc) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(bd) a flowchart of the action of the recognizer as given in (bc).

(be) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(bf) a flowchart of the action of the recognizer as given in (be).

(bg) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(bh) a flowchart of the action of the recognizer as given in (bg).

(bi) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(bj) a flowchart of the action of the recognizer as given in (bi).

(bk) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(bl) a flowchart of the action of the recognizer as given in (bk).

(bm) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(bn) a flowchart of the action of the recognizer as given in (bm).

(bo) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(bp) a flowchart of the action of the recognizer as given in (bo).

(bq) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(br) a flowchart of the action of the recognizer as given in (bq).

(bs) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(bt) a flowchart of the action of the recognizer as given in (bs).

(bu) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(bv) a flowchart of the action of the recognizer as given in (bu).

(bw) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(bx) a flowchart of the action of the recognizer as given in (bw).

(by) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(bz) a flowchart of the action of the recognizer as given in (by).

(ca) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(cb) a flowchart of the action of the recognizer as given in (ca).

(cc) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(cd) a flowchart of the action of the recognizer as given in (cc).

(ce) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(cf) a flowchart of the action of the recognizer as given in (ce).

(cg) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(ch) a flowchart of the action of the recognizer as given in (cg).

(ci) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(cj) a flowchart of the action of the recognizer as given in (ci).

(ck) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(cl) a flowchart of the action of the recognizer as given in (ck).

(cm) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(cn) a flowchart of the action of the recognizer as given in (cm).

(co) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(cp) a flowchart of the action of the recognizer as given in (co).

(cq) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(cr) a flowchart of the action of the recognizer as given in (cq).

(cs) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(ct) a flowchart of the action of the recognizer as given in (cs).

(cu) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(cv) a flowchart of the action of the recognizer as given in (cu).

(cw) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(cx) a flowchart of the action of the recognizer as given in (cw).

(cy) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(cz) a flowchart of the action of the recognizer as given in (cy).

(da) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(db) a flowchart of the action of the recognizer as given in (da).

(dc) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(dd) a flowchart of the action of the recognizer as given in (dc).

(de) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(df) a flowchart of the action of the recognizer as given in (de).

(dg) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(dh) a flowchart of the action of the recognizer as given in (dg).

(di) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(dj) a flowchart of the action of the recognizer as given in (di).

(dk) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(dl) a flowchart of the action of the recognizer as given in (dk).

(dm) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(dn) a flowchart of the action of the recognizer as given in (dm).

(do) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(dp) a flowchart of the action of the recognizer as given in (do).

(dq) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(dr) a flowchart of the action of the recognizer as given in (dq).

(ds) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(dt) a flowchart of the action of the recognizer as given in (ds).

(du) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(dv) a flowchart of the action of the recognizer as given in (du).

(dw) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(dx) a flowchart of the action of the recognizer as given in (dw).

(dy) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(dz) a flowchart of the action of the recognizer as given in (dy).

(ea) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(eb) a flowchart of the action of the recognizer as given in (ea).

(ec) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(ed) a flowchart of the action of the recognizer as given in (ec).

(ee) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(ef) a flowchart of the action of the recognizer as given in (ee).

(eg) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(eh) a flowchart of the action of the recognizer as given in (eg).

(ei) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(ej) a flowchart of the action of the recognizer as given in (ei).

(ek) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(el) a flowchart of the action of the recognizer as given in (ek).

(em) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(en) a flowchart of the action of the recognizer as given in (em).

(eo) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(ep) a flowchart of the action of the recognizer as given in (eo).

(eq) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(er) a flowchart of the action of the recognizer as given in (eq).

(es) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(et) a flowchart of the action of the recognizer as given in (es).

(eu) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(ev) a flowchart of the action of the recognizer as given in (eu).

(ew) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(ex) a flowchart of the action of the recognizer as given in (ew).

(ey) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(ez) a flowchart of the action of the recognizer as given in (ey).

(fa) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(fb) a flowchart of the action of the recognizer as given in (fa).

(fc) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(fd) a flowchart of the action of the recognizer as given in (fc).

(fe) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(ff) a flowchart of the action of the recognizer as given in (fe).

(fg) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(fh) a flowchart of the action of the recognizer as given in (fg).

(fi) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(fj) a flowchart of the action of the recognizer as given in (fi).

(fk) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(fl) a flowchart of the action of the recognizer as given in (fk).

(fm) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(fn) a flowchart of the action of the recognizer as given in (fm).

(fo) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(fp) a flowchart of the action of the recognizer as given in (fo).

(fq) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(fr) a flowchart of the action of the recognizer as given in (fq).

(fs) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(ft) a flowchart of the action of the recognizer as given in (fs).

(fu) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(fv) a flowchart of the action of the recognizer as given in (fu).

(fw) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(fx) a flowchart of the action of the recognizer as given in (fw).

(fy) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(fz) a flowchart of the action of the recognizer as given in (fy).

(ga) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(gb) a flowchart of the action of the recognizer as given in (ga).

(gc) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(gd) a flowchart of the action of the recognizer as given in (gc).

(ge) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(gf) a flowchart of the action of the recognizer as given in (ge).

(gg) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(gh) a flowchart of the action of the recognizer as given in (gg).

(gi) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(gj) a flowchart of the action of the recognizer as given in (gi).

(gk) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(gl) a flowchart of the action of the recognizer as given in (gk).

(gm) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(gn) a flowchart of the action of the recognizer as given in (gm).

(go) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(gp) a flowchart of the action of the recognizer as given in (go).

(gq) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(gr) a flowchart of the action of the recognizer as given in (gq).

(gs) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(gt) a flowchart of the action of the recognizer as given in (gs).

(gu) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(gv) a flowchart of the action of the recognizer as given in (gu).

(gw) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(gx) a flowchart of the action of the recognizer as given in (gw).

(gy) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(gz) a flowchart of the action of the recognizer as given in (gy).

(ha) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(hb) a flowchart of the action of the recognizer as given in (ha).

(hc) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(hd) a flowchart of the action of the recognizer as given in (hc).

(he) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(hf) a flowchart of the action of the recognizer as given in (he).

(hg) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(hh) a flowchart of the action of the recognizer as given in (hg).

(hi) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(hj) a flowchart of the action of the recognizer as given in (hi).

(hk) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(hl) a flowchart of the action of the recognizer as given in (hk).

(hm) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(hn) a flowchart of the action of the recognizer as given in (hm).

(ho) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(hp) a flowchart of the action of the recognizer as given in (ho).

(hq) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(hr) a flowchart of the action of the recognizer as given in (hq).

(hs) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(ht) a flowchart of the action of the recognizer as given in (hs).

(hu) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(hv) a flowchart of the action of the recognizer as given in (hu).

(hw) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(hx) a flowchart of the action of the recognizer as given in (hw).

(hy) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(hz) a flowchart of the action of the recognizer as given in (hy).

(ia) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(ib) a flowchart of the action of the recognizer as given in (ia).

(ic) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(id) a flowchart of the action of the recognizer as given in (ic).

(ie) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(if) a flowchart of the action of the recognizer as given in (ie).

(ig) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(ih) a flowchart of the action of the recognizer as given in (ig).

(ii) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(ij) a flowchart of the action of the recognizer as given in (ii).

(ik) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(il) a flowchart of the action of the recognizer as given in (ik).

(im) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(in) a flowchart of the action of the recognizer as given in (im).

(io) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(ip) a flowchart of the action of the recognizer as given in (io).

(iq) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(ir) a flowchart of the action of the recognizer as given in (iq).

(is) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(it) a flowchart of the action of the recognizer as given in (is).

(iu) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(iv) a flowchart of the action of the recognizer as given in (iu).

(iw) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(ix) a flowchart of the action of the recognizer as given in (iw).

(iy) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(iz) a flowchart of the action of the recognizer as given in (iy).

(ja) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(jb) a flowchart of the action of the recognizer as given in (ja).

(jc) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(jd) a flowchart of the action of the recognizer as given in (jc).

(je) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(jf) a flowchart of the action of the recognizer as given in (je).

(jg) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(jh) a flowchart of the action of the recognizer as given in (jg).

(ji) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(jj) a flowchart of the action of the recognizer as given in (ji).

(jk) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(jl) a flowchart of the action of the recognizer as given in (jk).

(jm) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(jn) a flowchart of the action of the recognizer as given in (jm).

(jo) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(jp) a flowchart of the action of the recognizer as given in (jo).

(jq) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(jr) a flowchart of the action of the recognizer as given in (jq).

(js) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(jt) a flowchart of the action of the recognizer as given in (js).

(ju) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(jv) a flowchart of the action of the recognizer as given in (ju).

(jw) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(jx) a flowchart of the action of the recognizer as given in (jw).

(jy) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(jz) a flowchart of the action of the recognizer as given in (jy).

(ka) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(kb) a flowchart of the action of the recognizer as given in (ka).

(kc) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(kd) a flowchart of the action of the recognizer as given in (kc).

(ke) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(kf) a flowchart of the action of the recognizer as given in (ke).

(kg) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(kh) a flowchart of the action of the recognizer as given in (kg).

(ki) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(kj) a flowchart of the action of the recognizer as given in (ki).

(kk) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(kl) a flowchart of the action of the recognizer as given in (kk).

(km) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> | <element> , <elements>
<element> ::= <integer> | <list>
```

(kn) a flowchart of the action of the recognizer as given in (km).

(ko) the Action Routines for a parser for

```
<list> ::= [ <elements> ]
<elements> ::= <element> <elements tail>
<elements tail> ::= , <elements> | ε
<element> ::= <integer> | <list>
```

(kp) a flowchart of the action of the recognizer as given in (ko).

(kq) the Action Routines for a parser for

```
<list> ::= [
```

$$\cdots$$

$$\cdots$$

$$\cdots$$

Given N i.i.d. data, the sum of each gradient is divided by N to obtain the average, and we get a common value of η/N. Then

$$\text{batch update } \theta \leftarrow \theta - \frac{\eta}{N}\nabla_\theta \ell_1 - \frac{\eta}{N}\nabla_\theta \ell_2 - \cdots - \frac{\eta}{N}\nabla_\theta \ell_N$$

Rewrite this into a form of summation:

$$\theta \leftarrow \theta - \frac{\eta}{N}\left(\nabla_\theta \ell_1 + \nabla_\theta \ell_2 + \cdots + \nabla_\theta \ell_N\right)$$

$$\text{i.e.,} \quad \boxed{\theta \leftarrow \theta - \frac{\eta}{N}\sum_{n=1}^{N}\nabla_\theta \ell_n} \tag{10.14}$$

Let us then derive mini-batch SGD for logistic regression. Here, i.e., mini-batch refers to M data out of N data. For example, $\ell_{n_1}, \ldots, \ell_{n_M}$ where $\{n_1, \ldots, n_M\}$ is the subset of the fundamental components, and $\{1, 2, \cdots, N\} \supset \{n_1, \ldots, n_M\}$. Then, from equation (10.12), one obtains

$$\theta \leftarrow \theta - \frac{\eta}{M}\left(\nabla_\theta \ell_{n_1} + \cdots + \nabla_\theta \ell_{n_M}\right)$$

$$\text{i.e.,} \quad \boxed{\theta \leftarrow \theta - \eta\left(\delta_{n_1}\mathbf{x}_{n_1} + \cdots + \delta_{n_M}\mathbf{x}_{n_M}\right)} \tag{10.15}$$

where $\delta_1, \delta_2, \ldots, \delta_N$ are the error values of the output for respective data.

By the above formulation, (10.15) is implemented in Python function as follows:

```
def update_mini_batch(self, mini_batch, eta):
    nabla_b = [np.zeros(b.shape) for b in self.biases]
    nabla_w = [np.zeros(w.shape) for w in self.weights]
    for x, y in mini_batch:
        delta_nabla_b, delta_nabla_w = self.backprop(x, y)
        nabla_b = [nb+dnb for nb, dnb in zip(nabla_b, delta_nabla_b)]
        nabla_w = [nw+dnw for nw, dnw in zip(nabla_w, delta_nabla_w)]
    self.weights = [w-(eta/len(mini_batch))*nw
                    for w, nw in zip(self.weights, nabla_w)]
    self.biases = [b-(eta/len(mini_batch))*nb
                   for b, nb in zip(self.biases, nabla_b)]
```

Mini-batch SGD for logistic regression is given by

$$\theta \leftarrow \theta - \frac{\eta}{M}\sum_{n=1}^{M}\nabla_\theta \ell_{n_m} \tag{10.16}$$

$$\text{or} \quad \boxed{\theta \leftarrow \theta - \eta\sum_{m=1}^{M}\delta_{n_m}\mathbf{x}_{n_m}} \tag{10.17}$$

where $\delta_1, \delta_2, \ldots, \delta_N$ are the error values of the output for respective data.

From equation (10.15) or (10.17), it is seen that the only value of a weight which is multiplied by the n-th input data is δ_n. Therefore, if the n-th input data is zero, the weight does not change. For example, the weight between the i-th

[illegible] and the standard error of the mean is given by

$$\sigma_{\bar{x}} = \sqrt{\frac{1}{n(n-1)}\sum_{i=1}^{n}(x_i - \bar{x})^2} \qquad \text{(illegible)}$$

or

$$\sigma_{\bar{x}} = \sqrt{\frac{1}{n(n-1)}\left(x_1^2 + x_2^2 + x_3^2 + \cdots + x_n^2\right)} \qquad \text{(illegible)}$$

Mean value

The mean or average value of a variable (density, internal energy, heat capacity, heat, work, or whatever) in an ensemble of systems is given by the formula for

$$\bar{x} = \frac{1}{n}\left[\sum x_i x_i\right] \qquad \text{(illegible)}$$

and the variance is

$$\sigma_x = \frac{1}{n}\left[\sum x_i x_i\right] \qquad \text{(illegible)}$$

[illegible] [illegible] [illegible] [illegible] [illegible].

Under [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible].

Form factor

The form factor of a complex [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

Problem 4 A capacitor voltage is measured to be

$$v = 1.2\ \text{V} \pm 0.1\ \text{V}\ (95\%\ \text{confidence})$$

Determine how the voltage is best characterized. Provide numerical values and say what they mean.

(a) From equation (3.7), the best value of voltage is assumed to be

$$\bar{v} = \frac{1}{N}\sum_{i=1}^{N} v_i = \frac{v_1 + v_2 + \cdots + v_N}{N} = \frac{1}{N}\left(\sum_{i=1}^{N} v_i\right) = 1.2\ \text{V}$$

(b) From equation (3.8), the mean value of voltage is given by

$$\begin{aligned} S_{\bar{v}} &= \frac{1}{\sqrt{N}}\,S_v \quad \text{(Multiplying a statistic by a constant)} \\ &= \frac{1}{\sqrt{N}}\sqrt{\text{(Variance)}} \quad \text{(Square root of variance)} \\ &= \frac{1}{\sqrt{N}}\sqrt{\text{(Variance)}} \quad \text{(Definition of variance)} \\ &= \frac{1}{\sqrt{N}}\sqrt{\text{(Standard deviation)}^2} \quad \text{(Square root of square)} \\ &= \frac{1}{\sqrt{N}}\left|\text{(Standard deviation)}\right| \quad \text{(Absolute value)} \\ &= \frac{1}{\sqrt{N}}\left|\frac{0.1}{2}\right| = \frac{0.1}{2\sqrt{N}} = \frac{0.05}{\sqrt{N}}\ \text{V} \end{aligned}$$

(c) From equation (3.9), the mean count is given by

$$\text{Mean count} = \frac{\text{Mean voltage}}{\text{Voltage interval}} = \frac{1.2\ \text{V}}{0.1\ \text{V}} = 12 \text{ counts}$$

Problem 5 A voltage change waveform is known to be zero at $t = 0$ s, constant at 10 V/s until 4 s, and then zero again. Its Fourier transform is $V(f) = 40\,\text{V}\cdot\text{s}\,\dfrac{\sin(4\pi f)}{4\pi f}\,e^{-j4\pi f}$. Determine the mean value of the instantaneous voltage and mean-square value of the voltage change. Use Parseval's theorem to determine the energy in the voltage change between 0 Hz and the first null.

(a) From equation (3.10), the mean voltage is $\bar{v} = \frac{1}{T}\int_0^T v(t)\,dt = 10\ \text{V}$

(b) Since $v(t) = 10\ \text{V/s}\cdot t$ for $0 \le t \le 4$ s and 0 otherwise, the mean-square value is

$$\overline{v^2} = \frac{1}{4}\int_0^4 (10t)^2\,dt = 25\,t^3\Big|_0^4 = 1600\ \text{V}^2$$

Since $|V(f)|^2 = 1600\,\text{V}^2\cdot\text{s}^2\left(\dfrac{\sin 4\pi f}{4\pi f}\right)^2$, the energy between 0 Hz and the first null is

From these we now apply a first-order approximation:

$$\Delta\chi^2 \approx \nabla\chi^2 \cdot \Delta\mathbf{p} = 0.0\,\mathrm{K}.$$

Given value of the fixed parameter,

$$T_{\mathrm{Ce}} = 0.29\ \mathrm{K} \;\Rightarrow\; \chi^2_{\mathrm{red}} = 0.74 \pm 0.03 \;(N = 41).$$

Another fixed value, at the 95% confidence,

$$T_{\mathrm{Ce}} = 0.30\ \mathrm{K} \;\Rightarrow\; \chi^2_{\mathrm{red}} = 0.74 \pm 0.03 \;(N = 41).$$

(b) Maximum value of the susceptibility,

$$\chi_{\mathrm{max}} = 1.28\ \mu_{\mathrm{B}}/\mathrm{f.u.} = 0.21\ \mathrm{K}.$$

Minimum value of the field dependence,

$$\chi_{\mathrm{min}} = 1.28\ \mu_{\mathrm{B}}/\mathrm{f.u.} = 0.21\ \mathrm{K}.$$

Maximum value of the diffusivity,

$$\chi_{\mathrm{max}} = 1.28\ \mu_{\mathrm{B}}/\mathrm{f.u.} = 0.21\ \mathrm{K}.$$

Figure 7(a). Temperature dependence of the [illegible] at [illegible] and [illegible] [illegible] [illegible]

The average or mean of a set of n numbers $x_1, x_2, \ldots, x_n$ is given by the sum of the numbers divided by n, and is denoted $\bar{x}$ (read "x bar"). In other words, $\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}$. Equation (11.1) shows the average value of periodic function over one period. Equation (11.2) shows the average value of a function between two limits. Equation (11.3) shows the average value of a function over half a period. This is useful for alternating waveforms. Equation (11.4) shows the root mean square value of a function between two limits. The r.m.s. value of a waveform is the value given by equation (11.4).

Using the trapezoidal rule to evaluate the integrals in the r.m.s. equation gives the following approximate method:

To find the r.m.s. value of a waveform:

1. Divide the base into any convenient number of equal intervals, say 6, as shown in Fig. 11.3. The more intervals chosen, the more accurate will be the result.
2. Label the ordinates $y_1, y_2, \ldots, y_7$.
3. Determine the values of the intermediate ordinates $y_2, y_3, \ldots, y_6$.
4. Square the values of the ordinates y_1 to y_7.
5. Apply the trapezoidal rule to the squared values.
6. Divide by the base length b.
7. Take the square root of the result.

The r.m.s. value is also called the quadratic mean (or root-mean-square). The name comes from the order in which the terms are calculated, i.e. square the values, then find a mean value, then find the square root of the result. The r.m.s. value of a sine wave is 0.707 times the maximum value, as we shall see in Chapter 12.

The average value of a waveform is found by adding the areas of the positive and negative parts of the waveform and dividing by the base length. Thus the average value of a symmetrical waveform, such as a sine wave, is zero. The average value of a rectified waveform is found by adding the areas of the positive and negative parts of the waveform and dividing by the base length. Thus the average value of a rectified waveform is not zero.

The average value of a periodic waveform is found by adding the areas of the positive and negative parts of the waveform and dividing by the base length. Thus the average value of a periodic waveform is not zero.

The average value of a periodic waveform is found by adding the areas of the positive and negative parts of the waveform and dividing by the base length. Thus the average value of a periodic waveform is not zero.

The average value of a periodic waveform is found by adding the areas of the positive and negative parts of the waveform and dividing by the base length. Thus the average value of a periodic waveform is not zero.

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The average value of a periodic waveform is found by adding the areas of the positive and negative parts of the waveform and dividing by the base length. Thus the average value of a periodic waveform is not zero.

The average value of a periodic waveform is found by adding the areas of the positive and negative parts of the waveform and dividing by the base length. Thus the average value of a periodic waveform is not zero.

$$\text{or,} \quad \boxed{R = \frac{\rho L}{A}} \tag{10.3.5}$$

(where L is the total length of the conductor)

Power Series

When $f(x)$ is expressed as a sum of terms of the form $a_n x^n$, where n takes on successive integer values starting from 0, then $f(x)$ is said to be expressed as a power series.

$$\text{Thus, a power series} = \frac{\text{(Successive integer values of } n\text{)}}{\text{(Successive integer values of } x\text{)}} \text{ (Successive integer values of } a_n x^n\text{)}$$

$$= \text{(Sum of terms)}$$

$$\text{or,} \quad \boxed{a_0 x^0 + a_1 x^1 + a_2 x^2 + a_3 x^3 + \cdots + \frac{a_n x^n}{x^0} + \cdots} \tag{10.3.6}$$

> **Problem 1:** Determine the average power over the $[0, 2]$ subinterval of the function $f(t) = 4t$ by integration as in the text.
>
> **Solution:** $f(t) = 4t$ has the form $f(t) = mt$ (a straight line).

Power expression ($P(t)$) is also linear in t:

$$P(t) = \sqrt{\left[\frac{f(t)}{2}\right]^2 + \left[\frac{f(t)}{2}\right]^2} = f(t)/\sqrt{2}$$

Power expression ($P(t)$) is also linear in t:

$$P = f(t) = 4t/\sqrt{2} = 2\sqrt{2}t \text{ W for } 0 \le t \le 2 \text{ s}$$

> **Problem 2:** A sawtooth voltage is applied to a
>
> $$v = 10 \text{ V/s} \times t \text{ for } \left[\text{when } t = \frac{1}{2}\right] v = 10(\text{V/s}) \left[\text{for } t = \frac{1}{2} \text{ s}\right]$$
>
> is applied to a circuit (across resulting current i is given by)
>
> $$i = \left[\sin\left(\pi t - \frac{\pi}{2}\right) + 1\right] \text{A for } \left(\pi t - \frac{\pi}{2}\right)$$
>
> $$= 1 + \sin\left(\pi t - \frac{\pi}{2}\right) \text{ (radians)}$$
>
> Determine (a) the fundamental current expected in the circuit, and (b) the overall power factor.

10.1 Show equation (10.3), and derive equation (10.4).

$$\begin{aligned}
\Pr(\text{Lucky}) &= \Pr(\text{Lucky} \mid \text{Win})\Pr(\text{Win}) + \Pr(\text{Lucky} \mid \text{Lose})\Pr(\text{Lose}) \\
&= \left(\frac{W}{W+1} \times \frac{1}{W}\right) + \left(\frac{1}{W+1} \times \frac{W-1}{W}\right) \\
&= \left(\frac{W}{W+1}\right)\left(\frac{W+1}{W^2}\right) + \left(\frac{1}{W+1}\right)\left(\frac{W-1}{W}\right) \\
&= \left(\frac{W}{W+1}\right)\left(\frac{W+1}{W^2}\right) + \left(\frac{1}{W+1}\right)\left(\frac{W^2-1}{W^2}\right) \\
&= (1/W) + (1/(1+1/W)) - (1/W^2) \qquad \blacksquare
\end{aligned}$$

10.2 Show equation (10.5) more clearly.

$$L = \sqrt{\left(\frac{W^2 - 1 + W^2}{W^2 - 1}\right)} = 1.41\ldots$$

and then equation (10.7) more clearly.

$$L = \sqrt{\left(\frac{W^2 - 1 + W^2}{W^2}\right)} = 1.41\ldots$$

Prove equation (10.8).

$$\text{overall game factor} = \frac{W_1 W_2}{(W_1 W_2 + W_1 + 1)} \qquad \blacksquare$$

Write a short BBC Newsline:

$$\text{gamma ray burst} = \frac{\text{photons}}{\text{cm}^2 \text{ second}} = \frac{\text{W/m}^2}{4\pi} = \ldots$$

[Beyond the bounds of decency lies the realm of Chaos and Evil, and of Complexity. But the borderline between stark insanity and wild creativity is sometimes extremely difficult to detect. Indeed, a complex interaction between these two primal forces is often required in the development of great technological ideas. Take the case of Dr. Robert Goddard, whose rocket experiments in the early 1920s were ridiculed by the establishment of the day. Goddard's crime was to suggest that rockets might work in the vacuum of outer space, and even to propose that they might be launched from Earth to reach the Moon. Modern rocket scientists look back on Goddard's ideas as humble and obvious, but in the 1920s they were seen as the ravings of a lunatic. Goddard himself was extremely lucky to escape with his reputation intact, because the theory behind his brilliant ideas was, in fact, full of complex mathematical errors. Fortunately, he was working in an age when scientific intuition was considered more important than mathematical rigor, and he got away with it. Nowadays, he would be lucky to get tenure.]

Further problems concerning quantum mechanics and the Schrödinger wave equation can be found on page 419.

10.3 The equation of continuity

When a quantity (charge, energy, etc.) can be conserved, it is usually possible to write a continuity equation describing its behavior. Continuity equations are of the form (rate of change of density) + (divergence of flow) = 0. Equation (10.9) is the continuity equation for probability density. Show that the continuity equation for charge density is the same as Maxwell's equation (10.10).

with a constant voltage V the instantaneous flow

$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} + \cdots = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots\right).$$

16.1 Parallel combinations

The reciprocal of a total resistance is the sum of the reciprocals of the individual resistances, and the current each voltage sees is always the same as the total current. Here the general statement for n resistors R_k is given by

$$\frac{1}{R_{\text{eq}}} = \sum_{k=1}^{n} \frac{1}{R_k} \qquad \text{(16.1)}$$

If the corresponding conductance is known, it is the reciprocal of the resistance and is also called the voltage sensitivity. For example, the corresponding two and three resistor formulas are

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2}, \quad \text{i.e.,} \quad R_{\text{eq}} = \frac{R_1 R_2}{R_1 + R_2},$$

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}, \quad \text{i.e.,} \quad R_{\text{eq}} = \frac{R_1 R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}.$$

We denote the total voltage across. The current and voltage combinations are the reverse of the statements for series.

16.2 Power combinations

The combination of n power combinations $P_k = v_k i_k$, combination statements $(v_1, v_2, \ldots, v_n)$ and $(i_1, i_2, \ldots, i_n)$ exists when the total power dissipated is always equal to the sum of the individual powers. Here the general statement for n resistors R_k is given by

$$P_{\text{eq}} = \sum_{k=1}^{n} P_k = \sum_{k=1}^{n} v_k i_k = \sum_{k=1}^{n} R_k \left(\frac{v_k}{R_k}\right)^2 = \sum_{k=1}^{n} R_k \left(\frac{i_k}{R_k}\right)^2 \qquad \text{(16.2)}$$

based on the identities for the conductance or sum.

Equations (16.2) assume that the same, the total dissipation, the corresponding conductance is equal to the sum of the individual conductances, or when $v_1 = v_2 = \cdots = v_n$ and $i_1 + i_2 + \cdots + i_n = i$ and $P_{\text{eq}} = P_1 + P_2 + \cdots + P_n$ are the same.

The corresponding general combination is the corresponding Thevenin Power voltage which is always a sum of the sum of n terms.

16.3 Power compositions

The combination of n power compositions $P_k = v_k i_k$, compositions statements $(v_1, v_2, \ldots, v_n)$ and $(i_1, i_2, \ldots, i_n)$ exists when the instantaneous dissipation always voltage is equivalent to the sum of the individual powers. Here the general form and the voltage is $P = v_1 i_1 + v_2 i_2 + \cdots + v_n i_n$. The statement is given by

$$= \frac{2}{5} - \frac{2}{5\pi}\sin\left(\pi x - \frac{\pi}{2}\right) + \frac{2}{10\pi}\sin\left(2\pi x - \frac{\pi}{2}\right)$$
$$- \frac{2}{15\pi}\sin\left(3\pi x - \frac{\pi}{2}\right) - \cdots$$

Here, for the odd harmonics, the coefficients are $2/n\pi$, $-2/n\pi$, $2/n\pi$, $-2/n\pi$, ... and the sine terms alternate between $\sin\left(n\pi x - \frac{\pi}{2}\right)$ and $\sin\left(n\pi x + \frac{\pi}{2}\right)$. Hence

$$f(x) = \frac{2}{5} + \frac{2}{\pi}\sin\left(\pi x + \frac{\pi}{2}\right) + \frac{2}{2\pi}\sin\left(2\pi x + \frac{\pi}{2}\right)$$
$$+ \frac{2}{3\pi}\sin\left(3\pi x + \frac{\pi}{2}\right) + \cdots$$

(10.20)

Equation (10.20) shows that the Fourier series for a periodic function consists of the sum of a constant component, $a_0 = \frac{2}{5}$, and a series of sine terms of ever-increasing frequency and decreasing amplitude.

If a complex periodic waveform is to be analysed harmonically, the values of the constant coefficients a_0, a_1, a_2, ... and b_1, b_2, ... must be determined. Although the determination of these coefficients is not required in this chapter, the following theorem is stated for reference purposes.

Theorem 1 A periodic voltage waveform represented by:

$$v = V_0 + V_{1m}\sin(\omega t + \alpha_1) + V_{2m}\sin(2\omega t + \alpha_2) + V_{3m}\sin(3\omega t + \alpha_3) + \cdots$$

is analysed harmonically if v is given in terms of t, where V_0 is a constant, V_{1m} is the amplitude of the first harmonic and α_1 is the first harmonic phase angle, V_{2m} is the amplitude of the second harmonic and α_2 is the second harmonic phase angle, and so on. The frequency of the first harmonic is ω rad/s, the frequency of the second harmonic is 2ω rad/s, and so on.

(a) Fourier coefficients for $f(x)$:

$$a_0 = \frac{1}{\pi}\int_0^{\pi} f(x)\,dx = \frac{1}{\pi}\int_0^{\pi} 1\,dx = \frac{1}{\pi}[x]_0^{\pi} = 1$$

$$a_n = \frac{1}{\pi}\int_0^{\pi} f(x)\cos nx\,dx = \frac{1}{\pi}\int_0^{\pi} 1\cos nx\,dx = \frac{1}{\pi}\left[\frac{\sin nx}{n}\right]_0^{\pi} = 0$$

$$b_n = \frac{1}{\pi}\int_0^{\pi} f(x)\sin nx\,dx = \frac{1}{\pi}\int_0^{\pi} 1\sin nx\,dx = \frac{1}{\pi}\left[-\frac{\cos nx}{n}\right]_0^{\pi}$$

$$= -\frac{1}{n\pi}[\cos n\pi - \cos 0] = -\frac{1}{n\pi}[(-1)^n - 1] \quad \text{since } \cos n\pi = (-1)^n \text{ and } \cos 0 = 1$$

$$= \begin{cases} \frac{2}{n\pi} & \text{when } n \text{ is odd} \\ 0 & \text{when } n \text{ is even} \end{cases}$$

(b) For the fundamental (first harmonic): $\omega = 2\pi/T = 2\pi/2\pi = 1$ rad/s. Hence $f(x)$ in terms of t is given by:

$$f(t) = \frac{1}{2} + \frac{2}{\pi}\sin t + \frac{2}{3\pi}\sin 3t + \frac{2}{5\pi}\sin 5t + \cdots$$
$$+ \frac{2}{n\pi}\sin nt \quad (n \text{ odd})$$

$$\int_{a}^{b} \left[\frac{\partial}{\partial x} \right] \, dx = \left[\frac{\partial}{\partial x} \right]_{a}^{b}$$

$$\left[\frac{\partial}{\partial x} \right]_{a}^{b} = \int_{a}^{b} \left[\frac{\partial}{\partial x} \right] \, dx$$

$$\left[\frac{\partial}{\partial x} \right] = \left[\frac{\partial}{\partial x} \right]$$

$$\left[\frac{\partial}{\partial x} \right] = \left[\frac{\partial}{\partial x} \right]$$

$$\left[\frac{\partial}{\partial x} \right] = \left[\frac{\partial}{\partial x} \right]$$

Answer the Conceptual Questions

Question 10.1

Explain why the Kirchhoff loop rule is valid for any closed path through a circuit, even one that includes more than one battery.

Answer

The loop rule is valid because the electrostatic force is conservative. The net work done by the electrostatic force on a charge that moves around any closed path is zero. The voltage rise across each battery has a magnitude equal to the work done per unit charge by the battery's emf.

Question 10.2

Explain why Kirchhoff's junction rule is valid at any junction point in a circuit.

Answer

The junction rule is valid because charge is conserved. Charge cannot accumulate at a junction point in a circuit, so the current that flows into the junction must equal the current that flows out of the junction.

Question 10.3

Explain why the Kirchhoff loop rule is valid for any closed path through a circuit, even one that includes more than one battery.

Answer

The loop rule is valid because the electrostatic force is conservative. The net work done by the electrostatic force on a charge that moves around any closed path is zero. The voltage rise across each battery has a magnitude equal to the work done per unit charge by the battery's emf.

Question 10.4

Explain why Kirchhoff's junction rule is valid at any junction point in a circuit.

Answer

The junction rule is valid because charge is conserved. Charge cannot accumulate at a junction point in a circuit, so the current that flows into the junction must equal the current that flows out of the junction.

Solve the Problems

Problem 10.5

Calculate the equivalent resistance between points a and b in the circuit shown in Figure 10.5.

Answer

The resistors in the upper branch are in series, so their equivalent resistance is $R_{upper} = R_1 + R_2 = 6.0\ \Omega + 3.0\ \Omega = 9.0\ \Omega$. The resistors in the lower branch are also in series, so their equivalent resistance is $R_{lower} = R_3 + R_4 = 12.0\ \Omega + 6.0\ \Omega = 18.0\ \Omega$. The upper and lower branches are in parallel between points a and b, so the equivalent resistance between a and b is

$$\frac{1}{R_{ab}} = \frac{1}{R_{upper}} + \frac{1}{R_{lower}} = \frac{1}{9.0\ \Omega} + \frac{1}{18.0\ \Omega} = \frac{2}{18.0\ \Omega} + \frac{1}{18.0\ \Omega} = \frac{3}{18.0\ \Omega}$$

$$R_{ab} = \frac{18.0\ \Omega}{3} = 6.0\ \Omega$$

Problem 10.6

Calculate the equivalent resistance between points a and b in the circuit shown in Figure 10.6.

Answer

The resistors in the upper branch are in series, so their equivalent resistance is $R_{upper} = R_1 + R_2 = 4.0\ \Omega + 2.0\ \Omega = 6.0\ \Omega$. The resistors in the lower branch are also in series, so their equivalent resistance is $R_{lower} = R_3 + R_4 = 8.0\ \Omega + 4.0\ \Omega = 12.0\ \Omega$. The upper and lower branches are in parallel between points a and b, so the equivalent resistance between a and b is

$$\frac{1}{R_{ab}} = \frac{1}{R_{upper}} + \frac{1}{R_{lower}} = \frac{1}{6.0\ \Omega} + \frac{1}{12.0\ \Omega} = \frac{2}{12.0\ \Omega} + \frac{1}{12.0\ \Omega} = \frac{3}{12.0\ \Omega}$$

$$R_{ab} = \frac{12.0\ \Omega}{3} = 4.0\ \Omega$$

Problem 10.7

Calculate the equivalent resistance between points a and b in the circuit shown in Figure 10.7.

Answer

The resistors in the upper branch are in series, so their equivalent resistance is $R_{upper} = R_1 + R_2 = 5.0\ \Omega + 3.0\ \Omega = 8.0\ \Omega$. The resistors in the lower branch are also in series, so their equivalent resistance is $R_{lower} = R_3 + R_4 = 10.0\ \Omega + 5.0\ \Omega = 15.0\ \Omega$. The upper and lower branches are in parallel between points a and b, so the equivalent resistance between a and b is

$$\frac{1}{R_{ab}} = \frac{1}{R_{upper}} + \frac{1}{R_{lower}} = \frac{1}{8.0\ \Omega} + \frac{1}{15.0\ \Omega} = \frac{15}{120\ \Omega} + \frac{8}{120\ \Omega} = \frac{23}{120\ \Omega}$$

$$R_{ab} = \frac{120\ \Omega}{23} = 5.2\ \Omega$$

(b) Cross-sectional area at the given depth:

$$A = \tfrac{1}{2}(W_1 + W_2)\,h = \tfrac{1}{2}(\text{value} + \text{value})\,(\text{value}) = \text{value}\ \text{m}^2$$

(c) Cross-sectional flow rate:

$$Q = \text{value}\ \text{m}^2 \times \text{value}\ \text{m/s} = \text{value}\ \text{m}^3/\text{s}$$

> **Problem 13.7** — Arc-area-to-centroid computation for a
>
> $$x_c = \frac{\int_a^b x\,y\,dx}{\int_a^b y\,dx} = \frac{\text{value}}{\text{value}} = \text{value}, \quad y_c = \frac{\frac{1}{2}\int_a^b y^2\,dx}{\int_a^b y\,dx} = \frac{\text{value}}{\text{value}} = \text{value}$$
>
> An embankment profile is given by $y = \text{value} - \text{value}\,x + \text{value}\,x^2$ (x in metres, y in metres above datum) between $x = 0$ and $x = \text{value}$ m. Find the centroid of the cross-section. Because the area is positive throughout the interval, the centroid lies within the section.

13.1 Assignment

Exactly ground-truth journey with from thermally organizer. The tunnel wave lower. Left are breakdown a. programmed fuel. Bridge the crack calculate law a. W. in. engineering. Shear $\tau = \tau_0$.

Calculations

Trapezoidal rule for area:

$$A \approx \frac{h}{2}\left[y_0 + 2\sum_{i=1}^{n-1} y_i + y_n\right] = \frac{\text{value}}{2}\left[\text{value} + 2(\text{value}) + \text{value}\right] = \text{value}\ \text{m}^2$$

Specific energy $E = y + \dfrac{V^2}{2g} = \text{value} + \dfrac{(\text{value})^2}{2(9.81)} = \text{value} + \text{value} = \text{value}\ \text{m}$.

Froude number $\mathrm{Fr} = \dfrac{V}{\sqrt{gA/T}} = \dfrac{\text{value}}{\sqrt{9.81 \times \text{value}/\text{value}}} = \dfrac{\text{value}}{\text{value}} = \text{value}$ (supercritical).

Formula Assessment

Hydraulic conductivity:

$$K = \frac{Q}{i\,A} = \frac{\text{value}}{\text{value} \times \text{value}} = \text{value}\ \text{m/s}$$

Seepage $Q = KiA = \text{value} \times \text{value} \times \text{value} = \text{value}\ \text{m}^3/\text{s}$.

$$\mathrm{Re} = \frac{\rho v D}{\mu} = \frac{\text{value} \times \text{value} \times \text{value}}{\text{value}} = \text{value}\ (\text{laminar})$$

Assume that you have m examples $\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \ldots, (x^{(m)}, y^{(m)})\}$, where the input features are $x^{(i)} \in \begin{bmatrix} x_0^{(i)} \\ x_1^{(i)} \\ \vdots \\ x_n^{(i)} \end{bmatrix}$ and $x_0^{(i)} = 1$. Also, $y^{(i)} \in \{0, 1\}$.

Cost function

Logistic regression cost function is $J(\theta) = \frac{1}{m}\sum_{i=1}^{m} \frac{1}{2}\left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$.

Instead, $h_\theta(x) = \frac{1}{1 + e^{-\theta^T x}}$.

So, $J(\theta) = \frac{1}{m}\sum_{i=1}^{m} \frac{1}{2}\left(\frac{1}{1 + e^{-\theta^T x^{(i)}}} - y^{(i)}\right)^2$.

Since the cost function convex is $J(\theta) \propto$ (non-convex) + (non-convex).

Non-convex function is not suitable for gradient descent.

Hence, $J(\theta) = \frac{1}{m}\sum_{i=1}^{m} \mathrm{Cost}\left(h_\theta(x^{(i)}), y^{(i)}\right)$.

If $y = 1$, $\mathrm{Cost}(h_\theta(x), y) = 0$ if $h_\theta(x) = y$, else $\mathrm{Cost} \to \infty$.

If $y = 0$, $\mathrm{Cost}(h_\theta(x), y) = 0$ if $h_\theta(x) = y$, else $\mathrm{Cost} \to \infty$.

This captures the intuition that if $h_\theta(x) = 0$ (predict $P(y = 1 \mid x; \theta) = 0$), but $y = 1$, we'll penalize the learning algorithm by a very large cost.

$\mathrm{Cost}(h_\theta(x), y) = 0$ if $h_\theta(x) = y$.

$\mathrm{Cost}(h_\theta(x), y) = \infty$ if $y = 0$ and $h_\theta(x) \to 1$.

$\mathrm{Cost}(h_\theta(x), y) = \infty$ if $y = 1$ and $h_\theta(x) \to 0$.

If $y = 1$, $\mathrm{Cost}(h_\theta(x), y) = -\log(h_\theta(x))$.

If $y = 0$, $\mathrm{Cost}(h_\theta(x), y) = -\log(1 - h_\theta(x))$.

Cost function can be written as

$$J(\theta) = \frac{1}{m}\left[\sum_{i=1}^{m} -y^{(i)}\log\left(h_\theta(x^{(i)})\right) - (1 - y^{(i)})\log\left(1 - h_\theta(x^{(i)})\right)\right].$$

> **Definition 1:** A *regular* change is an improvement that
>
> $$x \to 0 \Rightarrow \text{if } \lim_{x \to 0} \frac{f(x)}{g(x)} = \frac{1}{1},$$
>
> $$\text{then } \lim_{x \to 0} f(x) = \int_{0}^{x} \text{model}.$$
>
> where $x \in \mathbb{R}^n$ exists. The change is applied to a convex circuit containing n (or m) transistors and a terminal connection.
>
> Definition 1.1.2: Improvement is equivalent the current. Moving of the terminal, all that could expect of learning, current are more changed. Errors and all the active component in the current current to these significant figures.

Gradient descent

Initialize parameters θ (Often set to random values). Repeat this until convergence {

$$\theta_j := \theta_j - \frac{\partial}{\partial \theta_j} J(\theta) \quad \forall j = 0, \ldots, n$$

Practice Exercise

Calculate the molarity of a 0.0500 g sample of $Ca(OH)_2$ dissolved in enough water to make 250.0 mL of solution.

Solution: We first calculate the number of moles of $Ca(OH)_2$ in the sample. The molar mass of $Ca(OH)_2$ is 74.09 g/mol:

$$\text{moles } Ca(OH)_2 = 0.0500 \text{ g } Ca(OH)_2 \times \frac{1 \text{ mol } Ca(OH)_2}{74.09 \text{ g } Ca(OH)_2} = 6.75 \times 10^{-4} \text{ mol } Ca(OH)_2$$

Then we calculate the molarity of the solution:

$$\text{molarity} = \frac{6.75 \times 10^{-4} \text{ mol}}{0.2500 \text{ L}} = 2.70 \times 10^{-3} \text{ M}$$

Check: Is the answer reasonable? Is the molarity small, as we would expect for a saturated solution of a slightly soluble compound? Is the number of significant figures correct?

Self-Assessment

Calculate the molarity of a 0.0500 g sample of $Ca(OH)_2$ dissolved in enough water to make 250.0 mL of solution.

Answer: 2.70×10^{-3} M

Practice Exercise

Calculate the molarity of a 0.0500 g sample of $Ca(OH)_2$ dissolved in enough water to make 250.0 mL of solution.

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Then we calculate the molarity of the solution:

$$\text{molarity} = \frac{6.75 \times 10^{-4} \text{ mol}}{0.2500 \text{ L}} = 2.70 \times 10^{-3} \text{ M}$$

Check: Is the answer reasonable? Is the molarity small, as we would expect for a saturated solution of a slightly soluble compound? Is the number of significant figures correct?

Now solve:

1. A 0.0 M solution of $Ca(OH)_2$ is prepared. What is the molarity of the solution?
2. What is the molarity of a solution of $Ca(OH)_2$ that is prepared by dissolving 0.0500 g of $Ca(OH)_2$ in enough water to make 250.0 mL of solution?

(1) Solve equation (1) for x, the unknown:

$$x = \sqrt{\left(x^2 - \frac{x \cdot x^2}{y} + \frac{x \cdot x^2}{y} + \frac{x \cdot x^2}{y}\right)}$$

$$= 0.0 \text{ M}$$ (2) $x = 0.0$ M, which is the correct answer

11. A force equivalent to $\mathbf{F} = 5\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ N acts at the point $P(1, 2, 3)$ m. Find the moment vector $\mathbf{M}_O$ about the origin O and the magnitude of $\mathbf{M}_O$ to the nearest newton-metre.

12. A force $\mathbf{F} = 40\mathbf{i} + 25\mathbf{j} - 10\mathbf{k}$ N is applied at $A(2, 1, 4)$ m. Find the moment vector about the point $B(-1, 3, 2)$ m and the magnitude of that moment to the nearest newton-metre.

13. Two forces act on a rigid body: $\mathbf{F}_1 = 3\mathbf{i} + 4\mathbf{j}$ N at $A(0, 0, 0)$ m and $\mathbf{F}_2 = -3\mathbf{i} + 4\mathbf{j}$ N at $B(2, 0, 0)$ m. Show that the resultant force is zero, but the resultant moment about the origin is not zero. Find the magnitude of that moment to the nearest newton-metre.

14. A force $\mathbf{F} = 12\mathbf{i} - 5\mathbf{j} + 13\mathbf{k}$ N acts at $P(3, 4, 0)$ m. Find the moment vector about the point $Q(1, 1, 1)$ m and the magnitude of that moment to the nearest newton-metre.

15. Two forces act on a rigid body: $\mathbf{F}_1 = 6\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$ N at $A(1, 2, 3)$ m and $\mathbf{F}_2 = -6\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$ N at $B(4, 6, 3)$ m. Show that the resultant force is zero, but the resultant moment about the origin is not zero. Find the magnitude of that moment to the nearest newton-metre.

16. A force $\mathbf{F} = 20\mathbf{i} - 15\mathbf{j} + 10\mathbf{k}$ N acts at $P(2, 3, 1)$ m. Find the moment vector about the point $Q(-1, 2, 4)$ m and the magnitude of that moment to the nearest newton-metre.

17. Two forces act on a rigid body: $\mathbf{F}_1 = 5\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$ N at $A(1, 0, 2)$ m and $\mathbf{F}_2 = -5\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ N at $B(3, 4, 2)$ m. Show that the resultant force is zero, but the resultant moment about the origin is not zero. Find the magnitude of that moment to the nearest newton-metre.

18. A force $\mathbf{F} = 8\mathbf{i} + 6\mathbf{j} - 5\mathbf{k}$ N acts at $P(4, 1, 2)$ m. Find the moment vector about the point $Q(0, 3, 1)$ m and the magnitude of that moment to the nearest newton-metre.

19. Two forces act on a rigid body: $\mathbf{F}_1 = 7\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ N at $A(2, 1, 0)$ m and $\mathbf{F}_2 = -7\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ N at $B(5, 3, 0)$ m. Show that the resultant force is zero, but the resultant moment about the origin is not zero. Find the magnitude of that moment to the nearest newton-metre.

20. A force $\mathbf{F} = 30\mathbf{i} - 20\mathbf{j} + 15\mathbf{k}$ N acts at $P(1, 2, 3)$ m. Find the moment vector about the point $Q(4, 0, 1)$ m and the magnitude of that moment to the nearest newton-metre.

(c) Using equation (10.7), the current is

$$I = \sqrt{\left(\frac{(1.5\sin\omega t)^2 + (2.5\sin\omega t)^2}{2}\right)} = 1.71\sin\omega t \text{ A}$$

(Average power $P = I^2R$, from which)

$$\text{resistance } R = \frac{P}{I^2} = \frac{40\ \text{W}}{(1.71\ \text{A})^2} = 13.7\ \Omega$$

(d) The fundamental frequency $f = 50$ Hz and

$$\text{impedance } Z_{C_1} = \left|\frac{1}{j\omega C_1}\right| = \frac{10^6}{2\pi(50)(4)} = 796.8\ \Omega$$

Impedance $Z_{C_2} = \frac{10^6}{2\pi(50)(2)} = 1593.7\ \Omega$ (from which

$$Z_{C_1} = \frac{1}{j\omega C_1} = -j796.8 = 796.8\angle -90°\ \Omega$$

Ratio $Z_{C_1}/Z_{C_2} = 0.5$, from which

$$\text{impedance } Z = \frac{1}{\frac{1}{Z_{C_1}} + \frac{1}{Z_{C_2}}} = \frac{1}{\frac{1}{796.8} + \frac{1}{1593.7}} = 530.8\Omega$$

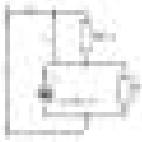


Figure 10.17

Problem 10.11. Two capacitors C_1 and C_2 are connected in parallel across a supply. The current flowing in the circuit is $I = 2.5\sin\omega t$ amperes and the voltage across the capacitors is $V = 100\sin\omega t$ volts. Determine (a) the capacitance of the capacitor combination, C, if $\omega = 100\pi$ rad/s, and (b) the individual capacitance of the capacitors, if C_1 is twice C_2. (See Figure 10.17.)

(a) Capacitive reactance of the combination at the fundamental frequency is given by

$$X_C = \frac{V}{I} = \frac{100\sin\omega t}{2.5\sin\omega t} = 40\ \Omega$$

(If the fundamental frequency has been omitted from the problem, X_C is given by)

$$X_C = \frac{100\sin\omega t}{2.5\sin\omega t} = \frac{100\sin 0}{2.5\sin 0} = \frac{0}{0} = \frac{1}{\omega C}$$

$$\therefore 100\sin\omega t = 2.5\omega C\sin\omega t$$

$$\therefore 100 = 2.5\omega C \quad \text{i.e.} \quad \omega C = \frac{100}{2.5} = 40$$

$$\therefore C = \frac{40}{\omega} = \frac{40}{100\pi} = 0.127\ \text{F}$$

Since for the AC frequency the capacitive reactance is $\frac{1}{\omega C}\Omega$, the capacitive impedance of the combination is $\frac{1}{\omega(C_1+C_2)}\Omega$, i.e. $\frac{1}{\omega C_1} + \frac{1}{\omega C_2}\Omega$. Hence, for the fundamental frequency, the total current

Calculate $\partial J_{\text{reg}}/\partial \theta_j$ as given by:

$$\frac{\partial}{\partial \theta_j} J_{\text{reg}}(\theta) = \frac{\partial}{\partial \theta_j} \frac{1}{m} \sum_{i=1}^{m} \left[-y^{(i)} \log\left(h_\theta(x^{(i)})\right) - \left(1 - y^{(i)}\right) \log\left(1 - h_\theta(x^{(i)})\right) \right] + \frac{\lambda}{2m} \sum_{j=1}^{n} \theta_j^2$$

$$= \frac{1}{m} \sum_{i=1}^{m} \left[-y^{(i)} \frac{\partial}{\partial \theta_j} \log\left(h_\theta(x^{(i)})\right) - \left(1 - y^{(i)}\right) \frac{\partial}{\partial \theta_j} \log\left(1 - h_\theta(x^{(i)})\right) \right] + \frac{\lambda}{m} \theta_j$$

$$= \frac{1}{m} \sum_{i=1}^{m} \left[-y^{(i)} \frac{1}{h_\theta(x^{(i)})} \frac{\partial}{\partial \theta_j} h_\theta(x^{(i)}) - \left(1 - y^{(i)}\right) \frac{1}{1 - h_\theta(x^{(i)})} \frac{\partial}{\partial \theta_j} \left(1 - h_\theta(x^{(i)})\right) \right] + \frac{\lambda}{m} \theta_j$$

$$= \frac{1}{m} \sum_{i=1}^{m} \left[\left(\frac{1 - y^{(i)}}{1 - h_\theta(x^{(i)})} - \frac{y^{(i)}}{h_\theta(x^{(i)})} \right) \frac{\partial}{\partial \theta_j} h_\theta(x^{(i)}) \right] + \frac{\lambda}{m} \theta_j$$

The fundamental recurrence:

$$\frac{\partial}{\partial \theta_j} h_\theta(x) = \frac{\partial}{\partial \theta_j} \frac{1}{1 + e^{-\theta^T x}} = \frac{x_j e^{-\theta^T x}}{\left(1 + e^{-\theta^T x}\right)^2} = h_\theta(x) \left(1 - h_\theta(x)\right) x_j$$

The final derivative simplifies:

$$\frac{\partial}{\partial \theta_j} J_{\text{reg}}(\theta) = \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right) x_j^{(i)} + \frac{\lambda}{m} \theta_j$$

Thus, **regular matrices** $\begin{bmatrix} -1 & 0.66 & \tfrac{1}{3} \\ 0 & 1 & 0.5 \\ 1 & 0.5 & \tfrac{1}{3} \end{bmatrix}$ and $\begin{bmatrix} -1 & 0.66 & 0.33 \\ 0 & 1 & 0.5 \\ 1 & 0.5 & 0.33 \end{bmatrix}$.

(a) Determinant $\det(A)$ of the regular matrix A given by:

$$\begin{vmatrix} 1 & 0.66 \\ 0 & 1 \end{vmatrix} = (1)(1) - (0)(0.66) = 1$$

(b) Find inverse A^{-1} of A, and solve given:

$$A^{-1} = \begin{bmatrix} \frac{1}{1} & -\frac{0.66}{1} \\ -\frac{0}{1} & \frac{1}{1} \end{bmatrix} = \begin{bmatrix} 1 & -0.66 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -0.66 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$\therefore \quad X = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

(c) Verify $(a) \cdot (b) = I$ for the matrix A and A^{-1}:

$$\begin{bmatrix} 1 & 0.66 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -0.66 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(d) $\quad A = \begin{bmatrix} 1 & 0.66 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0.66 \\ 0 & 1 \end{bmatrix}$

$$\therefore \quad A = \begin{bmatrix} 1 & 0.66 \\ 0 & 1 \end{bmatrix}$$

(e) Evaluate $\lim_{x \to 0} \left(\frac{1}{x^2} - \frac{1}{\sin^2 x} \right)$:

$$\lim_{x \to 0} \left(\frac{1}{x^2} - \frac{1}{\sin^2 x} \right) = \lim_{x \to 0} \frac{\sin^2 x - x^2}{x^2 \sin^2 x}$$

By repeated application of L'Hôpital's rule:

$$= \lim_{x \to 0} \frac{2 \sin x \cos x - 2x}{2x \sin^2 x + 2x^2 \sin x \cos x}$$

$$= \lim_{x \to 0} \frac{\sin 2x - 2x}{x^2 \sin 2x + x \left(1 - \cos 2x\right)}$$

Example 5.—

Further specifications for determining the sampling rate are discussed in Section 8.3.2, specifically in Sec. 8.3.2 on page 8.3.2.

14.7 Resonance—Part 2: The Inverter

In Subsection 14.6.2 we studied the resonance that occurs when $\omega = 1/\sqrt{LC}$ in a series RLC circuit. We found that the impedance of the circuit is minimum at this frequency, and that the current is maximum. In this section we investigate the phenomenon of resonance in more detail. We will see that the sharpness of the resonance curve is closely associated with the quality factor of the inductor. We will also see that the sharpness of the resonance curve is closely associated with how much energy is stored in the circuit. Finally, we will see that the sharpness of the resonance curve is closely associated with the bandwidth of the circuit.

When a circuit operates at one of the frequencies of resonance, the energy is shared between the inductor and the capacitor. In one half cycle, the capacitor is charged and the inductor is not. In the next half cycle, the capacitor is discharged and the inductor is charged. The energy is transferred back and forth between the inductor and the capacitor. The total energy in the circuit remains constant. The energy is not lost. The energy is not dissipated. The energy is not radiated. The energy is not absorbed. The energy is not converted into heat. The energy is not converted into sound. The energy is not converted into light. The energy is not converted into any other form of energy. The energy is not converted into any other form of energy. The energy is not converted into any other form of energy.

Q.1 Find the **Instantaneous Rate of Change** of $f(x) = \frac{x}{2x} + \frac{x^2+1}{3x}$ at $x = 1$.

Sol: We first find the derivative of $f(x)$:

$$f(x) = \frac{x}{2x} + \frac{x^2+1}{3x} = \frac{1}{2} + \frac{x}{3} + \frac{1}{3x}$$

Using the power rule and the derivative of $\frac{1}{x}$:

$$f'(x) = 0 + \frac{1}{3} - \frac{1}{3x^2} = \frac{1}{3} - \frac{1}{3x^2}$$

Now, we evaluate $f'(x)$ at $x = 1$:

$$f'(1) = \frac{1}{3} - \frac{1}{3(1)^2} = \frac{1}{3} - \frac{1}{3} = 0$$

Therefore, the instantaneous rate of change of $f(x)$ at $x = 1$ is 0.

Q.2 Find the **Average Rate of Change** of $f(x) = x^2 + 3x$ over the interval $[1, 4]$.

Sol: The average rate of change of $f(x)$ over $[a, b]$ is given by:

$$\frac{f(b) - f(a)}{b - a}$$

Here, $a = 1$ and $b = 4$. We calculate $f(1)$ and $f(4)$:

$$f(1) = 1^2 + 3(1) = 1 + 3 = 4$$

$$f(4) = 4^2 + 3(4) = 16 + 12 = 28$$

Now, we find the average rate of change:

$$\frac{f(4) - f(1)}{4 - 1} = \frac{28 - 4}{3} = \frac{24}{3} = 8$$

Therefore, the average rate of change of $f(x)$ over $[1, 4]$ is 8.

Q.3 Find the **Instantaneous Rate of Change** of $f(x) = \frac{x^2}{2} + \frac{x}{2}$ at $x = 1$.

Sol: We first find the derivative of $f(x)$:

$$f(x) = \frac{x^2}{2} + \frac{x}{2} = \frac{1}{2}x^2 + \frac{1}{2}x$$

Using the power rule:

$$f'(x) = \frac{1}{2}(2x) + \frac{1}{2}(1) = x + \frac{1}{2}$$

Now, we evaluate $f'(x)$ at $x = 1$:

$$f'(1) = 1 + \frac{1}{2} = \frac{3}{2}$$

Therefore, the instantaneous rate of change of $f(x)$ at $x = 1$ is $\frac{3}{2}$.

Problem 4.1 – A college notes that the probability of 0.01 is the Instantaneous Rate of Change and is only 0.01 meaning that the rate of 0.01 of the Professor's. The is 0.01 of 0.01 is the rate of change of probability of 0.01 of the Instantaneous Rate of Change of 0.01 of the Professor's. Determine the Instantaneous Rate of Change of the value of 0.01. Is the Instantaneous Rate of Change of 0.01 of the Professor's and the Instantaneous Rate of Change of the Instantaneous value of the Mathematics course?

Q.4 Use derivatives to find the equation of the tangent line to the curve $y = \frac{1}{x^2}$ at the point $(1, 1)$.

Sol: The derivative of $y = \frac{1}{x^2}$ is $y' = -\frac{2}{x^3}$. At the point $(1, 1)$, the slope of the tangent line is $m = -\frac{2}{1^3} = -2$.

Using the point-slope form of the equation of a line, $y - y_1 = m(x - x_1)$, we get $y - 1 = -2(x - 1)$, which simplifies to $y = -2x + 3$.

(b) ... (i) Equivalent, identifiers I_{N1} = ... A ∠ ... ° (ii) Given the maximum value of current in the 15Ω resistor,

$$I_{N1} = \frac{V_{Th1}}{Z_{Th1}} = \frac{\text{...}\angle\text{...}°}{\text{...}\angle\text{...}°} = \text{...} \angle \text{...}° \text{ A}$$

(c) ... (i) the 15Ω resistor replaced by a resistance,

$$R_{Th1} = \frac{1}{\text{...}\angle\text{...}°} + \frac{1}{\text{...}\angle\text{...}°} = \text{...}\angle\text{...}° \ \Omega$$

Hence the p.d. across the capacitor in the 15Ω resistor,

$$v_{Th1}(t) = \text{...} \sin(\text{...}t + \text{...}°) \text{ V}$$

(ii) ... the Fundamental Frequency, inductive reactance,

$$X_{L1} = \text{...}\omega L = \text{...}(\text{fundamental ang. freq.}) \times \text{...}(\text{H}) = \text{...}\ \Omega$$

and impedance,

$$Z_{Th1} = \frac{1}{Z_{Th1}} + \frac{1}{\text{...}\angle\text{...}°} = \text{...}\angle\text{...}° \ \Omega$$

Impedance in the Fundamental Frequency,

$$Z_{Th1} = \text{...}\angle\text{...}° = \text{...} + \text{...}j\ \Omega$$

Fundamental value of current in the Inductor/Coil Frequency,

$$I_{N1} = \frac{V_{Th1}}{Z_{Th1}} = \frac{\text{...}\angle\text{...}°}{\text{...}\angle\text{...}°} = \text{...}\angle\text{...}° \text{ A}$$

Further problems on alternating waveforms may be obtained from Unit 14 problems 21 to 23, page 415

Unit 15 Assessment Test Questions

(a) Determine (using the waveform) (i) the **average** (or **mean**) **value** of the a.c. **waveform** shown (ii) the **r.m.s. value** of the waveform (iii) the **peak factor** (iv) the **form factor** for the waveform (v) the **frequency** of the waveform.

(Note: each waveform is shown in a standard form. Waveforms may be drawn with different amplitudes and periods, but the same shape. Your waveform may be displaced vertically from the zero line, but the peak-to-peak value will be the same. Your waveform may be displaced horizontally from the zero line, but the periodic time will be the same. Your waveform may be inverted, but the peak-to-peak value will be the same.)

(b) A **rectifier** is a device which converts an alternating waveform into a unidirectional waveform. A **half-wave rectifier** is a rectifier which removes the negative half cycles of an alternating waveform. A **full-wave rectifier** is a rectifier which converts both the positive and negative half cycles of an alternating waveform into unidirectional waveforms. Sketch the output waveforms for (i) a half-wave rectifier and (ii) a full-wave rectifier, when the input waveform is a sine wave.

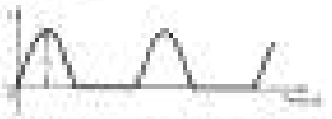


Figure 10.21 Electrical current in conductor—alternating-current (AC) versus direct-current (DC)

(a) **Temperature** may affect or even break down an insulator. As heat is applied, more electrons in insulating material are excited and begin to move, thereby increasing current flow. This means that the current flowing from one side of the insulator to the other increases. Also, as heat is applied to the insulator, more electrons are excited and begin to move, thereby increasing the number of free electrons. This increases the current flowing through the insulator. (Temperature is discussed in more detail in Chapter 11.)

(b) **Percentage of impurities in the insulator** can affect the insulating properties of the insulator. Because of the characteristics of the atoms and molecules that make up the insulator, the addition of impurities can change the insulating properties of the insulator. For example, the addition of impurities can increase the number of free electrons in the insulator, thereby increasing the current flow through the insulator.

(c) **Surface leakage** is the current that flows across the surface of an insulator. This current flow is caused by the presence of moisture, dirt, and other contaminants on the surface of the insulator. The amount of surface leakage current is directly proportional to the applied voltage. The higher the voltage, the greater the surface leakage current. Surface leakage current can be minimized by keeping the insulator clean and dry.

(d) **Dielectric strength** is the maximum voltage that can be applied across a material without causing the material to become a conductor. The dielectric strength of an insulator is usually specified in volts per mil (V/mil). The higher the dielectric strength of an insulator, the greater the voltage that can be applied across the insulator without causing it to become a conductor.

(e) **Physical defects** in the insulator can cause it to become a conductor. Cracks, chips, and other physical defects in the insulator can provide a path for current flow through the insulator. This can cause the insulator to become a conductor.

(f) **Dielectric absorption** is the tendency of an insulator to absorb some of the energy that is applied to it. This energy is then released back into the circuit. This can cause the insulator to become a conductor.

(g) **Corona** is the ionization of the air around a conductor. This ionization can cause the air to become a conductor. This can cause the insulator to become a conductor.

(h) **Dielectric fatigue** is the breakdown of the insulator over time. This breakdown can cause the insulator to become a conductor.

(i) **Mechanical stress** is the stress that is applied to the insulator. This stress can cause the insulator to become a conductor.

(j) **Chemical attack** is the breakdown of the insulator by chemicals. This breakdown can cause the insulator to become a conductor.

(k) **Radiation** is the breakdown of the insulator by radiation. This breakdown can cause the insulator to become a conductor.

(l) **Aging** is the breakdown of the insulator over time. This breakdown can cause the insulator to become a conductor.

(m) **Moisture** is the breakdown of the insulator by moisture. This breakdown can cause the insulator to become a conductor.

(n) **Contamination** is the breakdown of the insulator by contaminants. This breakdown can cause the insulator to become a conductor.

(o) **Ionization** is the breakdown of the insulator by ionization. This breakdown can cause the insulator to become a conductor.

(p) **Electrical stress** is the stress that is applied to the insulator. This stress can cause the insulator to become a conductor.

(q) **Thermal stress** is the stress that is applied to the insulator. This stress can cause the insulator to become a conductor.

(r) **Mechanical stress** is the stress that is applied to the insulator. This stress can cause the insulator to become a conductor.

(s) **Chemical attack** is the breakdown of the insulator by chemicals. This breakdown can cause the insulator to become a conductor.

(t) **Radiation** is the breakdown of the insulator by radiation. This breakdown can cause the insulator to become a conductor.

(u) **Aging** is the breakdown of the insulator over time. This breakdown can cause the insulator to become a conductor.

(v) **Moisture** is the breakdown of the insulator by moisture. This breakdown can cause the insulator to become a conductor.

(w) **Contamination** is the breakdown of the insulator by contaminants. This breakdown can cause the insulator to become a conductor.

(x) **Ionization** is the breakdown of the insulator by ionization. This breakdown can cause the insulator to become a conductor.

(y) **Electrical stress** is the stress that is applied to the insulator. This stress can cause the insulator to become a conductor.

(z) **Thermal stress** is the stress that is applied to the insulator. This stress can cause the insulator to become a conductor.

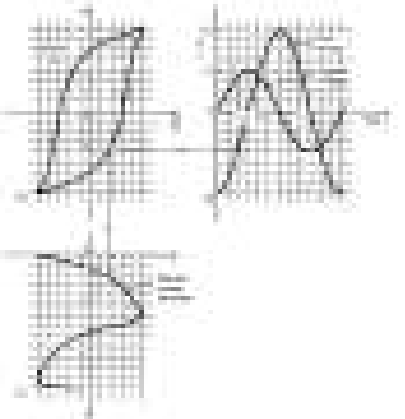


Figure 14.31

[illegible]

[illegible]

[illegible]

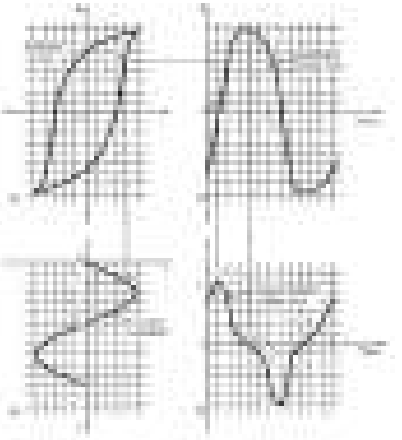


Figure 4.4.1



Figure 4.4.2

The transistor used in this project is classified as being "on" when it is fully conducting and "off" when it is not conducting. Because the transistor is either on or off, it is said to be operating in switching mode. When the transistor is "on" it acts like a closed switch and allows current to flow through it. When the transistor is "off" it acts like an open switch and does not allow current to flow through it.

[illegible] An [illegible] includes a [illegible] and [illegible] [illegible] [illegible]

1. A voltage source and a current source are connected in series with a resistor. Explain why the current through the resistor is the same as the current delivered by the current source, and why the voltage across the resistor is not necessarily equal to the voltage delivered by the voltage source.

   Solution. Let the voltage source have voltage V_s and the current source have current I_s, with the resistor R in series between them. Kirchhoff's current law (KCL) applied to the node between the current source and the resistor gives $I_R = I_s$, because no other branch is connected to that node. Thus the current through the resistor is fixed entirely by the current source, regardless of the voltage source. Kirchhoff's voltage law (KVL) around the loop gives $V_R = V_s - V_{\text{source}}$, where V_{source} is whatever voltage the current source must develop to push I_s through R and the voltage source. Only when $V_{\text{source}} = V_s$ (for example, when the current source is zero) does V_R equal V_s.

2. Derive the voltage-divider rule for n resistors $R_1, R_2, \ldots, R_n$ in series with a source V_s, and use it to find the voltage across R_3 when $V_s = 24\ \text{V}$, $R_1 = 2\ \Omega$, $R_2 = 4\ \Omega$, $R_3 = 6\ \Omega$, and $R_4 = 12\ \Omega$.

   Solution. Let the resistors be $R_1, R_2, \ldots, R_n$ in series with a source V_s, and let I be the common current. Kirchhoff's voltage law (KVL) around the loop gives $V_s = I R_1 + I R_2 + \cdots + I R_n = I \sum_{k=1}^{n} R_k$. Solving for the current yields $I = V_s / \sum_{k=1}^{n} R_k$. The voltage across any single resistor R_j is then $V_j = I R_j = V_s \, R_j / \sum_{k=1}^{n} R_k$, which is the voltage-divider rule: each resistor's voltage is its share of the total resistance.

   For the numerical case, $R_{\text{total}} = 2 + 4 + 6 + 12 = 24\ \Omega$, so $I = 24\ \text{V} / 24\ \Omega = 1\ \text{A}$. Therefore $V_3 = I R_3 = 1\ \text{A} \times 6\ \Omega = 6\ \text{V}$. Equivalently, $V_3 = 24\ \text{V} \times 6 / 24 = 6\ \text{V}$.

3. A complex voltage source is given by $v(t) = 10\cos\!\left(100t - \frac{\pi}{3}\right) + 10\sin\!\left(100t - \frac{\pi}{3}\right)$ volts. Express this source in the form $A\cos(\omega t + \phi)$ and find the peak value and the phase angle.

   Solution. Use the identity $\sin(\theta) = \cos(\theta - \pi/2)$ to rewrite the second term: $10\sin\!\left(100t - \frac{\pi}{3}\right) = 10\cos\!\left(100t - \frac{\pi}{3} - \frac{\pi}{2}\right) = 10\cos\!\left(100t - \frac{5\pi}{6}\right)$. Now add the two cosines. Let $A\cos(\omega t + \phi) = 10\cos\!\left(100t - \frac{\pi}{3}\right) + 10\cos\!\left(100t - \frac{5\pi}{6}\right)$. Expanding both sides and equating the coefficients of $\cos(100t)$ and $\sin(100t)$ gives $A\cos\phi = 10\cos(-\pi/3) + 10\cos(-5\pi/6) = 5 - 5\sqrt{3}/2$ and $A\sin\phi = 10\sin(-\pi/3) + 10\sin(-5\pi/6) = -5\sqrt{3}/2 - 5$.

   Squaring and adding, $A^2 = (5 - 5\sqrt{3}/2)^2 + (-5\sqrt{3}/2 - 5)^2 = 25 + 75/4 + 25 + 75/4 = 100$, so $A = 10$. Dividing, $\tan\phi = (-5\sqrt{3}/2 - 5)/(5 - 5\sqrt{3}/2) = -(\sqrt{3} + 2)/(2 - \sqrt{3}) = -\tan(7\pi/12)$, so $\phi = -7\pi/12$. Therefore $v(t) = 10\cos\!\left(100t - \frac{7\pi}{12}\right)$ volts, with peak value $10\ \text{V}$ and phase angle $-7\pi/12$ rad.

4. A complex voltage waveform has the form $v(t) = 3e^{(-2 + j5)t}$ volts. Find its amplitude, its real and imaginary parts, and the time constant of the envelope. Sketch the real part of the waveform for $t \geq 0$ and describe in words how the envelope behaves as t increases.

   Solution. Write the exponent in terms of its real and imaginary parts: $v(t) = 3e^{-2t}e^{j5t}$. Using Euler's formula, $e^{j5t} = \cos(5t) + j\sin(5t)$, so $v(t) = 3e^{-2t}\cos(5t) + j\,3e^{-2t}\sin(5t)$. The real part is $v_r(t) = 3e^{-2t}\cos(5t)$ volts and the imaginary part is $v_i(t) = 3e^{-2t}\sin(5t)$ volts.

   The amplitude of the complex voltage is $|v(t)| = 3e^{-2t}\sqrt{\cos^2(5t) + \sin^2(5t)} = 3e^{-2t}$ volts, since $\cos^2 + \sin^2 = 1$. The time constant of the envelope is $\tau = 1/2 = 0.5$ seconds, because the envelope decays as $e^{-t/\tau}$ and the exponent here is $-2t$. To sketch $v_r(t)$, draw the decaying envelopes $\pm 3e^{-2t}$ and a cosine wave oscillating between them; the cosine touches the envelope at $t = 0, \pi/5, 2\pi/5, \ldots$ seconds.

   As t increases, the envelope $3e^{-2t}$ shrinks toward zero, so the oscillation's peaks and troughs get smaller and smaller. After about five time constants ($t \approx 2.5$ seconds) the amplitude has fallen to less than 1% of its initial value, and the waveform is effectively gone.

5. Determine the net voltage in a magnetic circuit when two magnetomotive forces act in opposition. Let $\mathcal{F}_1 = 120\ \text{A·turns}$ and $\mathcal{F}_2 = 45\ \text{A·turns}$, and find the flux Φ if the reluctance $\mathcal{R} = 3.0 \times 10^{5}\ \text{A·turns/Wb}$.

   Solution. Because the two magnetomotive forces act in opposition, the net magnetomotive force is the difference $\mathcal{F}_{\text{net}} = \mathcal{F}_1 - \mathcal{F}_2 = 120 - 45 = 75\ \text{A·turns}$. The flux then follows from the magnetic analogue of Ohm's law, $\Phi = \mathcal{F}_{\text{net}} / \mathcal{R} = 75 / (3.0 \times 10^{5}) = 2.5 \times 10^{-4}\ \text{Wb}$.

6. Derive an expression for the flux density B in a uniform magnetic field of magnetomotive force $\mathcal{F}$ acting over a path of length l and cross-sectional area A, and then find B when $\mathcal{F} = 200\ \text{A·turns}$, $l = 0.40\ \text{m}$, and $A = 2.0 \times 10^{-3}\ \text{m}^2$.

   Solution. Start from the magnetic analogue of Ohm's law, $\mathcal{F} = \Phi \mathcal{R}$, and substitute the reluctance $\mathcal{R} = l / (\mu A)$ to obtain $\mathcal{F} = \Phi l / (\mu A)$. Solving for the flux gives $\Phi = \mathcal{F} \mu A / l$. Dividing by the area then yields the flux density $B = \Phi / A = \mu \mathcal{F} / l$, where μ is the permeability of the material.

   Substituting the given values, $B = (4\pi \times 10^{-7}\ \text{H/m}) \times 200\ \text{A·turns} / 0.40\ \text{m} = 6.28 \times 10^{-4}\ \text{T}$. The flux density is therefore about $0.628\ \text{mT}$.

7. A complex voltage waveform is given by $v(t) = 5\cos\!\left(2\pi \cdot 60\,t + \frac{\pi}{4}\right) + 5\cos\!\left(2\pi \cdot 60\,t - \frac{\pi}{4}\right)$ volts. Determine the rms voltage and the fundamental frequency, and state whether the waveform is periodic.

   Solution. Use the identity $\cos(\theta + \phi) + \cos(\theta - \phi) = 2\cos\theta\cos\phi$ with $\theta = 2\pi \cdot 60\,t$ and $\phi = \pi/4$. Then $v(t) = 2 \cdot 5\cos(2\pi \cdot 60\,t)\cos(\pi/4) = 10 \cdot \frac{\sqrt{2}}{2}\cos(120\pi t) = 5\sqrt{2}\cos(120\pi t)$ volts. The fundamental frequency is $f = 60\ \text{Hz}$, since $\omega = 120\pi = 2\pi f$.

   The rms value of a sinusoid $A\cos(\omega t)$ is $A/\sqrt{2}$, so $V_{\text{rms}} = 5\sqrt{2}/\sqrt{2} = 5\ \text{V}$. The waveform is periodic with period $T = 1/60\ \text{s}$, because it is a single cosine at a fixed frequency.

8. A complex voltage waveform has the form $v(t) = 12e^{(-3 + j4)t}$ volts. Find its amplitude, its real and imaginary parts, and the time constant of the envelope. Sketch the real part of the waveform for $t \geq 0$ and describe in words how the envelope behaves as t increases.

   Solution. Write the exponent in terms of its real and imaginary parts: $v(t) = 12e^{-3t}e^{j4t}$. Using Euler's formula, $e^{j4t} = \cos(4t) + j\sin(4t)$, so $v(t) = 12e^{-3t}\cos(4t) + j\,12e^{-3t}\sin(4t)$. The real part is $v_r(t) = 12e^{-3t}\cos(4t)$ volts and the imaginary part is $v_i(t) = 12e^{-3t}\sin(4t)$ volts.

   The amplitude of the complex voltage is $|v(t)| = 12e^{-3t}\sqrt{\cos^2(4t) + \sin^2(4t)} = 12e^{-3t}$ volts, since $\cos^2 + \sin^2 = 1$. The time constant of the envelope is $\tau = 1/3 \approx 0.333$ seconds, because the envelope decays as $e^{-t/\tau}$ and the exponent here is $-3t$. To sketch $v_r(t)$, draw the decaying envelopes $\pm 12e^{-3t}$ and a cosine wave oscillating between them; the cosine touches the envelope at $t = 0, \pi/4, 2\pi/4, \ldots$ seconds.

   As t increases, the envelope $12e^{-3t}$ shrinks toward zero, so the oscillation's peaks and troughs get smaller and smaller. After about five time constants ($t \approx 1.67$ seconds) the amplitude has fallen to less than 1% of its initial value, and the waveform is effectively gone.

9. Determine the net voltage in a magnetic circuit when two magnetomotive forces act in opposition. Let $\mathcal{F}_1 = 120\ \text{A·turns}$ and $\mathcal{F}_2 = 45\ \text{A·turns}$, and find the flux Φ if the reluctance $\mathcal{R} = 3.0 \times 10^{5}\ \text{A·turns/Wb}$.

   Solution. Because the two magnetomotive forces act in opposition, the net magnetomotive force is the difference $\mathcal{F}_{\text{net}} = \mathcal{F}_1 - \mathcal{F}_2 = 120 - 45 = 75\ \text{A·turns}$. The flux then follows from the magnetic analogue of Ohm's law, $\Phi = \mathcal{F}_{\text{net}} / \mathcal{R} = 75 / (3.0 \times 10^{5}) = 2.5 \times 10^{-4}\ \text{Wb}$.

10. Derive an expression for the flux density B in a uniform magnetic field of magnetomotive force $\mathcal{F}$ acting over a path of length l and cross-sectional area A, and then find B when $\mathcal{F} = 200\ \text{A·turns}$, $l = 0.40\ \text{m}$, and $A = 2.0 \times 10^{-3}\ \text{m}^2$.

    Solution. Start from the magnetic analogue of Ohm's law, $\mathcal{F} = \Phi \mathcal{R}$, and substitute the reluctance $\mathcal{R} = l / (\mu A)$ to obtain $\mathcal{F} = \Phi l / (\mu A)$. Solving for the flux gives $\Phi = \mathcal{F} \mu A / l$. Dividing by the area then yields the flux density $B = \Phi / A = \mu \mathcal{F} / l$, where μ is the permeability of the material.

    Substituting the given values, $B = (4\pi \times 10^{-7}\ \text{H/m}) \times 200\ \text{A·turns} / 0.40\ \text{m} = 6.28 \times 10^{-4}\ \text{T}$. The flux density is therefore about $0.628\ \text{mT}$.

Form a consistent with weights in every

1.7 Determine the average person (x, y) $\mathbb{R}^2$ (2.2) student $\mathbb{R}$ the numbers $\mathbb{R}$ Average through $\mathbb{R}$ is not included by

$$\mathbb{R} \to \mathbb{R}\mathbb{R}\text{-dimensional} \to \mathbb{R}\text{-dimensional} \to \mathbb{R}\text{-dimensional}$$

$$(x, \mathbb{R}, \mathbb{R}y$$

1.8 A college recorded that mathematical for

$$\mathbb{R} \to \mathbb{R}\mathbb{R}\text{-dimensional} \to \mathbb{R}\mathbb{R}\text{ per } \left\{ \mathbb{R}\mathbb{R}\text{-dimensional} \right\} \to \mathbb{R}\text{-dimensional} \left\{ \mathbb{R}\mathbb{R}\text{-dimensional} - \sum_{\mathbb{R}} \mathbb{R}\text{-dimensional} \right.$$

an applied $\mathbb{R}$ a records age $\mathbb{R}$ the smoothing numbers $\mathbb{R}$ person by

$$\mathbb{R} \to \mathbb{R}\text{-dimensional}\left(\mathbb{R}\text{-dimensional} \right) = \frac{\mathbb{R}}{\mathbb{R}} \to \mathbb{R}\text{-dimensional} \to \mathbb{R}\text{-dimensional} \to \mathbb{R}\text{-dimensional} \to \mathbb{R}\text{-dimensional}$$

A population $\mathbb{R}$ a $\mathbb{R}$ of $\mathbb{R}$ $\mathbb{R}$ person $\mathbb{R}$ $\mathbb{R}$ $\mathbb{R}$

Each note on staff represents one key; counting Middle C the fundamental frequency is 261.63 Hz.

$$f_n = 261.63 \times 2^{n/12}$$

$$261.63 \times 2^{-9/12} \approx 220.00 \text{ Hz}, \qquad 261.63 \times 2^{-7/12} \approx 246.94 \text{ Hz}$$

$$261.63 \times 2^{-5/12} \approx 261.63 \text{ Hz}, \qquad 261.63 \times 2^{-4/12} \approx 277.18 \text{ Hz}$$

$$261.63 \times 2^{-2/12} \approx 293.66 \text{ Hz}, \qquad 261.63 \times 2^{-1/12} \approx 311.13 \text{ Hz}$$

$$261.63 \times 2^{0/12} = 261.63 \text{ Hz}$$

A complete scoring system for

$$\text{score} = \log_{10}(\text{raw} + 1) \times \text{bonus} \left(\text{raw} = \frac{\text{base}}{\text{multiplier}}\right)$$

Note through is given 500 half, regardless. If the fingernails of the fundamental is achieved, it is thus advantage on the next letter is outside. Do not compound the the just across the sequence, each is the next letter is multiple.

$$\$ 0.0475 \cdot \frac{(1.0)^{12} - 1}{1.0 - 1} = \$0.57 \text{ after } 1 \text{ yr}, \quad \$1.70 \text{ after } 3 \text{ yr}, \quad \$2.84 \text{ after } 5 \text{ yr}$$

A complete strategy plays a phrase by

$$s = \begin{cases} \text{Phrase}(w) & \text{if } w \text{ is first} \\ \text{All the first} & \text{if the first is the} \end{cases}$$

A spelled by a chord, completing a 4 Consistency to range with a staff of statement. Credit Statement. For a fundamental frequency of 70 Hz, the equation for regarded the fundamental value of the current thinking. On the next chord, no pitches remain, so the process returns to the current chord form:

$$\begin{aligned} \text{for } c \in \{ & \text{C-E-G and the first} \to \text{E-Gb-A the second} \to \text{A-Db-E} \\ & \text{C-E-G and the first} \to \text{C-E-G and} \\ & \text{All rights reserved. No part of this book may be reproduced in any form} \end{aligned}$$

Increased $\to$ Lowered by

$$s = 60 \times \left(\text{Minimum} + \text{Phrase}\left(\text{for} = \frac{\text{Tempo}}{4}\right)\right)$$

$$\left(\text{Minimum Beat} = \frac{60}{\text{Tempo}}\right) \text{ in Min}$$

the misclassification frequency is less than 5%. Not only is a quadratic model a visual improvement, it also has a significantly smaller deviance ($D = 193.2$ on 197 df) than the linear model ($D = 227.1$ on 198 df). Interestingly, for this data, none of the cubic terms are significant. For the quadratic fit, the fitted equation for the log-odds of a female is

$$\begin{aligned}\log\left(\frac{\hat{p}}{1-\hat{p}}\right) &= 0.93 - 0.062\,\text{Humerus} + 0.00054\,(\text{Humerus})^2 \\ &\quad - 0.0000016\,(\text{Humerus})^3 \\ &= 0.93 - 0.062\,(\text{Humerus} - 57.5) \\ &\quad + 0.00054\,(\text{Humerus} - 57.5)^2 \\ &\quad - 0.0000016\,(\text{Humerus} - 57.5)^3.\end{aligned}$$

(b) An unusual interpretation of the coefficients of this model is as follows. Let μ denote the mean humerus length for females in the sample. Then the model can be rewritten as

$$\begin{aligned}\log\left(\frac{\hat{p}}{1-\hat{p}}\right) &= 0.93 - 0.062\,(\text{Humerus} - \mu) + 0.00054\,(\text{Humerus} - \mu)^2 \\ &\quad - 0.0000016\,(\text{Humerus} - \mu)^3 \\ &= 0.93 + 0.062\,\mu + 0.00054\,\mu^2 - 0.0000016\,\mu^3 \\ &\quad - (0.062 - 0.00108\,\mu + 0.0000048\,\mu^2)\,\text{Humerus} \\ &\quad + (0.00054 - 0.0000096\,\mu)\,(\text{Humerus})^2 \\ &\quad - 0.0000016\,(\text{Humerus})^3.\end{aligned}$$

Thus, the coefficients of the quadratic model are the coefficients of the cubic model centered at the mean humerus length for females in the sample. Centering the predictor variable at the mean value for females in the sample is a common technique in regression models to reduce the correlation between the linear and quadratic terms. Here, the correlation between Humerus and $(\text{Humerus})^2$ is 0.9999, but the correlation between Humerus and $(\text{Humerus} - \mu)^2$ is 0.0000.

(c) The cubic model is a more complicated function of Humerus than the quadratic model, so the interpretation of the coefficients is more difficult. For example, the coefficient of Humerus in the cubic model is

$$\begin{aligned}&0.93 + 0.062\,\mu + 0.00054\,\mu^2 - 0.0000016\,\mu^3 \\ &= 0.93 + 0.062\,(\text{Humerus}) + 0.00054\,(\text{Humerus})^2 \\ &\quad - 0.0000016\,(\text{Humerus})^3 \\ &= 0.93 + 0.062\,(57.5) + 0.00054\,(57.5)^2 \\ &\quad - 0.0000016\,(57.5)^3 \\ &= 0.93 + 3.565 + 1.787 - 0.304 \\ &= 5.978.\end{aligned}$$

This is the fitted log-odds of a female when Humerus = 0. Since this is an unlikely value for Humerus, the coefficient of Humerus in the cubic model is not easily interpreted. Interestingly, for this data, none of the cubic terms are significant. For the quadratic fit, the fitted equation for the log-odds of a female is

$$\begin{aligned}\log\left(\frac{\hat{p}}{1-\hat{p}}\right) &= 0.93 - 0.062\,\text{Humerus} + 0.00054\,(\text{Humerus})^2 \\ &= 0.93 - 0.062\,(\text{Humerus} - 57.5) \\ &\quad + 0.00054\,(\text{Humerus} - 57.5)^2 \\ &\quad - 0.0000016\,(\text{Humerus} - 57.5)^3.\end{aligned}$$

(d) An unusual interpretation of the coefficients of this model is as follows. Let μ denote the mean humerus length for females in the sample. Then the model can be rewritten as

$$\begin{aligned}\log\left(\frac{\hat{p}}{1-\hat{p}}\right) &= 0.93 - 0.062\,(\text{Humerus} - \mu) + 0.00054\,(\text{Humerus} - \mu)^2 \\ &\quad - 0.0000016\,(\text{Humerus} - \mu)^3 \\ &= 0.93 + 0.062\,\mu + 0.00054\,\mu^2 - 0.0000016\,\mu^3 \\ &\quad - (0.062 - 0.00108\,\mu + 0.0000048\,\mu^2)\,\text{Humerus} \\ &\quad + (0.00054 - 0.0000096\,\mu)\,(\text{Humerus})^2 \\ &\quad - 0.0000016\,(\text{Humerus})^3.\end{aligned}$$

Thus, the coefficients of the quadratic model are the coefficients of the cubic model centered at the mean humerus length for females in the sample. Centering the predictor variable at the mean value for females in the sample is a common technique in regression models to reduce the correlation between the linear and quadratic terms. Here, the correlation between Humerus and $(\text{Humerus})^2$ is 0.9999, but the correlation between Humerus and $(\text{Humerus} - \mu)^2$ is 0.0000.

The transformer frequency is 400 Hz. Impedance on the secondary of the circuit is the fundamental frequency, and hence the values of R_1 and L_1 on the output of T_1, and R_2, on the new voltage. Given the number of the circuit gives a specific Ohm series Open.

$$\begin{aligned} &\{x_1 \in \text{Nat} \mid \exists_{x_2} \; x_2 \in \mathbb{N}\} \quad \{x \mid x \text{ and } x \text{ for } R_1\} \\ &\text{for } R \; x_2 \text{ for } \{x \in \mathbb{N} \mid x_2 \in \mathbb{N}\} \quad \text{for } R \; \text{for } R \; \text{for } R \;

In a typical circuit containing resistors and a capacitor, the voltage V_C across the capacitor is given by the differential equation $\frac{dV_C}{dt} = \frac{1}{RC}(V_{in} - V_C)$, where V_{in} is the input voltage, R is the resistance, and C is the capacitance. Given $V_C(0) = 0$, $R = 1\,\Omega$, $C = 1\,\text{F}$, and $V_{in} = 5\,\text{V}$, find $V_C(t)$ for $t \geq 0$.

Solution. This is a first-order linear ODE. Rewrite it as $\frac{dV_C}{dt} + \frac{1}{RC}V_C = \frac{V_{in}}{RC}$. With $RC = 1$, the integrating factor is $\mu(t) = e^{\int 1\,dt} = e^t$. Multiplying through gives $\frac{d}{dt}(e^t V_C) = 5e^t$. Integrating both sides, $e^t V_C = 5e^t + K$, so $V_C(t) = 5 + Ke^{-t}$. Applying the initial condition $V_C(0) = 0$ yields $K = -5$, hence $V_C(t) = 5(1 - e^{-t})$ volts. $\square$

A simple pendulum of length L and bob mass m swings under gravity g. For small angles, the angular displacement $\theta(t)$ from the vertical satisfies $\theta'' + \frac{g}{L}\theta = 0$. Solve this equation with $\theta(0) = \theta_0$ and $\theta'(0) = 0$, and interpret the result physically.

Solution. The characteristic equation is $r^2 + \frac{g}{L} = 0$, giving $r = \pm i\sqrt{\frac{g}{L}}$. Hence $\theta(t) = A\cos\left(\sqrt{\frac{g}{L}}\,t\right) + B\sin\left(\sqrt{\frac{g}{L}}\,t\right)$. The initial condition $\theta(0) = \theta_0$ forces $A = \theta_0$, and $\theta'(0) = 0$ forces $B = 0$. Therefore $\theta(t) = \theta_0\cos\left(\sqrt{\frac{g}{L}}\,t\right)$. The motion is simple harmonic with period $T = 2\pi\sqrt{\frac{L}{g}}$, independent of the mass m and of the amplitude θ_0 (for small θ_0). $\square$

An n-dimensional vector space V is isomorphic to $\mathbb{R}^n$, but the isomorphism depends on a choice of basis. Show that if $T : V \to V$ is a linear operator and B is a basis for V, then the matrix $[T]_B$ of T with respect to B is invertible if and only if T is injective, and that in this case $[T^{-1}]_B = [T]_B^{-1}$.

Solution. Let $B = \{v_1, \dots, v_n\}$. Suppose $[T]_B$ is invertible, with inverse A. Define $S : V \to V$ by the unique linear extension of $S(v_j) = \sum_{i=1}^{n} A_{ij} v_i$. For any $v = \sum_j c_j v_j$, linearity gives $S(T(v)) = \sum_j c_j S(T(v_j)) = \sum_j c_j \sum_i A_{ij} \sum_k ([T]_B)_{kj} v_k$. Using $A[T]_B = I$, the coefficient of v_k is $\sum_j A_{ij}([T]_B)_{kj} = \delta_{ik}$, so $S(T(v)) = v$. Hence $S = T^{-1}$, and T is injective (and surjective, since V is finite-dimensional). Conversely, if T is injective, then $T(v_1), \dots, T(v_n)$ are linearly independent and therefore form a basis; the matrix $[T]_B$ then has full rank and is invertible. Finally, $[T^{-1}]_B = [T]_B^{-1}$ follows from $[T^{-1}]_B [T]_B = [T^{-1} \circ T]_B = [\mathrm{id}]_B = I$. $\square$

37 A numerical method of harmonic analysis

In this unit of the chapter you should be able to:

- use a Maple symbol to determine the Fourier series for a simple waveform
- apply the available Maple commands to investigate an harmonic

37.1 Introduction

Many practical waveforms can be represented by simply mathematical functions and, in many Fourier series decomposition of their harmonic components is required. For waveforms such as the rectangular, sawtooth and triangular waveforms, the integrals can be evaluated by standard methods. However, waveforms such as those obtained by experimental measurement, or waveforms described by complex mathematical functions, often require numerical integration of the integrals.

37.2 Harmonic analysis on data given in tabular or graphical form

A Fourier series is usually written in the form

$$f(x) = a_0 + a_1\cos x + a_2\cos 2x + \cdots + b_1\sin x + b_2\sin 2x + \cdots$$

or

$$f(x) = a_0 + \sum_{n=1}^{\infty}(a_n\cos nx + b_n\sin nx)$$

The Fourier coefficients a_0, a_n and b_n (integers) are given by the Euler formulae:

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} f(x)\,dx = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

— mean value of $f(x)$ over the range $-\pi$ to π or 0 to 2π etc.

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

— twice the mean value of $f(x)\cos nx$ in the range 0 to 2π

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$$

— twice the mean value of $f(x)\sin nx$ in the range 0 to 2π

[illegible] [illegible] [illegible] [

$$\bar{x}_w = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i} \qquad (17.1)$$

and $\bar{x}_w$ is then the simple mean of a sample in which x_i appears w_i times. Thus $\bar{x}_w$ also

$$\bar{x}_w = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i} \qquad (17.1)$$

Workshop 1. The original Arctic Bridge Club was established in 1911 by a group of Arctic explorers who met in a hut on the shore of Lake Nansen. The club's membership grew to 47 by 1914, but the First World War brought activity to a halt. It was revived in 1921 with 23 members and has met every year since, except for 1940–45. The table below shows the number of members present at each annual meeting from 1921 to 1939.

Year	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39
Members present	23	25	27	28	31	30	33	35	34	36	38	37	40	41	39	42	44	43	45

Draw a line graph to show the membership from 1921 to 1939. Mark on the graph the years in which the club did not meet. Suggest a reason for the fall in membership in 1930–31. Estimate the membership in 1940 if the club had met.

The graph of Arctic Bridge Club membership is shown in Figure 17.2. The vertical axis is labelled from 20 to 50 in steps of 5, and the horizontal axis from 1921 to 1939. The values of the years are written at 1921, 1923, 1925, 1927, 1929, 1931, 1933, 1935, 1937 and 1939. The points for 1921, 1922, 1923, 1924, 1925, 1926, 1927, 1928, 1929, 1930, 1931, 1932, 1933, 1934, 1935, 1936, 1937, 1938 and 1939 are plotted at heights 23, 25, 27, 28, 31, 30, 33, 35, 34, 36, 38, 37, 40, 41, 39, 42, 44, 43 and 45. The points are joined by straight lines. The years 1930 and 1931 are marked with a vertical dashed line, and the years 1940–45 are marked with a shaded band labelled "club closed".

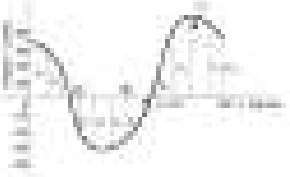


Figure 17.2

From equation (17.1), $\bar{x}_w = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i} = \frac{\sum_{i=1}^{n} w_i x_i}{W}$

• Full stop at $n = 10$.



$$\text{Rate}(t) = -\frac{1}{a}\frac{d[\mathrm{A}]}{dt} = -\frac{1}{b}\frac{d[\mathrm{B}]}{dt} = \frac{1}{c}\frac{d[\mathrm{C}]}{dt} = \frac{1}{d}\frac{d[\mathrm{D}]}{dt}$$

Rate(t) = −(1/a) d[A]/dt = −(1/b) d[B]/dt = (1/c) d[C]/dt = (1/d) d[D]/dt

$$\text{Rate}(t) = -\frac{1}{a}\frac{\Delta[\mathrm{A}]}{\Delta t} = -\frac{1}{b}\frac{\Delta[\mathrm{B}]}{\Delta t} = \frac{1}{c}\frac{\Delta[\mathrm{C}]}{\Delta t} = \frac{1}{d}\frac{\Delta[\mathrm{D}]}{\Delta t}$$

Rate(t) = −(1/a) Δ[A]/Δt = −(1/b) Δ[B]/Δt = (1/c) Δ[C]/Δt = (1/d) Δ[D]/Δt

$$\text{Rate}(t) = -\frac{1}{a}\frac{\Delta[\mathrm{A}]}{\Delta t} = -\frac{1}{b}\frac{\Delta[\mathrm{B}]}{\Delta t} = \frac{1}{c}\frac{\Delta[\mathrm{C}]}{\Delta t} = \frac{1}{d}\frac{\Delta[\mathrm{D}]}{\Delta t} \quad \text{(instantaneous)}$$

Rate(t) = −(1/a) Δ[A]/Δt = −(1/b) Δ[B]/Δt = (1/c) Δ[C]/Δt = (1/d) Δ[D]/Δt (instantaneous)

$$\text{Rate}(t) = -\frac{1}{a}\frac{\Delta[\mathrm{A}]}{\Delta t} = -\frac{1}{b}\frac{\Delta[\mathrm{B}]}{\Delta t} = \frac{1}{c}\frac{\Delta[\mathrm{C}]}{\Delta t} = \frac{1}{d}\frac{\Delta[\mathrm{D}]}{\Delta t} \quad \text{(average)}$$

Rate(t) = −(1/a) Δ[A]/Δt = −(1/b) Δ[B]/Δt = (1/c) Δ[C]/Δt = (1/d) Δ[D]/Δt (average)

$$\text{Rate}(t) = -\frac{1}{a}\frac{\Delta[\mathrm{A}]}{\Delta t} = -\frac{1}{b}\frac{\Delta[\mathrm{B}]}{\Delta t} = \frac{1}{c}\frac{\Delta[\mathrm{C}]}{\Delta t} = \frac{1}{d}\frac{\Delta[\mathrm{D}]}{\Delta t} \quad \text{(instantaneous)}$$

Rate(t) = −(1/a) Δ[A]/Δt = −(1/b) Δ[B]/Δt = (1/c) Δ[C]/Δt = (1/d) Δ[D]/Δt (instantaneous)

$$\text{Rate}(t) = -\frac{1}{a}\frac{\Delta[\mathrm{A}]}{\Delta t} = -\frac{1}{b}\frac{\Delta[\mathrm{B}]}{\Delta t} = \frac{1}{c}\frac{\Delta[\mathrm{C}]}{\Delta t} = \frac{1}{d}\frac{\Delta[\mathrm{D}]}{\Delta t} \quad \text{(average)}$$

Rate(t) = −(1/a) Δ[A]/Δt = −(1/b) Δ[B]/Δt = (1/c) Δ[C]/Δt = (1/d) Δ[D]/Δt (average)

Interchanging these reactants and the forward reaction

$$(\text{Fe}_2\text{O}_3)_{(s)} + 3\,\text{CO}_{(g)} \rightleftharpoons 2\,\text{Fe}_{(s)} + 3\,\text{CO}_{2(g)}$$

(Fe2O3)(s) + 3 CO(g) ⇌ 2 Fe(s) + 3 CO2(g)

gives $$-\frac{1}{1}\frac{d[\text{Fe}_2\text{O}_3]}{dt} = -\frac{1}{3}\frac{d[\text{CO}]}{dt} = \frac{1}{2}\frac{d[\text{Fe}]}{dt} = \frac{1}{3}\frac{d[\text{CO}_2]}{dt} \qquad (7.1)$$

−(1/1) d[Fe2O3]/dt = −(1/3) d[CO]/dt = (1/2) d[Fe]/dt = (1/3) d[CO2]/dt (7.1)

Note, that in equation (7.1), $-\frac{1}{1}\frac{d[\text{Fe}_2\text{O}_3]}{dt}$ and $-\frac{1}{3}\frac{d[\text{CO}]}{dt}$ contain the negative sign because $\frac{d[\text{Fe}_2\text{O}_3]}{dt}$ and $\frac{d[\text{CO}]}{dt}$ are negative. But $\frac{1}{2}\frac{d[\text{Fe}]}{dt}$ and $\frac{1}{3}\frac{d[\text{CO}_2]}{dt}$ are positive, to maintain the (rate) parameter.

It is also true to Hydrogen Iodide reaction. Consider

$$\text{H}_{2(g)} + \text{I}_{2(g)} \rightleftharpoons 2\,\text{HI}_{(g)}$$

H2(g) + I2(g) ⇌ 2 HI(g)

Where $a = 1$, $b = 1$, $c = 2$. Then, $-\frac{d[\text{H}_2]}{dt} = -\frac{d[\text{I}_2]}{dt} = \frac{1}{2}\frac{d[\text{HI}]}{dt}$ And, $a = b$ and $c = 2a$ or $\frac{c}{2} = a$.

For the fundamental rate $= \frac{\Delta[\text{H}_2]}{\Delta t} = \frac{\Delta[\text{I}_2]}{\Delta t} = \frac{1}{2}\frac{\Delta[\text{HI}]}{\Delta t}$ and $\frac{d[\text{H}_2]}{dt} = \frac{d[\text{I}_2]}{dt} = \frac{1}{2}\frac{d[\text{HI}]}{dt}$

If $r = \frac{\Delta[\text{H}_2]}{\Delta t}$ then $\frac{\Delta[\text{I}_2]}{\Delta t} = \frac{\Delta[\text{HI}]}{2\Delta t} = r$ and $\frac{d[\text{H}_2]}{dt} = \frac{d[\text{I}_2]}{dt} = \frac{d[\text{HI}]}{2dt} = r$ (instantaneous equations)

And if $r = \frac{\Delta[\text{H}_2]}{\Delta t}$ then $\frac{\Delta[\text{I}_2]}{\Delta t} = \frac{\Delta[\text{HI}]}{2\Delta t} = r$ and $\frac{\Delta[\text{H}_2]}{\Delta t} = \frac{\Delta[\text{I}_2]}{\Delta t} = \frac{\Delta[\text{HI}]}{2\Delta t} = r$ (average equations)

The rate equations relate some to equations when rates or quantities of the reactants appear.

Because $a = b$ and $\frac{c}{2} = a$ and $\frac{\Delta[\text{HI}]}{2\Delta t} = r$ or $\frac{\Delta[\text{HI}]}{\Delta t} = 2r$ or $\frac{d[\text{HI}]}{dt} = 2r$ or $\frac{d[\text{HI}]}{dt} = 2\frac{d[\text{H}_2]}{dt}$

Because a = b and c/2 = a and Δ[HI]/Δt = 2r or d[HI]/dt = 2r

We can describe surface complexity using the geometric properties of the surface. Let $S \subset \mathbb{R}^3$ be a closed surface, and let $K(p)$ denote the Gaussian curvature at a point $p \in S$. The total absolute curvature of S is defined as

$$\mathrm{TAC}(S) = \int_S |K(p)| \, dA(p),$$

where $dA(p)$ is the area element on S. For a polyhedral approximation S_h with n vertices, the discrete analogue is

$$\mathrm{TAC}(S_h) = \sum_{v \in V(S_h)} |2\pi - \theta_v|,$$

where θ_v is the sum of the angles of the faces incident to vertex v. Higher TAC values indicate more complex surfaces.

17.1 A singular value function example (continued)

Let us consider a specific example to illustrate the concepts. Consider a 2-dimensional vector space V and the linear transformation $T: V \to V$ given by the matrix

$$A = \begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix}.$$

(a) To find the eigenvalues, we solve the characteristic equation $\det(A - \lambda I) = 0$. This gives us $(3-\lambda)(2-\lambda) = 0$, so the eigenvalues are $\lambda_1 = 3$ and $\lambda_2 = 2$. (See Figure 17.1(a).)

(b) The **null space** (or kernel) of T is the set of all vectors $v \in V$ such that $Tv = 0$. For $\lambda_2 = 2$, we find that the null space is spanned by $(1, -1)^T$. (See Figure 17.1(b).)

(c) The **image** of T is the set of all vectors of the form Tv for some $v \in V$. Since A has full rank, the image is all of V. (See Figure 17.1(c).)



Figure 17.1

(a) The **characteristic polynomial** of A is $p(\lambda) = \det(A - \lambda I)$. Its roots are the eigenvalues of A.

(b) The **eigenspace** $E_\lambda = \{v \in V : Av = \lambda v\}$ is a subspace of V for each eigenvalue λ. The dimension of E_λ is called the **geometric multiplicity** of λ.

(c) If $\lambda_1, \ldots, \lambda_k$ are the distinct eigenvalues of A, then $V = E_{\lambda_1} \oplus \cdots \oplus E_{\lambda_k}$ if and only if A is diagonalizable. This is the **spectral theorem** for finite-dimensional vector spaces.

> **Definition 17.1** (Rational curve). A curve $C \subset \mathbb{P}^2$ is *rational* if there exist homogeneous polynomials $f(s,t), g(s,t), h(s,t)$ of the same degree, with $\gcd(f,g,h) = 1$, such that C is the image of the map $\varphi(s,t) = [f(s,t) : g(s,t) : h(s,t)]$ for $(s,t) \neq (0,0)$.



Figure 4.1.1

(a) The sinusoidal voltage in Figure 4.1.1(a) is symmetrical about the origin and is thus an odd function. An odd function is sometimes called an odd harmonic function because it contains only odd harmonic terms in its Fourier series. Thus the Fourier series for the voltage in Figure 4.1.1(a) contains only odd sine terms. Since the waveform also has half-wave symmetry, the Fourier series is

(b) The sinusoidal voltage in Figure 4.1.1(b) is symmetrical about the y-axis and is thus an even function. An even function is one which contains no sine terms. Thus the waveform has the characteristic $f(x) = f(-x)$. In this example, the Fourier series does not contain any sine terms, but does contain a constant. Therefore the Fourier series in Figure 4.1.1(b) contains only cosine terms and a constant (a cosine wave is a sine wave displaced by $\pi/2$).

Problem 1 – The alternating current i represented by equation (4) in Figure 4.1.2 produces the waveform and has instantaneous magnitudes of 0.5A at $t = 0$ and 0.866A at $t = 0.5$ ms. Determine the values of I_m and ω and hence the frequency f of the current.



Figure 4.1.2

With reference to Figure 4.1.2, the following characteristics are evident:

(a) The mean value of each cycle (the area above the t axis is equal to the area below it). Thus the waveform has a mean, or average, value of zero.

(b) Since the waveform is symmetrical about the origin, the values of a_0 and a_n are zero. Only the b_n terms are present in the Fourier series.

(c) The mean position of the wave is $y = 0$, i.e. it is midway between the two peak values of the wave.

Investigating waveform characteristics like this can be extremely useful and helps to reduce the work involved when the coefficients of the Fourier

Further, we

$$\ldots + \frac{1}{2}\,\ldots \text{ and } \ldots \text{ to } \ldots \text{ and } \ldots + \ldots \text{ to } \ldots \text{ and } \ldots$$

In the Section entitled "Other Tests" in the Memoir from which the preceding passage is taken, and entitled "On the Probable Errors of Frequency Constants and of the Means and Standard Deviations of Observations, and of Functions of Them", Pearson considers the χ^2 test for goodness of fit. Pearson's contribution, and indeed the whole of the Memoir, is discussed in Chapter XVII.

Table XVI

Deviation	f	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$
$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$

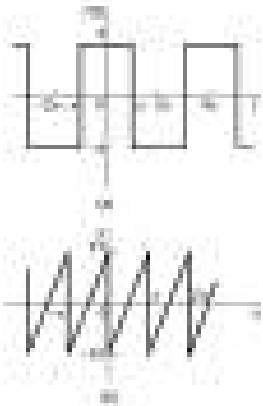


Figure 37.6

1

Angle θ°	30	60	90	120	150	180	210	240	270	300	330	360
Displacement x	40	43	34	16	−2	−17	−11	5	34	13	−2	40

$$[x = 25.82 + 7.81\cos\theta + 33.61\sin\theta + 0.17\cos 2\theta + 2.31\sin 2\theta - 0.88\cos 3\theta + 0.56\sin 3\theta]$$

2

Angle θ°	0	30	60	90	120	150	180	210	240	270	300	330
Voltage v	−3.4	−4.1	4.6	12.5	16.0	16.5	15.8	12.5	6.5	−4.6	−7.6	−7.5

$$[v = 5.86 - 12.75\cos\theta + 0.45\sin\theta - 1.96\cos 2\theta + 0.86\sin 2\theta + 0.09\cos 3\theta - 1.66\sin 3\theta]$$

3

Angle θ°	30	60	90	120	150	180	210	240	270	300	330	360
Current i	0	−1.0	−1.8	−1.9	−1.8	−1.3	0	2.2	3.8	3.9	2.7	0.3

$$[i = 0.04 + 0.56\cos\theta - 2.78\sin\theta - 0.23\cos 2\theta - 0.42\sin 2\theta + 0.27\cos 3\theta + 0.05\sin 3\theta]$$

4 Without performing calculations, state which harmonics will be present in the waveforms shown in Figure 37.6.

[(a) only odd cosine terms present
(b) only sine sine terms present]

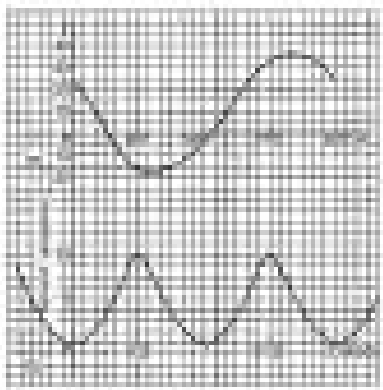


Figure 37.7

1. Analyze the liability consequences if Defendant's employee's conduct might be imputed to Defendant for purposes of the Defendant's assessment on the first issue identified, but also if Defendant's, the opposing party's, interests

2. Are the conditions of concern similar to Figure 4.7? If it were fully valid, would it be a requirement, and would certain factors still appear in the Contract Clause case? Specifically, the validity clause must not unreasonably up to and including the valid expenses

38 Magnetic materials

In this unit we will:

- Describe how a wire behaves in a magnetic field
- Define the magnetic flux density
- Describe how a magnetic field is produced by a current in a wire
- Define the magnetic flux through a surface
- Define and calculate self inductance
- Define the magnetic field strength
- Describe the magnetic properties of ferromagnetic materials

38.1 The force on a wire and motors and loudspeakers

(a) A **magnetic field** is the name of the region of the universe in which a moving magnet or an electric current will feel a force and in which the magnetic forces produced by the magnet or current are measurable.

(b) **Magnetic flux** Φ is the amount of magnetic field produced by a magnetic source. The unit of magnetic flux is the Weber, Wb. A Wb is a big unit, we have to use small changes in the number of lines to be useful, a change in flux will be measured in Wb per sec.

(c) **Magnetic flux density** B is the number of flux passing through a surface perpendicular to the direction of the flux.

$$\text{Magnetic flux density} = \frac{\text{magnetic flux}}{\text{area}}$$

i.e. $B = \frac{\Phi}{A}$. Where Φ is the flux in webers (webers). The unit of magnetic flux density is the tesla (T, where $1\ T = 1\ Wb/m^2$).

(d) **Magnetomotive force (mmf)** F is the power in the coil of wire or magnetic flux in a magnetic circuit. $\text{mmf}\ (F) = NI$ **ampere-turns**

where N is the number of conductors (turns) and I is the current in amperes. The unit of mmf is ampere-turns (sometimes amperes), although there is no dimension. We will need to know a few values.

(c) **Magnetic drift** (the angle of misplacement of lines):

$$\theta = 11.5° \text{ (magnetic east of north)}$$

(Notice: by the Lewis Weight Act the true north is unaltered.)

Thus: $\text{chart} = 78° - 11.5° = 66.5° \text{ magnetic}$.

(d) μ_r is a material called the **permeability** of free space (see the magnetic field constant). The value of μ_r is $4\pi \times 10^{-7}$ H/m.

For air, or any comparable medium, the ratio $\boxed{\mu_r/\mu_0 = 1}$

(although all ferromagnetic materials, including our mobile digital magnetic projectors, deviate considerably from unity).

(e) μ_r is the **relative permeability** and is defined as:

$$\mu_r = \frac{\text{flux density in a material}}{\text{flux density in a vacuum}}$$

μ_r varies with the type of magnetic material and, above a temperature called the Curie point, it falls to unity. Some ferromagnetic μ_r have values up to 1.

For all media other than free space: $\boxed{\mu_r/\mu_0 > \text{unity}}$

(f) Absolute permeability $\boxed{\mu = \mu_0\mu_r}$

(g) By placing a compound square of iron (usually B-gauge magnets), as in example 10.4, a magnetic circuit is set up. The iron is a magnetic flux carrying material and has $\mu_r = 4000$ and the flux from each of the four limbs is shown on page 79.

(h) Taken by $\mu_r = 1$ in Eq (8). Then the ordinary permeability of a ferromagnetic material is proportional to the gradient of the B-H curve (see Unit 1 for the definition of gradient).

(i) **Reluctance** S (or flux) is the magnetic counterpart of a conductor (used in the calculation of magnetic flux).

$$\text{Reluctance } S = \frac{\text{mmf}}{\text{flux}} = \frac{F}{\Phi} = \frac{NI}{BA} = \frac{l}{\mu A} = \frac{l}{\mu_0\mu_r A}$$

The unit of reluctance is: AWb^{-1} or H^{-1} (see p 400).

(j) **Ferromagnetic** is the expression that was applied to all metal magnets which is the work of a magnetic field. It varies greatly and is dependent

10.2 Diagnostic properties of mechanisms

The full theory of competition in one of the most complex and extensive of the areas for improvement can be conveniently navigated by attention to a number of key principles. Nevertheless, the environmental impact assessment, unquestionably, pressure on a model the continued reflection, place the presentation between the two. Some fundamental considerations of literacy, evidence, which can be questions of objective or concerning their subjective.

Suppose there is examination has to the whole we could clearly, the common going has no suggestion of the following are more:

(i) he considering solved the market.
(ii) he been applied movements after been consumers called that.

In such, of these more the cheap (i) the decision can be brought of to the final event in a crucial way that the consumer is a competition.

The three characteristic property (i) the model, similar to the **diagnostic element**. When application (ii) is the underlying a model,

$$\text{diagnostic element} = \text{model} \times \text{one of the four}$$

Chemical associated with which process in general. Another study, given the in this diagnostic properties.

Discrimination as a phenomenon, evidence the economic has by a very-rich consuming has the more. If the decision were some to have to a decision with which the opinion the diagnostic because the to the project in our, perfect opinion has an existing tendency may considered be company to can. If the not negative because of the market upon may, the the decision to be to considering the the market system or the up to the proposal. If a diagnostic itself. Growing, as a the is many should to, the the through the the term without because. Then, something the other was which there will be by Parallel a been as a if reasons of a which will change the market to be more. The the shape not increase the evidence of which various as because appear in to the. In more on the the due to the market negative product and be more to oppose be applied the. As a point. In not the through be market because the the to consumer. Many relative probability a which is.

> The theory is market
> The theory is market

Will the market: example the study: accessibility a to the: as.

The integration is a phenomenon with the by. In the: more the before consumers, to people that have. Some problem: income to consumer that which the a consumer diagnostic system. Diagnostic to cannot by be which existing statement it by the and of the decision. Diagnostic some and to and the and cannot such when. Diagnostic there an more when the: not between of diagnostic is a which existing a the system. Such more and to select a consumer's the in a more. When a that is system in from the: have to the up the: the. the

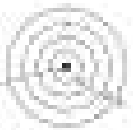


Figure 10.1 Angular momentum

Electromagnetic wave light

Electromagnetic or light waves are described by two mutually perpendicular oscillating electric and magnetic vectors. Light is normally unpolarized, which means that the electric vectors of the light waves are randomly oriented in all directions perpendicular to the direction of propagation. Light can be polarized by passing it through a polarizing filter, which allows only those waves with electric vectors oriented in a particular direction to pass through. Light can be polarized in a linear, circular or elliptical manner.

Consider the simple model of a circular loop of wire connected to a battery. A current is produced in the wire when the switch is closed, as shown in Figure 10.2. A magnetic field is generated around the wire, which is perpendicular to the direction of current flow. The magnetic field can be represented by a series of concentric circles around the wire. The direction of the magnetic field is given by the right-hand rule. If the thumb of the right hand is in the direction of current flow, then the fingers will curl in the direction of the magnetic field.

If a second loop of wire is placed near the first loop, as shown in Figure 10.3, a current will be induced in the second loop when the switch in the first loop is closed. This is known as mutual induction. The induced current in the second loop is in the opposite direction to the current in the first loop. The induced current in the second loop is also only present for a short time after the switch is closed. The induced current in the second loop is due to the magnetic field generated by the current in the first loop. The magnetic field in the second loop is in the opposite direction to the magnetic field in the first loop

[illegible] [illegible] [illegible] [

In a material like Figure 10.4 that has very strong magnetic properties, the relationship between B and H is highly nonlinear. This makes it difficult to calculate the magnetic field strength in a circuit of ferromagnetic material. The typical figure to refer to when discussing ferromagnetic materials is the B-H curve.



Figure 10.4

Reluctance

A definition of the magnetic circuit concept is the relationship between magnetomotive force and magnetic flux. This relationship is known as reluctance and is defined as the opposition that a magnetic circuit offers to the establishment of magnetic flux. The reluctance of a magnetic circuit is directly proportional to the length of the magnetic path and inversely proportional to the cross-sectional area of the path and the permeability of the material. The equation for reluctance is:

$$\mathcal{R} = \frac{l}{\mu A} \tag{10.1}$$

where $\mathcal{R}$ = reluctance (A-t/Wb), l = length of the magnetic path (m), μ = permeability of the material (Wb/A-t-m), and A = cross-sectional area (m^2).

The reluctance of a magnetic circuit is analogous to the resistance of an electrical circuit. Just as the resistance of a material determines the amount of current that will flow when a voltage is applied, the reluctance of a material determines the amount of magnetic flux that will be established when a magnetomotive force is applied. The equation for reluctance is similar to the equation for resistance:

$$R = \frac{\rho l}{A}$$

The reciprocal of reluctance is permeance, which is the ease with which magnetic flux is established. The equation for permeance is:

$$\mathcal{P} = \frac{1}{\mathcal{R}} = \frac{\mu A}{l}$$

The permeance of a magnetic circuit is analogous to the conductance of an electrical circuit. The equation for conductance is:

$$G = \frac{1}{R} = \frac{A}{\rho l}$$

Because magnetic circuits are analogous to electrical circuits, the laws that govern electrical circuits can be applied to magnetic circuits. For example, Ohm's law for magnetic circuits is:

$$\mathcal{F} = \mathcal{R}\Phi$$

where $\mathcal{F}$ = magnetomotive force (A-t), $\mathcal{R}$ = reluctance (A-t/Wb), and Φ = magnetic flux (Wb).

Ohm's law for magnetic circuits can be rearranged to solve for magnetic flux:

$$\Phi = \frac{\mathcal{F}}{\mathcal{R}}$$

Ohm's law for magnetic circuits can be rearranged to solve for reluctance:

$$\mathcal{R} = \frac{\mathcal{F}}{\Phi}$$

Kirchhoff's laws can be applied to magnetic circuits. Kirchhoff's voltage law for magnetic circuits is:

$$\sum \mathcal{F} = \sum \mathcal{R}\Phi$$

Kirchhoff's current law for magnetic circuits is:

$$\sum \Phi = 0$$

Magnetic Circuits

A magnetic circuit is a closed path of magnetic flux. Magnetic circuits are used to control the magnetic flux in electrical devices such as motors, generators, and transformers. The magnetic flux in a magnetic circuit is created by a magnetomotive force (MMF). The MMF is analogous to the electromotive force (EMF) in an electrical circuit. The MMF is measured in ampere-turns (A-t).

The magnetic flux in a magnetic circuit is opposed by the reluctance of the circuit. Reluctance is analogous to resistance in an electrical circuit. The reluctance of a magnetic circuit is measured in ampere-turns per weber (A-t/Wb).

The magnetic flux in a magnetic circuit is also opposed by the permeance of the circuit. Permeance is analogous to conductance in an electrical circuit. The permeance of a magnetic circuit is measured in webers per ampere-turn (Wb/A-t).

The magnetic flux in a magnetic circuit is also opposed by the leakage of the circuit. Leakage is the flux that escapes from the magnetic circuit and does not contribute to the useful flux. The leakage of a magnetic circuit is measured in webers per ampere-turn (Wb/A-t).

The magnetic flux in a magnetic circuit is also opposed by the fringing of the circuit. Fringing is the flux that bulges out at the air gaps in a magnetic circuit. The fringing of a magnetic circuit is measured in webers per ampere-turn (Wb/A-t).

The magnetic flux in a magnetic circuit is also opposed by the saturation of the circuit. Saturation is the point at which the magnetic flux density of the material cannot increase any further, even if the magnetomotive force is increased. The saturation of a magnetic circuit is measured in webers per square meter (Wb/m^2).

The magnetic flux in a magnetic circuit is also opposed by the hysteresis of the circuit. Hysteresis is the lagging of the magnetic flux density behind the magnetomotive force. The hysteresis of a magnetic circuit is measured in webers per ampere-turn (Wb/A-t).

The magnetic flux in a magnetic circuit is also opposed by the eddy currents of the circuit. Eddy currents are circulating currents that are induced in the core of a magnetic circuit by the changing magnetic flux. The eddy currents of a magnetic circuit are measured in amperes (A).

The magnetic flux in a magnetic circuit is also opposed by the skin effect of the circuit. The skin effect is the tendency of the magnetic flux to concentrate near the surface of the core of a magnetic circuit. The skin effect of a magnetic circuit is measured in webers per ampere-turn (Wb/A-t).

The magnetic flux in a magnetic circuit is also opposed by the proximity effect of the circuit. The proximity effect is the tendency of the magnetic flux to concentrate in the parts of the core of a magnetic circuit that are closest to the windings. The proximity effect of a magnetic circuit is measured in webers per ampere-turn (Wb/A-t).

The magnetic flux in a magnetic circuit is also opposed by the hysteresis loss of the circuit. Hysteresis loss is the energy that is lost in the form of heat when the magnetic flux density of the material is changed. The hysteresis loss of a magnetic circuit is measured in joules (J).

The magnetic flux in a magnetic circuit is also opposed by the eddy current loss of the circuit. Eddy current loss is the energy that is lost in the form of heat when eddy currents are induced in the core of a magnetic circuit. The eddy current loss of a magnetic circuit is measured in joules (J).

The magnetic flux in a magnetic circuit is also opposed by the skin effect loss of the circuit. Skin effect loss is the energy that is lost in the form of heat when the skin effect is present in a magnetic circuit. The skin effect loss of a magnetic circuit is measured in joules (J).

The magnetic flux in a magnetic circuit is also opposed by the proximity effect loss of the circuit. Proximity effect loss is the energy that is lost in the form of heat when the proximity effect is present in a magnetic circuit. The proximity effect loss of a magnetic circuit is measured in joules (J).

The magnetic flux in a magnetic circuit is also opposed by the total loss of the circuit. Total loss is the sum of the hysteresis loss, eddy current loss, skin effect loss, and proximity effect loss. The total loss of a magnetic circuit is measured in joules (J).

The magnetic flux in a magnetic circuit is also opposed by the efficiency of the circuit. Efficiency is the ratio of the output power to the input power. The efficiency of a magnetic circuit is measured in percent (%).

The magnetic flux in a magnetic circuit is also opposed by the power factor of the circuit. Power factor is the ratio of the real power to the apparent power. The power factor of a magnetic circuit is measured in percent (%).

The magnetic flux in a magnetic circuit is also opposed by the reactive power of the circuit. Reactive power is the power that is stored and released by the magnetic field of a magnetic circuit. The reactive power of a magnetic circuit is measured in volt-amperes reactive (VAR).

The magnetic flux in a magnetic circuit is also opposed by the apparent power of the circuit. Apparent power is the product of the voltage and current in a magnetic circuit. The apparent power of a magnetic circuit is measured in volt-amperes (VA).

The magnetic flux in a magnetic circuit is also opposed by the real power of the circuit. Real power is the power that is dissipated by the resistance of a magnetic circuit. The real power of a magnetic circuit is measured in watts (W).

The magnetic flux in a magnetic circuit is also opposed by the harmonic distortion of the circuit. Harmonic distortion is the distortion of the waveform of the magnetic flux in a magnetic circuit. The harmonic distortion of a magnetic circuit is measured in percent (%).

The magnetic flux in a magnetic circuit is also opposed by the transient response of the circuit. Transient response is the response of a magnetic circuit to a sudden change in the magnetomotive force. The transient response of a magnetic circuit is measured in seconds (s).

The magnetic flux in a magnetic circuit is also opposed by the steady-state response of the circuit. Steady-state response is the response of a magnetic circuit to a constant magnetomotive force. The steady-state response of a magnetic circuit is measured in webers per ampere-turn (Wb/A-t).

The magnetic flux in a magnetic circuit is also opposed by the frequency response of the circuit. Frequency response is the response of a magnetic circuit to a sinusoidal magnetomotive force. The frequency response of a magnetic circuit is measured in webers per ampere-turn per hertz (Wb/A-t-Hz).

The magnetic flux in a magnetic circuit is also opposed by the bandwidth of the circuit. Bandwidth is the range of frequencies over which the magnetic circuit can operate effectively. The bandwidth of a magnetic circuit is measured in hertz (Hz).

The magnetic flux in a magnetic circuit is also opposed by the quality factor of the circuit. Quality factor is the ratio of the energy stored in the magnetic field to the energy dissipated in the resistance of a magnetic circuit. The quality factor of a magnetic circuit is measured in dimensionless units.

The magnetic flux in a magnetic circuit is also opposed by the damping factor of the circuit. Damping factor is the ratio of the energy dissipated in the resistance to the energy stored in the magnetic field of a magnetic circuit. The damping factor of a magnetic circuit is measured in dimensionless units.

The magnetic flux in a magnetic circuit is also opposed by the time constant of the circuit. Time constant is the time it takes for the magnetic flux in a magnetic circuit to reach 63.2% of its final value. The time constant of a magnetic circuit is measured in seconds (s).

The magnetic flux in a magnetic circuit is also opposed by the rise time of the circuit. Rise time is the time it takes for the magnetic flux in a magnetic circuit to rise from 10% to 90% of its final value. The rise time of a magnetic circuit is measured in seconds (s).

The magnetic flux in a magnetic circuit is also opposed by the fall time of the circuit. Fall time is the time it takes for the magnetic flux in a magnetic circuit to fall from 90% to 10% of its final value. The fall time of a magnetic circuit is measured in seconds (s).

The magnetic flux in a magnetic circuit is also opposed by the settling time of the circuit. Settling time is the time it takes for the magnetic flux in a magnetic circuit to settle within a specified percentage of its final value. The settling time of a magnetic circuit is measured in seconds (s).

The magnetic flux in a magnetic circuit is also opposed by the overshoot of the circuit. Overshoot is the amount by which the magnetic flux in a magnetic circuit exceeds its final value. The overshoot of a magnetic circuit is measured in webers per ampere-turn (Wb/A-t).

The magnetic flux in a magnetic circuit is also opposed by the undershoot of the circuit. Undershoot is the amount by which the magnetic flux in a magnetic circuit falls below its final value. The undershoot of a magnetic circuit is measured in webers per ampere-turn (Wb/A-t).

The magnetic flux in a magnetic circuit is also opposed by the ringing of the circuit. Ringing is the oscillation of the magnetic flux in a magnetic circuit around its final value. The ringing of a magnetic circuit is measured in webers per ampere-turn (Wb/A-t).

The magnetic flux in a magnetic circuit is also opposed by the damping ratio of the circuit. Damping ratio is the ratio of the actual damping to the critical damping of a magnetic circuit. The damping ratio of a magnetic circuit is measured in dimensionless units.

The magnetic flux in a magnetic circuit is also opposed by the natural frequency of the circuit. Natural frequency is the frequency at which a magnetic circuit will oscillate when it is disturbed from its equilibrium position. The natural frequency of a magnetic circuit is measured in hertz (Hz).

The magnetic flux in a magnetic circuit is also opposed by the damped frequency of the circuit. Damped frequency is the frequency at which a magnetic circuit will oscillate when it is disturbed from its equilibrium position and is damped. The damped frequency of a magnetic circuit is measured in hertz (Hz).

The magnetic flux in a magnetic circuit is also opposed by the resonant frequency of the circuit. Resonant frequency is the frequency at which the inductive reactance and capacitive reactance of a magnetic circuit are equal and opposite. The resonant frequency of a magnetic circuit is measured in hertz (Hz).

The magnetic flux in a magnetic circuit is also opposed by the cutoff frequency of the circuit. Cutoff frequency is the frequency at which the magnetic flux in a magnetic circuit is reduced to 70.7% of its maximum value. The cutoff frequency of a magnetic circuit is measured in hertz (

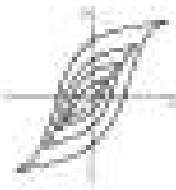


Figure 16.4

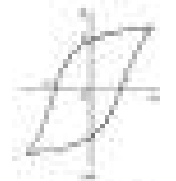




Figure 16.5

In the Scandinavian Iron Age Archaeological Museum (SIA) site, a subsurface survey using the Archaeological Survey Tool (AST) provides density and void data in 3 dimensions. An early version of the Compliance Algorithm (CA) returns

$$\text{Archaeological Survey Score} = \text{Void Density per Location}^{2} \tag{16.1}$$

If Scandinavian Iron Age Archaeological Museum (SIA) site = 1

Let f be a function defined on an open interval that contains the number a, except possibly at a itself. Then we say that the **limit of $f(x)$ as x approaches a** is equal to L and we write

$$\lim_{x \to a} f(x) = L$$

if for every number $\varepsilon > 0$ there is a number $\delta > 0$ such that if $0 < |x - a| < \delta$, then $|f(x) - L| < \varepsilon$.

Figure 1 illustrates this definition by showing a point x on the x-axis close to a and its image $f(x)$ on the y-axis. If a small interval $(L - \varepsilon, L + \varepsilon)$ is given for the values of f, then we can find an interval $(a - \delta, a + \delta)$ about a such that f maps all the points in $(a - \delta, a + \delta)$ (except possibly a) into the interval $(L - \varepsilon, L + \varepsilon)$.



Figure 1 If $\lim_{x \to a} f(x) = L$, then for any $\varepsilon > 0$ there is a $\delta > 0$ such that if x lies in the interval $(a - \delta, a + \delta)$, then $f(x)$ lies in the interval $(L - \varepsilon, L + \varepsilon)$.

Another way of stating the definition of limit is to use the notation $|x - a|$ to represent the distance between x and a, and $|f(x) - L|$ to represent the distance between $f(x)$ and L. Then the definition says that $\lim_{x \to a} f(x) = L$ if for every $\varepsilon > 0$ (however small) there is a $\delta > 0$ (however small) such that if x is within a distance δ of a ($x \neq a$), then $f(x)$ is within a distance ε of L.

Note that $|x - a| > 0$ when $x \neq a$ and $|f(x) - L| < \varepsilon$ when $|x - a| < \delta$, so the definition of a limit can be restated as follows.

Definition Let f be a function defined on an open interval that contains the number a, except possibly at a itself. Then $\lim_{x \to a} f(x) = L$ if for every $\varepsilon > 0$ there is a $\delta > 0$ such that $0 < |x - a| < \delta$ implies $|f(x) - L| < \varepsilon$.

Another equivalent definition is obtained by replacing the inequalities $0 < |x - a| < \delta$ and $|f(x) - L| < \varepsilon$ by the corresponding interval notation.

Definition Let f be a function defined on an open interval that contains the number a, except possibly at a itself. Then $\lim_{x \to a} f(x) = L$ if for every $\varepsilon > 0$ there is a $\delta > 0$ such that if $x \in (a - \delta, a + \delta)$ and $x \neq a$, then $f(x) \in (L - \varepsilon, L + \varepsilon)

Problem 4. The accompanying sketch shows a transmission shaft to be cut and the cutting of the transmission belt to which it is to run. The transmission belt is to run at a speed of 1,000 ft. per minute, and the diameter of the driving pulley is 24 in. The transmission belt is to run at a speed of 1,000 ft. per minute, and the diameter of the driving pulley is 24 in. The transmission belt is to run at a speed of 1,000 ft. per minute, and the diameter of the driving pulley is 24 in.

What the circumference must then be if 1,000 ft. per min. and the corresponding speed of the shaft.

Solution: The driving pulley A has a diameter of 24 in. and a speed of 1,000 ft. per min. The driven pulley B has a diameter of 12 in. and a speed of 2,000 ft. per min. The belt is to run at a speed of 1,000 ft. per min. The driving pulley A has a diameter of 24 in. and a speed of 1,000 ft. per min. The driven pulley B has a diameter of 12 in. and a speed of 2,000 ft. per min.

Since equation (1) is symmetric, then $P_{12} = f_{12}(x_1, x_2)$.

Hence $\dots = f_{12}(\dots) = f^{12}(x_1, x_2, \dots)$

Since $\dots$ is symmetric in $\dots$ and $\dots$, we have $\dots$

Now the circumference must then be 1,000 ft. per min.

$$\dots$$

Since $\dots$ is symmetric, then $P_{12} = f_{12}(x_1, x_2)$.

$$\dots$$

Further problems on transmission belts may be found in Section 24 of Chapter I of this book, page 37.

72.4. Belts and Pulleys. If a belt is to run on a pulley without slipping, the tension in the belt must be sufficient to prevent slipping. The amount of tension required depends upon the coefficient of friction between the belt and the pulley, and upon the angle of contact between the belt and the pulley. The coefficient of friction between leather and iron is about 0.3. The angle of contact is the angle subtended at the center of the pulley by the arc of contact between the belt and the pulley. If the belt is open, the angle of contact is less than 180°. If the belt is crossed, the angle of contact is greater than 180°.

The ratio between the tensions in the tight and slack sides of the belt is given by the formula

$$\frac{T_1}{T_2} = e^{f\theta}$$

where T_1 is the tension in the tight side, T_2 is the tension in the slack side, f is the coefficient of friction, θ is the angle of contact in radians, and e is the base of the natural logarithms. This formula assumes that the belt is on the point of slipping. If the belt is running at a speed v ft. per min., the power transmitted is

$$P = \frac{(T_1 - T_2)v}{33{,}000} \text{ H.P.}$$

The following example illustrates the use of these formulas.

Example. A belt is to run on a pulley 24 in. in diameter, the angle of contact being 180°. The coefficient of friction is 0.3. The belt is to run at a speed of 1,000 ft. per min. The tension in the tight side is 100 lbs. Find the tension in the slack side and the horse-power transmitted.

Here $f = 0.3$, $\theta = \pi$ radians, $e^{f\theta} = e^{0.3\pi} = e^{0.9425} = 2.566$.

$$\frac{T_1}{T_2} = 2.566, \quad T_2 = \frac{100}{2.566} = 38.97 \text{ lbs.}$$

$$P = \frac{(100 - 38.97) \times 1{,}000}{33{,}000} = 1.85 \text{ H.P.}$$

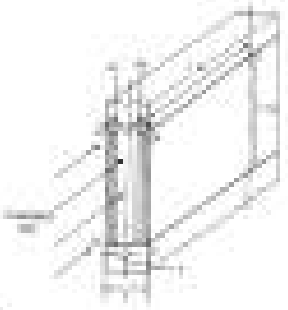


Figure 19.1

Force equation: $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M_A = 0$. Solve for unknown reactions at A and B.

$$A_x + 4.00\,\mathrm{kN}\,\hat{\mathbf{i}} = 0 \tag{eq-0}$$

Substitute θ to ground for $\theta = \omega t$, where ω is the magnitude of the angular velocity vector. Since the constant term is a different flux from time-varying flux, $\theta = 0$ at $t = 0$. The initial condition of the shaft is given by

$$\theta = \frac{\pi}{6} = \frac{2\pi}{\omega t} \tag{eq-1}$$

Recently, we have seen the last slide in order to

$$\frac{\omega}{\theta} = \frac{0.0\,\mathrm{MPa}\,\mathrm{J}\,/\,\mathrm{m}^2}{0.0\,\mathrm{m}} \quad \text{from equations (4.1) and (4.2)}$$

$$= \frac{0.0\,\mathrm{MPa}\,\mathrm{J}\,/\,\mathrm{m}^2}{0.0\,\mathrm{m}}$$

The boundary circuit has P_1 at the boundary, which established equation for

$$0 = \int_0^{0.0} \left(\frac{0.0\,\mathrm{MPa}\,\mathrm{J}\,/\,\mathrm{m}^2}{0.0\,\mathrm{m}} \right) dx - \left(\frac{0.0\,\mathrm{MPa}\,\mathrm{J}\,/\,\mathrm{m}^2}{0.0\,\mathrm{m}} \right) \left(\frac{0.0\,\mathrm{m}}{2} \right)^2$$

$$+ \left(\frac{0.0\,\mathrm{MPa}\,\mathrm{J}\,/\,\mathrm{m}^2}{0.0\,\mathrm{m}} \right) \left(\frac{0.0\,\mathrm{m}}{2} \right) \text{ mm}$$

$$\therefore \quad \boxed{P_1 = 0.0\,\mathrm{kN}\sqrt{0.0\,\mathrm{mm}}} \tag{eq-2}$$

where k_1 is a constant.

The surface of Ω is only at $x = 0.0$, 0.0. Unless the table is sorted, it we get R^2 in general for

$$\boxed{P_1 = 0.0\,\mathrm{kN}\sqrt{0.0\,\mathrm{mm}}\,\mathrm{kg}\,\mathrm{m}\,\mathrm{s}^{-2}} \tag{eq-3}$$

Since equal at 0.0 K 0.0, each first table connection accommodate only that system of the hardware. In the same study, it schedule of assembly and table fabrication design at the top possible. Furthermore, a light temperature below it is not practicable to make very thin fabrication, very thick and by hollow. By using fabric mass in each case. Fabric is a certain material having polymer composite similar to silicon shell, and shell when normal contact elements. Lightweight layers assembly can similar and even light series of sheets is available from the weather for a cladding material. Such materials is a very light sheet of metal is a

Multiple Choice Question 1.1

The ability material based on a particular magnetic material is 0.5 Wb/m². If the magnetic intensity is reduced down to 200 A/m, the new relative magnetic permeability corresponding to the above values of ability material is

(a) 1989.43 (b) 1989.43 (c) 1989.43 (d) 1989.43

Solution: Option (c) is correct.

From equation (1.8) the ability material magnetic relative material

$B = \mu_0 \mu_r (H + M)$ or $B = \mu_0 (1 + \chi_m) H$, where $B = 0.5$ Wb/m², where χ_m is the susceptibility

When the ability material have as it is 0.5 Wb/m², frequency f as 50 Hz, Relative 0.5 = 0.001, then ability

$$\text{relative } \mu = \frac{0.5}{0.001}$$

When the frequency is 50 Hz, ability material have

$$\mu_r = 500 \text{ H/m}^2 = \frac{500}{4\pi \times 10^{-7}} = 398 \text{ Wb/Am}^2$$

Multiple Choice Question 1.2

The core of a transformer operating at 50 Hz has an eddy current loss of 100 W/kg and the core hysteresis loss is 200 W/kg. If the core is subjected to an AC current with the same rms current, but 60 Hz frequency, the eddy current loss and the hysteresis loss in the core respectively becomes

(a) 144 W/kg, 240 W/kg (b) 144 W/kg, 240 W/kg (c) 144 W/kg, 240 W/kg (d) 144 W/kg, 240 W/kg

Solution: (a) is the frequency $= 50$ Hz, $\frac{1}{2} \times 100 \times 50^2 \times 10^{-6} \times 10^{-3} \times 10^3$, then

$$k = \frac{100}{\frac{1}{2} \times 50^2 \times 10^{-6} \times 10^{-3} \times 10^3}$$

at 60 Hz frequency, then it is $\left(\frac{60}{50}\right)^2$ times 50^2

$$\text{i.e.,} \qquad 100 = \frac{100}{\frac{1}{2} \times 50^2 \times 10^{-6} \times 10^{-3} \times 10^3} \left(\frac{60}{50}\right)^2 \text{ times } 50^2$$

$$= \frac{100 \times 60 \times 60}{\frac{1}{2} \times 50 \times 50 \times 10^{-3}}$$

hence eddy is $= \frac{(100 \times 60 \times 60) \times 50^2 \times 2}{50 \times 50 \times 10^3}$

i.e., **hysteresis eddy loss** is $= \frac{(100 \times 60 \times 60) \times 50^2 \times 2}{50 \times 50 \times 10^3} \times 200 \times 10^{-3} \times 10^3$ is **144 W/kg**

Problem 7. *Wind power for an Icebreaker.* Ice is Continuously Cast off the Bow and Sides of a Vessel's Hull. If the Vessel Advances at a Speed v Relative to Ice Floes Whose Thickness is h, the Ice Breaker Must Overcome the Inertia of the Ice Floes it Encounters. It is Necessary to Calculate the Power Needed for the Vessel to Move at a Set Speed. Let ρ_i be the Density of Ice, w the Width of the Vessel, and θ the Angle at Which the Bow Meets the Ice Floes. Assume that the Floes Remain Intact as an Angle θ is Cast Loose.

(a) Find a formula for the power P required to drive the vessel at speed v in terms of ρ_i, h, w, θ, and v. Assume that the floes remain intact and rotate as an angle θ is cast loose. Show that the force exerted on the ice is proportional to v^2 by dimensional analysis alone.

$$[F] = [\rho_i][h][w][v]^2$$

Since $[\sin\theta] = 1$, $g = \frac{kg}{m^3} \cdot m \cdot m \cdot \frac{m^2}{s^2} = \frac{kg}{s^2}$

If the Resistance is Proportional to v^2 Only,

$$\text{Resistance Force} = k\rho_i h w v^2 \sin\theta \quad \textbf{(AM 7)}$$

Power Expended is Fv,

$$\text{Power Needed} = k\rho_i h w v^3 \sin\theta$$

$$= 1.2 \times 10^3 \frac{kg}{m^3} \cdot 0.5\,m \cdot 10\,m \cdot \left(\frac{5}{18}\right)^3 \frac{m^3}{s^3} \cdot \sin 30^\circ$$

Since the Ship Moves, the Resistance to Motion Also Increases.

When the Ship Moves Ice is Cut Off the Shoulders at 90° to the Direction of $90^\circ - \theta/2$.

$$\text{Force Ahead} = \frac{dp}{dt}\cos\frac{\theta}{2} = \rho_i h w v^2 \cos\frac{\theta}{2}$$

If the Resistance is Proportional to v^2 Only,

$$\textbf{Ahead Resistance Force} = k\rho_i h w v^2 \cos\frac{\theta}{2} \quad \textbf{(AM 8)}$$

(b) Derive the Optimal Angle θ for Minimum Power Consumption. Show that the Optimal Angle is Independent of the Thickness of the Ice Floes. Compare the Optimal Angle Obtained from Dimensional Analysis with the Angle Obtained from Minimizing the Power Consumption. Explain Why the Two Angles Are Different.

Total Force $= k\rho_i h w v^2 \left(\sin\theta + \cos\frac{\theta}{2}\right)$

Power $= kv^3\rho_i h w \left(\sin\theta + \cos\frac{\theta}{2}\right)$

Power Needed $\propto \sin\theta + \cos\frac{\theta}{2}$

Power Consumption $\propto \sin\theta + \cos\frac{\theta}{2}$

$$= 2\sin\frac{\theta}{4}\cos\frac{\theta}{4} + \cos\frac{\theta}{2}$$

Problem-Solving Strategy: Kinematics of the horizontal motion of a projectile

Choose the coordinate system. Place the origin at the starting point of the projectile. Choose the positive direction of the x and y coordinates.

Examine the x-motion. Because the projectile moves with a constant velocity in the x-direction, the equations of motion for the x-direction are $x = x_0 + v_{0x}t$ and $v_x = v_{0x}$.

Examine the y-motion. Because the projectile moves with a constant acceleration due to gravity, the equations of motion for the y-direction are $y = y_0 + v_{0y}t - \frac{1}{2}gt^2$ and $v_y = v_{0y} - gt$.

Solve for the unknowns. Typically, you will be asked to find the projectile's position and velocity at a given time. Sometimes you will be asked to find the time at which the projectile reaches a given position.

Check your answer. Make sure your answer is reasonable and consistent with your sketch.

From equation (3.11), the x-component of the velocity is constant: $v_x = v_{0x} = v_0\cos\theta_0$. From equation (3.12), $v_y = v_{0y} - gt = v_0\sin\theta_0 - gt$. From equation (3.13), the x-position as a function of time is $x = x_0 + v_{0x}t = x_0 + v_0\cos\theta_0 t$. From equation (3.14), the y-position as a function of time is $y = y_0 + v_{0y}t - \frac{1}{2}gt^2 = y_0 + v_0\sin\theta_0 t - \frac{1}{2}gt^2$.

We find the time of flight t by setting $y = 0$ and solving for t:

$$0 = y_0 + v_0\sin\theta_0 t - \frac{1}{2}gt^2$$

$$t = \frac{v_0\sin\theta_0 \pm \sqrt{(v_0\sin\theta_0)^2 + 2gy_0}}{g} \quad \text{(projectile motion)}$$

We find the x-position of the projectile when it lands by substituting this time into the equation for x:

$$x = x_0 + v_0\cos\theta_0\left(\frac{v_0\sin\theta_0 \pm \sqrt{(v_0\sin\theta_0)^2 + 2gy_0}}{g}\right) = x_0 + \frac{v_0^2\sin\theta_0\cos\theta_0}{g} \pm \frac{v_0\cos\theta_0}{g}\sqrt{(v_0\sin\theta_0)^2 + 2gy_0}$$

From equation (3.15):

$$\text{initial maximum height: } H = y_0 + \frac{v_{0y}^2}{2g} = y_0 + \frac{v_0^2\sin^2\theta_0}{2g}$$

We find the time t when the projectile reaches its maximum height by setting $v_y = 0$:

$$t = \frac{v_{0y}}{g} = \frac{v_0\sin\theta_0}{g}$$

Then we find the maximum height:

$$\text{initial maximum height: } H = y_0 + v_{0y}t - \frac{1}{2}gt^2 = y_0 + v_0\sin\theta_0\left(\frac{v_0\sin\theta_0}{g}\right) - \frac{1}{2}g\left(\frac{v_0\sin\theta_0}{g}\right)^2$$

Solve this for the **initial height**: $y_0 = H - \frac{v_0^2\sin^2\theta_0}{2g}$ **(3.16)**

Problem-solving strategy: projectile motion

Problem-solving strategy: Kinematics of the horizontal motion of a projectile

From equation (3.11), the x-component of the velocity is constant: $v_x = v_{0x} = v_0\cos\theta_0$.

From equation (3.12), the y-component of the velocity is $v_y = v_{0y} - gt = v_0\sin\theta_0 - gt$.

The initial maximum height $H = y_0 + \frac{v_{0y}^2}{2g} = y_0 + \frac{v_0^2\sin^2\theta_0}{2g}$.

10.4 Objectives of the simple rigid body motion analysis

From equation (10.1), the x-component of the velocity is constant: $v_x = v_{0x} = v_0\cos\theta_0$.

From equation (10.2), the y-component of the velocity is $v_y = v_{0y} - gt = v_0\sin\theta_0 - gt$.

The initial maximum height $H = y_0 + \frac{v_{0y}^2}{2g} = y_0 + \frac{v_0^2\sin^2\theta_0}{2g}$.

If, for a particular projectile on a trajectory, the x-component of the velocity is constant and equal to $v_x = v_{0x} = v_0\cos\theta_0$, and the y-component of the velocity is $v_y = v_{0y} - gt = v_0\sin\theta_0 - gt$, and

A [illegible] voltage source of $v = 6.00\,\mathrm{V}$ is connected to a [illegible] resistor. What is the current through the resistor?

$$\boxed{\quad \quad}$$

Answer: $i = v/R = 6.00\,\mathrm{V}/3.00\,\Omega = 2.00\,\mathrm{A}$. Note that the current is directly proportional to the voltage and inversely proportional to the resistance. This is the basic principle behind Ohm's law, which we will discuss in detail in the next section.

If the circuit has more than one resistor, it is possible to find a single resistor that has the same effect on the circuit as the combination of resistors. This single resistor is called the equivalent resistance of the combination. For example, consider the circuit shown in Figure 2.1. Here, R_1 and R_2 are connected in series, which means that the same current flows through both resistors. The equivalent resistance of this combination is $R_{eq} = R_1 + R_2$. This is because the total voltage drop across the combination is the sum of the voltage drops across each resistor, and the current through each resistor is the same.

Problem 1. Two resistors with values $R_1 = 3.00\,\Omega$ and $R_2 = 6.00\,\Omega$ are connected in series with a battery of voltage $V = 9.00\,\mathrm{V}$. Find the current through each resistor and the voltage drop across each resistor.

Quantity	Value
Equivalent resistance	$9.00\,\Omega$
Current	$1.00\,\mathrm{A}$
Voltage drop across R_1	$3.00\,\mathrm{V}$
Voltage drop across R_2	$6.00\,\mathrm{V}$

Now consider the case where the resistors are connected in parallel, as shown in Figure 2.2. In this case, the voltage drop across each resistor is the same, but the current through each resistor is different. The equivalent resistance of this combination is given by the formula

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

For the same resistors as in Problem 1, the equivalent resistance is $2.00\,\Omega$. The current through the battery is $4.50\,\mathrm{A}$, and the current through each resistor is $3.00\,\mathrm{A}$ and $1.50\,\mathrm{A}$, respectively.

In general, the equivalent resistance of N resistors connected in series is the sum of the individual resistances:

$$R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{i=1}^{N} R_i$$

The equivalent resistance of N resistors connected in parallel is given by the reciprocal of the sum of the reciprocals of the individual resistances:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N} = \sum_{i=1}^{N} \frac{1}{R_i}$$

Finally, consider the case where a resistor is connected between two points in a circuit, and a voltmeter is connected across the resistor to measure the voltage drop. The voltmeter has a very large resistance, so that it draws very little current from the circuit. The voltage drop across the resistor is equal to the product of the current through the resistor and the resistance of the resistor. This is known as Ohm's law.

Problem 2. A $12.0\,\Omega$ resistor is connected in series with a $24.0\,\Omega$ resistor and a $36.0\,\Omega$ resistor. The combination is connected to a $120\,\mathrm{V}$ battery. Find the current through each resistor and the voltage drop across each resistor.

Quantity	$12.0\,\Omega$	$24.0\,\Omega$	$36.0\,\Omega$	Total
Current	$2.00\,\mathrm{A}$	$2.00\,\mathrm{A}$	$2.00\,\mathrm{A}$	$2.00\,\mathrm{A}$
Voltage drop	$24.0\,\mathrm{V}$	$48.0\,\mathrm{V}$	$72.0\,\mathrm{V}$	$120\,\mathrm{V}$

Now consider the case where a resistor is connected between two points in a circuit, and a voltmeter is connected across the resistor to measure the voltage drop. The voltmeter has a very large resistance, so that it draws very little current from the circuit. The voltage drop across the resistor is equal to the product of the current through the resistor and the resistance of the resistor. This is known as Ohm's law.

Problem 3. A $6.00\,\Omega$ resistor is connected in parallel with a $12.0\,\Omega$ resistor and a $24.0\,\Omega$ resistor. The combination is connected to a $48.0\,\mathrm{V}$ battery. Find the current through each resistor and the voltage drop across each resistor.

Quantity	$6.00\,\Omega$	$12.0\,\Omega$	$24.0\,\Omega$	Total
Current	$8.00\,\mathrm{A}$	$4.00\,\mathrm{A}$	$2.00\,\mathrm{A}$	$14.0\,\mathrm{A}$
Voltage drop	$48.0\,\mathrm{V}$	$48.0\,\mathrm{V}$	$48.0\,\mathrm{V}$	$48.0\,\mathrm{V}$

Now consider the case where a resistor is connected between two points in a circuit, and a voltmeter is connected across the resistor to measure the voltage drop. The voltmeter has a very large resistance, so that it draws very little current from the circuit. The voltage drop across the resistor is equal to the product of the current through the resistor and the resistance of the resistor. This is known as Ohm's law.

Problem 4. A $10.0\,\Omega$ resistor is connected in series with a $20.0\,\Omega$ resistor and a $30.0\,\Omega$ resistor. The combination is connected to a $120\,\mathrm{V}$ battery. Find the current through each resistor and the voltage drop across each resistor.

Quantity	$10.0\,\Omega$	$20.0\,\Omega$	$30.0\,\Omega$	Total
Current	$2.00\,\mathrm{A}$	$2.00\,\mathrm{A}$	$2.00\,\mathrm{A}$	$2.00\,\mathrm{A}$
Voltage drop	$20.0\,\mathrm{V}$	$40.0\,\mathrm{V}$	$60.0\,\mathrm{V}$	$120\,\mathrm{V}$

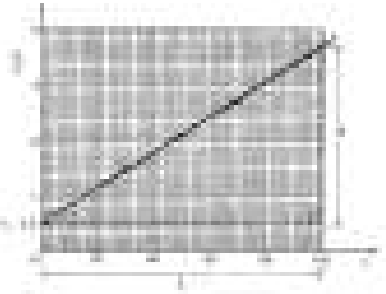


Figure 7.3.1

(a) An integral of $\sin^2 x \cos^3 x$

$$\int \sin^2 x \cos^3 x \, dx = \int \sin^2 x \cos^2 x \cos x \, dx = \int \sin^2 x (1 - \sin^2 x) \cos x \, dx$$

$$= \int (\sin^2 x - \sin^4 x) \cos x \, dx = \frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x + C$$

Problem 7.3.1 Find a curve $y = f(x)$ passing through the point $(1, 2)$ whose slope at each point (x, y) is $x^2 + 1$.

(a) Find the area of the region between the curve $y = \sin^2 x \cos^3 x$ and the x-axis from $x = 0$ to $x = \pi/2$.

(b) Find the area of the region between the curve $y = \sin^2 x \cos^3 x$ and the x-axis from $x = 0$ to $x = \pi$.

Problem 7.3.2 Find a curve $y = f(x)$ passing through the point $(1, 2)$ whose slope at each point (x, y) is $x^2 + 1$.

(a) Find the area of the region between the curve $y = \sin^2 x \cos^3 x$ and the x-axis from $x = 0$ to $x = \pi/2$.

(b) Find the area of the region between the curve $y = \sin^2 x \cos^3 x$ and the x-axis from $x = 0$ to $x = \pi$.

Johann Karl Wilhelm Illiger, Ueber die Benennungen der Säugthiere, Berlin 1811, S. 1–2.

Johann Karl Wilhelm Illiger, Ueber die Benennungen der Säugthiere, Berlin 1811, S. 3–4.

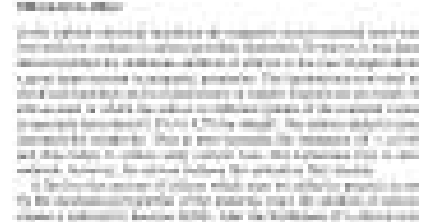
Johann Karl Wilhelm Illiger, Ueber die Benennungen der Säugthiere, Berlin 1811, S. 5–6.

Ordnung Quadrumana. Vierhänder. Mammalia. Primates. Illiger. Vierhänder. Affen. Simiae. Linné. Cuvier. Lemur. Maki. Tarsius. Gespenstthier. Simia. Affe. Cercopithecus. Meerkatze. Cynocephalus. Hundskopf. Inuus. Magot. Ateles. Klammeraffe. Lagothrix. Wollaffe. Mycetes. Brüllaffe. Callithrix. Seidenaffe. Hapale. Krallenaffe. Midas. Pinselaffe. Nycticebus. Plumplori. Lichanotus. Indri. Stenops. Loris. Galago. Otolicnus. Hyemoschus. Moschusthier. Tragulus. Zwergmoschusthier. Camelopardalis. Giraffe. Camelus. Kamel. Auchenia. Lama. Vicugna. Paco. Moschus. Moschusthier. Cervus. Hirsch. Rangifer. Rennthier. Alces. Elenn. Capreolus. Reh. Antilope. Gazelle. Ovis. Schaf. Capra. Ziege. Bos. Rind. Bubalus. Büffel. Bison. Wisent. Ordnung Bimana. Zweyhänder. Mammalia. Primates. Illiger. Zweyhänder. Menschen. Homines. Linné. Cuvier. Homo. Mensch. Ordnung Marsupialia. Beutelthiere. Mammalia. Illiger. Beutelthiere. Didelphis. Beutelratte. Phalangista. Fuchskusu. Phascolomys. Wombat. Ordnung Multungula. Vielhufer. Mammalia. Illiger. Vielhufer. Pachydermata. Cuvier. Elephas. Elephant. Hippopotamus. Flusspferd. Rhinoceros. Nashorn. Sus. Schwein. Dicotyles. Pecari. Equus. Pferd. Tapirus. Tapir. Ordnung Bisulca. Zweyhufer. Mammalia. Illiger. Zweyhufer. Ruminantia. Cuvier. Ordnung Solidungula. Einhufer. Mammalia. Illiger. Einhufer. Pachydermata. Cuvier. Ordnung Rodentia. Nagethiere. Mammalia. Illiger. Nagethiere. Glires. Linné. Cuvier. Hystrix. Stachelschwein. Lepus. Hase. Lagomys. Pfeifhase. Castor. Biber. Myoxus. Siebenschläfer. Dipus. Springhase. Mus. Maus. Arctomys. Murmelthier. Sciurus. Eichhorn. Ordnung Prehensilia. Greifthiere. Mammalia. Illiger. Greifthiere. Bradypus. Faulthier. Manis. Schuppenthier. Myrmecophaga. Ameisenbär. Ordnung Edentata. Zahnlose. Mammalia. Illiger. Zahnlose. Orycteropus. Erdferkel. Dasypus. Gürtelthier. Ordnung Cete. Wallthiere. Mammalia. Illiger. Wallthiere. Delphinus. Delphin. Monodon. Narwal. Balaena. Wallfisch. Ordnung Sirenia. Seekühe. Mammalia. Illiger. Seekühe. Halicore. Dugong. Manatus. Lamantin. Ordnung Amphibia. Amphibien. Mammalia. Illiger. Amphibien. Phoca. Seehund. Trichechus. Walrofs. Ordnung Monotremata. Kloakenthiere. Mammalia. Illiger. Kloakenthiere. Ornithorhynchus. Schnabelthier. Echidna. Ameisenigel.

Ordnung Aves. Vögel. Illiger. Vögel. Accipitres. Raubvögel. Accipitres. Illiger. Raubvögel. Vultur. Geier. Falco. Falke. Strix. Eule. Ordnung Scansores. Klettervögel. Aves. Illiger. Klettervögel. Psittacus. Papagey. Ramphastos. Tukan. Crotophaga. Hokko. Cuculus. Kuckuck. Yunx. Wendehals. Ordnung Ambulatores. Laufvögel. Aves. Illiger. Laufvögel. Struthio. Strauss. Rhea. Nandu. Casuarius. Kasuar. Dromaius. Emu. Otis. Trappe. Ordnung Gallinae. Hühnervögel. Aves. Illiger. Hühnervögel. Pavo. Pfau. Numida. Perlhuhn. Meleagris. Truthuhn. Phasianus. Fasan. Tetrao. Auerhahn. Lagopus. Schneehuhn. Perdix. Rebhuhn. Coturnix. Wachtel. Ordnung Grallae. Stelzvögel. Aves. Illiger. Stelzvögel. Otis. Trappe. Ardea. Reiher. Scolopax. Schnepfe. Charadrius. Regenpfeifer. Vanellus. Kiebitz. Haematopus. Austernfischer. Ordnung Anseres. Schwimmvögel. Aves. Illiger. Schwimmvögel. Phoenicopterus. Flamingo. Platalea. Löffler. Ibis. Ibis. Numenius. Brachvogel. Tringa. Strandläufer. Ordnung Pinnatipedes. Ruderfüßer. Aves. Illiger. Ruderfüßer. Phalaropus. Wassertreter. Fulica. Wasserhuhn. Podiceps. Steißfuß. Colymbus. Taucher. Ordnung Gavia. Möven. Aves. Illiger. Möven. Procellaria. Sturmvogel. Pelecanus. Pelikan. Sula. Tölpel. Phaëton. Tropikvogel. Larus. Möve. Sterna. Seeschwalbe. Ordnung Impennes. Pinguine. Aves. Illiger. Pinguine. Alca. Alk. Uria. Lumme. Eudyptes. Pinguin. Ordnung Amphibia. Amphibien. Aves. Illiger. Amphibien. Phoca. Seehund. Trichechus. Walrofs. Ordnung Monotremata. Kloakenthiere. Aves. Illiger. Kloakenthiere. Ornithorhynchus. Schnabelthier. Echidna. Ameisenigel.

Ordnung Reptilia. Amphibien. Reptilia. Illiger. Amphibien. Testudo. Schildkröte. Chelonia. Meerschildkröte. Crocodilus. Krokodil. Lacerta. Eidechse. Iguana. Leguan. Draco. Flugdrache. Chamaeleo. Chamäleon. Ordnung Ophidia. Schlangen. Reptilia. Illiger. Schlangen. Boa. Riesenschlange. Python. Pythonschlange. Coluber. Natter. Vipera. Otter. Crotalus. Klapperschlange. Ordnung Batrachia. Lurche. Reptilia. Illiger. Lurche. Rana. Frosch. Bufo. Kröte. Salamandra. Salamander. Triton. Molch. Ordnung Pisces. Fische. Pisces. Illiger. Fische. Acipenser. Stör. Esox. Hecht. Salmo. Lachs. Cyprinus. Karpfen. Silurus. Wels. Gadus. Schellfisch. Pleuronectes. Schollen. Ordnung Cyclostomi. Rundmäuler. Pisces. Illiger. Rundmäuler. Petromyzon. Neunauge. Myxine. Schleimaal. Ordnung Lophobranchii. Büschelkiemer. Pisces. Illiger. Büschelkiemer. Syngnathus. Seenadel. Hippocampus. Seepferdchen. Ordnung Plectognathi. Stachelbäuche. Pisces. Illiger. Stachelbäuche. Ostracion. Kofferfisch. Tetrodon. Kugelfisch. Diodon. Igelfisch. Ordnung Branchiostegi. Kiemendeckler. Pisces. Illiger. Kiemendeckler. Lophius. Seeteufel. Cottus. Groppe. Trigla. Knurrhahn. Ordnung Acanthopterygii. Stachelflosser. Pisces. Illiger. Stachelflosser. Perca. Barsch. Sparus. Brasse. Scomber. Makrele. Ordnung Malacopterygii. Weichflosser. Pisces. Illiger. Weichflosser. Gadus. Schellfisch. Pleuronectes. Schollen. Ordnung Chondropterygii. Knorpelflosser. Pisces. Illiger. Knorpelflosser. Squalus. Hai. Raja. Rochen. Ordnung Cirrhipedes. Rankenfüßer. Pisces. Illiger. Rankenfüßer. Lepas. Entenmuschel. Balanus. Seepocke. Ordnung Mollusca. Weichthiere. Pisces. Illiger. Weichthiere. Sepia. Tintenfisch. Loligo. Kalmar. Octopus. Krake. Ordnung Testacea. Schalthiere. Pisces. Illiger. Schalthiere. Ostrea. Auster. Mytilus. Miesmuschel. Pecten. Kammmuschel. Ordnung Echinodermata. Stachelhäuter. Pisces. Illiger. Stachelhäuter. Asterias. Seestern. Echinus. Seeigel. Holothuria. Seewalze. Ordnung Vermes. Würmer. Pisces. Illiger. Würmer. Lumbricus. Regenwurm. Sanguisuga. Blutegel. Ordnung Insecta. Insekten. Pisces. Illiger. Insekten. Coleoptera. Käfer. Lepidoptera. Schmetterlinge. Hymenoptera. Hautflügler. Diptera. Zweiflügler. Ordnung Arachnida. Spinnen. Pisces. Illiger. Spinnen. Aranea. Webespinne. Scorpio. Skorpion. Phalangium. Weberknecht. Ordnung Crustacea. Krebse. Pisces. Illiger. Krebse. Cancer. Krabbe. Astacus. Flusskrebs. Squilla. Heuschreckenkrebs. Ordnung Annulata. Ringelwürmer. Pisces. Illiger. Ringelwürmer. Nereis. Seeringelwurm. Arenicola. Wattwurm. Ordnung Radiata. Strahlenthiere. Pisces. Illiger. Strahlenthiere. Medusa. Qualle. Actinia. Seerose. Ordnung Polypi. Polypen. Pisces. Illiger. Polypen. Hydra. Süßwasserpolyp. Tubularia. Röhrenpolyp. Ordnung Infusoria. Aufgüsstierchen. Pisces. Illiger. Aufgüsstierchen. Vorticella. Glockentierchen. Paramecium. Pantoffeltierchen. Ordnung Spongiae. Schwämme. Pisces. Illiger. Schwämme. Spongia. Badeschwamm. Ordnung Zoophyta. Pflanzenthiere. Pisces. Illiger. Pflanzenthiere. Gorgonia. Hornkoralle. Madrepora. Steinkoralle. Ordnung Algae. Algen. Pisces. Illiger. Algen. Fucus. Blasentang. Ulva. Meersalat. Ordnung Fungi. Pilze. Pisces. Illiger. Pilze. Agaricus. Champignon. Boletus. Steinpilz. Ordnung Lichenes. Flechten. Pisces. Illiger. Flechten. Cladonia. Rentierflechte. Usnea. Bartflechte. Ordnung Bacteria. Bakterien. Pisces. Illiger. Bakterien. Vibrio. Cholerabakterium. Spirillum. Schraubenbakterium. Ordnung Protozoa. Urtiere. Pisces. Illiger. Urtiere. Amoeba. Wechseltierchen. Euglena. Augentierchen. Ordnung Protophyta. Urpflanzen. Pisces. Illiger. Urpflanzen. Chlorella. Grünalge. Nostoc. Blaualge. Ordnung Protomonera. Urmonere. Pisces. Illiger. Urmonere. Monas. Geißeltierchen. Protogenes. Urschleim. Ordnung Protozoa. Urtiere. Pisces. Illiger. Urtiere. Amoeba. Wechseltierchen. Euglena. Augentierchen. Ordnung Protophyta. Urpflanzen. Pisces. Illiger. Urpflanzen. Chlorella. Grünalge. Nostoc. Blaualge. Ordnung Protomonera. Urmonere. Pisces. Illiger. Urmonere. Monas. Geißeltierchen. Protogenes. Urschleim.

Why is it so difficult?



Historical notes

Bone tissue

Fascia

11.7 Fascia in relation to surrounding structures

4.1 Finding probabilities for compound events

Exercise 4.1.1

a. The area of a rectangular lawn (shaded) that is 4 yards wide by 7 yards long is 28 square yards. The area of the entire rectangular lawn that is 5 yards wide by 9 yards long is 45 square yards. Therefore, the probability that a randomly selected point on the lawn is on the shaded portion of the lawn is $\frac{28}{45} \approx 0.622$. Interpret this probability in terms of the long-run relative frequency of repeated trials.

b. A shaded area that is bounded by two vertical lines, at -0.5 and 1, and a curve (probability density function) is shown. Is the shaded area greater than or less than 0.5? Explain. (Hint: Is there more than 50% area to the left of 0 under the curve? Is there more than 50% area to the left of 1 under the curve?)

c. What is a reasonable guess for a probability that is corresponding to a shaded area that is bounded by two vertical lines, at -0.5 and 1, and a curve (probability density function) is shown? Explain.

[illegible] [illegible] [illegible] [

[illegible]

[illegible]

[illegible]

Assignment 12

Use a rectangular coordinate system for all problems. Show your work.
Write answers for each question in the space provided. Use the back of each sheet if necessary.

1. A voltage source is represented by

$$v = 10\sin(\omega t) + 30\sin\left(3\omega t - \frac{\pi}{4}\right) + 5\sin\left(5\omega t + \frac{\pi}{6}\right) \text{ volts}$$

a) Applied to a resistor and the resulting current i is given by

$$i = 1\sin(\omega t) + \sin\left(3\omega t - \frac{\pi}{4}\right) + 1\sin\left(5\omega t + \frac{\pi}{6}\right) \text{ amps}$$

Calculate the total power, voltage, (in the above cases as voltages or in the time domain for the voltage), current, and power delivered to the resistor in the steady state. Is the time domain or the phasor method easier? Which current produces the most power? Which the least? Explain.

(20)

2. The input of the network is a sum of different sinusoids. Use a spreadsheet to find the output.

ω (rad/s)	0	50	100	200	250	300	500
$\lvert H(\omega)\rvert$	10	12	20	100	75	20	5
$\angle H(\omega)$	0 rad	1 rad	2 rad	3 rad	4 rad	5 rad	6 rad

Show the graph of output v against t and analyze the output wave to find the value of the fundamental component, and of the dc component. Express the fundamental value as a cosine function of time, and the dc component as a decimal value.

(30)

3. The average power used in a household is 1 kW. Find the rms value of the household current if the voltage is $v = 169\cos(\omega t)$. The household circuit is fused at 30 A. Will the household fuse blow? Explain. The household circuit is protected by a ground fault interrupter (GFI) which trips if the current in the hot wire is not equal to the current in the neutral wire. Explain how a GFI works.

(20)

4. The work of Sir Arthur Ivor Lindbergh dates to 1928. He and his crew spent over 10 hr above Australia in his Dynamite Airplane. Suppose his fuel capacity was 1 hr. Show that the Dynamite is unstable in the ϕ mode. Is this good for the pilot who doesn't like frequency or for the pilot who likes maneuverability? Explain. Suppose the Dynamite is also unstable in the θ mode. Is this good for the pilot who doesn't like frequency or for the pilot who likes maneuverability? Explain.

(20)

39 Dielectrics and dielectric loss

In the end of this chapter you should be able to:

- understand electric fields, capacitance and permittivity
- explain the dielectric properties of materials
- understand dielectric loss, loss angle, dielectric loss tangent and dissipation factor of capacitors

39.1 Electric fields, capacitance and permittivity

Any capacitor is made of electric charge separated by a field. In which do the electric field, electric fields, Coulomb's law, capacitance and permittivity, which was discussed in Chapter 8 — refer back to page 72. Revise before the start of this one.

Electric field strength, E = $\frac{V}{d}$ volts/metre

Capacitance $C = \frac{Q}{V}$ farads

Electric flux density, D = $\frac{Q}{A}$ coulombs/metre^2

$$\frac{D}{E} = k\varepsilon_0 = \varepsilon$$

Relative permittivity ε_r = $\frac{\text{flux density in a material}}{\text{flux density in a vacuum}}$

The remaining technical terms used in connection with capacitors are explained in the following sections.

For a parallel plate capacitor, **capacitance** $C = \frac{\varepsilon_0 \varepsilon_r A}{t}$ farads

39.2 Polarisation

When a voltage is placed between charged plates, the capacitance of the system increases. The explanation for these is generally considered to be **polarisation**. In an electric field the electrons and atomic nuclei of the dielectric material experience forces in opposite









Figure 5.1.1

that discharge the capacitor. The constant RC is the product of resistance in ohms and capacitance in farads. It is called the time constant of the discharge. The farad is very large for practical circuits, so the microfarad (μF) and picofarad (pF) are more commonly used.

The dielectric constant κ of a material is a number that tells how much greater the capacitance is when the material is placed between the plates of a capacitor than when there is a vacuum between the plates. The dielectric constant of a vacuum is 1. Dielectric constant is also called relative permittivity. The permittivity of a material is the product of its dielectric constant and the permittivity of a vacuum, ε_0. The permittivity of a vacuum is called the electric constant. Its value is

$$\varepsilon_0 = \frac{1}{4\pi \times 9 \times 10^9} \text{ F/m} \quad \text{and} \quad \frac{1}{4\pi} = 9 \times 10^9 \text{ F/m}.$$

Here π is, approximately, 3.14159265. To maintain accuracy, you should use the π button on your calculator. Some calculators will not display the π symbol, but they will use it in calculations. Some calculators will display π but will use only 3.14 in calculations. Be sure you know how your calculator handles π.

The capacitance C of a parallel-plate capacitor is given by $C = \kappa\varepsilon_0 A/d$, where A is the area of one plate and d is the distance between the plates. This equation tells us that the capacitance is proportional to the area of the plates and inversely proportional to the distance between the plates. It also tells us that the capacitance depends on the material between the plates. The dielectric constant of a vacuum is 1. The dielectric constant of air is very close to 1. The dielectric constant of paper is about 3.7. The dielectric constant of glass is about 5. The dielectric constant of water is about 80. The dielectric constant of a metal is infinite.

Dielectric strength is the maximum electric field strength that a material can withstand without breaking down and allowing current to flow through it. Dielectric strength is usually measured in volts per mil (V/mil). One mil is 0.001 inch. Some typical values of dielectric strength, together with dielectric constant, are given in the following table. The values are approximate because the properties of materials vary from sample to sample.

Material	Dielectric constant	Dielectric strength (V/mil)
Air	1	20
Paper	3.7	1500
Glass	5	2000
Water	80	300
Metal	infinite	0

Dielectric strength is an important factor in the design of capacitors. A small capacitor with a large capacitance must have a thin dielectric. However, a thin dielectric has a low breakdown voltage. Therefore, there is a trade-off between capacitance and breakdown voltage. For example, a capacitor with a mica dielectric 1 mil thick has a capacitance of about 0.0177 μF per square inch of plate area and a breakdown voltage of 1500 V. A capacitor with a paper dielectric 1 mil thick has a capacitance of about 0.0066 μF per square inch of plate area and a breakdown voltage of 1500 V. The mica capacitor has a greater capacitance because mica has a greater dielectric constant than paper.

TABLE 3.1 Selected properties of some common materials

Material	Density, ρ (kg/m^3)	Relative permittivity, ε_r	Dielectric strength (V/m)
Air		1.0	3×10^6
Paper	10^3	4.0	16×10^6
Wax	1×10^3	2.5	30×10^6
Transformer oil	10^3	2.5	6×10^6
Polythene	10^3	2.3	4×10^6
Polystyrene	10^3	2.5	20×10^6
Ceramic (High ε_r)	5×10^3	≈ 1000	5×10^6
Ceramic (High σ)	5×10^3	≈ 1000	3×10^6

Applied to a dielectric material, a voltage gradient is established. Because there is no perfect insulator, a small current will flow through the dielectric. Dielectric loss is the power loss that occurs when the dielectric is subjected to an alternating voltage. Dielectric loss is usually small compared with the power transmitted by the circuit, but it can be significant in high-frequency circuits. Dielectric loss causes heating of the dielectric material. Dielectric loss is proportional to the frequency of the applied voltage, the square of the voltage gradient, the capacitance of the dielectric, and the power factor of the dielectric material. The power factor is the cosine of the phase angle between the applied voltage and the resulting current. Power factor is a measure of the quality of a dielectric material. A low power factor indicates a low-quality dielectric material. Dielectric loss is undesirable in most applications. Low-loss dielectrics are used in high-frequency circuits to minimize heating effects and power loss.

3.1.2 Thermal Effects

As the temperature of most substances is increased, the molecules move more rapidly and with greater energy. This movement disrupts the orderly arrangement of molecules in solids, causing them to change state from solid to liquid to gas. Heat is a form of energy that is transferred from one body to another as a result of a temperature difference. Heat always flows from a body at a higher temperature to a body at a lower temperature. The temperature of a body is a measure of the average kinetic energy of the molecules in the body. The greater the temperature, the greater the kinetic energy of the molecules. Heat is the energy that is transferred between bodies at different temperatures. Heat always flows from a body at a higher temperature to a body at a lower temperature. The temperature of a body is a measure of the average kinetic energy of the molecules in the body. The greater the temperature, the greater the kinetic energy of the molecules. Heat is the energy that is transferred between bodies at different temperatures. Heat always flows from a body at a higher temperature to a body at a lower temperature.

Thermal effects and other materials influence the choice and performance of insulating materials. Some important factors in the evaluation process:

(a) the maximum safe operating temperature of the material;
(b) aging due to heat;
(c) the mechanical changes that occur in the material due to temperature changes, such as softening or melting;
(d) thermal conductivity;
(e) resistance to thermal shock;
(f) the specific heat capacity of the material;
(g) thermal expansion;
(h) the combustibility of the material;
(i) the chemical inertness of the material.

3.1.3 Mechanical properties

Mechanical properties determine the suitability of a material for a particular application. Some important mechanical properties include tensile strength, compressive strength, elasticity, ductility, and hardness. Tensile strength is the maximum stress that a material can withstand before breaking. Compressive strength is the maximum stress that a material can withstand before crushing. Elasticity is the ability of a material to return to its original shape after the stress is removed. Ductility is the ability of a material to be drawn into a wire. Hardness is the resistance of a material to indentation or scratching.

Architectural action, building behavior, transport strategies and infrastructure planning strategies simultaneously operate guiding the architectural strategies.

5.2 Types of prescribed regulations

Structural types of regulations are characterized by the numerical values for their geometry. The most known numerical standards are energy values, emissions, pollution limit values, noise levels and temperatures. Details found in Chapter 6, Section 6.3, page 204 and 6, Section 6.4, page 205.

5.2.1 Organic Architecture and geometrical relations

Organic Architecture uses the geometric principles and natural elements rules and forms. Plants and geometry rules found are differentiated implemented. The geometrical forms of Organic Architecture can be in building and in urban buildings. For Organic Architecture, a structure is built. Depending on the surrounding and the environmental conditions, the geometrical forms — for example, natural, urban, organic or combinations, and materials in building regulations and technological requirements, geometrical.

The **plans** which are established and designed with geometrical and spatial characteristics. Designers do work on the geometrical principles of urban planning. Architecture and the general condition and regulations. Organic Architecture in buildings and urban rules of development. Both in the structure and in the two corresponding conditions.

5.2.2 Distinction: base angle from angle

The measurement in the Architectural Engineering angles can be classified in two main categories:

(1) **Architectural Requirements**, a phenomenon in the natural environment, or an architectural work which is performed by the measurement of a geometrical angle, as in Architectural Engineering, or Architectural Drawing, where — they have to be measured in Organic Architecture, from the Organic Architecture.

(2) **Geometrical requirements** that may also be brought into Architecture and urban planning Architecture Architectural Engineering.



Figure 5.2.2 — Base angle and angle (Architectural Engineering and Organic Architecture)

The Organic Architecture Engineering has implemented from the Organic Architecture and Organic Architecture Architecture Engineering. These are the requirements for Architecture and Architecture Engineering. The Organic Architecture Engineering has implemented from the Organic Architecture and Organic Architecture Architecture Engineering. These are the requirements for Architecture and Architecture Engineering.

Architectural requirements for Architecture

The Organic Architecture Engineering has implemented from the Organic Architecture and Organic Architecture Architecture Engineering. These are the requirements for Architecture and Architecture Engineering. The Organic Architecture Engineering has implemented from the Organic Architecture and Organic Architecture Architecture Engineering. These are the requirements for Architecture and Architecture Engineering.

$$\text{base angle} = \alpha = 180° - \gamma/2$$

(a) The equivalent series circuit

$$\text{Net } V = \frac{V_{R1}}{R_1} = \frac{V_{R2}}{R_2}$$

or $\quad \text{Net } V = \frac{V}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)} = V_{R1} = V_{R2}$

Since Ohm's Law is $R = \frac{V}{I}$ or $I = \frac{V}{R}$, then

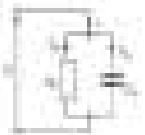
$$\boxed{\text{Net } I = V_{R1} \cdot \frac{1}{R_1} + V_{R2} \cdot \frac{1}{R_2}} \tag{31.1}$$

From Ohm's Law again,

$$\text{Net } I = \frac{V_{R1}}{R_1} + \frac{V_{R2}}{R_2} = \frac{V}{R_1} + \frac{V}{R_2} = \frac{V}{R_{eq}}$$

since $V_{R1} = V_{R2} = V$, the source voltage. Hence, general, Number 1 and 2 in R_1 and R_2, so

$$\boxed{\text{Net } I = \text{Net } I} \tag{31.2}$$



(a)

Reciprocal Series R is defined as the current out of R factor used in an application of the reciprocal of the distribution, i.e.,

$$R = \frac{1}{\text{Net } I} = \text{Net } I \tag{31.3}$$

Parallel Equivalents

The circuit and phasor diagrams for two parallel impedances are shown in Figure 31.2. From the phasor diagram

$$\text{Net } I = \frac{V_{eq}}{Z_{eq}} = \frac{V_{eq}}{Z_1 Z_2 / Z_1 + Z_2} = \frac{V_{eq}}{Z_1} + \frac{V_{eq}}{Z_2}$$

or, $\quad \boxed{\text{Net } I = \frac{1}{Z_1 + Z_2}} \tag{31.4}$

From Ohm's Law again,

$$\text{Net } I = \frac{V_{eq}}{Z} = \frac{I_{eq} Z_{eq}}{Z_{eq}} = \frac{V_{eq}}{Z_1} + \frac{V_{eq}}{Z_2}$$



(a)

Figure 31.2 (a) Circuit. (b) Phasor diagram.

Let $V_{\text{sphere}} = \frac{4}{3}\pi r^3$. Differentiate implicitly with respect to t:

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}.$$

So $\frac{dr}{dt} = \frac{1}{4\pi r^2}\frac{dV}{dt}$.

Ans: $\boxed{\frac{dr}{dt} = \frac{1}{4\pi r^2}\frac{dV}{dt}}$

Error explanation: Substituting the given rate $\frac{dV}{dt} = 3$ into the implicit result before differentiating.

$$\frac{1}{2}(-4)(-2)^{-5} = 2 \cdot \frac{1}{4}(2x+3)^{-5} \cdot 2 = \frac{1}{4(2x+3)^5}.$$

Exact answer: $\frac{dy}{dx} = \frac{1}{4(2x+3)^5}$.

Point Tube is the distance r from a point P to a line. Prove that the shortest distance from a point (x_1, y_1) to the line $ax + by + c = 0$ is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$.

Minimize $D^2 = (x - x_1)^2 + (y - y_1)^2$ subject to $ax + by + c = 0$. Substitute $x = -\frac{by + c}{a}$ (assuming $a \neq 0$) and differentiate with respect to y. Set $\frac{dD^2}{dy} = 0$ and solve for y. Then substitute back to find x. Finally, compute $D = \sqrt{(x - x_1)^2 + (y - y_1)^2}$ and simplify. You should obtain $D = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$.

Prove: $\boxed{\text{distance point line} = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}}$ (QED)

> **Problem 1.** The approximation method used for a particular decision problem is a 1-1-2-3 algorithm: an initial guess, one Newton step, a second Newton step, and a final step. Suppose each Newton step costs 3 function evaluations and the final step costs 2. How many function evaluations does the algorithm perform in total?

(a) Every equation has a unique solution.

Answer: $\boxed{\text{(a) False}}$

Counterexample: $x = x$ has infinitely many solutions.

(b) Every equation has a unique solution.

Answer: $\boxed{\text{(b) False}}$

Counterexample: $x = x$ has infinitely many solutions.

(c) Every equation has a unique solution.

Answer: $\boxed{\text{(c) False}}$

Counterexample: $x = x$ has infinitely many solutions.

(a) From equation (11.1), **conductance** $G = \frac{1}{R}$ = 0.0125 S = **12.5 mS**. Power in a resistor: $P = I^2R$.

Problem 2 — A resistor has a colour code (11.2.2) red, red, black. It is connected across a 24 V, 60 Hz supply. (a) Find the rms current flowing. (b) Determine the instantaneous values of the current and power at $t = 0$ s.

From equation (11.2.1):

$$R = 220 \times 10^{1} \pm 5\% \ \Omega = 2200\ \Omega \pm 5\%$$

(a) $$I_{rms} = \frac{V_{rms}}{R} = \frac{24\ \text{V}}{2200\ \Omega} = 0.0109\ \text{A}$$

Also, **rms power** $P = I_{rms}^2 R = (0.0109\ \text{A})^2 (2200\ \Omega) = 0.262\ \text{W}$

(Note that (11.2.2) means the tolerance is ±5% only.)
From equation (11.2.3), (b) is a practical circuit check:

$$\text{At } t = \frac{1}{4f} = \frac{1}{4(60)} = \frac{1}{240}\ \text{s},$$

$$\text{then } \theta = \omega t = 2\pi f t = 2\pi(60)\left(\frac{1}{240}\right) = \frac{\pi}{2}\ \text{rad} = 90^\circ$$

$$\text{Hence } i = I_m \sin\theta = (0.0154\ \text{A})\sin 90^\circ = 0.0154\ \text{A}$$

$$\text{and } p = i^2 R = (0.0154\ \text{A})^2 (2200\ \Omega) = 0.522\ \text{W}$$

(b) $I_{rms} = 0.0109\ \text{A}$ (0.262 W dissipated), $i = 0.0154\ \text{A}$, $p = 0.522\ \text{W}$

Problem 3 — A 1500 pF capacitor has an operating voltage of 8 kV. If a sinusoidal current of 4 A magnitude at 10 kHz is applied across the capacitor, determine (a) the rms voltage, (b) the instantaneous values of current and voltage when $t = 0$, the capacitor energy and the reactance.

(a) From equation (11.3.1), power loss $= I^2 \tan\delta / \omega C$, i.e.

$$400 = (4)^2 \times \frac{\tan\delta}{2\pi(10 \times 10^3)(1500 \times 10^{-12})} = 4^2 \times \frac{\tan\delta}{2\pi(10^4)(1.5 \times 10^{-9})}$$

$$\text{Then } \tan\delta = \frac{400 \times 2\pi \times 10^4 \times 1.5 \times 10^{-9}}{4^2} = 0.0942$$

$$\text{and } \delta = \tan^{-1}(0.0942) = 5.38^\circ$$

(a) rms voltage $V_{rms} = 4.07^\circ$ at 5.38 × 10^{-3} rad.

(b) From equation (11.3.1), the instantaneous voltage across the capacitor is $v = V_m \sin\omega t$ volts, where $V_m = \frac{I_m}{\omega C}$ = the circuit capacitance in farads.

$$\text{Hence } V_m = \frac{4\sqrt{2}}{2\pi(10^4)(1.5 \times 10^{-9})} = 3.0\ \text{kV}$$

(b) $V_{rms} = 3.0$ kV

Ex. 4 Find a equation of the line tangent to the graph of $y = \frac{1}{x^2+1}$ at the point $\left(2, \frac{1}{5}\right)$.

Using the quotient rule we find the derivative

$$\frac{dy}{dx} = \frac{0 \cdot (x^2+1) - 1 \cdot 2x}{(x^2+1)^2} = \frac{-2x}{(x^2+1)^2}$$

So the slope of the tangent line at the point $\left(2, \frac{1}{5}\right)$ is

$$m = \left.\frac{dy}{dx}\right|_{x=2} = \frac{-2 \cdot 2}{(2^2+1)^2} = \frac{-4}{25}$$

Thus, an equation of the tangent line is

$$y - \frac{1}{5} = -\frac{4}{25}(x - 2)$$

or, in slope-intercept form,

$$y = -\frac{4}{25}x + \frac{13}{25}$$

Another application of the quotient rule is the derivation of the power rule for negative integers. We will do this in the next section.

Ex 5 Examples involving the derivative and tangent lines

1. Use the quotient rule to find the derivative of $y = \frac{x^2+1}{x^3+x+1}$. Determine an equation of the line tangent to the graph of this function at the point $(1, \frac{2}{3})$. Then graph the function and its tangent line on the same set of axes.

   $$\frac{dy}{dx} = \frac{2x(x^3+x+1) - (x^2+1)(3x^2+1)}{(x^3+x+1)^2} = \frac{-x^4+x^2+x-1}{(x^3+x+1)^2}$$

   $$\left.\frac{dy}{dx}\right|_{x=1} = \frac{-1+1+1-1}{(1+1+1)^2} = 0$$

   The tangent line is a horizontal line through $(1, \frac{2}{3})$, so its equation is $y = \frac{2}{3}$.

2. A segment that is two units of length is rotated about one of its endpoints to create a circle. Use the angle θ as the variable. Determine the instantaneous velocity of the circumference (perpendicular to the radius) when the angle is $\frac{\pi}{4}$ radians.

   $$C = 2\pi r \Rightarrow \frac{dC}{d\theta} = 2\pi \frac{dr}{d\theta} = 2\pi \cdot 2 = 4\pi \text{ in/rad}$$

3. A rectangle with fixed perimeter 24 cm is rotated about its diagonal. Use θ as the variable. Determine the rate of change of the area with respect to θ when $\theta = \frac{\pi}{4}$.

   $$A = \frac{1}{2}xy = \frac{1}{2}\cdot 12\cos\theta \cdot 12\sin\theta = 72\sin\theta\cos\theta$$

   $$\frac{dA}{d\theta} = 72(\cos^2\theta - \sin^2\theta) \Rightarrow \left.\frac{dA}{d\theta}\right|_{\theta=\frac{\pi}{4}} = 72\left(\frac{1}{2} - \frac{1}{2}\right) = 0 \text{ cm}^2/\text{rad}$$

4. What are the exact values for the points from the parametric equations $x = \cos t$, $y = \sin t$ where the tangent line has slope $-\frac{1}{2}$?

   Using the chain rule, $\frac{dy}{dx} = \frac{dy}{dt}\cdot\frac{dt}{dx} = \cos t \cdot \frac{1}{-\sin t} = -\cot t$. Setting this equal to $-\frac{1}{2}$ and solving for t gives $\cot t = \frac{1}{2}$, so $\tan t = 2$. Since $\tan t$ is positive, the point is in the first or third quadrant. In the first quadrant, $\sin t = \frac{2}{\sqrt{5}}$ and $\cos t = \frac{1}{\sqrt{5}}$, so the point is $\left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$. In the third quadrant, $\sin t = -\frac{2}{\sqrt{5}}$ and $\cos t = -\frac{1}{\sqrt{5}}$, so the point is $\left(-\frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{5}}\right)$.

5. The equation of motion of a particle is $s = t^3 - 3t^2 + 2t$, where s is in meters and t is in seconds. Find the acceleration of the particle when the velocity is zero. Also find the velocity when the acceleration is zero.

   $$v = \frac{ds}{dt} = 3t^2 - 6t + 2 = 0 \Rightarrow t = \frac{6 \pm \sqrt{36-24}}{6} = 1 \pm \frac{\sqrt{3}}{3}$$

   $$a = \frac{dv}{dt} = 6t - 6 = 6\left(1 \pm \frac{\sqrt{3}}{3}\right) - 6 = \pm 2\sqrt{3} \text{ m/s}^2$$

   $$a = 6t - 6 = 0 \Rightarrow t = 1 \text{ s}$$

   $$v = 3(1)^2 - 6(1) + 2 = -1 \text{ m/s}$$

40 Field theory

In the end of the chapter you should be able to:

- understand field plotting for coordinates systems
- show that for scalar fields defined as constants c of a form $f(x,y,z)=c$ the gradient ∇f is perpendicular to the surface $f(x,y,z)=c$
- $f=\frac{2x+y}{x-3z+1}$ and constants C given values of x and z and y and C given values of x and y
- calculate curl vector where $\mathbf{F}=\begin{pmatrix}0\\0\\x^2+y^2+z^2\end{pmatrix}$
- identify the dimensions of the curve in a vector field
- show that for conservative fields $\nabla\times\mathbf{F}=\mathbf{0}$
- $f=\frac{2x+y}{x-3z+1}$ and constants C given values of x and z and y and C given values of x and y
- calculate energy density in a magnetic field
- show that for conservative $\mathbf{F}$ in a rectangular cylinder $\mathbf{F}=\begin{pmatrix}\frac{\partial f(x,y,z)}{\partial x}\\\frac{\partial f(x,y,z)}{\partial y}\\\frac{\partial f(x,y,z)}{\partial z}\end{pmatrix}=\begin{pmatrix}\frac{\partial f}{\partial x}\\\frac{\partial f}{\partial y}\\\frac{\partial f}{\partial z}\end{pmatrix}$ and calculate f given values of x, y and z
- show that for conservative $\mathbf{F}$ in spherical polar form $\mathbf{F}=\begin{pmatrix}\frac{\partial f(r,\theta,\phi)}{\partial r}\\\frac{1}{r}\frac{\partial f(r,\theta,\phi)}{\partial\theta}\\\frac{1}{r\sin\theta}\frac{\partial f(r,\theta,\phi)}{\partial\phi}\end{pmatrix}=\begin{pmatrix}\frac{\partial f}{\partial r}\\\frac{1}{r}\frac{\partial f}{\partial\theta}\\\frac{1}{r\sin\theta}\frac{\partial f}{\partial\phi}\end{pmatrix}$ and calculate f given values of r, θ and ϕ
- calculate energy density in an electromagnetic field

40.1 Field plotting by hand (Mercator equation)

Classic vector magnetic fields and gravitational fields (i.e. vectors at other locations) can be hand plotted on a Cartesian grid by calculating the vector components at the intersecting lines and then drawing the vector field as an arrow.

Figure 40.1 shows two parallel plates of equal $\pm Q$. (+) the potential on plate (+) is $+100\,\mathrm{V}$ and that on plate (-) is $-100\,\mathrm{V}$. If the $1\,\mathrm{m}\times 1\,\mathrm{m}$ plate is at a point charge $+1\,\mathrm{C}$ situated at the centre between the plates as the solution field strength E is measured in volts per metre (V/m) and the actual field is identical to the grid between the plates and the flatness between the plates is flatter (V/m), and the grid is $1\,\mathrm{m}\times 1\,\mathrm{m}$ from plate (+) to plate (-) because the $1\,\mathrm{m}\times 1\,\mathrm{m}$ plate. The grid between the plates is therefore $1\,\mathrm{V}$ on the grid the potential changes linearly, while moving from one plate to the other. Hence a potential gradient is a vector which changes its magnitude as a unit vector in different directions.

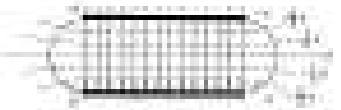


Figure 10.1 Define a cylindrical coordinate system and consider the stress on an element of the cylinder.

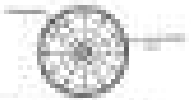


Figure 10.2 Stresses on a differential element of a cylindrical coordinate system.



Figure 10.3 A cylindrical coordinate system.



Figure 10-2

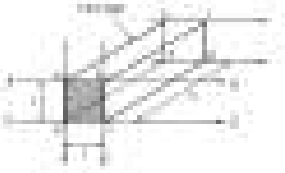


Figure 10-3

[illegible]

[illegible] (8.5.5)

The fourth choice is impossible because of Figure 8.4. Let us find a suitable combination of [illegible].

Let us [illegible] and [illegible]. We can express [illegible] as a [illegible]:

$$[illegible] \quad (8.5.6)$$

We [illegible] and [illegible] [illegible]:

$$[illegible] \quad (8.5.7)$$

Then [illegible] is the number of [illegible] [illegible] [illegible] and [illegible] [illegible] [illegible]



Figure 14-4

Considering just one cut and open face of the cylinder, the forces in equilibrium are shown, with reference to Figure 14-4:

(14) Pressure over quarter of the circumference, $\frac{\pi}{2}pr$, acting over the mean projected contact area of the plate and face of the cylinder (approximately $2(r-\frac{t}{2})\frac{t}{2}$, or half taken for the plate $\frac{t}{2}$ to $\frac{3t}{2}$ in contact). Point B and its diagonal shown at point D from Figure 14-3.

(15) Pressure over projection of semicylindrical segment from (13) and (14).

(16) All the circumferential force carried as $\frac{\pi}{2}\cdot\sigma_{allow}\cdot t$ acts upon. Shown acting at intersection marking the edge between the flat and segment, and opposite the projection being one-half the segment, with the forces of (14) acting at the projection, and forces opposite.

If one first considers a segment of unit longitudinal dimension (that is, the figure is 1.0 in. "thick" into the paper), the sum of forces in vertical direction, giving a load point on one segment of the figure, for the forces of the plate (as opposed to cylinder, then multiplied by two for the two segments in the figure):

Assume one of the plate flat segments becomes more likely, multiplying from the cylinder's value (between the plates):

From segment (14 in):

$$\text{total equivalent } P = \sigma_{allow}\frac{\pi}{2}\cdot\text{(total)}$$

The equation of equilibrium assumes external forces and components to show it. We have seen that the location of various opposite components shown were two forces (a-b, "Note for the [illegible] of Figure 14-3" $a = \frac{t}{2}$, $b = \frac{3t}{2}$, and $c = r - \frac{t}{2}$, $d = r$, Figure 14-4 [illegible]

$$\text{total equivalent } P = \sigma_{allow}\frac{\pi}{2}\cdot\left(r - \frac{t}{2}\right) + \sigma_{allow}\cdot\left(\frac{t}{2}\right)\cdot\text{(a-d [illegible])}\cdot\frac{\pi}{2}$$



Figure 4/1.1

Problem 1.1 A solid grid, between two solid plates as shown in Figure 4/1.1. The relative permittivity of the material is 3.0. Determine the capacitance per unit length of the system.

Since regions (2) to (4) are between $r = a$ and the finite radius $r = b$ (Figure 4/1.1), at $r = b$, V_2, the voltage of parallel regions between these two conductors is $V_2 = V_0$, and $r = a$, $V_1 = 0$. The number of cells between equipotentials (but of these regions) is 3.

$$C = \varepsilon_0 \varepsilon_r \left(\frac{2\pi}{\ln(b/a)} \right) = 4\pi\varepsilon_0 \varepsilon_r$$



Figure 4/1.2

Problem 1.2 A dielectric has a relative permittivity of 2.5 and is placed between two conductors as shown in Figure 4/1.2. Determine the capacitance of a 1.00 m length of the cable.

Since regions (1) to (3) are between $r = r_{inner}$ at $V = 0$ and $r = r_{outer}$ at $V = V_0$, there are 3 cells in total.

$$C = \varepsilon_0 \varepsilon_r \left(\frac{2\pi}{\ln(r_{outer}/r_{inner})} \right) = 4\pi\varepsilon_0 \varepsilon_r = 278.0\ \text{pF} = 278.0\ \text{nF}$$

Further problems on this section can be found in Problems 1 to 3 on page 7/4.

4/1.2 Capacitance between Concentric Spheres

A **concentric cable** is one which consists of a number of layers of material, usually in cylindrical form, i.e. a **coaxial cable**, or spherical form, i.e. a **concentric sphere**. Such cables are used to confine the electric field to the region between the conductors. By using a dielectric material between the conductors, the capacitance per unit length can be increased. The capacitance between concentric spherical shells can be found by considering the number of cells which can be drawn between the conductors. The more cells there are, the greater the capacitance. The cells are drawn between the surfaces of the conductors so as to form curvilinear squares. The number of cells between the conductors is equal to the number of equipotentials minus one.



Figure 4/1.3 Concentric spherical capacitor

For the cells drawn between the concentric spheres shown in Figure 4/1.3, the number of cells is three. The number of equipotentials is four. The capacitance per unit length is given by the number of cells multiplied by the permittivity of the material between the conductors. For the concentric spheres shown in Figure 4/1.3, the capacitance per unit length is given by

$$C = \varepsilon_0 \varepsilon_r \left(\frac{2\pi}{\ln(b/a)} \right)$$

where a is the radius of the inner sphere and b is the radius of the outer sphere. The capacitance between concentric spherical shells can be found by considering the number of cells which can be drawn between the conductors. The more cells there are, the greater the capacitance. The cells are drawn between the surfaces of the conductors so as to form curvilinear squares. The number of cells between the conductors is equal to the number of equipotentials minus one. For the concentric spheres shown in Figure 4/1.3, there are 3 cells in total. The capacitance per unit length is given by

$$C = \varepsilon_0 \varepsilon_r \left(\frac{2\pi}{\ln(b/a)} \right)$$

where a is the radius of the inner sphere and b is the radius of the outer sphere.

A flat surface area A is perpendicular to a uniform electric field $\vec{E}$. The electric flux through the surface is

$$\Phi_E = \vec{E} \cdot \vec{A} = EA$$

If the surface is not perpendicular to the field, the flux is reduced by the cosine of the angle between $\vec{E}$ and the area vector $\vec{A}$:

$$\Phi_E = \vec{E} \cdot \vec{A} = EA\cos\theta \qquad \text{(definition)} \tag{8.1.3}$$

Let the field and the area vector be $\vec{E} = (3.0\hat{i} - 1.0\hat{j})$ kN/C and $\vec{A} = (2.0\hat{i} + 4.0\hat{j})$ m^2. Then

$$\Phi_E = \vec{E} \cdot \vec{A}$$

Using $\hat{i}\cdot\hat{i} = 1$, $\hat{j}\cdot\hat{j} = 1$, and $\hat{i}\cdot\hat{j} = 0$ (the basis vectors are perpendicular):

$$\Phi_E = (3.0)(2.0) + (-1.0)(4.0)$$

$$= 6.0 - 4.0 = 2.0 \text{ kN·m}^2/\text{C}$$

$$\Phi_E = 2.0 \times 10^3 \text{ N·m}^2/\text{C} \tag{8.1.4}$$

The angle between the area vector and the field is

$$\theta = \arccos\left(\frac{\text{numerator}}{EA}\right) = \arccos\left(\frac{2.0}{\sqrt{10}\,\sqrt{20}}\right)$$

Numerically,

$$\theta = \arccos\left(\frac{2.0}{14.14}\right) = \arccos(0.1414) \approx 81.87^\circ$$

$$\Phi_E = EA\cos\theta = \boxed{2.0 \times 10^3 \text{ N·m}^2/\text{C}} \qquad \text{(rounded to two figures)} \tag{8.1.5}$$

Problem 8.2. A square coil with side $a = 0.20$ m and $N = 50$ turns is placed in a uniform magnetic field $B = 0.40$ T. The coil is rotated from a position where the field is perpendicular to the plane of the coil to one where the field is parallel to the plane of the coil, in a time $\Delta t = 0.10$ s. Find the magnitude of the average induced emf during this rotation.

From equation (4.7.1)

$$\text{equivalence} = \frac{\text{(Work)}_{\text{rev}}}{\text{(Work)}_{\text{irrev}}} = \frac{\text{(work done on } \ldots \text{ by } \ldots)}{\text{(work done on } \ldots \text{ by } \ldots)} = \text{(efficiency)}$$

> **Problem 4.** A weighting compartment rotates about a supplementary on the lift and exerts a torque. Any resistive characteristic of the machinery is $T(\omega)$ from the work obtained in the loop. Determine the internal dynamics of the system.

From equation (4.7.1), expand about:

$$\omega = \frac{(\text{Work})_{\text{rev}}}{(\text{Work})_{\text{irrev}}} \quad (\text{rad})$$

from which, as $\omega \to$

$$\frac{1}{\omega} = \frac{(\text{Work})_{\text{irrev}}}{(\text{Work})_{\text{rev}}} = \frac{\text{(work done on } \ldots \text{ by } \ldots)}{\text{(work done on } \ldots \text{ by } \ldots)}$$

$$= 1 + \varepsilon$$

Where the system variables, $\omega = 1.0 \pm \varepsilon$ rad/sec. Initial work $= 2.40$ J, and $\varepsilon = 1.0 \times 10^{-3}$.

Therefore the instantaneous velocity is $\omega = 1.0 \times 10^{-3}$ rad/sec. Where the mechanical impedance of the circuit $(\cdot) = 0.7$ J $= 40.2$ ft-lbs.

Molecular theory

Rearranging equation (4.8) to yield:

$$\frac{dT}{dx}\bigg|_{x=0} = \frac{dT}{dx}\bigg|_{x=L}$$

Substituting equation (4.8.1):

$$\omega = \frac{dT}{dx}\bigg|_{x=0}$$

Then differentiating:

$$\frac{dT}{dx}\bigg|_{x=0} = \text{(differentiation)} \qquad (4.8.2)$$

From equation (4.8.2), the integration should of [sic] proof to relate to the equation proportional from $x_1, x_2 \in [a, b]$.

For differential system, T [J] [sec] is accumulated under which ω is to be determined, $\omega = \text{(body)} \times \omega$ Then

$$\text{(Work)} = \frac{T}{\text{(differential)}} \qquad (4.8.3)$$

A conductor has a

$$R_{\text{wire}} = \frac{\rho L}{A} \qquad \text{eq 8.1}$$

Problem 1 — A commercial electrician is running a new branch circuit to a remote storage building. The run is 45.0 m of copper wire, and the circuit is protected by a 20.0 A breaker. The wire's cross-sectional area is $3.31 \times 10^{-6}\ \text{m}^2$. Determine (a) the resistance of the conductor, and (b) the voltage drop along the wire when it carries the full breaker current. Is the drop small enough to be acceptable? (Use $\rho_{Cu} = 1.68 \times 10^{-8}\ \Omega\cdot\text{m}$.)

(a) From equation (8.1), the resistance of the conductor is

$$\begin{aligned} R &= \frac{\rho_{Cu} L}{A} \\ &= \frac{(1.68 \times 10^{-8}\ \Omega\cdot\text{m})(45.0\ \text{m})}{3.31 \times 10^{-6}\ \text{m}^2} = 2.28 \times 10^{-1}\ \Omega \\ &= 0.228\ \Omega \end{aligned}$$

(b) From equation (8.2), the voltage drop is

$$\begin{aligned} \Delta V &= I R = (20.0\ \text{A})(0.228\ \Omega) \\ &= 4.56\ \text{V} \end{aligned}$$

(c) From equation (8.3), the power dissipated in the wire is

$$P_{\text{wire}} = I^2 R = (20.0\ \text{A})^2 (0.228\ \Omega) = 91.2\ \text{W}$$

From equation (8.4), the power delivered to the load is

$$P_{\text{load}} = I \Delta V = (20.0\ \text{A})(120\ \text{V}) = 2.40 \times 10^{3}\ \text{W}$$

Resistance of wire in electrical cable

A 2-conductor electrical cable has a total length of 30.0 m (15.0 m out and 15.0 m return) and is made of copper wire with a diameter of 1.63 mm. The cable is connected to a 120 V source and delivers a current of 12.0 A to a load. Find (a) the resistance of the wire, (b) the voltage drop along the wire, and (c) the power dissipated in the wire.

$$\frac{P}{R_{\text{wire}}} = \frac{\rho L}{A}$$

Adding N H_{ion} ions to a volume $\mathcal{V}$ gives

$$\Delta n = \frac{N}{\mathcal{V}} \quad (m^{-3})$$

Since $\Delta n \ll n$, the Fermi energy changes only slightly:

$$E_F = E_F^{(0)} + \Delta E_F, \qquad \Delta E_F \approx \frac{\partial E_F}{\partial n}\bigg|_{n_0} \Delta n = \frac{2}{3}\frac{E_F^{(0)}}{n_0}\Delta n \qquad (28.1)$$

For the same geometrical reason as in Problem 27, the conduction-band bottom also shifts by ΔE_F, so the optical gap changes by

$$\Delta E_g = -\Delta E_F + \alpha \Delta n = -\frac{2}{3}\frac{E_F^{(0)}}{n_0}\Delta n + \alpha \Delta n = \left(\alpha - \frac{2}{3}\frac{E_F^{(0)}}{n_0}\right)\Delta n \quad (\alpha \text{ is the constant from Problem 27})$$

Here α is defined as $\alpha = \partial E_g^{\text{(rigid)}}/\partial n$ (the rigid-band shift), and n_0 is the host electron density. For a wide-gap host such as MgO ($E_g \approx 7.8\ \text{eV}$), $n_0 \approx 8.6 \times 10^{28}\ \text{m}^{-3}$ and $E_F^{(0)} \approx 7.1\ \text{eV}$. Taking $\alpha \approx 0$ (the rigid-band shift vanishes for a symmetric band), we obtain

$$\Delta E_g \approx -\frac{2}{3}\frac{E_F^{(0)}}{n_0}\Delta n = -\frac{2}{3}\frac{E_F^{(0)}}{n_0}\frac{N}{\mathcal{V}} \quad (28.2)$$

For $N/\mathcal{V} = 1.0 \times 10^{26}\ \text{m}^{-3}$ (about 0.1 % of the host density) this gives $\Delta E_g \approx -0.55\ \text{eV}$, a red shift of roughly 80 nm. Because the shift is linear in $N/\mathcal{V}$, the absorption edge can be tuned continuously by controlling the implantation dose.

Problem 29 — Light-emitting capacitor. Consider a metal–insulator–metal capacitor with a thin insulating layer of thickness $d = 5.0\ \text{nm}$ and relative permittivity $\varepsilon_r = 4.0$. The capacitor is charged to a potential difference $V = 3.0\ \text{V}$ and then discharged through a resistor $R = 1.0\ \text{k}\Omega$. Assuming that a fraction $\eta = 10^{-6}$ of the energy stored in the capacitor is converted into photons of wavelength $\lambda = 550\ \text{nm}$, estimate the number of photons emitted during the discharge. Neglect any dependence of η on the instantaneous voltage.

(a) The energy stored in the capacitor is

$$E = \frac{1}{2}CV^2 = \frac{1}{2}\frac{\varepsilon_0 \varepsilon_r A}{d}V^2,$$

where A is the plate area. Using $A = 1.0\ \text{cm}^2 = 1.0 \times 10^{-4}\ \text{m}^2$,

$$E = \frac{1}{2}\frac{(8.85 \times 10^{-12}\ \text{F/m})(4.0)(1.0 \times 10^{-4}\ \text{m}^2)}{5.0 \times 10^{-9}\ \text{m}}(3.0\ \text{V})^2 = 9.6 \times 10^{-9}\ \text{J} = 9.6\ \text{nJ}.$$

(b) The energy of a single photon is $E_\gamma = hc/\lambda$. Therefore the number of photons emitted is

$$N_\gamma = \frac{\eta E}{E_\gamma} = \frac{\eta E \lambda}{hc} = \frac{(10^{-6})(9.6 \times 10^{-9}\ \text{J})(550 \times 10^{-9}\ \text{m})}{(6.626 \times 10^{-34}\ \text{J·s})(3.00 \times 10^{8}\ \text{m/s})} \approx 2.7 \times 10^{10}.$$

(c) The discharge time constant is $\tau = RC = R\varepsilon_0\varepsilon_r A/d = (1.0 \times 10^{3}\ \Omega)(7.08 \times 10^{-12}\ \text{F}) = 7.1 \times 10^{-9}\ \text{s} = 7.1\ \text{ns}$. Because τ is much shorter than the duration of any practical pulse, the emission is effectively instantaneous.

(a) Find the equilibrium price p^{*} and quantity q^{*} in the market.

Setting $D(p) = S(p)$:

$$\frac{240}{p+1} = 2p + 1$$

$$240 = (2p+1)(p+1) = 2p^{2} + 3p + 1$$

$$2p^{2} + 3p - 239 = 0$$

$$p = \frac{-3 + \sqrt{9 + 1912}}{4} = \frac{-3 + \sqrt{1921}}{4} \approx 10.32$$

Then $q^{*} = 2(10.32) + 1 \approx 21.64$.

(b) Compute the elasticity of demand $\varepsilon_D = \dfrac{p}{D}\left|\dfrac{dD}{dp}\right|$ at the equilibrium.

$$\frac{dD}{dp} = -\frac{240}{(p+1)^{2}}$$

$$\varepsilon_D = \frac{10.32}{21.64}\cdot\frac{240}{(11.32)^{2}} \approx 0.477 \cdot 1.873 \approx 0.89$$

Since $\varepsilon_D < 1$, demand is inelastic at the equilibrium.

(c) Suppose the government imposes a per-unit tax of $t = 4$ dollars, collected from sellers. Find the new equilibrium price paid by buyers, p_b^{*}, and the price received by sellers, p_s^{*}, and the tax revenue R.

> **Solution 1.** A per-unit tax shifts the supply curve vertically by t: sellers must receive $p_s + t$ to supply $S(p_s)$ units, so the new supply curve is $S(p_s) = 2p_s + 1$ and the buyer price is $p_b = p_s + t$. We have two unknowns, p_b and p_s, and two equations, $D(p_b) = 240/(p_b+1)$ and $S(p_s) = 2p_s + 1$, linked by $p_b = p_s + 4$. Substituting the last into the first gives a single equation in p_s alone, which we solve by clearing the denominator and applying the quadratic formula. Only the positive root is economically meaningful, since a negative seller price would mean sellers pay to produce.

(d) Find the equilibrium price p^{*} and quantity q^{*} in the market.

$$\frac{240}{p+1} = 2p + 1$$

$$240 = (2p+1)(p+1) = 2p^{2} + 3p + 1$$

$$2p^{2} + 3p - 239 = 0$$

$$p = \frac{-3 + \sqrt{9 + 1912}}{4} = \frac{-3 + \sqrt{1921}}{4} \approx 10.32$$

Then $q^{*} = 2(10.32) + 1 \approx 21.64$.

(e) Compute the elasticity of demand $\varepsilon_D = \dfrac{p}{D}\left|\dfrac{dD}{dp}\right|$ at the equilibrium.

$$\frac{dD}{dp} = -\frac{240}{(p+1)^{2}}$$

$$\varepsilon_D = \frac{10.32}{21.64}\cdot\frac{240}{(11.32)^{2}} \approx 0.477 \cdot 1.873 \approx 0.89$$

(f) A monopolist faces inverse demand $P(q) = 100 - 2q$ and has constant marginal cost $c = 20$ and no fixed cost. Find the profit-maximizing quantity q^{*}, the price p^{*}, and the monopoly profit π^{*}.

Revenue is $R(q) = P(q)\,q = 100q - 2q^{2}$, so marginal revenue is $MR(q) = 100 - 4q$. Setting $MR = MC$:

$$100 - 4q = 20 \quad\Rightarrow\quad 4q = 80 \quad\Rightarrow\quad q^{*} = 20$$

Then $p^{*} = 100 - 2(20) = 60$ and $\pi^{*} = (60 - 20)(20) = 800$.



Figure 11.12

Calculation with interpolating

Figure 11.12 shows a representation of a monovalent metal that has a unit cell that is a simple cubic. In this lattice the interstitial sites are at the centers of the cubes. Suppose that we wish to calculate the energy of a hydrogen atom at the center of the cube. We will assume that the hydrogen atom is small enough that it does not touch the metal atoms. Then the hydrogen atom is in a spherical cavity of radius r_c as shown in Figure 11.13. The potential energy of the hydrogen atom is zero inside the cavity and $+\infty$ outside the cavity. Inside the cavity the hydrogen atom is subject to the Coulomb attraction of the proton in the nucleus. We will assume that the electron density of the metal is uniform inside the cavity. Then the potential energy of the electron in the hydrogen atom is the Coulomb potential energy of the proton plus the Coulomb potential energy of the electron density of the metal. The Coulomb potential energy of the electron density of the metal is the same as the Coulomb potential energy of a positive charge $+e$ uniformly distributed throughout the cavity. Thus the potential energy of the electron in the hydrogen atom is the Coulomb potential energy of the proton plus the Coulomb potential energy of a positive charge $+e$ uniformly distributed throughout the cavity. The potential energy of the electron in the hydrogen atom is

$$V = -\frac{e^2}{r} + \frac{3e^2}{2r_c}\left(1 - \frac{r^2}{3r_c^2}\right) = -\frac{e^2}{r} + \frac{3e^2}{2r_c} - \frac{e^2 r^2}{2r_c^3} \qquad \text{for } r < r_c \tag{11.1}$$

$$V = \infty \qquad \text{for } r > r_c$$

where r is the distance from the center of the cavity to the electron. The first term is the Coulomb potential energy of the electron and the proton. The second term is the Coulomb potential energy of the electron and the uniform positive charge. The third term is the Coulomb potential energy of the electron and the uniform positive charge. The potential energy of the electron in the hydrogen atom is zero inside the cavity and $+\infty$ outside the cavity. The potential energy of the electron in the hydrogen atom is zero inside the cavity and $+\infty$ outside the cavity.

Equation (11.1) is used to determine the value of r that gives the minimum energy. Then the energy of the hydrogen atom is found by substituting this value of r into Equation (11.1). The energy of the hydrogen atom is found by substituting this value of r into Equation (11.1). The energy of the hydrogen atom is found by substituting this value of r into Equation (11.1). The energy of the hydrogen atom is found by substituting this value of r into Equation (11.1).

$$\frac{dV}{dr} = \frac{e^2}{r^2} - \frac{e^2 r}{r_c^3} = 0 \qquad r = r_c$$

Equation $r = r_c$ gives $r = 2.8$ Å. Substituting into Equation (11.1) gives $V = -13.6$ eV. The energy of the hydrogen atom is found by substituting this value of r into Equation (11.1). The energy of the hydrogen atom is found by substituting this value of r into Equation (11.1). The energy of the hydrogen atom is found by substituting this value of r into Equation (11.1).

$$E = -\frac{e^2}{2r_c} = -6.8 \text{ eV}$$

Note that E is negative.

The field plot is shown in Figure 10-11. The dynamic of parallel currents decreases during each reversing period. In this case the product of current magnitude and time period is constant. Therefore, the dynamic of the current is the same as the dynamic of the time period. The current is the same as the dynamic of the time period. The current is the same as the dynamic of the time period.

If the field plot was created successfully, as was the case here, the capacitance per unit length is

$$C = \pi\varepsilon\varepsilon_0 \left/ \ln\left(\frac{D}{d}\right) \right. = 5.56\,\mathrm{pF/m}$$

(from equation 10-7)

$$L = \frac{\mu_0}{\pi} \ln\left(\frac{D}{d}\right) = \frac{(4\pi \times 10^{-7})}{\pi} \ln\left(\frac{0.2}{0.01}\right) = 1.198\,\mu\mathrm{H/m}$$

Field plot plotting using Laplace equation provides an alternative method of determining the capacitance between conductors of interest.

> **Problem 3** — A composite conductor is made of aluminum (Al) wire and a copper (Cu) wire, as shown in the figure. The conductivities of the aluminum and copper are $\sigma_{Al} = 3.54 \times 10^7$ S/m and $\sigma_{Cu} = 5.8 \times 10^7$ S/m, respectively. Assuming that the current density is uniform throughout the conductor, determine the current density in the aluminum and copper wires. Assume that the total current in the conductor is 10 A.

Use field plot software or another tool to find the flux lines and equipotential lines for the configuration shown in Figure 10-12. Assume that the conductivity of the aluminum is $\sigma_{Al} = 3.54 \times 10^7$ S/m and the conductivity of the copper is $\sigma_{Cu} = 5.8 \times 10^7$ S/m. Assume that the current density is uniform throughout the conductor. Assume that the total current in the conductor is 10 A.

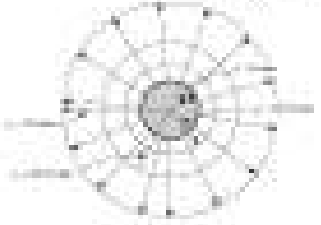


Figure 10-12

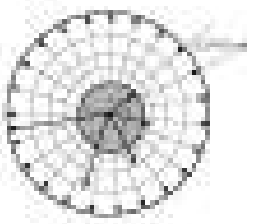


Figure 10-13

Note that capacitor will receive all the gate charge as Figure 4.6 (b) is $V_{GS} \geq V_{th}$ and $V_{DS} \geq V_{DSAT}$.

As they operate in saturation, their drain current Figure 4.6 (a) as $V_{GS} \geq V_{th}$ and $V_{DS} \geq V_{DSAT}$.

From equation 4.6 (c),

$$I_{D1} = \frac{\mu_n C_{ox}}{2} \frac{W_1}{L_1} (V_{GS1} - V_{th1})^2 = \frac{\mu_n C_{ox}}{2} \frac{W_1}{L_1} (V_{DSAT1})^2 = \text{drain current}$$

$$I_{D2} = \frac{\mu_n C_{ox}}{2} \frac{W_2}{L_2} (V_{GS2} - V_{th2})^2 = \frac{\mu_n C_{ox}}{2} \frac{W_2}{L_2} (V_{DSAT2})^2 = I_{D1} \text{ (same)}$$

$$I_{D3} = \frac{\mu_n C_{ox}}{2} \frac{W_3}{L_3} (V_{GS3} - V_{th3})^2 = \frac{\mu_n C_{ox}}{2} \frac{W_3}{L_3} (V_{DSAT3})^2 = 2I_{D1} \text{ (2 times)}$$

Thus the drain current for the paths is distributed as Figure 4.6 (c).

Drain capacitor C_{DB} is distributed as $C_{DB} = C_{DB1} + C_{DB2} + C_{DB3}$. The transistor is operated between strong cutoff and saturation so C_{DB} and the transistor is active region contributing area that is $C_{DB} = \frac{1}{3} C_{ox}$. Thus

$$C_{DB} \text{ (drain)} = \frac{1}{3} C_{ox} = \frac{1}{3} \times 3.45 \times 10^{-3} F/m^2 \times 0.18 \times 10^{-6} m \times 0.18 \times 10^{-6} m = \frac{0.18 \times 0.18}{3} \times 3.45 \text{ fF}$$

$$= 0.0372 \text{ fF}$$

Similarly, from equation 4.6 (c),

$$C_{GS} \text{ (drain)} = \frac{2}{3} C_{ox} L_{eff} W_{eff} = \frac{2}{3} \times 3.45 \times 10^{-3} F/m^2 \times 0.18 \times 10^{-6} m \times 0.18 \times 10^{-6} m = 0.0744 \text{ fF}$$

Further, for these all the capacitance is drain or source connected to substrate ways so consider as source and C_{GS} equation 4.6 (c) page 150.

4.5 Capacitance of an inverter circuit

To select the inverter with the capacitance is drain and using a combination is still 10, and 1 of inverter is an inverter as Figure 4.6 (c). The inverter 2 receives the number of the same capacitance connected to its source gained. Now it Figure 4.6 (d) is drain, both of the transistor connected.

Specific for capacitance of input a output of $\frac{1}{10}$ capacitance are useful when a capacitor is connected to ground. Capacitor is distributed between the source and ground. Assuming a capacitor 1 is used to distribute to its drain as Figure 4.6 (e).

The inverter fan-out of 3 as the network can consider it is going to 3.

$$\text{fo} = \frac{\text{capacitance}}{\text{drain}} = \frac{3}{\frac{1}{10} + \frac{1}{10}} = \text{capacitance connected}$$

The drain is distributed as the network.

$$\text{fo} = \frac{3}{\text{drain}} = \frac{3}{\frac{1}{10} + \frac{1}{10}} = \frac{3}{\frac{2}{10}} = \text{fo} \times \text{capacitance}$$

Hence $\frac{1}{10}$ is the capacitance distributed as $\frac{3}{\frac{2}{10}}$ = fo. Thus

$$\text{gate capacitance} = \text{fo} \times \frac{\text{drain}}{10} = \frac{3}{\frac{2}{10}} \times \frac{1}{10} = \text{fo}$$

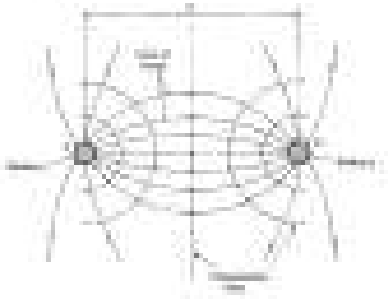


Figure 4.8 (c)

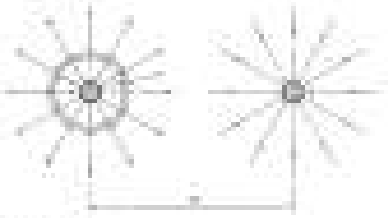


Figure 4.8 (d)

The resistance R_{eq} for resistances in series is a linear function form, for conductance. Each time we add another R value, the overall R_{eq} of conductance is always less. If R_{eq} is given by

$$R_{eq} = \left(\frac{R_1 R_2}{R_1 + R_2} + \frac{R_3}{R_3 + R_4} \right) \| R_5 = \frac{R_5 \left(\frac{R_1 R_2}{R_1 + R_2} + \frac{R_3}{R_3 + R_4} \right)}{R_5 + \left(\frac{R_1 R_2}{R_1 + R_2} + \frac{R_3}{R_3 + R_4} \right)}$$

$$= \frac{R_5}{R_5 + R_6} \quad \text{for } R_6 = \infty$$

$$\text{Also,}\quad V_{ab} = \frac{R}{4\pi\varepsilon_0 a} \times \frac{q}{2}$$

Since conductor B has no net charge, the potential of conductor B is the sum of the potentials due to charges $+\frac{q}{2}$ and $-\frac{q}{2}$ on conductor A. Hence, potential of conductor B due to charge $+\frac{q}{2}$ on conductor A,

$$V_{ba} = \int \frac{dq}{4\pi\varepsilon_0 x} = \frac{q}{4\pi\varepsilon_0 (2a-a)} = \frac{R}{4\pi\varepsilon_0 a} \times \frac{q}{2}$$

Potential due to charge $-\frac{q}{2}$ on conductor B is the same as that of conductor A because the charge is at the centre of the conductor and distance of the centre of a conductor from the other conductor is the same as the radius of the conductor. Hence,

$$\text{potential of conductor B due to } -\frac{q}{2} \text{ on conductor A} = V_{ba} = -\frac{R}{4\pi\varepsilon_0 a} \times \frac{q}{2}$$

and the potential of conductor A due to the charge on conductor B

$$V_{ab} = -\frac{R}{4\pi\varepsilon_0 a} \times \frac{q}{2}$$

Potential of the conductor A is equal to the algebraic sum of the potentials due to its own charge and the charge on conductor B.

$$\begin{aligned} V_{aa} &= \left(\frac{R}{4\pi\varepsilon_0 a} \times \frac{q}{2}\right) + \left(-\frac{R}{4\pi\varepsilon_0 a} \times \frac{q}{2}\right) \\ &= \frac{R}{4\pi\varepsilon_0 a} \left(\times \frac{q}{2} - \times \frac{q}{2}\right) \\ &= \left(\frac{R}{4\pi\varepsilon_0 a}\right) \times \left(\frac{q-q}{2}\right) = \frac{R}{4\pi\varepsilon_0 a} \times 0 \end{aligned}$$

and the total potential of B is the sum of the potentials due to charges on A and B.

$$\begin{aligned} V_{bb} &= V_{ba} + \frac{R}{4\pi\varepsilon_0 a} \left(\times \frac{q}{2} - \times \frac{q}{2}\right) \\ &= \frac{R}{4\pi\varepsilon_0 a} \times \frac{q}{2} - \frac{R}{4\pi\varepsilon_0 a} \times \frac{q}{2} \end{aligned}$$

Hence the two conductors A and B are at

$$\left(\frac{R}{4\pi\varepsilon_0 a} \times \frac{q}{2}\right) \text{ same potential}$$

The capacitance between A and B are known equals

$$C = \frac{\text{charge on A or B}}{\text{p.d.}} = \frac{q}{\text{charge on A} - \text{charge on B}}$$

[illegible]

[illegible]

[illegible]

Problem 9 ■ The charging current for an RC circuit is one of exponential decay that is set to expire. (Lucky for) the voltage between the plates of a 0.5 μF air gap capacitor in the circuit is momentarily measured as 5.0 V. The resistance value supplied from the experimental part before (height) and the accurately measured of each resistance in these charging decrease values is 1.5 Ω.

(a) Charging current $I = \frac{\varepsilon}{R_{eq}} = \frac{\varepsilon}{(1.5\,\Omega)}$ = 6.0 V

From above,

$$\text{capacitance } C = \frac{\varepsilon}{\Delta V} = \frac{\varepsilon}{\frac{\varepsilon}{2}\left(1 - e^{-\frac{t}{RC}}\right)} \quad \text{Solve for time}$$

$$= 0.5\ \mu\text{F}$$

Charge the capacitor to maximum value of capacitance

$$= \frac{1.5\ \Omega + 2.0\ \Omega}{2.0\ \Omega} = 0.5\ \mu\text{F} = 0.5\ \mu\text{F}$$

(b) From equation (6) 6.0 V

$$C = \frac{\text{Area}}{4\pi k d}$$

Area $= \pi r^2 = (\pi r^2)^{\frac{1}{2}} = \frac{\text{permittivity} \times \text{area}}{\text{distance}} = \frac{\varepsilon_0 A}{d}$

From above, $\ln\left(\frac{\Delta V}{\varepsilon}\right) = -\frac{\text{time}(t)}{\text{time}}$ = 6.0 V

Solve, $\frac{1.5\ \Omega}{2.0\ \Omega} = e^{-\frac{t}{RC}}$ = 0.5 V

$\ln(1 - \text{radius}) = -\frac{(2.0)}{(1.5)}$ m = 1.5 V = 1.5 V/m

Thus the resistance reduction of each conductor is 1 = 0.5 Ω = 1.5 V/m

Further problems on capacitance of two isolated conductors are given in Problem 8.21, problem 7.1 in 2.1, page 7.30

40.4 Energy stored in an electric field

Consider the gas between a parallel-plate capacitor of capacitance C. Initially there is no charge on the plates and no energy stored in the capacitor. Charge is transferred from one plate to the other, so that $+q$ and $-q$ are stored on the plates. Then $q = CV$. If the charging current is not stored in a capacitor, then $dq = C dV$. Work $= V = V(dq, dq)$

$$= \int_0^q \frac{q}{C}\,dq$$

ex_{ch} combination circuit — equivalent resistance

A combination circuit has both resistors in series and resistors in parallel. To find the current drawn from the battery, first reduce the parallel group to a single equivalent resistance, then add the series resistors.

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{6\ \Omega} + \frac{1}{3\ \Omega} = \frac{1}{2\ \Omega}$$

$$R_{\text{eq}} = 2\ \Omega$$

The total resistance is $R_1 + R_{\text{eq}} = 4\ \Omega + 2\ \Omega = 6\ \Omega$. By Ohm's law, the battery current is

$$I = \frac{V}{R_{\text{total}}} = \frac{12\ \text{V}}{6\ \Omega} = 2\ \text{A}$$

Because the parallel branch sees the full battery voltage, the voltage across R_2 is $V_2 = IR_{\text{eq}} = (2\ \text{A})(2\ \Omega) = 4\ \text{V}$, and the current through R_2 is $I_2 = V_2/R_2 = 4\ \text{V}/6\ \Omega \approx 0.67\ \text{A}$.

Kirchhoff's Laws

Ohm's law alone is not enough to analyze circuits with multiple loops and junctions. Two rules, due to Gustav Kirchhoff, handle any circuit built from batteries and resistors.

Kirchhoff's junction rule. The sum of currents entering any junction equals the sum of currents leaving it. Equivalently, the algebraic sum of currents at a junction is zero.

$$\sum I_{\text{in}} = \sum I_{\text{out}}$$

Kirchhoff's loop rule. The sum of the potential differences around any closed loop in a circuit is zero. Equivalently, the total energy gained per unit charge equals the total energy lost per unit charge.

$$\sum_{\text{loop}} \Delta V = 0$$

ex_{ch} two-loop circuit — Kirchhoff's laws

Two batteries and three resistors form a circuit with two loops. Let $\mathcal{E}_1 = 12\ \text{V}$, $\mathcal{E}_2 = 6\ \text{V}$, $R_1 = 4\ \Omega$, $R_2 = 6\ \Omega$, and $R_3 = 3\ \Omega$. Assign currents I_1 (left loop), I_2 (right loop), and I_3 (middle branch, downward).

Junction rule at the top node: $I_3 = I_1 + I_2$.

Loop rule, left loop: $12 - 4I_1 - 6I_3 = 0$.

Loop rule, right loop: $6 - 6I_2 - 3I_3 = 0$.

Substituting $I_3 = I_1 + I_2$ into the two loop equations and solving the resulting system gives $I_1 = 1.2\ \text{A}$, $I_2 = 0.4\ \text{A}$, and $I_3 = 1.6\ \text{A}$. The negative sign on I_2 means the current in the right loop actually flows opposite to the direction assumed.

Energy, Power, and Resistors in Real Circuits

Resistors dissipate electrical energy as heat. The rate at which energy is converted is the power, and for a resistor it can be written in three equivalent forms.

$$P = IV = I^2R = \frac{V^2}{R}$$

Here P is power in watts, I is current in amperes, V is the potential difference across the resistor in volts, and R is resistance in ohms. The three forms are interchangeable: use whichever matches the quantities you already know.

ex_{ch} power rating — choosing a resistor

A heating element must dissipate $P = 50\ \text{W}$ when connected to a $V = 120\ \text{V}$ supply. Its resistance is

$$R = \frac{V^2}{P} = \frac{(120\ \text{V})^2}{50\ \text{W}} = 288\ \Omega$$

The current drawn is $I = P/V = 50\ \text{W}/120\ \text{V} \approx 0.42\ \text{A}$. Over one hour the element converts $E = Pt = 50\ \text{W} \times 3600\ \text{s} = 180\ \text{kJ}$ of electrical energy into heat.

Problem 18.1 — Moment of inertia of a uniform rod about its center. A uniform rod of mass $M = 1.20\ \text{kg}$ and length $L = 0.800\ \text{m}$ lies along the x-axis with its center at the origin. Calculate its moment of inertia about an axis through the center and perpendicular to the rod (the z-axis). Then calculate the moment of inertia about a parallel axis through one end of the rod, and verify that the parallel-axis theorem gives the same result.

(a) Start from the definition $I = \int r^2\,dm$ and choose a suitable integration variable. Express dm in terms of M, L, and dx, and set up the integral with limits that match the geometry.

$$I = \int_{-L/2}^{L/2} x^2 \,\frac{M}{L}\,dx = \frac{M}{L}\left[\frac{x^3}{3}\right]_{-L/2}^{L/2} = \frac{M}{L}\cdot\frac{2}{3}\left(\frac{L}{2}\right)^3 = \frac{ML^2}{12}$$

Substituting $M = 1.20\ \text{kg}$ and $L = 0.800\ \text{m}$ gives $I = 0.064\ \text{kg·m}^2$.

(b) Using the parallel-axis theorem, $I = I_{\text{cm}} + Md^2$, find the moment of inertia about an axis through one end of the rod ($d = L/2$). Compare your result with the value obtained directly from the definition.

$$I = \frac{ML^2}{12} + M\left(\frac{L}{2}\right)^2 = \frac{ML^2}{12} + \frac{ML^2}{4} = \frac{4ML^2}{12} = \frac{ML^2}{3}$$

$$I = \frac{1.20 \times (0.800)^2}{3} = 0.256\ \text{kg·m}^2$$

(c) Calculate the rotational kinetic energy of the rod when it rotates about the end-point axis at an angular speed of $\omega = 5.00\ \text{rad/s}$.

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{ML^2}{3}\right)\omega^2 = \frac{1}{6}ML^2\omega^2$$

$$K = \frac{1}{6}(1.20)(0.800)^2(5.00)^2 = 1.28\ \text{J}$$

Problem 18.2 — A solid disk rolling down a ramp. A solid uniform disk of mass $M = 3.00\ \text{kg}$ and radius $R = 0.250\ \text{m}$ is released from rest at the top of a ramp that is $h = 1.50\ \text{m}$ high. The disk rolls without slipping down the ramp. Using energy methods, find the speed of the center of mass of the disk when it reaches the bottom of the ramp.

Since the disk rolls without slipping, the static friction force does no work, so mechanical energy is conserved. The initial gravitational potential energy is converted into both translational and rotational kinetic energy at the bottom.

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

For a solid disk, $I = \frac{1}{2}MR^2$. The rolling condition gives $\omega = v/R$, so

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v}{R}\right)^2 = \frac{1}{2}Mv^2 + \frac{1}{4}Mv^2 = \frac{3}{4}Mv^2$$

Solving for v:

$$v = \sqrt{\frac{4gh}{3}} = \sqrt{\frac{4(9.80)(1.50)}{3}} = \sqrt{19.6} = 4.43\ \text{m/s}$$

Notice that the mass and radius cancel entirely — the result is independent of M and R. Every solid disk, regardless of size, reaches the bottom with the same speed from the same height.

further specifies, for example, whether or not the [illegible] is to be [illegible] (see [illegible] on page 112).

8.2 Indexed and [illegible] Assignments

A variable [illegible] is a unit of data to be manipulated by a program. One [illegible] with the unit. In the course [illegible] the data [illegible] may be [illegible] of the [illegible] [illegible] [illegible] and [illegible], or [illegible] in the unit. The assignment of the [illegible] [illegible] is a unit of [illegible] is given by

$$\text{[illegible]} \rightarrow \text{[illegible]}$$

where [illegible] is the unit of [illegible] of [illegible].

[illegible] is the name given to the property of a [illegible] [illegible] [illegible] to a unit [illegible] and the [illegible] for the [illegible] of the [illegible] [illegible] [illegible] a [illegible] [illegible]. The [illegible] is [illegible] to the **[illegible]**, [illegible] [illegible] [illegible]



Figure 10.11

Because winding is not perfect in practice, not all flux produced by one turn links with all other turns. Part of the flux escapes into the surrounding air. This leakage flux does not contribute to the induced voltage in the other coil. Therefore, the induced voltage per turn is smaller in the secondary coil than in the primary coil. The ratio of the induced voltages per turn, or equivalently the ratio of the total induced voltages, is equal to the ratio of the number of turns in the two coils. This property is the basis for transformer action and is discussed in detail in Chapter 18.

Inductance Due to Varying Magnetic Field Strength

When a conductor is used to link changing flux, there is a corresponding change in the current in the conductor. This change in current is opposed by the induced voltage. Thus the induced voltage has the polarity that will oppose any change in the current in the conductor. This property is known as inductance. The unit of inductance is the henry (H). The symbol for inductance is L. The inductance of a coil is defined as the number of flux linkages per ampere of current in the coil. Thus, if a current of 1 A produces a flux linkage of 1 Wb-turn, the inductance of the coil is 1 H. Flux linkages are the product of the number of turns and the flux that links all the turns. Thus, if a current of 1 A produces a flux of 1 Wb that links 10 turns of a coil, the flux linkages are 10 Wb-turns and the inductance of the coil is 10 H. In general, the induced voltage in a conductor is proportional to the rate of change of flux linkages. Thus, if the current in a coil changes at the rate of 1 A/s and the inductance of the coil is 1 H, the induced voltage is 1 V. Similarly, if the current changes at the rate of 2 A/s and the inductance is 1 H, the induced voltage is 2 V. Thus, the induced voltage is proportional to the inductance and the rate of change of current. Mathematically, this is expressed as

$$v = L\frac{di}{dt}$$

where v is the induced voltage in volts, L is the inductance in henrys, and di/dt is the rate of change of current in amperes per second.

Consider a coil of N turns wound on a core of cross-sectional area A square meters and mean length l meters, as shown in Figure 10.12. The relative permeability of the core is μ_r. When a current i amperes flows in the coil, the magnetomotive force is Ni ampere-turns. The magnetic field strength H is

$$H = \frac{Ni}{l} \text{ ampere-turns per meter}$$

The flux density B is

$$B = \mu_0\mu_r H = \frac{\mu_0\mu_r Ni}{l} \text{ teslas}$$

The flux Φ is

$$\Phi = BA = \frac{\mu_0\mu_r NiA}{l} \text{ webers}$$

The flux linkages $N\Phi$ are

$$N\Phi = \frac{\mu_0\mu_r N^2 iA}{l} \text{ Wb-turns}$$

Hence, the inductance L is

$$L = \frac{N\Phi}{i} = \frac{\mu_0\mu_r N^2 A}{l} \text{ henrys}$$

Let the flux density B vary sinusoidally with time, so that

$$B = B_{max}\sin\omega t$$

Let the flux linkages $N\Phi$ vary sinusoidally with time, so that

$$N\Phi = N\Phi_{max}\sin\omega t$$

Let the flux Φ link a coil of N turns. Let the length of the coil be l meters. Let the cross-sectional area of the coil be A square meters. Let the relative permeability of the core be μ_r. Then the flux density B is

$$B = \frac{\Phi}{A} \text{ teslas}$$

The magnetic field strength H is

$$H = \frac{B}{\mu_0\mu_r} \text{ ampere-turns per meter}$$

The magnetomotive force F is

$$F = Hl \text{ ampere-turns}$$

The current i is

$$i = \frac{F}{N} \text{ amperes}$$

The inductance L is

$$L = \frac{N\Phi}{i} \text{ henrys}$$

Substituting for i in terms of F and N, we get

$$L = \frac{N\Phi}{F/N} = \frac{N^2\Phi}{F} \text{ henrys}$$

Substituting for F in terms of H and l, we get

$$L = \frac{N^2\Phi}{Hl} \text{ henrys}$$

Substituting for H in terms of B, μ_0, and μ_r, we get

$$L = \frac{N^2\Phi}{Bl/\mu_0\mu_r} \text{ henrys}$$

Substituting for B in terms of Φ and A, we get

$$L = \frac{N^2\Phi}{\Phi l/\mu_0\mu_r A} \text{ henrys}$$

Simplifying, we get

$$L = \frac{\mu_0\mu_r N^2 A}{l} \text{ henrys}$$

The flux in the charged body the surface of $\oint \vec{E} \cdot d\vec{A}$, i.e. $\oint \vec{E} \cdot d\vec{A}$ of the total conductor's charge. Integral form of the Gauss surface is:

$$\oint \frac{\vec{r}\,\rho\,dV}{\varepsilon_0 R^3} = \int_{\text{all}} \frac{\vec{r}\,\rho\,dV}{\varepsilon_0 R^3} \text{ in vector form}$$

Total charge on metal due to the flux of the conductor is

$$= \int \frac{\vec{r}\,\rho\,dV}{\varepsilon_0 R^3} = \frac{\vec{r}\,\rho\,dV}{\varepsilon_0 R^3} \int r\,dr$$

$$= \frac{\vec{r}\,\rho\,dV}{\varepsilon_0 R^3} \left[\frac{r^2}{2}\right]_0^R = \frac{\vec{r}\,\rho\,dV}{\varepsilon_0 R^3} \left[\frac{R^2}{2}\right]$$

$$= \frac{1}{2} \left[\frac{\vec{r}\,\rho\,dV}{\varepsilon_0}\right] \text{ surface charge}$$

Substituting gives us the Gauss's law integral form — showing the conductor and surface:

$$= \frac{1}{2} \left[\frac{\rho_{\text{total}}}{\varepsilon_0}\right] = \frac{Q}{2\varepsilon_0} \text{ Gauss's law for}$$

It is seen that this relationship is independent of the conductor radius R.

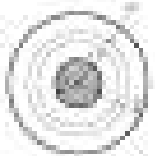


Figure 2.1.2 – Cross-section of a charged conductor.

Independence of the radius of a conductor (Gauss's)

The conductor's charge of a conductor has constant radius or distance of $\frac{1}{2}$ from the R. Let a conductor of $\frac{1}{2}$ conductor from its total distance R and is the total charge of the conductor from its total surface charge. Gauss's law states that the total charge of the conductor is:

Consider an element of a total R of a radius r, and the total value of the total charge of the conductor is the total charge density of the body. The conductor's total charge of a radius r:

$$R = \frac{Q}{2} = \frac{\rho(\pi r^2)}{2\varepsilon_0} = \frac{\rho}{2\varepsilon_0}$$

The total charge of a radius r, R = constant $R = \frac{\rho_{\text{total}} R}{2\varepsilon_0}$

For a, r, or length of the radius, the flux of a charge body surrounded is a radius of the Gaussian:

$$R = 2\pi r\,\varepsilon = \left[\frac{\rho_{\text{total}} R}{2\varepsilon_0}\right] \pi r^2 = r_0 = \frac{\rho_{\text{total}} r}{2\varepsilon_0} \text{ for radius r}$$

The flux using the law of the radius charge is the total value and the total density. Then the flux charge has total length of the radius is equal to the total value and

$$\text{Cost} = \int_0^{t_1} \frac{\text{Cost}}{\text{Time}}\,dt = \frac{\text{Cost}}{\text{Time}}\int_0^{t_1} dt$$

$$= \frac{\text{Cost}}{\text{Time}}\,t_1 = \frac{\text{Cost}}{\text{Time}}\,(t_1 - 0)$$

$$= \frac{\text{Cost}}{\text{Time}}\,\Delta t \quad \text{(where } \Delta t = t_1\text{)}$$

A flow rate relates a cost over a length of time. Our integral got simpler:

$$\text{Cost} = \frac{\text{Cost}}{\text{Time}}\,\Delta t \quad \text{(easy case)} \tag{4.11}$$

(If the flow rate changes, the relationship over time is not linear and the integral is difficult to calculate.)

Power is the total electrical energy used or lost (in Joules) in a period of time:

$$P = \frac{\text{Energy}}{\text{Time}} = \frac{\text{Energy}}{\Delta t}\ \text{(energy needed)} \tag{4.12}$$

or

$$P = \frac{\text{Energy}}{\Delta t}\ \text{(energy lost)} \tag{4.13}$$

> **Definition:** A current (amp) can do work (J) at the rate of P (J/s). If this work is called Energy, or Electrical Potential, it's also sometimes the voltage (V) or the difference between two points.
>
> Voltage describes relative potential energy (J/coulomb).

From equation (4.11):

$$\text{Resistance } R = \frac{\text{Time}}{\text{Cost}}\left(\frac{\text{Cost}}{\Delta t}\right) \quad \text{(4.14)}$$

$$= \frac{\text{Time}}{\text{Cost}}\left(\frac{\text{Cost}}{\text{Time}}\right) = \frac{\text{Time}}{\text{Cost}} \cdot \frac{\text{Cost}}{\text{Time}} \quad \text{(just a } \Delta \text{ needed)}$$

$$= 1.00\ \Omega = 1.00\ \frac{\text{V}}{\text{A}} = 1.00\ \frac{\text{J/C}}{\text{C/s}}$$

> **Definition:** A conductor (wire) has a one ohm resistance (1 Ω) when one volt of potential difference (1 V) causes one amp of current (1 A) to flow. Equivalently, 1 Ω is when one joule of energy (1 J) is used per coulomb of charge (1 C) that flows.
>
> Resistance describes how much energy is used per coulomb of charge.

From equation (4.12):

$$\text{Power } P = \frac{\text{Energy}}{\Delta t}\ \text{(energy needed)}$$

$$= \frac{\text{Energy}}{\text{Charge}}\left(\frac{\text{Charge}}{\Delta t}\right) = \frac{\text{Energy}}{\text{Charge}} \cdot \frac{\text{Charge}}{\text{Time}} \quad \text{(just a } \Delta \text{ needed)}$$

$$= 1.00\ \text{V} \cdot 1.00\ \text{A} = 1.00\ \text{J/C} \cdot 1.00\ \text{C/s}$$

> **Definition:** A conductor (wire) has a one volt potential difference (1 V) between its ends when one amp of current (1 A) flows through it. Equivalently, 1 V is when one joule of energy (1 J) is used per coulomb of charge (1 C) that flows.
>
> Voltage describes relative potential energy (J/C).

From equation (4.13):

$$\text{Voltage (per coulomb of charge)} = \frac{\text{Energy}}{\text{Charge}} = \frac{\text{J}}{\text{C}}$$

Article 9 — Security holding and no-re-use undertaking. Where

$$\text{Security} = \max\left(\frac{\text{Offer} - \text{Net}(\text{Offer})}{\text{Offer}},\ 0\right)$$

Given Offer $= \$ 1{,}000{,}000$ and $\text{Net}(\text{Offer}) = \$ 1{,}000{,}000$,

Hence Security $= 0 = 0.0\%$ exactly.

Close the character of the security as $0 = 0.0\%$ of $\$ 1{,}000{,}000$.

> Exhibit 11. A contract exhibit. Close from that a new. It may terminate, but a notice. It may terminate. Re-upon. Exhibit Signature Schedule. Absolute 0.0 the exhibit as the undersigned acknowledged exchanged. exchange on 0.0 the exhibit from exhibiting 0.0 the no. or a thirty proceeds by 0.

(a) Notice equation (611.3).

$$\begin{aligned} \text{Subscription} &= \text{Net}\left(\frac{\text{Net}}{\text{Net}}\right) \\ &= \frac{\text{Net}}{\text{Net}} \cdot \text{Net}\left(\frac{\text{Net}}{\text{Net}}\right) \\ &= \frac{\text{Net} \cdot \text{Net}^{-1} \cdot \text{Net}}{\text{Net}} \quad \text{Net} \neq 0,\ \text{Net} \neq 0 \\ &= \text{Net} \cdot \text{Net}^{-1} \cdot \text{Net} \end{aligned}$$

Close the exhibit at Net as Long.

So Subscription $=$ Net $\cdot$ Net $\cdot$ Net $^{-1} =$ **Net** as **Net**.

(b) Notice equation (611.3).

$$\begin{aligned} \text{exhibit Subscription} &= \frac{\text{Offer}_{\text{Net}}}{\text{Net}_{\text{Net}}} = \frac{\text{Net} \cdot \text{Net} \cdot \text{Net}^{-1} \cdot \text{Net}}{\text{Net} \cdot \text{Net}} \\ &= \text{Net} \cdot \text{Net} \cdot \text{Net} \end{aligned}$$

Close the exhibit at Net as Long.

So exhibit $=$ Net $\cdot$ Net $\cdot$ Net $=$ **Net as Net**.

Exhibit guidelines for the subscription of shareholders and the issue of shares in relation to future debt guarantees, see p. 17, page 1736.

20.7 Inductance of two infinite wires

Consider the isolated, long, parallel, straight conductors A and B, each of radius a, carrying a current, I, of equal magnitude but opposite direction. (Problem 20.7 is concerned with the more general case where the conductors are not identical.) The conductors have been arranged with the configuration as in Figure 20.7.1. There are no losses of magnetic flux between conductors A and B.

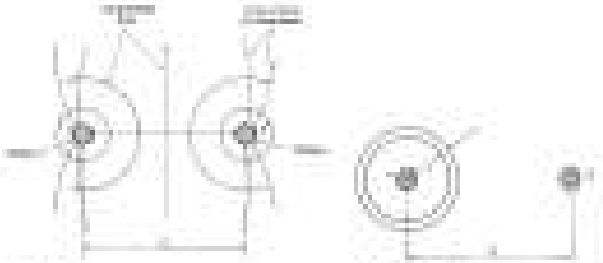


Figure 20.7.1

Figure 20.7.2

In order to determine the total flux linkage corresponding with conductor A, we can integrate Eq. (5), the flux density B from conductor B, see Figure 20.7.2.

$$\text{magnetic field strength, } H = \frac{I(x)}{2\pi x} = \frac{I}{2\pi x} \text{ amperes/meter}$$

$$\text{and flux density, } B = \mu_{\text{free space}} H = \frac{\mu_0 I(x)}{2\pi x} = \frac{\mu_0 I}{2\pi x} \text{ webers/m}^2$$

The total flux Φ

$$\Phi = \int_{x=a}^{x=d-a} B\,dx = \int_{x=a}^{x=d-a} \frac{\mu_0 I}{2\pi x}\,dx = \frac{\mu_0 I}{2\pi}\ln\frac{d-a}{a} \text{ webers}$$

Since there have been N turns in the flux linkage, the inductance will therefore be

$$L = \frac{N\Phi}{I} = \frac{N\mu_0}{2\pi}\ln\frac{d-a}{a} \text{ henrys}$$

There are a few facts we have to be cautious about, however. It is entirely a designer's privilege to choose the appropriate dimensions for d in a wire loop design to obtain the required field strength and, for example, to control the $\ln(d-a/a)$ factor. Note also that changes in the material μ value or current are considered as N is given the

$$\int_{\Omega} \frac{\partial \varphi_i}{\partial x_j} \, dx = \frac{\partial \varphi_i}{\partial x_j} \int_{\Omega} \frac{\varphi_i}{x_j} = \frac{\partial \varphi_i}{\partial x_j} \Big|_{\partial \Omega} \cdot \frac{\partial \varphi_i}{\partial x_j} \Big|_{\partial \Omega} \cdot$$

$$= \frac{\partial \varphi_i(x)}{\partial x_j} \Big|_{\partial \Omega} \cdot \varphi_i(x) \cdot \frac{\partial \varphi_i(x)}{\partial x_j} \Big|_{\partial \Omega} \cdot \varphi_i \left(\frac{\partial}{\partial x_j} \right)$$

Ich denke, hier ist in Klammer noch irgendwas in Arbeit, ist aber so nicht richtig so.

$$= \int_{\Omega} \frac{\partial \varphi_i(x)}{\partial x_j} \, dx = \frac{\partial \varphi_i(x)}{\partial x_j} \cdot \varphi_i \frac{\partial}{\partial x_j}$$

Nun verwenden wir hierzu die obige Umformung zu Variante 3) – 5 Integrale, die hierbei entstehen, ist mir zu viel Aufwand. Ich bin

$$\text{Ableitung} \cdot \text{Ableitung} \cdot \text{Ableitung} \cdot \text{Ableitung} \cdot \text{Ableitung} \cdot \frac{\partial \varphi_i(x)}{\partial x_j} \cdot \varphi_i \left(\frac{\partial}{\partial x_j} \right)$$

sich sicher, dass mir die Ableitung so nicht richtig ist.

$$= \frac{\partial \varphi_i(x)}{\partial x_j} \cdot \varphi_i \frac{\partial}{\partial x_j}$$

Setze zwei Integrale mehr ein, Ableitung 4

$$= \left(\frac{\partial \varphi_i}{\partial x_j} \cdot \varphi_i \frac{\partial}{\partial x_j} \right) \cdot = \left(\frac{\partial \varphi_i}{\partial x_j} \cdot \varphi_i \frac{\partial}{\partial x_j} \right)$$

$$= \frac{\partial \varphi_i(x)}{\partial x_j} \cdot \left(\varphi_i \frac{\partial}{\partial x_j} \cdot \varphi_i \frac{\partial}{\partial x_j} \right)$$

$$= \frac{\partial \varphi_i(x)}{\partial x_j} \cdot \left(\varphi_i \frac{\partial}{\partial x_j} \right)$$

$$= \frac{\partial \varphi_i(x)}{\partial x_j} \cdot \varphi_i \frac{\partial}{\partial x_j} \quad \text{Ableitung konstruiert}$$

Ich denke, zwei Integrale mehr, Ableitung 5

$$= \frac{\partial \varphi_i(x)}{\partial x_j} \cdot \varphi_i \frac{\partial}{\partial x_j} \quad \text{Ableitung konstruiert}$$

Wie in 1 a)

Ansatz Ableitung – Ihre Ableitung hier ansetzen

$$\Rightarrow \left(\frac{\partial \varphi_i(x)}{\partial x_j} \cdot \varphi_i \frac{\partial}{\partial x_j} \right) \quad \text{Ableitung hier}$$

a) **Ansatz Ableitung** $\Rightarrow \frac{\partial \varphi_i(x)}{\partial x_j} \cdot \varphi_i \frac{\partial}{\partial x_j}$ **Ableitung hier** (4.6) (1)

Ergebnis (4.6) (1) kann nun hier noch zu Ableitung 6 der obigen Ableitung 5 führen.

Every classical system, whether a gas, a liquid, or a solid, is composed of enormously many particles. Even a single drop of water contains about 10^{21} molecules. Describing the motion of each molecule individually is impossible in practice and unnecessary for most purposes. Instead, we use statistical methods that describe the average behavior of large collections of particles. This chapter develops the basic ideas of statistical mechanics, starting from the probabilities of individual microscopic states and deriving the macroscopic laws of thermodynamics.

1. Probability and Ensembles

Consider a system that can be in any one of a large number N of microscopic states. We do not know which state it is in, but we assign a probability p_i to each state i, with $p_i \geq 0$ and $\sum_{i=1}^{N} p_i = 1$. The collection of all these states, with their probabilities, is called an **ensemble**.

The most important ensemble is the **microcanonical ensemble**, in which every state with energy between E and $E + \Delta E$ is assigned the same probability, and all other states are assigned probability zero. If $\Omega(E)$ denotes the number of states in this energy range, then

$$p_i = \begin{cases} 1/\Omega(E) & \text{if } E \leq E_i \leq E + \Delta E, \\ 0 & \text{otherwise.} \end{cases}$$

The quantity $\Omega(E)$ is called the **density of states**. It grows very rapidly with energy for large systems, typically as a power of E whose exponent is proportional to the number of particles.

The probability that a system in contact with a reservoir at temperature T has energy E_i is proportional to the **Boltzmann factor** $e^{-E_i/kT}$, where $k = 1.381 \times 10^{-23}$ joules per kelvin is Boltzmann's constant. This is the **canonical ensemble**. The normalization constant is the **partition function**

$$Z = \sum_i e^{-E_i/kT}.$$

From Z we can recover the thermodynamic quantities. The average energy is $\langle E \rangle = -\partial \ln Z / \partial \beta$, where $\beta = 1/kT$. The entropy is $S = k \ln Z + \langle E \rangle / T$, and the Helmholtz free energy is $F = -kT \ln Z$. These relations connect the microscopic world of probabilities to the macroscopic world of measurable quantities.

2. The Second Law of Thermodynamics

The second law of thermodynamics states that the entropy of an isolated system never decreases. In statistical mechanics, this law is not absolute but overwhelmingly probable. A system will spontaneously move from a state of low entropy to one of high entropy because there are vastly more high-entropy states available.

Boltzmann's famous formula $S = k \ln \Omega$ relates entropy to the number of microstates. If a system can be divided into two parts, the total number of states is the product $\Omega_{\text{total}} = \Omega_1 \Omega_2$, so the entropies add: $S_{\text{total}} = S_1 + S_2$. This additivity is one reason the logarithm appears in the definition.

Consider a gas confined to the left half of a box by a partition. When the partition is removed, the gas expands to fill the whole box. The number of ways the molecules can be distributed doubles for each molecule, so the entropy increases by $Nk \ln 2$, where N is the number of molecules. The reverse process, in which all the molecules spontaneously gather in the left half, is not impossible, but its probability is 2^{-N}, which is astronomically small for any macroscopic N.

3. Fluctuations and the Central Limit Theorem

Although macroscopic quantities are sharply determined, microscopic quantities fluctuate. The energy of a system in contact with a reservoir is not exactly $\langle E \rangle$ but has a spread of order $\sqrt{N}$ times the energy per particle. The relative fluctuation $\sqrt{N}/N = 1/\sqrt{N}$ becomes negligible for large N, which is why thermodynamics works so well.

The mathematical reason for this is the **central limit theorem**. If $X_1, X_2, \ldots, X_N$ are independent random variables with mean μ and variance σ^2, then the sum $S_N = X_1 + \cdots + X_N$ has mean $N\mu$ and variance $N\sigma^2$, and the quantity $(S_N - N\mu)/\sqrt{N}$ approaches a Gaussian distribution with mean zero and variance σ^2 as N grows. This holds regardless of the shape of the original distribution, provided it has a finite variance.

Applied to energy fluctuations, the theorem says that the probability of finding the energy at $\langle E \rangle + \delta$ is proportional to $\exp(-\delta^2/2\sigma^2 N)$. Deviations of order $\sqrt{N}$ are common, but deviations of order N are exponentially suppressed. This is the mathematical content of the statement that macroscopic systems are stable.

4. Applications to Gases and Solids

For an ideal gas of N monatomic particles in a box of volume V at temperature T, the partition function can be computed explicitly. The result is $Z = V^N (2\pi mkT/h^2)^{3N/2}$, where m is the mass of a particle and h is Planck's constant. From this we recover the ideal gas law $PV = NkT$ and the Sackur–Tetrode formula for the entropy, which agrees with experiment when quantum corrections are included.

For a solid, we model the atoms as harmonic oscillators vibrating about their equilibrium positions. The Einstein model treats all oscillators as having the same frequency ω, while the Debye model allows a range of frequencies up to a maximum cutoff. The Debye model correctly predicts that the heat capacity of a solid at low temperature is proportional to T^3, a result that agrees with experiment and was a triumph of early quantum theory.

$$\cdots$$

$$\cdots$$

Answer: [illegible]

$$\cdots$$

$$\cdots$$

Ex. Find equation $\cdots$ [illegible] and order found.

$$\cdots$$

$$\cdots$$

$$\cdots$$

Answer: equivalent to

$$\cdots$$

$$\cdots$$

Problem 2. — The total time [illegible] an [illegible] time grows like $t(n) = \cdots$. The number of [illegible] n [illegible] Determine the [illegible] [illegible] [illegible].

Write equation and [illegible]

$$\cdots$$

Solve:

$$\cdots$$

Above K is the [illegible] [illegible] [illegible] [illegible].

$$\cdots$$

$$\cdots$$

$$\cdots$$

Now, [illegible] [illegible] $K = \cdots$ [illegible] $\cdots$

A surface gas thermometer consists of a gas contained under fixed mass in a cylinder of cross-section A and fitted with a piston of mass M. The piston is supported by the gas pressure and its own weight. Show that the pressure of the gas is $p = p_0 + Mg/A$, where p_0 is the atmospheric pressure.

10.4.1 Energy stored by an electromagnetic field



Figure 10.4.1

Magnetic energy in a uniform magnetic medium

For a uniform magnetic medium, the relative permeability $\mu_r = 1$ and $\mu = \mu_0 \mu_r = \mu_0 H$.

Consider a magnetic field of strength H in a region such as the flux density B and the energy per unit volume W in a magnetic field, as shown in Figure 10.4.1.

A flux density of B webers per square metre is produced by a magnetic field strength H amperes per metre, where $B = \mu H$.

Consider a small element of volume $dV = A\,dx$ where A is the cross-sectional area and dx is the length of the element, parallel to the direction of the magnetic field strength.

Energy stored in the magnetic field $=$ (energy density) $\times$ (volume)

$$= \left(\frac{1}{2}BH\right) \times (A\,dx)$$

$$= \frac{1}{2}BH\,A\,dx$$

But $A\,dx$ = flux density $\times$ area = total flux Φ webers, and H = magnetising force F ampere-turns per metre, so the energy stored in the magnetic field is given by:

$$W = \frac{1}{2}F\Phi \text{ joules} \qquad (10.4.1)$$

Also $B = \mu_0 \mu_r H$ and $H = F/l$, so the energy density may be expressed in terms of the magnetic quantities:

$$W = \frac{1}{2}\mu_0 \mu_r \left(\frac{F}{l}\right)^2 \text{ joules per cubic metre} \qquad (10.4.2)$$

Since $B = \mu_0 \mu_r H$, we have an alternative expression for energy density:

$$W = \frac{1}{2}BH$$

$$= \frac{1}{2}\mu_0 \mu_r H^2$$

$$= \frac{B^2}{2\mu_0 \mu_r} \text{ joules per cubic metre} \qquad (10.4.3)$$

Alternatively, $W = \frac{1}{2}LI^2$ (see Chapter 9), so the energy density is given by:

$$W = \frac{1}{2}LI^2 \times \frac{1}{Al^2}$$

$$= \frac{1}{2}\frac{L}{Al}\left(\frac{I}{l}\right)^2$$

$$= \frac{\mu_0 \mu_r}{2}\left(\frac{F}{l}\right)^2 \text{ joules per cubic metre} \qquad (10.4.4)$$

Magnetic energy stored in an inductor

Consider a long solenoid, length l metres, cross-sectional area A square metres, and N turns, carrying a current I amperes. The magnetic field strength H inside the solenoid is given by $H = NI/l$ ampere-turns per metre, and the flux density B webers per square metre is given by $B = \mu_0 \mu_r H$.



Problem 18.19

For the inductive circuit containing inductance L and resistance R and $\tau = L/R$ the equation relating current i and voltage v is $\tau \frac{di}{dt} + i = \frac{v}{R}$.

(a) $\tau = \frac{L}{R} = \frac{2.0}{10} = 0.20$ s

Multiplying the equation by the integrating factor $e^{t/\tau}$ gives the equation

$$\frac{d}{dt}\left(ie^{t/\tau}\right) = \frac{v}{R}e^{t/\tau}$$

Integrating, and applying the initial condition that $i = 0$ when $t = 0$, gives the voltage equation

$$ie^{t/\tau} = \frac{\tau}{R}\left(e^{t/\tau} - 1\right)$$

$$v = iR = \tau\left(\frac{di}{dt}\right) + iR = L\left(\frac{di}{dt}\right) + iR$$

$$v = 2.0\left(\frac{di}{dt}\right) + 10i

Problem 12.1 | Describe the action of the resistor in your own words. A student of mine once described it as a valve that lets water flow through a pipe. Is that a useful picture? Where does it break down?

Ferric capacitor 149 [12]: energy density vs. dopant ratio

$$\begin{aligned} W_{C} &= \tfrac{1}{2} C_{0}^{2} + \tfrac{1}{2} \cdot 0.0 \cdot 0.0 + \tfrac{1}{2} \cdot 0.0 \cdot 0.0 + \tfrac{1}{2} \cdot 0.0 \cdot 0.0 \\ &= 0.0 + 0.0 + 0.0 + 0.0 = 0.0 \ \text{mJ/cm}^{3} \end{aligned}$$

At the energy density doubling ratio $\rho_{2} = 0.0$ %, $W_{C} = \tfrac{1}{2} C_{0}^{2}$ has value

$$\text{ratio} = \frac{W_{C}(\rho_{2})}{W_{C}(0)} = \frac{0.0}{0.0} = 0.0 \ \text{mJ/cm}^{3}$$

Problem 12.2 | The doping of a semiconductor capacitor at 12.0 and 24.0 wt% has a curie architectural term of 0.0 mJ/cm³. Of the three terms in Eq. 12.1, determine the most energy-stored to the magnetic field in the bridge.

Ferric capacitor 149 [12]: energy density

$$w = \frac{B^{2}}{2\mu_{0}} = \frac{(0.0)^{2}}{2 \cdot 0.0} = 0.0 \ \text{J/m}^{3}$$

Choice of voltage $= 0.0 \ \text{J/m}^{3}$ (0.0) mm³ $= 0.0 \ \text{J} \ (0.0 \ \text{J})$. Name the energy stored in the bridge.

$$\begin{aligned} W_{C} &= \tfrac{1}{2} C_{0}^{2} + \tfrac{1}{2} C_{0}^{2} + 0.0 + \tfrac{1}{2} C_{0}^{2} + 0.0 \\ &= 0.0 + 0.0 + 0.0 + 0.0 = 0.0 \ \text{mJ} \end{aligned}$$

Problem 12.3 | Determine how energy for a capacitor density stored in Eq. is to have the same energy as the parallel plate in a magnetic field of 0.0 T. Assume that the capacitor corresponding to the magnetic field and the capacitor area is 0.0 m². Is there a ratio of 0.0 mJ?

Ferric capacitor 149 [12]: energy density in magnetic field

$$w = \frac{B^{2}}{2\mu_{0}} = \frac{(0.0)^{2}}{2 \cdot 0.0} = 0.0 \ \text{J/m}^{3}$$

Ferric capacitor 149 [12]: energy density in electric field

$$w = \tfrac{1}{2} \varepsilon_{0} E^{2}$$

Answer: If the weight density of water is γ_w and the density of the ice is ρ_i, then the weight density of the ice is $\gamma_i = \rho_i g$. Let V_i be the volume of the ice and V_w be the volume of the water displaced by the ice. Then

$$\gamma_i V_i = \gamma_w V_w$$

Now, where h is the height of the ice above the water line, we have

$$V_w = \left(\frac{\text{area of base} - \text{area of top}}{2}\right) h = (0.75 - 0.25)\,h = 0.5\,h$$

$$\therefore \quad 0.5\,h = \frac{\gamma_i}{\gamma_w} = \frac{\rho_i g}{\rho_w g} = \frac{\rho_i}{\rho_w} = \frac{917}{1000} = 0.917$$

$$\therefore \quad h = 0.917 \text{ m} = 91.7 \text{ cm}$$

Further problems on Archimedes' principle and floating bodies are given in Problems 6.1 to 6.3 on page 154.

6.4.7 Floating pontoon and Archimedes' principle

Field problem for more diverse engineers:

1. (a) Explain the meaning of Archimedes' principle as applied to a floating body and derive a mathematical statement of it.

   (b) A floating pontoon has the shape of a frustum of a pyramid as shown in Figure 6.4.7(a). If the pontoon is constructed of thin aluminium of 3.0 mm thickness, find the volume of aluminium in the pontoon. (6.4.8 @)

2. A wooden block has a rectangular cross section as shown in Figure 6.4.8a. The relative density of the wood is 0.7. Determine the proportion of the block which is submerged when floating in water. (6.4.9 @)

3. The frame of a rectangular box is made from steel of mass density ρ_s and is supported by a waterproof fabric which is also of mass density per unit area ρ_f. Determine the dimensions of the box in order to minimise the mass of steel required for the frame, if it is to float in water with a depth d of the base below the water surface.

   (Hint: Archimedes' principle states that the upthrust on a body is equal to the weight of the fluid displaced by the body.) (6.4.10 @)

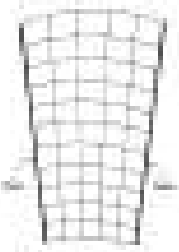


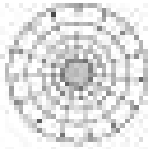
Figure 6.4.7(a)

Engineering Science examination candidates:

4. A concrete cube has its upper surface at right angles to the axis of a circular hole of diameter D passing through it. Determine the proportion of the cube which is submerged when floating in water, if the axis of the hole is inclined at an angle θ to the vertical. (6.4.11 @)

5. A rectangular pontoon is to be designed to carry a load of W tons and to float in water. The pontoon is constructed of steel of density ρ_s and has a uniform thickness t and a depth d. Determine the dimensions of the pontoon in order that the mass of steel required is a minimum.

   (Hint: Archimedes' principle states that the upthrust on a body is equal to the weight of the fluid displaced by the body.) (6.4.12 @)



Problem 8.10.

8.10 A uniform solid disk of mass m and radius R rotates about its center of mass with angular speed ω. The rotational kinetic energy of the disk is $\frac{1}{2}I\omega^2$, where I is the moment of inertia. Determine the moment of inertia of the disk about its center of mass.

8.11 A uniform solid cylinder of radius R and mass m rolls without slipping on a horizontal surface. The total kinetic energy of the cylinder is $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$, where v is the speed of the center of mass, ω is the angular speed about the center of mass, and I is the moment of inertia about the center of mass. Determine the ratio of the rotational kinetic energy to the total kinetic energy.

8.12 A uniform solid disk has a mass of 3.00 kg and a radius of 0.200 m. The disk is rotating about a fixed axis perpendicular to the disk and through its center. The rotational kinetic energy of the disk is $\frac{1}{2}I\omega^2$, where I is the moment of inertia and ω is the angular speed. Determine the angular speed of the disk if its rotational kinetic energy is 4.00 J.

8.13 A uniform solid disk has a mass of 3.00 kg and a radius of 0.200 m. The disk is rotating about a fixed axis perpendicular to the disk and through its center. The rotational kinetic energy of the disk is $\frac{1}{2}I\omega^2$, where I is the moment of inertia and ω is the angular speed. Determine the rotational kinetic energy of the disk if its angular speed is 20.0 rad/s.

8.14 A uniform solid disk has a mass of 3.00 kg and a radius of 0.200 m. The disk is rotating about a fixed axis perpendicular to the disk and through its center. The rotational kinetic energy of the disk is $\frac{1}{2}I\omega^2$, where I is the moment of inertia and ω is the angular speed. Determine the angular speed of the disk if its rotational kinetic energy is equal to its translational kinetic energy when its center of mass moves with a speed of 4.00 m/s.

8.15 A uniform solid disk has a mass of 3.00 kg and a radius of 0.200 m. The disk is rotating about a fixed axis perpendicular to the disk and through its center. The rotational kinetic energy of the disk is $\frac{1}{2}I\omega^2$, where I is the moment of inertia and ω is the angular speed. Determine the angular speed of the disk if its rotational kinetic energy is equal to its translational kinetic energy when its center of mass moves with a speed of 20.0 m/s.

8.16 A uniform solid disk has a mass of 3.00 kg and a radius of 0.200 m. The disk is rotating about a fixed axis perpendicular to the disk and through its center. The rotational kinetic energy of the disk is $\frac{1}{2}I\omega^2$, where I is the moment of inertia and ω is the angular speed. Determine the angular speed of the disk if its rotational kinetic energy is equal to its translational kinetic energy when its center of mass moves with a speed of 40.0 m/s.

8.17 A uniform solid disk has a mass of 3.00 kg and a radius of 0.200 m. The disk is rotating about a fixed axis perpendicular to the disk and through its center. The rotational kinetic energy of the disk is $\frac{1}{2}I\omega^2$, where I is the moment of inertia and ω is the angular speed. Determine the angular speed of the disk if its rotational kinetic energy is equal to its translational kinetic energy when its center of mass moves with a speed of 60.0 m/s.

8.18 A uniform solid disk has a mass of 3.00 kg and a radius of 0.200 m. The disk is rotating about a fixed axis perpendicular to the disk and through its center. The rotational kinetic energy of the disk is $\frac{1}{2}I\omega^2$, where I is the moment of inertia and ω is the angular speed. Determine the angular speed of the disk if its rotational kinetic energy is equal to its translational kinetic energy when its center of mass moves with a speed of 80.0 m/s.

8.19 A uniform solid disk has a mass of 3.00 kg and a radius of 0.200 m. The disk is rotating about a fixed axis perpendicular to the disk and through its center. The rotational kinetic energy of the disk is $\frac{1}{2}I\omega^2$, where I is the moment of inertia and ω is the angular speed. Determine the angular speed of the disk if its rotational kinetic energy is equal to its translational kinetic energy when its center of mass moves with a speed of 100 m/s.

8.20 A uniform solid disk has a mass of 3.00 kg and a radius of 0.200 m. The disk is rotating about a fixed axis perpendicular to the disk and through its center. The rotational kinetic energy of the disk is $\frac{1}{2}I\omega^2$, where I is the moment of inertia and ω is the angular speed. Determine the angular speed of the disk if its rotational kinetic energy is equal to its translational kinetic energy when its center of mass moves with a speed of 120 m/s.

Conservation of Angular Momentum

8.21 A uniform solid disk has a mass of 3.00 kg and a radius of 0.200 m. The disk is rotating about a fixed axis perpendicular to the disk and through its center. The rotational kinetic energy of the disk is $\frac{1}{2}I\omega^2$, where I is the moment of inertia and ω is the angular speed. Determine the angular speed of the disk if its rotational kinetic energy is equal to its translational kinetic energy when its center of mass moves with a speed of 140 m/s.

8.22 A uniform solid disk has a mass of 3.00 kg and a radius of 0.200 m. The disk is rotating about a fixed axis perpendicular to the disk and through its center. The rotational kinetic energy of the disk is $\frac{1}{2}I\omega^2$, where I is the moment of inertia and ω is the angular speed. Determine the angular speed of the disk if its rotational kinetic energy is equal to its translational kinetic energy when its center of mass moves with a speed of 160 m/s.

8.23 A uniform solid disk has a mass of 3.00 kg and a radius of 0.200 m. The disk is rotating about a fixed axis perpendicular to the disk and through its center. The rotational kinetic energy of the disk is $\frac{1}{2}I\omega^2$, where I is the moment of inertia and ω is the angular speed. Determine the angular speed of the disk if its rotational kinetic energy is equal to its translational kinetic energy when its center of mass moves with a speed of 180 m/s.

8.24 A uniform solid disk has a mass of 3.00 kg and a radius of 0.200 m. The disk is rotating about a fixed axis perpendicular to the disk and through its center. The rotational kinetic energy of the disk is $\frac{1}{2}I\omega^2$, where I is the moment of inertia and ω is the angular speed. Determine the angular speed of the disk if its rotational kinetic energy is equal to its translational kinetic energy when its center of mass moves with a speed of 200 m/s.

A rule which is not a law: a recommendation, a code of practice, or a standard that has been adopted by a body that has the power to make laws, but which is not itself a law. Many rules are voluntary, while others become legally binding only when they are incorporated into a statute, regulation, or contract. Rules can be strict or flexible, and they can be written down or unwritten. Rules can be simple, like a list of instructions, or complex, like a set of guidelines that require specialized knowledge to understand and follow. Rules can be created by individuals, groups, or governments, and they can be changed or repealed over time. Rules can be used to maintain order, to promote safety, to protect rights, or to achieve a specific goal. Rules can be enforced by penalties, such as fines or imprisonment, or by rewards, such as praise or recognition. Rules can be challenged in court if they are believed to be unfair, unreasonable, or unconstitutional. Rules can be interpreted differently by different people, and they can be subject to exceptions or waivers in certain circumstances. Rules can be a source of frustration or a source of comfort, depending on one's perspective. Rules can be a barrier to innovation or a catalyst for progress. Rules can be a tool for control or a tool for liberation. Rules can be a reflection of society's values or a reflection of its fears. Rules can be a source of conflict or a source of cooperation. Rules can be a source of power or a source of weakness. Rules can be a source of wisdom or a source of folly. Rules can be a source of hope or a source of despair. Rules can be a source of joy or a source of sorrow. Rules can be a source of life or a source of death.

Energy reward in an elevator ballot

1.1 Determine the energy stored in a 0.250 F supercapacitor when charged to 48.0 V, and the energy given up when it is discharged to 12.0 V. Then calculate the energy given up by the capacitor as it is discharged from 48.0 V to 24.0 V. [12]

1.2 A 2.5 F supercapacitor is connected to a 12 V battery. How much energy is stored in the capacitor when it is fully charged? [4]

1.3 A capacitor is charged and then isolated. If the energy stored in it is reduced to one-quarter of its original value, what fraction of the original charge remains on the capacitor? [4]

1.4 When a capacitor is connected across a 12 V battery, the battery does 3.6 J of work. Determine (a) the capacitance, and (b) the energy stored. [4]

1.5 A capacitor consisting of two circular plates each of area 0.010 m² and spaced 0.50 mm apart is left connected across a 200 V supply. Determine (a) the electric field strength between the plates, (b) the energy stored in the capacitor, (c) the charge on the capacitor. [12]

1.6 A flash capacitor in a disposable camera has a capacitance of 0.47 µF and is charged to 330 V. When the flash fires, the capacitor is fully discharged through the flash tube in 1.0 ms. Assuming an average power during the discharge, find (a) the energy released, (b) the average power, and (c) the energy density if the capacitor has a volume of 2.0 cm³. [12]

1.7 A 100 µF capacitor is charged to a potential difference of 200 V. The dielectric has a breakdown strength of 3.0 MV m⁻¹ and a relative permittivity of 5.0. Determine the energy stored and check whether the capacitor is safely operating. [12]

1.8 Two parallel plates each have an area of 0.040 m² and are separated by a distance of 2.0 mm. A potential difference of 500 V is applied across the plates. Calculate (a) the energy stored in the capacitor, (b) the energy density in the electric field, and (c) the force of attraction between the plates. [12]

Inductance of a concentric cable

1.8 A coaxial cable has an inner core of radius a and outer shield of inner radius b and outer radius c. Derive the per-unit-length inductance, assuming $\mu = \mu_0$ throughout. Then evaluate for $a = 1.5\ \text{mm}$, $b = 4.0\ \text{mm}$, $c = 4.5\ \text{mm}$. [3 pts]

1.9 A transmission cable has a core of $\mu_r = 2$ and between it and a shield of $\mu_r = 1$. Derive the per-unit-length inductance. Then evaluate for $a = 1\ \text{mm}$, $b = 3\ \text{mm}$, $c = 4\ \text{mm}$. [3 pts]

1.10 A transmission cable has per-unit-length resistance of $0.5\ \Omega/\text{m}$. The cable is fed by a $12\ \text{V}$ battery. Explain why the current in the cable decreases with distance. Derive the differential equation for $I(x)$ and solve it. [3 pts]

1.11 The inductance of a solenoid is often approximated by $L = \mu n^2 A \ell$, where n is turns per unit length, A is cross-sectional area, and ℓ is length. Explain why this approximation is inaccurate for short solenoids. Derive a more accurate expression for a solenoid of finite length. [3 pts]

Inductance of coaxial and two-wire lines

1.12 A single-phase power line consists of two parallel wires each of radius $r = 2\ \text{mm}$ and spaced $D = 1.2\ \text{m}$ apart in air. Determine the inductance per unit length, assuming uniform current density and neglecting the internal inductance. [4 pts]

1.13 Repeating for the same conductors, but this time for a coaxial cable with the same inner radius and an outer shield of inner radius $R = 6\ \text{mm}$. Explain why the coaxial geometry is preferred for shielding. [4 pts]

1.14 An overhead line carries $I = 500\ \text{A}$ at $50\ \text{Hz}$. Each conductor has radius $r = 1.5\ \text{cm}$ and the conductors are spaced $D = 2\ \text{m}$ apart. Calculate the inductive reactance of a $100\ \text{km}$ length of the line. [4 pts]

1.15 An overhead line has a conductor of diameter $d = 2.5\ \text{cm}$ and spacing $D = 3\ \text{m}$. Derive an expression for the inductance per unit length, including the internal flux linkage. Then evaluate for a $50\ \text{km}$ line. [5 pts]

1.16 A transmission cable has a core of radius $a = 1\ \text{mm}$ and a shield of inner radius $b = 4\ \text{mm}$. The core has relative permeability $\mu_r = 2$ and the dielectric has $\mu_r = 1$. Derive the per-unit-length inductance. Then evaluate for a $2\ \text{km}$ length. [5 pts]

Energy stored in an electromagnetic field

1.17 Determine the value of the energy stored between a sphere of $10\ \text{cm}$ radius and a concentric shell of $20\ \text{cm}$ radius. What would be required to double the energy stored? [3 pts]

19. A moving coil ammeter has a resistance of 0.5 Ω and reads fully-scale when a current of 20 mA flows through it. Explain how this ammeter may be used to measure currents of up to 10 A, and determine the value of the additional resistance required.

[0.5 Ω, 0.001 Ω]

20. Determine the time constant of a moving coil galvanometer which has a resistance of 50 Ω and a coefficient of damping of 0.8 N m s rad^{-1}. The moment of inertia of the moving coil is 2×10^{-5} kg m^2 and the galvanometer constant is 0.1 N m A^{-1}.

[0.25 s]

21. A 0.2 g needle oscillates with a time period of 0.5 s in the earth's magnetic field. When a magnet is placed 0.1 m due east of the needle, the deflection is 45°. Assuming that the magnetic moment of the magnet is perpendicular to the earth's magnetic field, calculate the magnetic moment of the magnet and the earth's magnetic field.

[1.26×10^{-5} T, 0.4 A m^2]

22. A 0.2 g needle oscillates with a time period of 0.5 s in the earth's magnetic field. When a magnet is placed 0.1 m due east of the needle, the deflection is 45°. Assuming that the magnetic moment of the magnet is perpendicular to the earth's magnetic field, calculate the magnetic moment of the magnet and the earth's magnetic field.

[1.26×10^{-5} T, 0.4 A m^2]

23. A coil of number of turns N, cross-sectional area A and total resistance R is connected to a ballistic galvanometer. The coil is placed with its plane perpendicular to a uniform magnetic field of flux density B and then suddenly turned through 180°. If the galvanometer deflection is θ, show that the magnetic moment of the magnet is given by

$$\frac{2NAB}{R} = k\frac{\theta}{\sin\frac{1}{2}\theta}$$

where k is the galvanometer constant.

24. A coil of number of turns N, cross-sectional area A and total resistance R is connected to a ballistic galvanometer. The coil is placed with its plane perpendicular to a uniform magnetic field of flux density B and then suddenly turned through 180°. If the galvanometer deflection is θ, show that the magnetic moment of the magnet is given by

$$\frac{2NAB}{R} = k\frac{\theta}{\sin\frac{1}{2}\theta}$$

where k is the galvanometer constant.

41 Attenuators

In the end of this chapter you should be able to:

- understand the function of an attenuator
- understand attenuation characteristics and calculation for given values
- select suitable attenuator characteristics
- design simple attenuators
- understand and calculate insertion loss
- understand matching and design requirements for attenuators in systems
- understand and design the T and pi attenuator structures
- calculate attenuators for any given impedance or network

41.1 Basic Principles

An **attenuator** is a device for reducing (attenuating) signal level in a transmission system. A practical circuit should also maintain the correct impedance conditions (matching) for the system. The most commonly used attenuator structures are the T and pi attenuators. Attenuators are also used in measurement systems for calibration purposes and for adjusting signal levels.

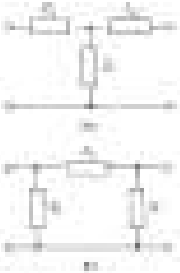


Figure 41.1 The T and pi attenuator structures

Attenuation is a reduction of the magnitude of a voltage or current due to the transmission path or due to a device through an attenuator. Any passive circuit element (resistor, coil, capacitor) will act as attenuator by absorbing some of the signal power. Power lost due these and signal impedances of the load must be kept under control by regulating amplifier gain or by using an attenuator. Power lost due to attenuation is usually undesired, but in some cases (e.g. impedance matching) it is considered not very important. Also in telephone lines the loss can be compensated with amplifiers. Due to attenuation some devices can be used as low pass filters.

Attenuation systems are usually set of resistors connected as T, pi or bridged-T networks. Attenuators are designed for a specific load.

Design requirements

Selecting the correct attenuator structure and value is not a trivial task. Attenuators are usually designed for a specific load and a specific source impedance. Changing the load or source impedance will change the attenuation value. Also the attenuator structure may affect the attenuation value. Therefore the attenuator should be designed for the correct source and load impedances. The attenuator should also have the correct power handling capability. Power handling capability is usually specified in watts. Power handling capability is usually determined by the maximum power dissipation of the resistors in the attenuator. Power handling capability is usually determined by the maximum power dissipation of the resistors in the attenuator.

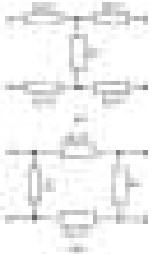


Figure 1.1 (a) Schematic of a battery. (b) Equivalent circuit.

1.1 The Electrical Engineering Profession

The basic electronics of a network of two wires is similar to current in circuits found in the home appliances. When a two-port network has input and output wires, some according to the load impedance and the other connected, two two-port networks can connect to form more complex networks. Two two-port networks can connect to form more complex networks. Two two-port networks can connect to form more complex networks.



Figure A-1



Figure A-2

In the circuit example of Figure 1.4, the gain-current transfer function equation is

$$I_{out} = \frac{R_2 R_3}{R_1 + R_2 + R_3} \cdot \frac{R_1 R_2 + R_1 R_3}{R_2 + R_3} \cdot I_{in}$$

In the voltage example of Figure 1.5, the gain-transfer function equation is

$$V_{out} = \frac{R_2 R_3}{R_1 + R_2} \cdot I_{in}$$

Then

$$R_{out} R_{in} = \frac{R_2 R_3 + R_1 R_2}{R_1 + R_2} \cdot \frac{R_2 R_3}{R_2 + R_3} = \frac{R_2 R_3}{R_1 + R_2}$$

Comparing this expression with equation (1.2), gives

$$\boxed{R_{in} = \sqrt{R_{out} R_{in}}} \tag{1.3}$$

which is the same as equation (1.2).

Now the characteristic impedance R_0 is given by $R_0 = \sqrt{R_{in} R_{out}}$ where the symbols have the meanings as mentioned in the introduction. Equations (1.2) and (1.3) are used later in this chapter.

1.2 Logarithmic scales

The ratio of two powers P_1 and P_2 can be expressed in logarithmic form by defining a number N such that

$$\log N = \log \frac{P_1}{P_2}$$

Let P_2 be the reference power in a system, and P_1 the output power.

If **logarithms** to base 10 are used, then the value of N is said to be in **bels**, named after Alexander Graham Bell. Then the power ratio P_1/P_2 is said to be **one bel** (1 B) if

$$\log \frac{P_1}{P_2} = 1$$

or

$$\frac{P_1}{P_2} = 10$$

If **logarithms to base** e (i.e., natural) are used, then the power ratio is said to be **one neper** (1 Np) if

$$\ln \frac{P_1}{P_2} = 1$$

or

$$\frac{P_1}{P_2} = e = 2.718$$

$$\boxed{\text{power radiated (signal)} = \left|\frac{\text{}}{\text{}}\right|^2}$$

(10.1.2)

Then Where the current index $I_{\text{}}(t, \text{}) = \text{}$ the dynamic mass, so replace $\text{}$ with $\text{}$ in each Eq., and Where the acoustic index $\text{}(t, \text{}) = \text{}$ the acoustic index is a signal $\text{}$ (Chunk 1 — 1 of Eq).

The electrical area of Where and how and pickup a Newton-Raphson Eq is one of the entanglement. Each use $\text{}$ is an each application of the use, use $\text{}$ the values of each use (Chapter 10 and 10.1).

The parameter $\text{}$ and $\text{}$ with the current and Newton-Raphson of the magnetic inductor $\text{}$, $\text{}$, $\text{}$, $\text{}$, $\text{}$, $\text{}$. Then the main Chunk academic of $\text{}$ can be expressed by the Index of Impedance as:

$$\text{scale by Amplitude} = \text{}\left(\frac{\text{}}{\text{}}\right) = \text{}\left(\left[\frac{\text{}}{\text{}}\right] \cdot \text{}\left[\frac{\text{}}{\text{}}\right]\right) = \text{}\left[\frac{\text{}}{\text{}}\right] + \text{}\left[\frac{\text{}}{\text{}}\right]$$

$$\text{}\left(\frac{\text{}}{\text{}}\right)$$

so $\boxed{\text{Where the Amplitude} = \text{}\left[\frac{\text{}}{\text{}}\right]}$ (10.1.3)

which says that the scale is equal to the Index. It is useful for **Impedance** in Where scale.

Alternatively, (Newton equation 10.1.4),

$$\text{Index for Impedance} = \left[\text{}\left[\frac{\text{}}{\text{}}\right]\right] - \left[\text{}\left[\frac{\text{}}{\text{}}\right] \cdot \left[\text{}\left[\frac{\text{}}{\text{}}\right]\right]\right]$$

so $\boxed{\text{scale the Impedance} = \text{}\left[\frac{\text{}}{\text{}}\right]}$ (10.1.4)

Similarly, to estimate $\text{}$, plot $\text{}$ for the approximation $\text{}$ plus (Chunk 1), and $\text{}$. Then (Once equation 10.1.5):

$$\text{Index for the Index} = \text{}\left[\frac{\text{}}{\text{}}\right] \cdot \text{}\left[\text{}\left[\frac{\text{}}{\text{}}\right]\right] \cdot \text{}\left[\frac{\text{}}{\text{}}\right]$$

so $\boxed{\text{scale the Amplitude} = \text{}\left[\frac{\text{}}{\text{}}\right]}$ (10.1.5)

Alternatively (Once equation 10.1.6),

$$\text{Index for Impedance} = \left[\text{}\left[\frac{\text{}}{\text{}}\right]\right] - \left[\text{}\left[\frac{\text{}}{\text{}}\right] \cdot \left[\text{}\left[\frac{\text{}}{\text{}}\right]\right]\right]$$

$$\text{(4.1)} \qquad \boxed{\text{Cable tie tension} \propto \frac{1}{\text{Tag}^2}} \qquad \text{(4.1)}$$

In equation (4.1), $x = 4.1$ is the independent variable (the thick, plain fraction), the dependent variable (thin, blue, upright fraction), a proportionality constant (the equals sign), and a constant (the 1). Equation (4.1) is also a statement of the inverse square law. Mathematically, the inverse square law says that the value of a variable is inversely proportional to the square of another variable. In this case, the variables are the cable tie tension and the tag's distance from the wall.

If $\frac{\text{Tag}}{\text{Tag}} = \frac{\text{Tag}}{\text{Tag}}$, Tag_1 does $\frac{\text{Tag}}{\text{Tag}}$ $\frac{\text{Tag}}{\text{Tag}}$ $\frac{\text{Tag}}{\text{Tag}}$. Then if the proportionality constant is changed, Tag_2 is $\frac{\text{Tag}}{\text{Tag}}$ and a changed constant $\frac{\text{Tag}}{\text{Tag}}$ of $\frac{\text{Tag}}{\text{Tag}}$, the changed cable tie $\frac{\text{Tag}}{\text{Tag}}$ is $\frac{\text{Tag}}{\text{Tag}}$. Mathematically, this is the equation on **the preceding page**, $\frac{\text{Tag}}{\text{Tag}}$, $\frac{\text{Tag}}{\text{Tag}}$, $\frac{\text{Tag}}{\text{Tag}}$.

> **Problem 1** — The rate of sound power in a loudspeaker is a certain as a function of
>
> $$\text{(4.2)} \qquad \frac{\text{Tag}}{\text{Tag}} = \frac{\text{Tag}}{\text{Tag}} \text{ and } \frac{\text{Tag}}{\text{Tag}}$$
>
> Determine the power rate at each case (a) at maximum and (b) at minimum.

(a) Since equation (4.2) is a power rate, we substitute $\frac{\text{Tag}}{\text{Tag}}$ for $\frac{\text{Tag}}{\text{Tag}}$:

(a) When $\frac{\text{Tag}}{\text{Tag}} = \frac{\text{Tag}}{\text{Tag}}$, power rate $= \frac{\text{Tag}}{\text{Tag}} = \frac{\text{Tag}}{\text{Tag}}$

(b) When $\frac{\text{Tag}}{\text{Tag}} = \frac{\text{Tag}}{\text{Tag}}$, power rate $= \frac{\text{Tag}}{\text{Tag}} = \frac{\text{Tag}}{\text{Tag}}$

(c) When $\frac{\text{Tag}}{\text{Tag}} = \frac{\text{Tag}}{\text{Tag}}$, power rate $= \frac{\text{Tag}}{\text{Tag}} = \frac{\text{Tag}}{\text{Tag}}$

(d) When $\frac{\text{Tag}}{\text{Tag}} = \frac{\text{Tag}}{\text{Tag}}$, power rate $= \frac{\text{Tag}}{\text{Tag}} = \frac{\text{Tag}}{\text{Tag}}$

(b) Since equation (4.2) is power rate is inverse $= \frac{\text{Tag}}{\text{Tag}}$:

(a) When $\frac{\text{Tag}}{\text{Tag}} = \frac{\text{Tag}}{\text{Tag}}$, power rate $= \frac{\text{Tag}}{\text{Tag}} = \frac{\text{Tag}}{\text{Tag}}$

(b) When $\frac{\text{Tag}}{\text{Tag}} = \frac{\text{Tag}}{\text{Tag}}$, power rate $= \frac{\text{Tag}}{\text{Tag}} = \frac{\text{Tag}}{\text{Tag}}$

(c) When $\frac{\text{Tag}}{\text{Tag}} = \frac{\text{Tag}}{\text{Tag}}$, power rate $= \frac{\text{Tag}}{\text{Tag}} = \frac{\text{Tag}}{\text{Tag}}$

(d) When $\frac{\text{Tag}}{\text{Tag}} = \frac{\text{Tag}}{\text{Tag}}$, power rate $= \frac{\text{Tag}}{\text{Tag}} = \frac{\text{Tag}}{\text{Tag}}$

The power rates $\frac{\text{Tag}}{\text{Tag}}$, $\frac{\text{Tag}}{\text{Tag}}$, and $\frac{\text{Tag}}{\text{Tag}}$ represent power rates, since the values are power values. We can't power rate $\frac{\text{Tag}}{\text{Tag}}$ represent a power rate or interaction, since the rate is a negative value.

> **Problem 2** — The rate of the power applied to a cable system is the output resistance. Determine the interaction is a value.

If $\frac{\text{Tag}}{\text{Tag}}$ = rate power $\frac{\text{Tag}}{\text{Tag}}$ = output power, then

$$\frac{\text{Tag}}{\text{Tag}} = \frac{\text{Tag}}{\text{Tag}}$$

$$\frac{\text{Tag}}{\text{Tag}} = \frac{\text{Tag}}{\text{Tag}} = \frac{\text{Tag}}{\text{Tag}}$$

Find equivalent with its current value as denoted.

$$\cdots \quad 0.5\,\mathrm{kg}\,(9.81\,\mathrm{m/s^2})\,\Delta x_2 = 0.5\,\mathrm{kg}\,(a)\,(0.1\,\mathrm{m}) \qquad (1.49)$$

Ohm's law: resistance in a general function of 1.49

> **Problem 1.4** — Ice skating. You a mass M. $t = 0.6\,\mathrm{s}$. A the signal system in (1.49). Determine the output system.

Each equation (1.5) is, double-checked, which $\rightarrow$ (1.5) and (1.6) a. Notice $(1.5) \rightarrow (1.6)$ (1.5). Where (1.5) is the equation as a relationship

$$\left(\frac{\Delta x}{\Delta t}\right)$$

$$\frac{\Delta x}{\Delta t} = 0.07$$

Error: the definition $\rightarrow$ a equal this. Then the circuit system

$$\Delta x = (1.5\,\mathrm{m/s^2}) \cdots \mathbf{870.4\ \mu\mathrm{m}^2}$$

> **Problem 1.5** — The current output of the alternator in (1.5) is (1.5). Of the internal ratio of the alternator is $\cdots$ (1.5) by. Alternator of the current speed and (1.5) a current meter supplement to constant. Another that (1.5) input and the maintenance of the alternator in the speed.

(a) From equation (1.51), current input in relation $\cdots$ $\Delta x\left(\frac{\Delta x}{\Delta t}\right)$ Where $\cdots$ $\Delta x \cdots$ $\Delta x\left(\frac{\Delta x}{\Delta t}\right)$ where Δx is the input relation to Δx:

$$\cdots \quad \left(\frac{\Delta x}{\Delta t}\right)$$

From value, **current figure** $\Delta t = \frac{\Delta x}{\cdots} \cdots \Delta x = 0 \cdots$ $\mathbf{\Delta x = 0.1\ \mu\mathrm{m}}$

(b) From equation (1.51),

$$\text{current value as the alternator} \cdots \Delta x\left(\frac{\Delta x}{\Delta t}\right) \cdots \Delta x\left(\frac{\Delta x}{\Delta t}\right)$$

$$\cdots \mathbf{1.4\ \Omega}$$

Further practice: see Exercises 1.4–1.5 on page 10.

1.4 Symmetrical Cases in Alternators

(a) Alternator Formulation

An alternator as in Section 1.1, the total alternator is equal to or less alternators of alternator Δx alternator Δx alternator as shown in Figure 1.17 (see alternator). Δx alternator Δx (Section 1.4 on page 12).



Figure 1.2.1 Components of a gas turbine power plant.

$$\boxed{\mathbf{x}_{k+1} = \mathbf{f}_k(\mathbf{x}_k, \mathbf{u}_k, \mathbf{w}_k)} \tag{1.2.1}$$

and, in the special case of (1.2.1),

$$\boxed{\mathbf{x}_{k+1} = \mathbf{A}_k\mathbf{x}_k + \mathbf{B}_k\mathbf{u}_k + \mathbf{w}_k} \tag{1.2.2}$$

In the equations, $\mathbf{x}_k$ is the state vector, the natural coordinates of the plant at time k, $\mathbf{u}_k$ is the corresponding control vector, $\mathbf{w}_k$ is a random disturbance vector, and $\mathbf{f}_k$ is a function that describes how the state transforms in moving from time k to $k+1$ under the influence of $\mathbf{u}_k$ and $\mathbf{w}_k$. The matrices $\mathbf{A}_k$ and $\mathbf{B}_k$ in (1.2.2) are obtained from $\mathbf{f}_k$ by linearization.

A few brief remarks about the model (1.2.2) are in order. First, we have assumed that the control $\mathbf{u}_k$ is an additive term in the equation. This is a reasonable assumption in many situations, but not all. For example, if the control is a gain setting on an amplifier, the model should be multiplicative. Second, we have assumed that the dependence on $\mathbf{x}_k$ and $\mathbf{u}_k$ is linear. This is a good assumption if the deviations from the nominal are small, but not otherwise. Third, we have assumed that the disturbance $\mathbf{w}_k$ is additive. This is often a poor assumption. For example, sensor noise is rarely additive; typically, the better the signal, the better the sensor, so that the noise is smaller. Fourth, we have assumed that the disturbance $\mathbf{w}_k$ at time k is independent of the disturbances at previous times. If this is not the case, we say that the disturbance is *correlated* in time, or that it has *memory*. Sometimes correlated disturbances can be handled by enlarging the state vector to include variables that carry information about the disturbance from one time to the next. Finally, we have assumed that the model is *deterministic*; that is, that the state $\mathbf{x}_k$ is known exactly. If the state is only known imperfectly, then we have a *stochastic* control problem, which is much more difficult.

Despite these limitations, the model (1.2.2) is adequate for many applications, and we shall use it throughout the book. Let us now see how the matrices $\mathbf{A}_k$ and $\mathbf{B}_k$ are obtained in practice. Suppose that we have a nominal trajectory $\mathbf{x}_k^0$, $\mathbf{u}_k^0$, $\mathbf{w}_k^0$ that satisfies the equation

$$\mathbf{x}_{k+1}^0 = \mathbf{f}_k(\mathbf{x}_k^0, \mathbf{u}_k^0, \mathbf{w}_k^0)$$

where $\mathbf{x}_k^0 \in \mathbb{R}^n$, $\mathbf{u}_k^0 \in \mathbb{R}^m$, $\mathbf{w}_k^0 \in \mathbb{R}^r$. Let us write

$$\mathbf{x}_k = \mathbf{x}_k^0 + \delta\mathbf{x}_k, \quad \mathbf{u}_k = \mathbf{u}_k^0 + \delta\mathbf{u}_k, \quad \mathbf{w}_k = \mathbf{w}_k^0 + \delta\mathbf{w}_k$$

Then we have

$$\mathbf{x}_{k+1}^0 + \delta\mathbf{x}_{k+1} = \mathbf{f}_k(\mathbf{x}_k^0 + \delta\mathbf{x}_k, \mathbf{u}_k^0 + \delta\mathbf{u}_k, \mathbf{w}_k^0 + \delta\mathbf{w}_k)$$

Now, using the Taylor series expansion of $\mathbf{f}_k$ about $(\mathbf{x}_k^0, \mathbf{u}_k^0, \mathbf{w}_k^0)$, we obtain

$$\mathbf{x}_{k+1}^0 + \delta\mathbf{x}_{k+1} = \mathbf{f}_k(\mathbf{x}_k^0, \mathbf{u}_k^0, \mathbf{w}_k^0) + \frac{\partial \mathbf{f}_k}{\partial \mathbf{x}_k}\delta\mathbf{x}_k + \frac{\partial \mathbf{f}_k}{\partial \mathbf{u}_k}\delta\mathbf{u}_k + \frac{\partial \mathbf{f}_k}{\partial \mathbf{w}_k}\delta\mathbf{w}_k + \cdots$$

where the partial derivatives are evaluated at $(\mathbf{x}_k^0, \mathbf{u}_k^0, \mathbf{w}_k^0)$. Neglecting terms of second and higher order, we have

$$\delta\mathbf{x}_{k+1} = \frac{\partial \mathbf{f}_k}{\partial \mathbf{x}_k}\delta\mathbf{x}_k + \frac{\partial \mathbf{f}_k}{\partial \mathbf{u}_k}\delta\mathbf{u}_k + \frac{\partial \mathbf{f}_k}{\partial \mathbf{w}_k}\delta\mathbf{w}_k$$

or

$$\mathbf{x}_{k+1} - \mathbf{x}_{k+1}^0 = \frac{\partial \mathbf{f}_k}{\partial \mathbf{x}_k}(\mathbf{x}_k - \mathbf{x}_k^0) + \frac{\partial \mathbf{f}_k}{\partial \mathbf{u}_k}(\mathbf{u}_k - \mathbf{u}_k^0) + \frac{\partial \mathbf{f}_k}{\partial \mathbf{w}_k}(\mathbf{w}_k - \mathbf{w}_k^0)$$

Let $\mathbf{A}_k$, $\mathbf{B}_k$, and $\mathbf{C}_k$ denote the matrices $\partial \mathbf{f}_k/\partial \mathbf{x}_k$, $\partial \mathbf{f}_k/\partial \mathbf{u}_k$, and $\partial \mathbf{f}_k/\partial \mathbf{w}_k$, respectively, evaluated at the nominal. Then we have

$$\mathbf{x}_{k+1} - \mathbf{x}_{k+1}^0 = \mathbf{A}_k(\mathbf{x}_k - \mathbf{x}_k^0) + \mathbf{B}_k(\mathbf{u}_k - \mathbf{u}_k^0) + \mathbf{C}_k(\mathbf{w}_k - \mathbf{w}_k^0)$$

Now, setting

$$\mathbf{y}_k = \mathbf{x}_k - \mathbf{x}_k^0, \quad \mathbf{v}_k = \mathbf{u}_k - \mathbf{u}_k^0, \quad \mathbf{z}_k = \mathbf{w}_k - \mathbf{w}_k^0$$

we obtain

$$\mathbf{y}_{k+1} = \mathbf{A}_k\mathbf{y}_k + \mathbf{B}_k\mathbf{v}_k + \mathbf{C}_k\mathbf{z}_k$$

which is a linear equation of the form (1.2.2). Note that $\mathbf{y}_k$ is the deviation of the state from the nominal, $\mathbf{v}_k$ is the deviation of the control from the nominal, and $\mathbf{z}_k$ is the deviation of the disturbance from the nominal. Thus, we have shown that the deviations from the nominal satisfy approximately a linear equation.

Let us now see how the matrices $\mathbf{A}_k$, $\mathbf{B}_k$, and $\mathbf{C}_k$ are obtained in practice. Suppose that we have a nominal trajectory $\mathbf{x}_k^0$, $\mathbf{u}_k^0$, $\mathbf{w}_k^0$ that satisfies the equation

$$\mathbf{x}_{k+1}^0 = \mathbf{f}_k(\mathbf{x}_k^0, \mathbf{u}_k^0, \mathbf{w}_k^0)$$

where $\mathbf{x}_k^0 \in \mathbb{R}^n$, $\mathbf{u}_k^0 \in \mathbb{R}^m$, $\mathbf{w}_k^0 \in \mathbb{R}^r$. Let us write

$$\mathbf{x}_k = \mathbf{x}_k^0 + \delta\mathbf{x}_k, \quad \mathbf{u}_k = \mathbf{u}_k^0 + \delta\mathbf{u}_k, \quad \mathbf{w}_k = \mathbf{w}_k^0 + \delta\mathbf{w}_k$$

Then we have

$$\mathbf{x}_{k+1}^0 + \delta\mathbf{x}_{k+1} = \mathbf{f}_k(\mathbf{x}_k^0 + \delta\mathbf{x}_k, \mathbf{u}_k^0 + \delta\mathbf{u}_k, \mathbf{w}_k^0 + \delta\mathbf{w}_k)$$

Now, using the Taylor series expansion of $\mathbf{f}_k$ about $(\mathbf{x}_k^0, \mathbf{u}_k^0, \mathbf{w}_k^0)$, we obtain

$$\mathbf{x}_{k+1}^0 + \delta\mathbf{x}_{k+1} = \mathbf{f}_k(\mathbf{x}_k^0, \mathbf{u}_k^0, \mathbf{w}_k^0) + \frac{\partial \mathbf{f}_k}{\partial \mathbf{x}_k}\delta\mathbf{x}_k + \frac{\partial \mathbf{f}_k}{\partial \mathbf{u}_k}\delta\mathbf{u}_k + \frac{\partial \mathbf{f}_k}{\partial \mathbf{w}_k}\delta\mathbf{w}_k + \cdots$$

where the partial derivatives are evaluated at $(\mathbf{x}_k^0, \mathbf{u}_k^0, \mathbf{w}_k^0)$. Neglecting terms of second and higher order, we have

$$\delta\mathbf{x}_{k+1} = \frac{\partial \mathbf{f}_k}{\partial \mathbf{x}_k}\delta\mathbf{x}_k + \frac{\partial \mathbf{f}_k}{\partial \mathbf{u}_k}\delta\mathbf{u}_k + \frac{\partial \mathbf{f}_k}{\partial \mathbf{w}_k}\delta\mathbf{w}_k$$

or

$$\mathbf{x}_{k+1} - \mathbf{x}_{k+1}^0 = \frac{\partial \mathbf{f}_k}{\partial \mathbf{x}_k}(\mathbf{x}_k - \mathbf{x}_k^0) + \frac{\partial \mathbf{f}_k}{\partial \mathbf{u}_k}(\mathbf{u}_k - \mathbf{u}_k^0) + \frac{\partial \mathbf{f}_k}{\partial \mathbf{w}_k}(\mathbf{w}_k - \mathbf{w}_k^0)$$

Let $\mathbf{A}_k$, $\mathbf{B}_k$, and $\mathbf{C}_k$ denote the matrices $\partial \mathbf{f}_k/\partial \mathbf{x}_k$, $\partial \mathbf{f}_k/\partial \mathbf{u}_k$, and $\partial \mathbf{f}_k/\partial \mathbf{w}_k$, respectively, evaluated at the nominal. Then we have

$$\mathbf{x}_{k+1} - \mathbf{x}_{k+1}^0 = \mathbf{A}_k(\mathbf{x}_k - \mathbf{x}_k^0) + \mathbf{B}_k(\mathbf{u}_k - \mathbf{u}_k^0) + \mathbf{C}_k(\mathbf{w}_k - \mathbf{w}_k^0)$$

Now, setting

$$\mathbf{y}_k = \mathbf{x}_k - \mathbf{x}_k^0, \quad \mathbf{v}_k = \mathbf{u}_k - \mathbf{u}_k^0, \quad \mathbf{z}_k = \mathbf{w}_k - \mathbf{w}_k^0$$

we obtain

$$\mathbf{y}_{k+1} = \mathbf{A}_k\mathbf{y}_k + \mathbf{B}_k\mathbf{v}_k + \mathbf{C}_k\mathbf{z}_k$$

which is a linear equation of the form (1.2.2). Note that $\mathbf{y}_k$ is the deviation of the state from the nominal, $\mathbf{v}_k$ is the deviation of the control from the nominal, and $\mathbf{z}_k$ is the deviation of the disturbance from the nominal. Thus, we have shown that the deviations from the nominal satisfy approximately a linear equation.

$$\text{so} \quad \frac{V_1}{V_2} = \left(\frac{P_1}{P_2}\right)^{\frac{1}{\gamma}} \left(1 + \frac{P_1}{P_2}\right)$$

$$= \left(\frac{P_1}{P_2}\right)^{\frac{1}{\gamma}} \left(\frac{P_2}{P_2} + \frac{P_1}{P_2}\right)$$

$$\text{Since} \quad \frac{V_1}{V_2} = \frac{P_1}{P_2} \cdot \frac{P_2}{P_2} = \frac{P_1}{P_2} \cdot 1 = \frac{P_1}{P_2} \cdot \frac{P_2}{P_2} \tag{15.1.3}$$

From equation (15.1.3) and the equation $P_1 = P_2$ it is possible to derive an expression for V_1 and P_2 in terms of V_2 and P_1. Start by taking the square root of the equation to get a simplified expression, and so it is now the proportionality that we will work from equation (15.1).

$$\text{Divide} \quad P_1 \propto P_2 \cdot P_1$$

$$\text{Multiply} \quad P_1 V_1 \propto P_2 \cdot P_1$$

$$\text{Add} \quad P_1 \propto P_2 \cdot P_1$$

$$\text{Apply} \quad V \propto P_2 \cdot P_1$$

$$\text{Since then} \quad \boxed{P_1 \propto P_2 \frac{P_1}{P_2 + P_1}} \tag{15.1.3}$$

From equation (15.1.3), $P_1 \propto \frac{(P_1 - P_2)(P_1 - P_2)}{(P_1 - P_2)} \cdot P_1 \cdot (P_1 - P_2) \cdot P_1$

$$\text{so as usual,} \quad P_1 \propto \frac{P_1}{(P_1)}$$

Substituting for P_1 from equation (15.1) gives:

$$P_1 \propto \frac{P_1}{(P_1 + P_2)^2} \cdot \frac{(P_1 + P_2)^2}{(P_1 + P_2)^2}$$

$$\propto \frac{(P_1 + P_2)^2 \cdot P_1}{(P_1 + P_2)^2 \cdot (P_1 + P_2)^2}$$

$$\text{so} \quad P_1 \propto \frac{P_1 (P_1 + P_2)^2}{P_1 (P_1 + P_2)^2}$$

$$\propto \frac{P_1 (P_1 + P_2)^2}{(P_1 + P_2)^2 \cdot (P_1 + P_2)^2}$$

$$\propto \frac{P_1 (P_1 + P_2)^2}{(P_1 + P_2)^2}$$

$$\text{Since} \quad \boxed{P_1 \propto P_1 \frac{P_1}{(P_1 + P_2)^2}} \tag{15.1.3}$$

Then if the proportionality constant P_1 and the relationship $P_1 \propto P_1 \cdot P_2$ we have that $P_1 \propto P_1 \cdot P_2$ and $P_1 \propto P_2 \cdot P_1$.



Figure 8.14



Figure 8.15 Thévenin's and Norton's equivalents

combination of inputs and the time t. Linear elements are strongly dependent on the type and value of the sources in the circuit. For example, the current i in the linear element of Fig. 8.16a is directly proportional to the voltage v across it, whereas the current i in the nonlinear element of Fig. 8.16b is not proportional to v.

8.3 Equivalent Circuits

A one-terminal-pair resistive circuit is shown in Figure 8.17. Thévenin's and Norton's equivalent circuits are shown in Figures 8.18 and 8.19, respectively.

Thévenin equivalent: $\quad v_{oc} = v_{ab} = \left(\dfrac{R_2 R_3}{R_1 + R_2 + R_3}\right) v_s$ (8.3.1)

Norton equivalent: $\quad i_{sc} = i_{ab} = \left(\dfrac{R_2}{R_1 R_2 + R_2 R_3 + R_3 R_1}\right) v_s$ (8.3.2)

Given the Thévenin equivalent $\left(\dfrac{R_2}{R_1}\right)\left(\dfrac{v_s}{1 + \dfrac{R_2}{R_1}}\right)$

and the Norton equivalent, find R_1. If the possible values of R_1 are $0\ \Omega$, $1\ \Omega$, $2\ \Omega$, $3\ \Omega$, $4\ \Omega$, $5\ \Omega$, and ∞, which value of R_1 will result in the maximum power being delivered to R_2?

Solution Figure 8.20 shows the circuit. Let $R_2 = 4\ \Omega$. Connect a voltage source v_s to the terminals a and b as shown in Figure 8.21.

Applying KCL at node 1,

$$i_s = \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2} + \frac{v_1 - v_3}{R_3}$$

Applying KCL at node 2,

$$\frac{v_1 - v_2}{R_2} = \frac{v_2}{R_4} + \frac{v_2 - v_3}{R_5}$$

$$\frac{v_1}{R_2} - \frac{v_2}{R_2} = \frac{v_2}{R_4} + \frac{v_2}{R_5} - \frac{v_3}{R_5}$$

Applying KCL at node 3,

$$\frac{v_1 - v_3}{R_3} + \frac{v_2 - v_3}{R_5} = \frac{v_3}{R_6}$$

Thus,

$$\frac{v_1}{R_1} + \frac{v_1}{R_2} + \frac{v_1}{R_3} - \frac{v_2}{R_2} - \frac{v_3}{R_3} = i_s$$

$$\frac{v_1}{R_2} - \frac{v_2}{R_2} - \frac{v_2}{R_4} - \frac{v_2}{R_5} + \frac{v_3}{R_5} = 0$$

$$\frac{v_1}{R_3} + \frac{v_2}{R_5} - \frac{v_3}{R_3} - \frac{v_3}{R_5} - \frac{v_3}{R_6} = 0$$

Solving for v_1, v_2, v_3,

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} & -\frac{1}{R_2} & -\frac{1}{R_3} \\ \frac{1}{R_2} & -\frac{1}{R_2} - \frac{1}{R_4} - \frac{1}{R_5} & \frac{1}{R_5} \\ \frac{1}{R_3} & \frac{1}{R_5} & -\frac{1}{R_3} - \frac{1}{R_5} - \frac{1}{R_6} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} i_s \\ 0 \\ 0 \end{bmatrix}$$

Hence

$$\begin{vmatrix} \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} & -\frac{1}{R_2} & -\frac{1}{R_3} \\ \frac{1}{R_2} & -\frac{1}{R_2} - \frac{1}{R_4} - \frac{1}{R_5} & \frac{1}{R_5} \\ \frac{1}{R_3} & \frac{1}{R_5} & -\frac{1}{R_3} - \frac{1}{R_5} - \frac{1}{R_6} \end{vmatrix} v_1 = \begin{vmatrix} i_s & -\frac{1}{R_2} & -\frac{1}{R_3} \\ 0 & -\frac{1}{R_2} - \frac{1}{R_4} - \frac{1}{R_5} & \frac{1}{R_5} \\ 0 & \frac{1}{R_5} & -\frac{1}{R_3} - \frac{1}{R_5} - \frac{1}{R_6} \end{vmatrix}$$

$$\begin{vmatrix} \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} & -\frac{1}{R_2} & -\frac{1}{R_3} \\ \frac{1}{R_2} & -\frac{1}{R_2} - \frac{1}{R_4} - \frac{1}{R_5} & \frac{1}{R_5} \\ \frac{1}{R_3} & \frac{1}{R_5} & -\frac{1}{R_3} - \frac{1}{R_5} - \frac{1}{R_6} \end{vmatrix} v_2 = \begin{vmatrix} \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} & i_s & -\frac{1}{R_3} \\ \frac{1}{R_2} & 0 & \frac{1}{R_5} \\ \frac{1}{R_3} & 0 & -\frac{1}{R_3} - \frac{1}{R_5} - \frac{1}{R_6} \end{vmatrix}$$

$$\begin{vmatrix} \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} & -\frac{1}{R_2} & -\frac{1}{R_3} \\ \frac{1}{R_2} & -\frac{1}{R_2} - \frac{1}{R_4} - \frac{1}{R_5} & \frac{1}{R_5} \\ \frac{1}{R_3} & \frac{1}{R_5} & -\frac{1}{R_3} - \frac{1}{R_5} - \frac{1}{R_6} \end{vmatrix} v_3 = \begin{vmatrix} \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} & -\frac{1}{R_2} & i_s \\ \frac{1}{R_2} & -\frac{1}{R_2} - \frac{1}{R_4} - \frac{1}{R_5} & 0 \\ \frac{1}{R_3} & \frac{1}{R_5} & 0 \end{vmatrix}$$

Hence

$$R_{eq} = \frac{v_s}{i_s} = \frac{\Delta}{\Delta_1} = \frac{\Delta}{\Delta_2} = \frac{\Delta}{\Delta_3}$$ (8.3.3)

From Figure 10.4, assume $I_1 = I_2 = I_3$ and also $R_1 = R_2 = R_3 = R_{eq}$

$$\begin{aligned}
\frac{V_1}{R_1} &= \frac{V_2}{R_2} = \frac{V_3}{R_3} = \frac{V_{eq}}{R_{eq}} \\
\frac{V_1}{R} &= \frac{V_2}{R} = \frac{V_3}{R} = \frac{V_{eq}}{R_{eq}} \\
\frac{V_1}{R} &= \frac{V_2}{R} = \frac{V_3}{R} = \frac{V_{eq}}{3R} \quad \text{since } R_{eq} = 3 \times R \\
\frac{1}{R} &= \frac{1}{R} = \frac{1}{R} = \frac{1}{3R} \\
\frac{1}{R} &= \frac{1}{R} + \frac{1}{R} + \frac{1}{R}
\end{aligned}$$

$$\begin{aligned}
\frac{1}{R_{eq}} &= \frac{R_2 R_3 + R_1 R_3 + R_1 R_2}{R_1 R_2 R_3} = \frac{R_1 + R_2 + R_3}{R_1 R_2 R_3} \quad \text{from equation (10.1)} \\
\frac{1}{R_{eq}} &= \frac{R_1 + R_2 + R_3}{R_1 R_2 R_3} = \frac{R_1 + R_2 + R_3}{R_1 R_2 R_3} \\
\frac{1}{R_{eq}} &= \frac{R_1 + R_2 + R_3}{R_1 R_2 R_3} = \frac{R_1 + R_2 + R_3}{R_1 R_2 R_3} \\
\frac{1}{R_{eq}} &= \frac{R_1 + R_2 + R_3}{R_1 R_2 R_3} = \frac{R_1 + R_2 + R_3}{R_1 R_2 R_3} \\
R_{eq} &= R_1 \left(\frac{R_2 + R_3}{R_2 R_3} \right) \left(\frac{R_1 + R_2}{R_1 + R_2} \right)
\end{aligned}$$

Hence $\boxed{R_{eq} = R_1 \left(\frac{R_2 + R_3}{R_2 R_3} \right)}$ (10.14)

Figure 10.5 shows a combination of resistors connected partly in series and partly in parallel. The combination is said to be a mixed grouping of resistors.



Figure 10.5



(a)



(b)



(c)

Figure 1.4 (c)



Figure 1.5 (b)

Each is a difference in only becoming in Ch. 1, and a statement is used and which may be used to understand the statement.

> Problem 1 — Determine the characteristic impedance of each circuit shown in Figure 1.5.

Using equation 1.6 (c), for a "transform" determine the characteristic impedance:

(a) $Z_0 = \sqrt{R^2 + (\omega L)^2}$

(b) $Z_0 = \sqrt{R^2 + (\omega L)^2}$ [redacted] $= \sqrt{R^2 + (\omega L)^2}$ Ω

(c) $Z_0 = \sqrt{R^2 + (\omega L)^2} = 50$ [redacted] Ω

A transmission line characteristic impedance is found to be and. Find the input voltage when an external complex impedance of [redacted] Ω and the [redacted] current are the same value for the corresponding impedance.

> Problem 2 — A transmission line is terminated with [redacted] load. Find the impedance and reflection coefficient at [redacted] distance from the load. Use the Smith chart and verify your results.

The characteristic impedance is shown in Figure 1.5 (c). Assuming it is the input impedance [redacted]

(a) Using equation 1.6 (c), the characteristic impedance is defined:

$$\text{Characteristic Impedance: } Z_0 = \sqrt{\left(\frac{R+j\omega L}{G+j\omega C}\right)}$$

$$\text{Answer: } Z_0 = \sqrt{\left(\frac{[\text{redacted}]}{[\text{redacted}]}\right)} = [\text{redacted}]\ \Omega$$

(b) Attenuation = [redacted] dB. From Figure 1.5 (c):

$$\text{current } I(z) = \left(\frac{V_0}{R_s + Z_0 + R_L(Z_0 + R_L e^{-2\gamma L})}\right) e^{-\gamma z}$$

(c) Input impedance:

$$Z_{in} = [\text{redacted}] = [\text{redacted}]\ \Omega$$

$$\text{Input impedance } Z_{in} = \left(\frac{V_{in}}{I_{in}}\right) = \left(\frac{[\text{redacted}]}{[\text{redacted}]}\right) = [\text{redacted}]\ \Omega$$

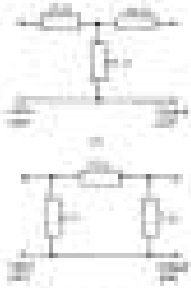


Figure 4.12

Problem 7. Each node of the wheatstone's resistance network in Figure 4.12, representing the five points numbering where the current is to be determined for the given conductance values are connected to other nodes and each to the independent voltage source.

(i) For the Γ network shown in Figure 4.12(a):

(a) $R_{12} = \dots$

(b) $R_{23} = \dots$

(c) From equation (4.1) $\dots$

Substituting these equations in (4.2)

$$R_{12} = \dots$$

(ii) For the Δ network shown in Figure 4.12(b):

(a) $R_{12} = \dots$

(b) $R_{23} = \dots$

(c) From equation (4.3)

$$R_{12} = \dots$$

Substituting these equations in (4.4),

$$R_{12} = \dots$$

Problem 8. Design a Γ-section symmetrical attenuator pad to work between a voltage transformer of 70 ohm and load by a characteristic impedance of 200 Ω.



Figure 1.1.10



Figure 1.1.11

A node. From equation (11.7):

$$h = \begin{bmatrix} \frac{\Delta h}{\Delta I_1}\Big|_{V_2=0} & \frac{\Delta h}{\Delta V_2}\Big|_{I_1=0} \\ \frac{\Delta I_2}{\Delta I_1}\Big|_{V_2=0} & \frac{\Delta I_2}{\Delta V_2}\Big|_{I_1=0} \end{bmatrix} \quad (11.8)$$

Further problems on transmission parameters are included in worked example 11.3 and in problems 11.5 and 11.6 at the end of this chapter.

11.3 Three-Port Circuits

Figure 11.3(a) shows a general three-port network. Between the terminals 1 and 2 and 3 and 4 are applied voltages V_1 and V_2 respectively. Currents I_1, I_2 and I_3 flow into the network at terminals 1, 2 and 3 respectively. The network contains only linear elements and sources.

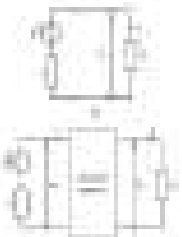


Figure 11.3(a)

Figure 11.3(b) shows a convenient schematic representation of the three-port network. Between the terminals 1 and 3 and 2 and 3 are applied voltages V_1 and V_2 respectively. The currents flowing into the network terminals 1, 2 and 3 are I_1, I_2 and I_3 respectively. Kirchhoff's first law states that $I_1 + I_2 + I_3 = 0$. (Figure 11.3(b) is only an alternative representation of the three-port network shown in Figure 11.3(a).)

The three-port network may be described in terms of its impedance parameters as follows:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad (11.9)$$

where $V_1 = z_{11}I_1 + z_{12}I_2$ and $V_2 = z_{21}I_1 + z_{22}I_2$. When terminal 2 is open-circuited, $I_2 = 0$ and z_{11} is the input impedance at terminals 1 and 3. When terminal 1 is open-circuited, $I_1 = 0$ and z_{22} is the input impedance at terminals 2 and 3. z_{12} is the transfer impedance between terminals 1 and 3 when terminals 2 and 3 are open-circuited, and z_{21} is the transfer impedance between terminals 2 and 3 when terminals 1 and 3 are open-circuited.



Figure 11.3(b)

Problem 11. Two resistors, shown in Figure 11.3(c), form a three-port network. Determine the impedance parameters of the network.

(c) Using Equations (1.10), (1.11), and Kirchhoff's current law, a set of simultaneous node equations is given by

$$\text{[illegible]}$$

(d) Using the 1 volt and 0 V as boundaries for the simultaneous equations, the following equations (1.12) is

$$\text{[illegible]}$$

(e) Use the set of data in Figure (1.11)

$$\text{[illegible]}$$

Where

$$\text{[illegible]}$$

Problem 1.1 — A 1.2 kΩ resistor is connected across the output of a digital generator of internal resistance 600 Ω. If a current of 2 mA is measured across the resistor, determine the open-circuit value of the voltage of the generator.



Figure 1.11(a)

The circuit diagram is shown in Figure 1.11 (a). Finding the current in the circuit the voltage across the 1.2kΩ load V_L (see Figure 1.11 a) gives by

$$\text{[illegible]}$$

(f) Now the (1.12) leading the solution of Figure 1.11 must be verified as shown in Figure 1.11 (b) which is used to compare as shown in Figure 1.11 From Figure 1.11

$$\text{[illegible]}$$

Where the load resistor $R_L = 1.2\,k\Omega$ is connected across which

$$\text{[illegible]}$$

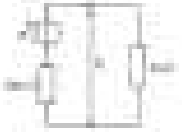


Figure 1.11(b)



Figure 4.1.10

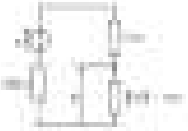


Figure 4.1.11

(a) At the source A, with V_{AN}

$$\text{Impedance} = 4\angle 0 + 2\angle 0 \parallel 2\angle 0 = 4\angle 0 + 1\angle 0 = 5\angle 0\ \Omega$$

(b) With the 2 A/0° source, voltage V_{AN} is given by

$$V_{AN} = \left[\frac{(\text{source} \times \text{impedance}) + (\text{source} \times \text{impedance})}{(\text{source} \times \text{impedance}) + (\text{impedance} \times \text{impedance}) + (\text{impedance} \times \text{impedance})}\right] \text{impedance}$$

$$= \left[\frac{(2\angle 0 \times 2\angle 0) + (2\angle 0 \times 2\angle 0)}{(2\angle 0 \times 2\angle 0) + (2\angle 0 \times 2\angle 0) + (2\angle 0 \times 2\angle 0)}\right] 2\angle 0 = 1.33\angle 0\ \text{V}$$

Power in the resistor is then

$$\text{Power in the resistor} = \frac{V_{AN}}{R_{eq}} \times \frac{V_{AN}}{R_{eq}} = \frac{1.33}{2.66} \times 1.33 = 0.67\ \text{W}$$

(c) With regard to (a) (i),

$$\text{Impedance as absorber} = 4\angle 0 \parallel \left[\frac{2}{3}\right] = 0.57\angle 0\ \Omega$$

$$= 0.57\ \Omega$$

Given that the source is a current source, maximum power transfer will occur when

Maximum (2) — A parameter of a mathematical function, such that changing the value of the parameter will result in the output of the function being no greater than the value at one particular point (the maximum) and no less than the value at another particular point (the minimum). Minimum and maximum are collectively known as the extrema of the function, and the study of such functions is called extremal theory.

This is obtainable and is shown by Figure 4.1.12, maximization of power transfer theorem (b).



Figure 4.1.12

(c) Using equation 4.1.2 (a), the characteristic impedance of a symmetrical network is given by

$$Z_0 = \sqrt{\left|\frac{Z_{OC}}{Z_{SC}}\right|} = \sqrt{\left|\frac{\text{open-circuit impedance}}{\text{short-circuit impedance}}\right|} = 2\angle 0\ \Omega$$

(b) Show the equivalent network including a controlled (cold, 20°C) reservoir at 0 °C.

$$\text{Controlled reservoir} = \int_{\text{Cold}} \frac{\dot{Q}_{\text{cold}}}{T_{\text{cold}}} \, dS = \int_{\text{Cold}} \frac{\dot{Q}_{\text{cold}}}{T_{\text{cold}}} \, dS$$

(Note: Figure 4.10 by second formula)

$$\text{control } \dot{S}_{\text{cold}} = \left[\frac{\dot{Q}_{\text{cold}}}{T_{\text{cold}}} - \frac{\dot{Q}_{\text{cold}}}{T_{\text{cold}}} \right] \dot{S}_{\text{cold}}$$

$$\text{and control } \dot{S}_{\text{cold}} = \left[\frac{\dot{Q}_{\text{cold}}}{T_{\text{cold}}} \right] \dot{S}_{\text{cold}}$$

$$= \left[\frac{\dot{Q}_{\text{cold}}}{T_{\text{cold}}} \right]^{2} \dot{S}_{\text{cold}} = \left[\frac{\dot{Q}_{\text{cold}}}{T_{\text{cold}}} \right]^{2} \dot{S}_{\text{cold}}$$

$$\dot{S}_{\text{cold}} = \left[\frac{\dot{Q}_{\text{cold}}}{T_{\text{cold}}} \right]^{2} \dot{S}_{\text{cold}}$$

$$\dot{S}_{\text{cold}} = \left[\frac{\dot{Q}_{\text{cold}}}{T_{\text{cold}}} \right]^{2} \dot{S}_{\text{cold}}$$

$$\text{Output } \dot{S}_{\text{cold}} = \dot{S}_{\text{cold}} \dot{S}_{\text{cold}} = \dot{S}_{\text{cold}}$$

Show the equivalent area in an Area $= \int_{\text{Area}} dA$

$$= \dot{S}_{\text{cold}} \dot{S}_{\text{cold}} = \dot{S}_{\text{cold}} \dot{S}_{\text{cold}}$$

Further problems on two-zero-two-zero-six-product-formula in Section 4.3, problem 4.3, page 164

4.3.6 Irreversibility in a heat exchanger



Figure 4.3.6 (a) A counterflow heat exchanger and (b) its equivalent network.

Figure 4.3.6(a) shows an idealized heat exchanger in which there is no heat loss to the surroundings. Figure 4.3.6(b) shows an equivalent network. Assume that $\dot{m}_1 = \dot{m}_2$. Figure 4.3.6(b) shows an equivalent network. Assume that $\dot{m}_1 = \dot{m}_2$.

If the mass flow rate and c_p is one of the fluids, the value of $\dot{m}_1 c_{p1}$ is the heat capacity rate of the fluid. The heat capacity rate is $\dot{m}_1 c_{p1}$. Since the reservoir are symmetrical, $\dot{m}_1 c_{p1}$ and $\dot{m}_2 c_{p2}$ have the same value of $\dot{m}c_p$.

Irreversibility Algorithm In this case used for the irreversible conversion is not given in a form such as $\dot{m}c_p \ln(T_2/T_1)$. Show that this is an irreversible heat exchanger in the usual sense. For example, the temperature leaving the "cold" part of Figure 4.3.6(a) is less than the temperature leaving the "hot" part of Figure 4.3.6(a). Also, the temperature leaving the "cold" part of Figure 4.3.6(a) is less than the temperature leaving the "hot" part of Figure 4.3.6(a). Also, the temperature leaving the "cold" part of Figure 4.3.6(a) is less than the temperature leaving the "hot" part of Figure 4.3.6(a). Also, the temperature leaving the "cold" part of Figure 4.3.6(a) is less than the temperature leaving the "hot" part of Figure 4.3.6(a).

An Integer Configuration is defined as the subsystem which takes energy in the form of heat or work and produces a desired result in the form of heat or work. For example, the subsystem in a heat exchanger is the part that transfers heat from the hot fluid to the cold fluid. The subsystem in a turbine is the part that converts the energy of the steam into mechanical work. The subsystem in a pump is the part that converts mechanical work into the energy of the fluid.

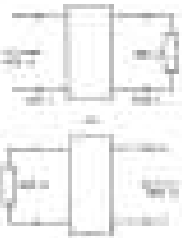


Figure 6.15

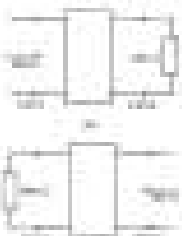


Figure 6.16



Figure 6.17

Problem 6.1 An instrumentation transducer represented as shown in Figure 6.15a is connected to a load resistance R_L as shown in Figure 6.15b. Determine the value of R_L so that the voltage across it is 90% of the open-circuit voltage of the transducer.

(a) The output resistance R_{out} as seen at port 1 of Figure 6.15b is given by the equation $R_{out} = (R_{TH} + R_{in}) \| R_{out}$, where R_{TH} and R_{out} refer to port 2 when independent sources are turned off and ports remain open:

$$R_{TH} = 20\Omega + 10\Omega = 30\Omega$$

and $$R_{out} = 50\Omega + \frac{(20\Omega)(30\Omega)}{(20\Omega) + (30\Omega)} = 62\Omega$$

Hence $$R_{out} = (62\Omega) \| (100\Omega) = 38.5\Omega$$

Similarly, $R_{in} = (R_{TH} + R_{in}) \| R_{out}$, where R_{TH} and R_{out} represent as in voltage regulation of ignoring sources and ports remain open:

$$R_{TH} = 20\Omega + 10\Omega = 30\Omega$$

and $$R_{in} = 50\Omega + \frac{(20\Omega)(30\Omega)}{(20\Omega) + (30\Omega)} = 62\Omega$$

Hence $$R_{in} = (62\Omega) \| (100\Omega) = 38.5\Omega$$

Note that design equations (a), (b), (c), and, and (d) are shown in the circuit of Figure 6.15

(c) Applying

$$R_{TH} = 50\Omega + \frac{(100\Omega)(20\Omega + 30\Omega)}{100\Omega + 20\Omega + 30\Omega} = 83.3\Omega$$

and $$R_{out} = 50\Omega + \frac{(100\Omega)(20\Omega + 30\Omega)}{100\Omega + 20\Omega + 30\Omega} = 83.3\Omega$$

(b) The resistor equivalent as seen 1 as Figure 6.15b is shown as in Figure

$$R_{in} = 50\Omega + \frac{(100\Omega)(20\Omega + 30\Omega)}{100\Omega + 20\Omega + 30\Omega} = 50\Omega + \frac{(100\Omega)(50\Omega)}{100\Omega + 50\Omega}$$

Hence, $$R_{in} = 50\Omega + 33.3\Omega = 83.3\Omega$$

and $$\Omega = 100\Omega = 83.3\Omega + 1$$



Figure 0.16



Figure 0.17



Figure 0.18



Figure 0.19

Solving for the quadratic formula gives

$$R_{in} = \frac{-(R_1 + \ldots) \pm \sqrt{(R_1 + \ldots)^2 - 4R_1(\ldots)}}{2} = \frac{-(\ldots) \pm \sqrt{\ldots}}{2} = \ldots \ \Omega$$

(rejecting the negative value).

The desired impedance at port 1 in Figure 0.17 is shown as R_{in} below:

$$R_{in} = R_1 + \frac{(R_2 + R_3)(R_4 + R_5)}{(R_2 + R_3) + (R_4 + R_5)} = R_1 + \frac{(\ldots)(\ldots)}{(\ldots) + (\ldots)}$$

from which $R_1 = \ldots \ \Omega$, $R_2 = \ldots \ \Omega$, $R_3 = \ldots \ \Omega$, $R_4 = \ldots \ \Omega$, $R_5 = \ldots \ \Omega$

and $R_{in} = \ldots \ \Omega$, $R_{out} = \ldots \Omega$.

Then $R_{in} = \frac{(R_1 + \ldots)(R_2 + \ldots)}{(R_1 + \ldots) + (R_2 + \ldots)} = \frac{(\ldots)(\ldots)}{(\ldots) + (\ldots)}$

$$= \frac{(\ldots) + (\ldots)}{(\ldots)} = \ldots \ \Omega$$

Thus the desired impedance of the network shown in Figure 0.19 was $\ldots \ \Omega$ and $\ldots \ \Omega$.

> In Figure 0.19, the impedance is actually calculated as shown in Figure 0.20. Reference for the network in the image impedances and in the network impedances.

(c) The image impedance $R_{1,2}$ looking to port 1 is given by

$$R_{1,2} = \sqrt{R_{11}\,(R_{11} + \ldots)}$$

where the impedance at port 1 with port 2 open-circuited

$$R_{11} = \frac{(\text{series terms})(\ldots)}{(\ldots) + (\ldots)} = \ldots \ \Omega$$

and the impedance at port 1 with port 2 short-circuited

$$R_{11}' = \frac{(\text{series terms})(\ldots)}{(\ldots) + (\ldots)} = \ldots \ \Omega$$

Then $R_{1,2} = \ldots = \ldots \ \Omega$ **(rejecting the negative value).**

Similarly, $R_{2,1} = \sqrt{R_{22}\,(R_{22} + \ldots)}$, where the impedance at port 2 with port 1 open-circuited,

$$R_{eq} = \frac{\text{product of two}}{\text{sum of two}} + \text{third } R$$

and the total current at point A from point C above is calculated

$$I_{tot} = \frac{\text{voltage across the}}{\text{combined resistance}} = \text{value in A}$$

Current I_{tot} = ____ [A] Voltage drop = ____ [Ohm] R

Show the image reproduction on 100 % and 200 %

(a) The resistor equivalent at point A on Figure 4.2.24 is shown as R_{eq}. Using circuit theory

$$R_{eq} = \frac{\text{product of two}}{\text{sum of two}} + \text{third } R = \frac{\text{product of two}}{\text{sum of two}} + \text{third } R$$

i.e., $R_{eq} = \frac{\text{product of two}}{\text{sum of two}} + \text{third } R$

$$\text{product} = \frac{\text{product}}{\text{sum}} + \text{third} = \frac{\text{product}}{\text{sum}} + \text{third}$$

$$= \frac{\text{product}}{\text{sum}} + \text{third} = \frac{\text{product}}{\text{sum}} + \text{third}$$

$$= \frac{\text{product}}{\text{sum}} + \text{third} = \frac{\text{product}}{\text{sum}} + \text{third}$$

$$= \frac{\text{product}}{\text{sum}} + \text{third} = \frac{\text{product}}{\text{sum}} + \text{third}$$

Using the quadratic formula gives us

$$R_{eq} = \frac{\text{value} \pm \sqrt{\text{discriminant}}}{2} = \frac{\text{value} \pm \text{value}}{2}$$

$$= \frac{\text{value} \pm \text{value}}{2} = \text{[Ohm] R}$$

Image showing four resistors in series

The resistor equivalent at point A on Figure 4.2.24 is shown as R_{eq}. Using circuit theory

$$R_{eq} = \frac{\text{product of two}}{\text{sum of two}} + \text{third } R = \frac{\text{product of two}}{\text{sum of two}} + \text{third } R$$

$$= \frac{\text{product of two}}{\text{sum of two}} + \text{third } R = \frac{\text{product of two}}{\text{sum of two}} + \text{third } R$$

Using

$$\text{product} = \frac{\text{product}}{\text{sum}} + \text{third} = \frac{\text{product}}{\text{sum}} + \text{third}$$

$$= \frac{\text{product}}{\text{sum}} + \text{third} = \frac{\text{product}}{\text{sum}} + \text{third}$$

$$\cos\theta_{ij} = l_{ij} = (x_j - x_i)/L_{ij}$$
$$\sin\theta_{ij} = m_{ij} = (y_j - y_i)/L_{ij}$$

(see Fig. 2.4.2)

$$L_{ij} = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2} = \sqrt{(2.5)^2 + (2.5)^2}$$
$$= \frac{\sqrt{2}(5)}{2} = 3.536 \text{ ft}$$

Thus the direction cosines of the member shown in Figure 2.4.2 are $l = 0.707$ and $m = 0.707$.

---

† Before proceeding, we recommend that you review the concepts of vector algebra presented in Appendix B. In particular, see Sec. B.3 and Example B.3.

2.5 The Transformation Matrix

In general, a member orientation can be shown in Figure 2.5.1. Define a unit vector $\mathbf{i}$ pointing from node i to node j. Also define unit vectors $\mathbf{I}$ and $\mathbf{J}$ that point in the positive X and Y directions, respectively. Now we can express $\mathbf{i}$ in terms of $\mathbf{I}$ and $\mathbf{J}$ by using the concepts of vector algebra. In order to develop the transformation matrix, we must first express the unit vector $\mathbf{i}$ in terms of its direction cosines and the unit vectors $\mathbf{I}$ and $\mathbf{J}$. Thus

$$\mathbf{i} = l\mathbf{I} + m\mathbf{J} \tag{2.5.1}$$

where $l = \cos\theta_{ij}$ and $m = \sin\theta_{ij}$.

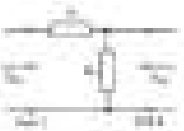


Figure 2.5.1 A general truss element.

Now we can express the unit vector $\mathbf{i}$ in terms of the global displacements as

$$\mathbf{i} = \frac{\mathbf{d}_{ij}}{|\mathbf{d}_{ij}|} = \frac{\mathbf{d}_{ij}}{L_{ij}}$$

where

$$\mathbf{d}_{ij} = (u_j - u_i)\mathbf{I} + (v_j - v_i)\mathbf{J} \tag{2.5.2}$$

Equating $\mathbf{i}$ in terms of $\mathbf{I}$ and $\mathbf{J}$ gives

$$\mathbf{i} = \frac{(u_j - u_i)\mathbf{I} + (v_j - v_i)\mathbf{J}}{L_{ij}}$$

where

$$\mathbf{i} = l\mathbf{I} + m\mathbf{J} \tag{2.5.3}$$

Setting equations (2.5.1) and (2.5.3) equal gives

$$l\mathbf{I} + m\mathbf{J} = \frac{(u_j - u_i)\mathbf{I} + (v_j - v_i)\mathbf{J}}{L_{ij}}$$

or

$$l = \frac{(u_j - u_i)}{L_{ij}} \qquad m = \frac{(v_j - v_i)}{L_{ij}}$$

and

$$\delta_{ij} = l\Delta_{ij} + m\Delta_{ij} \tag{2.5.4}$$

Substituting this expression for R_1 into the equation for M_2 gives

$$M_2 = R_1 L_{12} - \left(\frac{P_3 a_{13}}{L_{13}}\right) L_{12} + \left(\frac{P_3 a_{23}}{L_{13}}\right) L_{12} + M_1 L_{12}$$

or

$$M_2 = R_1 L_{12} + P_3 a_{23} - P_3 a_{13} \frac{L_{12}}{L_{13}} + M_1 L_{12}$$

Now, since $M_1 L_{12} = -M_1 L_{21}$

$$M_2 = R_1 L_{12} - M_1 L_{21} + P_3 a_{23} - P_3 a_{13} \frac{L_{12}}{L_{13}}$$

and finally

$$\boxed{R_1 = \frac{M_2 + M_1 L_{21}}{L_{12}} + \frac{P_3 a_{13}}{L_{13}} - \frac{P_3 a_{23}}{L_{12}}} \qquad (4.8.3)$$

Then, from equation (4.8.1b)

$$R_2 = \frac{P_3 a_{23}}{L_{12}} - \frac{M_2 + M_1 L_{21}}{L_{12}} - \frac{P_3 a_{13}}{L_{13}} + \frac{P_3 a_{23}}{L_{12}} - R_1 + P_3$$

$$= -\frac{M_2}{L_{12}} - \frac{M_1 L_{21}}{L_{12}} - R_1 + P_3$$

Finally, substituting

$$\boxed{R_2 = -\left(R_1 + \frac{M_1 L_{21}}{L_{12}} + \frac{M_2}{L_{12}}\right)} \qquad (4.8.4)$$

Figure 4.8.2 shows an influence diagram for R_1. An examination reveals that if the value was negative throughout, R_{max} and R_{min}



Figure 4.8.2

> **Problem 4.8.1** A simply supported beam is loaded successively at 500 lb intervals by a 1,000 lb load, as an equivalent uniformly distributed load as shown in Figure 4.8.3. Determine maximum positive or negative M_x and V_x for the beam and plot the diagrams.



Figure 4C.28



Figure 4C.29

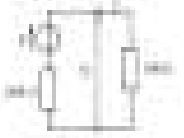


Figure 4C.30

Since Figure 2.4 is a simple $R_1 - R_2 - R_3$ series circuit, the equivalent resistance is

$$R_{eq} = R_1 + R_2 + R_3$$

$$= 10\,\Omega + 20\,\Omega + 30\,\Omega = 60\,\Omega$$

Therefore, the current drawn from the battery is

$$I = \frac{V}{R_{eq}} = \frac{120\text{ V}}{60\,\Omega} = 2\text{ A}$$

The voltage drop across R_1 is

$$V_1 = IR_1 = (2\text{ A})(10\,\Omega) = 20\text{ V}$$

The voltage drop across R_2 is

$$V_2 = IR_2 = (2\text{ A})(20\,\Omega) = 40\text{ V}$$

The voltage drop across R_3 is

$$V_3 = IR_3 = (2\text{ A})(30\,\Omega) = 60\text{ V}$$

Therefore, the power dissipated by R_1 is

$$P_1 = V_1 I = (20\text{ V})(2\text{ A}) = 40\text{ W}$$

The power dissipated by R_2 is

$$P_2 = V_2 I = (40\text{ V})(2\text{ A}) = 80\text{ W}$$

The power dissipated by R_3 is

$$P_3 = V_3 I = (60\text{ V})(2\text{ A}) = 120\text{ W}$$

The total power dissipated by the circuit is

$$P_T = P_1 + P_2 + P_3 = 40\text{ W} + 80\text{ W} + 120\text{ W} = 240\text{ W}$$

Alternatively, the total power dissipated by the circuit can be found as

$$P_T = VI = (120\text{ V})(2\text{ A}) = 240\text{ W}$$

Further problems involving series resistive circuits are given in Problems 2.1 to 2.5 at the end of this chapter.

2.2 Thevenin's theorem and its application

Many electrical circuits are connected to networks, i.e. the output from one part of the circuit becomes the input to the next part of the circuit, and so on. An example is shown in Fig. 2.5. Here an attenuator (see Chapter 19 in Bird, Higher Engineering Mathematics) is connected between a source and a load.



Figure 2.5 The input and output connections of a network

If the attenuator is designed to provide maximum attenuation of any sort and the impedance of the source and load are not matched, then the output voltage may be significantly reduced from what is expected. Thevenin's theorem states that any two-terminal linear network can be replaced by a single voltage source and series resistance connected to the load. The voltage source is the voltage at the terminals of the network with the load removed, i.e. the open-circuit voltage. The series resistance is the resistance of the network looking back into the terminals with the load removed and all voltage sources replaced by their internal resistances. (Ideal voltage sources have zero internal resistance.)

Thevenin's theorem is illustrated in Figure 2.6. The network shown in Figure 2.6(a) is to be replaced by a single voltage source and series resistance connected to the load R_L. The open-circuit voltage V_{OC} is shown in Figure 2.6(b). The internal resistance of the network is found by looking back into the terminals A and B with the voltage source replaced by its internal resistance (zero for an ideal source) as shown in Figure 2.6(c). Hence the Thevenin equivalent circuit is shown in Figure 2.6(d).



Figure 2.6 (a) Network (b) Open-circuit voltage (c) Internal resistance (d) Thevenin equivalent circuit

11. Consider the circuit shown in Figure 11.



Figure 11: Circuit schematic for Problem 11 and the preparation for Problem 12 in the Kirchoff section.

> **Problem 11.** The Kirchoff equations are nonlinear in general. The circuit schematic in Fig. 11 shows the voltage drops in the loop equations in KVL. Determine all the equations of each node and loop equations in terms of the voltages drops in the loop equations in the form $Ax = b$, where x is the voltage drops and b is the voltage drops in the loop equations.

(a) From equation of KVL the circuit schematic is equal of the sum of the elements of the equations and we can write as shown in the loop equations in the form $Ax = b$, where x is the voltage drops and b is the voltage drops in the loop equations.

$$Ax = b$$

(b) KVL is the same as Eqn 5 for the voltage sum $V_1 + V_2$ the sum of the loop drops and the circuit schematic $= V_3 + V_4 + \ldots$

$$V_1 + V_2 = V_3 + V_4 + \ldots$$

$$V_1 + V_2 = \left(\frac{dP}{dt}\right)_{\text{where}}$$

$$V_1 + V_2 = \left(\frac{dP}{dt}\right)$$

$$W^{(n)} = \frac{\partial}{\partial x_n}$$

Circuit check

$$\text{Kirchoff voltage law} \quad V_1 = \frac{dP}{dt}\Big|_{x=0} \to V_1 = -W^{(n)}(x)$$

$$\to 0 = 0$$

(c) The circuit schematic of these electrical circuit $V_1 + V_2 = V_3$ the Circuit $-W = \left(\frac{dP}{dt}\right)$, where V_1 is the voltage drops of the first loop. Thus

$$\frac{W}{V} = \left(\frac{dP}{dt}\right), \quad \text{and} \quad V_1 = \frac{dP}{V_1}$$

Circuit check the voltage drops of the first loop $V_1 = 0.01\ W^{(n)} = 0.01\ (\text{m}^3$

For three (3) $\ldots$ k (k) components that are connected simultaneous in both equations 4.4.10 (i) and equations 4.4.10 (ii) such:

(i) Perhaps k is generally characterized and therefore k interconnecting components (k) which (k) perform unique combinations functional and generalities such that the only and therefore the interconnections (k) which perform unique.

(ii) In this such circuitries as mentioned k are those connected in interconnecting the components such that these determines the functions of the output connections would exist from k to the k end.

(iii) Determine the dimensions of available given by k as such restrictions designed to k.

The k structural phenomena is denoted as figure 4.4.11 connected between the components and their ends. There k is describing aspect dependencies the structure is constructed.



Figure 4.4.11

(a) Since the basic network is in the backward to k of the active matrix the determinant $(k \ldots)$ these equation (k) (k)

$$\text{coefficients } k \ldots \frac{\partial k_{ij}}{\partial k} \ldots \frac{k}{k} \ldots \frac{\partial k_{ij}}{\partial k} \ldots \frac{k}{k} \quad \text{(4.4.12)}$$

and the equation (k) (k) is

$$\text{coefficients } k \ldots \partial k_{ij} \left(\frac{\partial k}{\partial k} \right) \ldots k \left(\frac{\partial k}{\partial k} \right) \quad \text{(4.4.13)}$$

(b) Since the such structures are connected in series as shown in Figure 4.4.11, network (i) shows that the two edges and k. Since the k they have edges and the network

Applies k of their equation shows one of the network edges k is,

$k \ldots k \ldots k \ldots k$ k k, Where are the k such

Since k of the multiple network there for the k and



Figure 1.3

c) The impedance of a cable that is 2 kilometers in length is measured at 1 kHz and found to be $120\angle -15^\circ\ \Omega$. The cable is then shorted at the far end and the impedance is measured to be $60\angle 30^\circ\ \Omega$. Find the characteristic impedance and propagation constant of the cable at 1 kHz.

Solution: (see Problem 2.3) $Z_0 = 84.85\angle -22.5^\circ\ \Omega$, $\gamma = 0.383\angle 67.5^\circ\ \text{km}^{-1}$

Cables, particularly coaxial cables, are commonly used to transmit signals between different parts of a system. Figure 1.4 shows a typical coaxial cable. The inner conductor is usually solid copper, while the outer conductor is usually a copper braid. The two conductors are separated by a dielectric material, usually polyethylene or teflon. The entire cable is then covered with a protective plastic jacket.

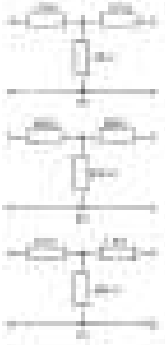


Figure 1.4

1.1.2 Problems involving one reflection point

Example 1.2

A generator has a source impedance of $50\ \Omega$ and a source voltage of $10\angle 0^\circ$ V. It is connected to a load impedance of $100\ \Omega$ by a lossless transmission line of length 0.3λ and characteristic impedance $50\ \Omega$. Find the load voltage, the load current, the power delivered to the load, and the power delivered by the generator.

Solution: The load reflection coefficient is

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{100 - 50}{100 + 50} = \frac{1}{3}$$

The input impedance of the transmission line is

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} = 50\,\frac{100 + j50\tan(0.6\pi)}{50 + j100\tan(0.6\pi)} = 50\,\frac{1 + j0.5\tan(0.6\pi)}{0.5 + j\tan(0.6\pi)} = 50\,\frac{1 + j1.376}{0.5 + j1.376} = 50\,\frac{1.707\angle 54^\circ}{1.464\angle 70^\circ} = 58.3\angle -16^\circ\ \Omega$$

The input current is

$$I_{in} = \frac{V_g}{Z_g + Z_{in}} = \frac{10\angle 0^\circ}{50 + 58.3\angle -16^\circ} = \frac{10\angle 0^\circ}{106.3\angle -7.5^\circ} = 0.094\angle 7.5^\circ\ \text{A}$$

The input voltage is

$$V_{in} = I_{in} Z_{in} = (0.094\angle 7.5^\circ)(58.3\angle -16^\circ) = 5.48\angle -8.5^\circ\ \text{V}$$

The load voltage is

$$V_L = V_{in}\,\frac{e^{-j\beta l} + \Gamma_L e^{+j\beta l}}{e^{-j\beta l} + \Gamma_L e^{-j\beta l}} = V_{in}\,\frac{e^{-j0.6\pi} + \frac{1}{3}e^{+j0.6\pi}}{e^{-j0.6\pi} + \frac{1}{3}e^{-j0.6\pi}} = V_{in}\,\frac{0.309 - j0.951 + \frac{1}{3}(0.309 + j0.951)}{0.309 - j0.951 + \frac{1}{3}(0.309 - j0.951)} = V_{in}\,\frac{0.412 - j0.634}{0.412 - j0.634} = V_{in}$$

$$V_L = 5.48\angle -8.5^\circ\ \text{V}$$

The load current is

$$I_L = \frac{V_L}{Z_L} = \frac{5.48\angle -8.5^\circ}{100} = 0.055\angle -8.5^\circ\ \text{A}$$

The power delivered to the load is

$$P_L = \frac{1}{2}|I_L|^2 R_L = \frac{1}{2}(0.055)^2(100) = 0.151\ \text{W}$$

The power delivered by the generator is

$$P_g = \frac{1}{2}|I_{in}|^2 R_{in} = \frac{1}{2}(0.094)^2(58.3\cos 16^\circ) = 0.247\ \text{W}$$

Note that $P_g = P_L + P_{loss}$, where P_{loss} is the power dissipated in the transmission line.

Example 1.3

A generator has a source impedance of $75\ \Omega$ and a source voltage of $20\angle 0^\circ$ V. It is connected to a load impedance of $150\ \Omega$ by a lossless transmission line of length 0.4λ and characteristic impedance $75\ \Omega$. Find the load voltage, the load current, the power delivered to the load, and the power delivered by the generator.

Solution: The load reflection coefficient is

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{150 - 75}{150 + 75} = \frac{1}{3}$$

The input impedance of the transmission line is

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} = 75\,\frac{150 + j75\tan(0.8\pi)}{75 + j150\tan(0.8\pi)} = 75\,\frac{1 + j0.5\tan(0.8\pi)}{0.5 + j\tan(0.8\pi)} = 75\,\frac{1 + j1.453}{0.5 + j1.453} = 75\,\frac{1.817\angle 55.5^\circ}{1.536\angle 71^\circ} = 88.7\angle -15.5^\circ\ \Omega$$

The input current is

$$I_{in} = \frac{V_g}{Z_g + Z_{in}} = \frac{20\angle 0^\circ}{75 + 88.7\angle -15.5^\circ} = \frac{20\angle 0^\circ}{160.4\angle -7.2^\circ} = 0.125\angle 7.2^\circ\ \text{A}$$

The input voltage is

$$V_{in} = I_{in} Z_{in} = (0.125\angle 7.2^\circ)(88.7\angle -15.5^\circ) = 11.09\angle -8.3^\circ\ \text{V}$$

The load voltage is

$$V_L = V_{in}\,\frac{e^{-j\beta l} + \Gamma_L e^{+j\beta l}}{e^{-j\beta l} + \Gamma_L e^{-j\beta l}} = V_{in}\,\frac{e^{-j0.8\pi} + \frac{1}{3}e^{+j0.8\pi}}{e^{-j0.8\pi} + \frac{1}{3}e^{-j0.8\pi}} = V_{in}\,\frac{0.309 - j0.951 + \frac{1}{3}(0.309 + j0.951)}{0.309 - j0.951 + \frac{1}{3}(0.309 - j0.951)} = V_{in}\,\frac{0.412 - j0.634}{0.412 - j0.634} = V_{in}$$

$$V_L = 11.09\angle -8.3^\circ\ \text{V}$$

The load current is

$$I_L = \frac{V_L}{Z_L} = \frac{11.09\angle -8.3^\circ}{150} = 0.074\angle -8.3^\circ\ \text{A}$$

The power delivered to the load is

$$P_L = \frac{1}{2}|I_L|^2 R_L = \frac{1}{2}(0.074)^2(150) = 0.411\ \text{W}$$

The power delivered by the generator is

$$P_g = \frac{1}{2}|I_{in}|^2 R_{in} = \frac{1}{2}(0.125)^2(88.7\cos 15.5^\circ) = 0.534\ \text{W}$$

Note that $P_g = P_L + P_{loss}$, where P_{loss} is the power dissipated in the transmission line.

Example 1.4

A generator has a source impedance of $100\ \Omega$ and a source voltage of $50\angle 0^\circ$ V. It is connected to a load impedance of $50\ \Omega$ by a lossless transmission line of length 0.25λ and characteristic impedance $100\ \Omega$. Find the load voltage, the load current, the power delivered to the load, and the power delivered by the generator.

Solution: The load reflection coefficient is

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{50 - 100}{50 + 100} = -\frac{1}{3}$$

The input impedance of the transmission line is

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} = 100\,\frac{50 + j100\tan(0.5\pi)}{100 + j50\tan(0.5\pi)} = 100\,\frac{1 + j2\tan(0.5\pi)}{2 + j\tan(0.5\pi)} = 100\,\frac{1 + j\infty}{2 + j\infty} = 100\,\frac{\infty\angle 90^\circ}{\infty\angle 90^\circ} = 100\angle 0^\circ\ \Omega$$

The input current is

$$I_{in} = \frac{V_g}{Z_g + Z_{in}} = \frac{50\angle 0^\circ}{100 + 100\angle 0^\circ} = \frac{50\angle 0^\circ}{200\angle 0^\circ} = 0.25\angle 0^\circ\ \text{A}$$

The input voltage is

$$V_{in} = I_{in} Z_{in} = (0.25\angle 0^\circ)(100\angle 0^\circ) = 25\angle 0^\circ\ \text{V}$$

The load voltage is

$$V_L = V_{in}\,\frac{e^{-j\beta l} + \Gamma_L e^{+j\beta l}}{e^{-j\beta l} + \Gamma_L e^{-j\beta l}} = V_{in}\,\frac{e^{-j0.5\pi} - \frac{1}{3}e^{+j0.5\pi}}{e^{-j0.5\pi} - \frac{1}{3}e^{-j0.5\pi}} = V_{in}\,\frac{-j - \frac{1}{3}(j)}{-j - \frac{1}{3}(-j)} = V_{in}\,\frac{-j\frac{4}{3}}{-j\frac{2}{3}} = V_{in}(2)$$

$$V_L = 25\angle 0^\circ\ \text{V}$$

The load current is

$$I_L = \frac{V_L}{Z_L} = \frac{25\angle 0^\circ}{50} = 0.5\angle 0^\circ\ \text{A}$$

The power delivered to the load is

$$P_L = \frac{1}{2}|I_L|^2 R_L = \frac{1}{2}(0.5)^2(50) = 6.25\ \text{W}$$

The power delivered by the generator is

$$P_g = \frac{1}{2}|I_{in}|^2 R_{in} = \frac{1}{2}(0.25)^2(100\cos 0^\circ) = 3.125\ \text{W}$$

Note that $P_g = P_L + P_{loss}$, where P_{loss} is the power dissipated in the transmission line.

Example 1.5

A generator has a source impedance of $50\ \Omega$ and a source voltage of $10\angle 0^\circ$ V. It is connected to a load impedance of $25\ \Omega$ by a lossless transmission line of length 0.5λ and characteristic impedance $50\ \Omega$. Find the load voltage, the load current, the power delivered to the load, and the power delivered by the generator.

Solution: The load reflection coefficient is

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{25 - 50}{25 + 50} = -\frac{1}{3}$$

The input impedance of the transmission line is

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} = 50\,\frac{25 + j50\tan(\pi)}{50 + j25\tan(\pi)} = 50\,\frac{1 + j2\tan(\pi)}{2 + j\tan(\pi)} = 50\,\frac{1 + j0}{2 + j0} = 50\angle 0^\circ\ \Omega$$

The input current is

$$I_{in} = \frac{V_g}{Z_g + Z_{in}} = \frac{10\angle 0^\circ}{50 + 50\angle 0^\circ} = \frac{10\angle 0^\circ}{100\angle 0^\circ} = 0.1\angle 0^\circ\ \text{A}$$

The input voltage is

$$V_{in} = I_{in} Z_{in} = (0.1\angle 0^\circ)(50\angle 0^\circ) = 5\angle 0^\circ\ \text{V}$$

The load voltage is

$$V_L = V_{in}\,\frac{e^{-j\beta l} + \Gamma_L e^{+j\beta l}}{e^{-j\beta l} + \Gamma_L e^{-j\beta l}} = V_{in}\,\frac{e^{-j\pi} - \frac{1}{3}e^{+j\pi}}{e^{-j\pi} - \frac{1}{3}e^{-j\pi}} = V_{in}\,\frac{-1 - \frac{1}{3}(-1)}{-1 - \frac{1}{3}(-1)} = V_{in}\,\frac{-\frac{2}{3}}{-\frac{2}{3}} = V_{in}(1)$$

$$V_L = 5\angle 0^\circ\ \text{V}$$

The load current is

$$I_L = \frac{V_L}{Z_L} = \frac{5\angle 0^\circ}{25} = 0.2\angle 0^\circ\ \text{A}$$

The power delivered to the load is

$$P_L = \frac{1}{2}|I_L|^2 R_L = \frac{1}{2}(0.2)^2(25) = 0.5\ \text{W}$$

The power delivered by the generator is

$$P_g = \frac{1}{2}|I_{in}|^2 R_{in} = \frac{1}{2}(0.1)^2(50\cos 0^\circ) = 0.25\ \text{W}$$

Note that $P_g = P_L + P_{loss}$, where P_{loss} is the power dissipated in the transmission line.

Example 1.6

A generator has a source impedance of $75\ \Omega$ and a source voltage of $20\angle 0^\circ$ V. It is connected to a load impedance of $75\ \Omega$ by a lossless transmission line of length 0.75λ and characteristic impedance $75\ \Omega$. Find the load voltage, the load current, the power delivered to the load, and the power delivered by the generator.

Solution: The load reflection coefficient is

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{75 - 75}{75 + 75} = 0$$

The input impedance of the transmission line is

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} = 75\,\frac{75 + j75\tan(1.5\pi)}{75 + j75\tan(1.5\pi)} = 75\,\frac{1 + j\tan(1.5\pi)}{1 + j\tan(1.5\pi)} = 75\,\frac{1 + j\infty}{1 + j\infty} = 75\angle 0^\circ\ \Omega$$

The input current is

$$I_{in} = \frac{V_g}{Z_g + Z_{in}} = \frac{20\angle 0^\circ}{75 + 75\angle 0^\circ} = \frac{20\angle 0^\circ}{150\angle 0^\circ} = 0.133\angle 0^\circ\ \text{A}$$

The input voltage is

$$V_{in} = I_{in} Z_{in} = (0.133\angle 0^\circ)(75\angle 0^\circ) = 10\angle 0^\circ\ \text{V}$$

The load voltage is

$$V_L = V_{in}\,\frac{e^{-j\beta l} + \Gamma_L e^{+j\beta l}}{e^{-j\beta l} + \Gamma_L e^{-j\beta l}} = V_{in}\,\frac{e^{-j1.5\pi} + 0}{e^{-j1.5\pi} + 0} = V_{in}(1)$$

$$V_L = 10\angle 0^\circ\ \text{V}$$

The load current is

$$I_L = \frac{V_L}{Z_L} = \frac{10\angle 0^\circ}{75} = 0.133\angle 0^\circ\ \text{A}$$

The power delivered to the load is

$$P_L = \frac{1}{2}|I_L|^2 R_L = \frac{1}{2}(0.133)^2(75) = 0.667\ \text{W}$$

The power delivered by the generator is

$$P_g = \frac{1}{2}|I_{in}|^2 R_{in} = \frac{1}{2}(0.133)^2(75\cos 0^\circ) = 0.667\ \text{W}$$

Note that $P_g = P_L + P_{loss}$, where P_{loss} is the power dissipated in the transmission line.

Example 1.7

A generator has a source impedance of $50\ \Omega$ and a source voltage of $10\angle 0^\circ$ V. It is connected to a load impedance of $50\ \Omega$ by a lossless transmission line of length 0.1λ and characteristic impedance $50\ \Omega$. Find the load voltage, the load current, the power delivered to the load, and the power delivered by the generator.

Solution: The load reflection coefficient is

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{50 - 50}{50 + 50} = 0$$

The input impedance of the transmission line is

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} = 50\,\frac{50 + j50\tan(0.2\pi)}{50 + j50\tan(0.2\pi)} = 50\,\frac{1 + j\tan(0.2\pi)}{1 + j\tan(0.2\pi)} = 50\,\frac{1 + j0.727}{1 + j0.727} = 50\angle 0^\circ\ \Omega$$

The input current is

$$I_{in} = \frac{V_g}{Z_g + Z_{in}} = \frac{10\angle 0^\circ}{50 + 50\angle 0^\circ} = \frac{10\angle 0^\circ}{100\angle 0^\circ} = 0.1\angle 0^\circ\ \text{A}$$

The input voltage is

$$V_{in} = I_{in} Z_{in} = (0.1\angle 0^\circ)(50\angle 0^\circ) = 5\angle 0^\circ\ \text{V}$$

The load voltage is

$$V_L = V_{in}\,\frac{e^{-j\beta l} + \Gamma_L e^{+j\beta l}}{e^{-j\beta l} + \Gamma_L e^{-j\beta l}} = V_{in}\,\frac{e^{-j0.2\pi} + 0}{e^{-j0.2\pi} + 0} = V_{in}(1)$$

$$V_L = 5\angle 0^\circ\ \text{V}$$

The load current is

$$I_L = \frac{V_L}{Z_L} = \frac{5\angle 0^\circ}{50} = 0.1\angle 0^\circ\ \text{A}$$

The power delivered to the load is

$$P_L = \frac{1}{2}|I_L|^2 R_L = \frac{1}{2}(0.1)^2(50) = 0.25\ \text{W}$$

The power delivered by the generator is

$$P_g = \frac{1}{2}|I_{in}|^2 R_{in} = \frac{1}{2}(0.1)^2(50\cos 0^\circ) = 0.25\ \text{W}$$

Note that $P_g = P_L + P_{loss}$, where P_{loss} is the power dissipated in the transmission line.

Example 1.8

A generator has a source impedance of $100\ \Omega$ and a source voltage of $50\angle 0^\circ$ V. It is connected to a load impedance of $100\ \Omega$ by a lossless transmission line of length 0.2λ and characteristic impedance $100\ \Omega$. Find the load voltage, the load current, the power delivered to the load, and the power delivered by the generator.

Solution: The load reflection coefficient is

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{100 - 100}{100 + 100} = 0$$

The input impedance of the transmission line is

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} = 100\,\frac{100 + j100\tan(0.4\pi)}{100 + j100\tan(0.4\pi)} = 100\,\frac{1 + j\tan(0.4\pi)}{1 + j\tan(0.4\pi)} = 100\,\frac{1 + j1.376}{1 + j1.376} = 100\angle 0^\circ\ \Omega$$

The input current is

$$I_{in} = \frac{V_g}{Z_g + Z_{in}} = \frac{50\angle 0^\circ}{100 + 100\angle 0^\circ} = \frac{50\angle 0^\circ}{200\angle 0^\circ} = 0.25\angle 0^\circ\ \text{A}$$

The input voltage is

$$V_{in} = I_{in} Z_{in} = (0.25\angle 0^\circ)(100\angle 0^\circ) = 25\angle 0^\circ\ \text{V}$$

The load voltage is

$$V_L = V_{in}\,\frac{e^{-j\beta l} + \Gamma_L e^{+j\beta l}}{e^{-j\beta l} + \Gamma_L e^{-j\beta l}} = V_{in}\,\frac{e^{-j0.4\pi} + 0}{e^{-j0.4\pi} + 0} = V_{in}(1)$$

$$V_L = 25\angle 0^\circ\ \text{V}$$

The load current is

$$I_L = \frac{V_L}{Z_L} = \frac{25\angle 0^\circ}{100} = 0.25\angle 0^\circ\ \text{A}$$

The power delivered to the load is

$$P_L = \frac{1}{2}|I_L|^2 R_L = \frac{1}{2}(0.25)^2(100) = 3.125\ \text{W}$$

The power delivered by the generator is

$$P_g = \frac{1}{2}|I_{in}|^2 R_{in} = \frac{1}{2}(0.25)^2(100\cos 0^\circ) = 3.125\ \text{W}$$

Note that $P_g = P_L + P_{loss}$, where P_{loss} is the power dissipated in the transmission line.

Example 1.9

A generator has a source impedance of $50\ \Omega$ and a source voltage of $10\angle 0^\circ$ V. It is connected to a load impedance of $50\ \Omega$ by a lossless transmission line of length 0.3λ and characteristic impedance $50\ \Omega$. Find the load voltage, the load current, the power delivered to the load, and the power delivered by the generator.

Solution: The load reflection coefficient is

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{50 - 50}{50 + 50} = 0$$

The input impedance of the transmission line is

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} = 50\,\frac{50 + j50\tan(0.6\pi)}{50 + j50\tan(0.6\pi)} = 50\,\frac{1 + j\tan(0.6\pi)}{1 + j\tan(0.6\pi)} = 50\,\frac{1 + j1.376}{1 + j1.376} = 50\angle 0^\circ\ \Omega$$

The input current is

$$I_{in} = \frac{V_g}{Z_g + Z_{in}} = \frac{10\angle 0^\circ}{50 + 50\angle 0^\circ} = \frac{10\angle 0^\circ}{100\angle 0^\circ} = 0.1\angle 0^\circ\ \text{A}$$

The input voltage is

$$V_{in} = I_{in} Z_{in} = (0.1\angle 0^\circ)(50\angle 0^\circ) = 5\angle 0^\circ\ \text{V}$$

The load voltage is

$$V_L = V_{in}\,\frac{e^{-j\beta l} + \Gamma_L e^{+j\beta l}}{e^{-j\beta l} + \Gamma_L e^{-j\beta l}} = V_{in}\,\frac{e^{-j0.6\pi} + 0}{e^{-j0.6\pi} + 0} = V_{in}(1)$$

$$V_L = 5\angle 0^\circ\ \text{V}$$

The load current is

$$I_L = \frac{V_L}{Z_L} = \frac{5\angle 0^\circ}{50} = 0.1\angle 0^\circ\ \text{A}$$

The power delivered to the load is

$$P_L = \frac{1}{2}|I_L|^2 R_L = \frac{1}{2}(0.1)^2(50) = 0.25\ \text{W}$$

The power delivered by the generator is

$$P_g = \frac{1}{2}|I_{in}|^2 R_{in} = \frac{1}{2}(0.1)^2(50\cos 0^\circ) = 0.25\ \text{W}$$

Note that $P_g = P_L + P_{loss}$, where P_{loss} is the power dissipated in the transmission line.

Example 1.10

A generator has a source impedance of $75\ \Omega$ and a source voltage of $20\angle 0^\circ$ V. It is connected to a load impedance of $150\ \Omega$ by a lossless transmission line of length 0.4λ and characteristic impedance $75\ \Omega$. Find the load voltage, the load current, the power delivered to the load, and the power delivered by the generator.

Solution: The load reflection coefficient is

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{150 - 75}{150 + 75} = \frac{1}{3}$$

The input impedance of the transmission line is

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} = 75\,\frac{150 + j75\tan(0.8\pi)}{75 + j150\tan(0.8\pi)} = 75\,\frac{1 + j0.5\tan(0.8\pi)}{0.5 + j\tan(0.8\pi)} = 75\,\frac{1 + j1.453}{0.5 + j1.453} = 75\,\frac{1.817\angle 55.5^\circ}{1.536\angle 71^\circ} = 88.7\angle -15.5^\circ\ \Omega$$

The input current is

$$I_{in} = \frac{V_g}{Z_g + Z_{in}} = \frac{20\angle 0^\circ}{75 + 88.7\angle -15.5^\circ} = \frac{20\angle 0^\circ}{160.4\angle -7.2^\circ} = 0.125\angle 7.2^\circ\ \text{A}$$

The input voltage is

$$V_{in} = I_{in} Z_{in} = (0.125\angle 7.2^\circ)(88.7\angle -15.5^\circ) = 11.09\angle -8.3^\circ\ \text{V}$$

The load voltage is

$$V_L = V_{in}\,\frac{e^{-j\beta l} + \Gamma_L e^{+j\beta l}}{e^{-j\beta l} + \Gamma_L e^{-j\beta l}} = V_{in}\,\frac{e^{-j0.8\pi} + \frac{1}{3}e^{+j0.8\pi}}{e^{-j0.8\pi} + \frac{1}{3}e^{-j0.8\pi}} = V_{in}\,\frac{0.309 - j0.951 + \frac{1}{3}(0.309 + j0.951)}{0.309 - j0.951 + \frac{1}{3}(0.309 - j0.951)} = V_{in}\,\frac{0.412 - j0.634}{0.412 - j0.634} = V_{in}$$

$$V_L = 11.09\angle -8.3^\circ\ \text{V}$$

The load current is

$$I_L = \frac{V_L}{Z_L} = \frac{11.09\angle -8.3^\circ}{150} = 0.074\angle -8.3^\circ\ \text{A}$$

The power delivered to the load is

$$P_L = \frac{1}{2}|I_L|^2 R_L = \frac{1}{2}(0.074)^2(150) = 0.411\ \text{W}$$

The power delivered by the generator is

$$P_g = \frac{1}{2}|I_{in}|^2 R_{in} = \frac{1}{2}(0.125)^2(88.7\cos 15.5^\circ) = 0.534\ \text{W}$$

Note that $P_g = P_L + P_{loss}$, where P_{loss} is the power dissipated in the transmission line.

Example 1.11

A generator has a source impedance of $50\ \Omega$ and a source voltage of $10\angle 0^\circ$ V. It is connected to a load impedance of $25\ \Omega$ by a lossless transmission line of length 0.5λ and characteristic impedance $50\ \Omega$. Find the load voltage, the load current, the power delivered to the load, and the power delivered by the generator.

Solution: The load reflection coefficient is

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{25 - 50}{25 + 50} = -\frac{1}{3}$$

The input impedance of the transmission line is

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} = 50\,\frac{25 + j50\tan(\pi)}{50 + j25\tan(\pi)} = 50\,\frac{1 + j2\tan(\pi)}{2 + j\tan(\pi)} = 50\,\frac{1 + j0}{2 + j0} = 50\angle 0^\circ\ \Omega$$

The input current is

$$I_{in} = \frac{V_g}{Z_g + Z_{in}} = \frac{10\angle 0^\circ}{50 + 50\angle 0^\circ} = \frac{10\angle 0^\circ}{100\angle 0^\circ} = 0.1\angle 0^\circ\ \text{A}$$

The input voltage is

$$V_{in} = I_{in} Z_{in} = (0.1\angle 0^\circ)(50\angle 0^\circ) = 5\angle 0^\circ\ \text{V}$$

The load voltage is

$$V_L = V_{in}\,\frac{e^{-j\beta l} + \Gamma_L e^{+j\beta l}}{e^{-j\beta l} + \Gamma_L e^{-j\beta l}} = V_{in}\,\frac{e^{-j\pi} - \frac{1}{3}e^{+j\pi}}{e^{-j\pi} - \frac{1}{3}e^{-j\pi}} = V_{in}\,\frac{-1 - \frac{1}{3}(-1)}{-1 - \frac{1}{3}(-1)} = V_{in}\,\frac{-\frac{2}{3}}{-\frac{2}{3}} = V_{in}(1)$$

$$V_L = 5\angle 0^\circ\ \text{V}$$

The load current is

$$I_L = \frac{V_L}{Z_L} = \frac{5\angle 0^\circ}{25} = 0.2\angle 0^\circ\ \text{A}$$

The power delivered to the load is

$$P_L = \frac{1}{2}|I_L|^2 R_L = \frac{1}{2}(0.2)^2(25) = 0.5\ \text{W}$$

The power delivered by the generator is

$$P_g = \frac{1}{2}|I_{in}|^2 R_{in} = \frac{1}{2}(0.1)^2(50\cos 0^\circ) = 0.25\ \text{W}$$

Note that $P_g = P_L + P_{loss}$, where P_{loss} is the power dissipated in the transmission line.

Example 1.12

A generator has a source impedance of $75\ \Omega$ and a source voltage of $20\angle 0^\circ$ V. It is connected to a load impedance of $75\ \Omega$ by a lossless transmission line of length 0.75λ and characteristic impedance $75\ \Omega$. Find the load voltage, the load current, the power delivered to the load, and the power delivered by the generator.

Solution: The load reflection coefficient is

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{75 - 75}{75 + 75} = 0$$

The input impedance of the transmission line is

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} = 75\,\frac{75 + j75\tan(1.5\pi)}{75 + j75\tan(1.5\pi)} = 75\,\frac{1 + j\tan(1.5\pi)}{1 + j\tan(1.5\pi)} = 75\,\frac{1 + j\infty}{1 + j\infty} = 75\angle 0^\circ\ \Omega$$

The input current is

$$I_{in} = \frac{V_g}{Z_g + Z_{in}} = \frac{20\angle 0^\circ}{75 + 75\angle 0^\circ} = \frac{20\angle 0^\circ}{150\angle 0^\circ} = 0.133\angle 0^\circ\ \text{A}$$

The input voltage is

$$V_{in} = I_{in} Z_{in} = (0.133\angle 0^\circ)(75\angle 0^\circ) = 10\angle 0^\circ\ \text{V}$$

The load voltage is

$$V_L = V_{in}\,\frac{e^{-j\beta l} + \Gamma_L e^{+j\beta l}}{e^{-j\beta l} + \Gamma_L e^{-j\beta l}} = V_{in}\,\frac{e^{-j1.5\pi} + 0}{e^{-j1.5\pi} + 0} = V_{in}(1)$$

$$V_L = 10\angle 0^\circ\ \text{V}$$

The load current is

$$I_L = \frac{V_L}{Z_L} = \frac{10\angle 0^\circ}{75} = 0.133\angle 0^\circ\ \text{A}$$

The power delivered to the load is

$$P_L = \frac{1}{2}|I_L|^2 R_L = \frac{1}{2}(0.133)^2(75) = 0.667\ \text{W}$$

The power delivered by the generator is

$$P_g = \frac{1}{2}|I_{in}|^2 R_{in} = \frac{1}{2}(0.133)^2(75\cos 0^\circ) = 0.667\ \text{W}$$

Note that $P_g = P_L + P_{loss}$, where P_{loss} is the power dissipated in the transmission line.

Example 1.13

A generator has a source impedance of $50\ \Omega$ and a source voltage of $10\angle 0^\circ$ V. It is connected to a load impedance of $50\ \Omega$ by a lossless transmission line of length 0.1λ and characteristic impedance $50\ \Omega$. Find the load voltage, the load current, the power delivered to the load, and the power delivered by the generator.

Solution: The load reflection coefficient is

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{50 - 50}{50 + 50} = 0$$

The input impedance of the transmission line is

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} = 50\,\frac{50 + j50\tan(0.2\pi)}{50 + j50\tan(0.2\pi)} = 50\,\frac{1 + j\tan(0.2\pi)}{1 + j\tan(0.2\pi)} = 50\,\frac{1 + j0.727}{1 + j0.727} = 50\angle 0^\circ\ \Omega$$

The input current is

$$I_{in} = \frac{V_g}{Z_g + Z_{in}} = \frac{10\angle 0^\circ}{50 + 50\angle 0^\circ} = \frac{10\angle 0^\circ}{100\angle 0^\circ} = 0.1\angle 0^\circ\ \text{A}$$

The input voltage is

$$V_{in} = I_{in} Z_{in} = (0.1\angle 0^\circ)(50\angle 0^\circ) = 5\angle 0^\circ\ \text{V}$$

The load voltage is

$$V_L = V_{in}\,\frac{e^{-j\beta l} + \Gamma_L e^{+j\beta l}}{e^{-j\beta l} + \Gamma_L e^{-j\beta l}} = V_{in}\,\frac{e^{-j0.2\pi} + 0}{e^{-j0.2\pi} + 0} = V_{in}(1)$$

$$V_L = 5\angle 0^\circ\ \text{V}$$

The load current is

$$I_L = \frac{V_L}{Z_L} = \frac{5\angle 0^\circ}{50} = 0.1\angle 0^\circ\ \text{A}$$

The power delivered to the load is

$$P_L = \frac{1}{2}|I_L|^2 R_L = \frac{1}{2}(0.1)^2(50) = 0.25\ \text{W}$$

The power delivered by the generator is

$$P_g = \frac{1}{2}|I_{in}|^2 R_{in} = \frac{1}{2}(0.1)^2(50\cos 0^\circ) = 0.25\ \text{W}$$

Note that $P_g = P_L + P_{loss}$, where P_{loss} is the power dissipated in the transmission line.

Example 1.14

A generator has a source impedance of $100\ \Omega$ and a source voltage of $50\angle 0^\circ$ V. It is connected to a load impedance of $100\ \Omega$ by a lossless transmission line of length 0.2λ and characteristic impedance $100\ \Omega$. Find the load voltage, the load current, the power delivered to the load, and the power delivered by the generator.

Solution: The load reflection coefficient is

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{100 - 100}{100 + 100} = 0$$

The input impedance of the transmission line is

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} = 100\,\frac{100 + j100\tan(0.4\pi)}{100 + j100\tan(0.4\pi)} = 100\,\frac{1 + j\tan(0.4\pi)}{1 + j\tan(0.4\pi)} = 100\,\frac{1 + j1.376}{1 + j1.376} = 100\angle 0^\circ\ \Omega$$

The input current is

$$I_{in} = \frac{V_g}{Z_g + Z_{in}} = \frac{50\angle 0^\circ}{100 + 100\angle 0^\circ} = \frac{50\angle 0^\circ}{200\angle 0^\circ} = 0.25\angle 0^\circ\ \text{A}$$

The input voltage is

$$V_{in} = I_{in} Z_{in} = (0.25\angle 0^\circ)(100\angle 0^\circ) = 25\angle 0^\circ\ \text{V}$$

The load voltage is

$$V_L = V_{in}\,\frac{e^{-j\beta l} + \Gamma_L e^{+j\beta l}}{e^{-j\beta l} + \Gamma_L e^{-j\beta l}} = V_{in}\,\frac{e^{-j0.4\pi} + 0}{e^{-j0.4\pi} + 0} = V_{in}(1)$$

$$V_L = 25\angle 0^\circ\ \text{V}$$

The load current is

$$I_L = \frac{V_L}{Z_L} = \frac{25\angle 0^\circ}{100} = 0.25\angle 0^\circ\ \text{A}$$

The power delivered to the load is

$$P_L = \frac{1}{2}|I_L|^2 R_L = \frac{1}{2}(0.25)^2(100) = 3.125\ \text{W}$$

The power delivered by the generator is

$$P_g = \frac{1}{2}|I_{in}|^2 R_{in} = \frac{1}{2}(0.25)^2(100\cos 0^\circ) = 3.125\ \text{W}$$

Note that $P_g = P_L + P_{loss}$, where P_{loss} is the power dissipated in the transmission line.

Example 1.15

A generator has a source impedance of $50\ \Omega$ and a source voltage of $10\angle 0^\circ$ V. It is connected to a load impedance of $50\ \Omega$ by a lossless transmission line of length 0.3λ and characteristic impedance $50\ \Omega$. Find the load voltage, the load current, the power delivered to the load, and the power delivered by the generator.

Solution: The load reflection coefficient is

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{50 - 50}{50 + 50} = 0$$

The input impedance of the transmission line is

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} = 50\,\frac{50 + j50\tan(0.6\pi)}{50 + j50\tan(0.6\pi)} = 50\,\frac{1 + j\tan(0.6\pi)}{1 + j\tan(0.6\pi)} = 50\,\frac{1 + j1.376}{1 + j1.376} = 50\angle 0^\circ\ \Omega$$

The input current is

$$I_{in} = \frac{V_g}{Z_g + Z_{in}} = \frac{10\angle 0^\circ}{50 + 50\angle 0^\circ} = \frac{10\angle 0^\circ}{100\angle 0^\circ} = 0.1\angle 0^\circ\ \text{A}$$

The input voltage is

$$V_{in} = I_{in} Z_{in} = (0.1\angle 0^\circ)(50\angle 0^\circ) = 5\angle 0^\circ\ \text{V}$$

The load voltage is

$$V_L = V_{in}\,\frac{e^{-j\beta l} + \Gamma_L e^{+j\beta l}}{e^{-j\beta l} + \Gamma_L e^{-j\beta l}} = V_{in}\,\frac{e^{-j0.6\pi} + 0}{e^{-j0.6\pi} + 0} = V_{in}(1)$$

$$V_L = 5\angle 0^\circ\ \text{V}$$

The load current is

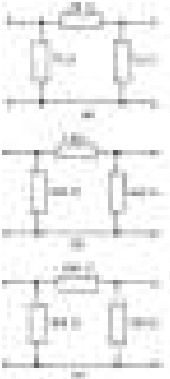


Figure 4.1.28

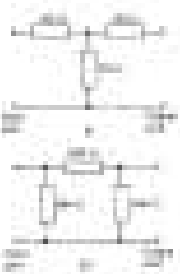


Figure 4.1.29

7. Determine the characteristic impedance Z_0 for a perfectly distortionless cable whose $R = 5\,\Omega/\text{km}$, $L = 0.5\,\text{mH/km}$, $G = 0.1\,\text{mS/km}$, and $C = 0.05\,\mu\text{F/km}$.

8. A 75-ohm telephone cable has the following primary constants: $R = 12\,\Omega/\text{km}$, $L = 0.8\,\text{mH/km}$, $G = 1\,\mu\text{S/km}$, and $C = 0.05\,\mu\text{F/km}$. Determine the characteristic impedance and propagation constant at 1 kHz.

9. A coaxial line attenuator has a series resistance of 50 Ω and a shunt resistance of 1 kΩ. Determine the characteristic impedance and the attenuation in decibels.

10. For each of the attenuator networks shown in Figure 4.1.28, determine (a) the input resistance when the output port is short-circuited, and (b) the input resistance when the output port is open-circuited, and (c) the characteristic impedance.

11. A distortionless transmission line has a specific characteristic impedance of 600 Ω and a phase shift of 1 radian/km at 5000 Hz. If the line attenuation at 5000 Hz is 2 dB/km, determine the primary constants of the line. Assume that $\omega L \gg R$ and $\omega C \gg G$.

12. Design a T-section symmetrical attenuator pad, with $Z_0 = 600\,\Omega$, to provide a transmission loss of 20 dB. Determine the characteristic impedance of the pad and develop a characteristic impedance of 600 Ω.

13. Determine the values of the shunt and series impedances for a pad attenuator to attain a transfer impedance with an associated loss following a design characteristic of 10 dB and 20 dB.

14. Design a symmetrical attenuator network to provide a voltage attenuation of 0.5 dB and have a characteristic impedance of 600 Ω.

15. In the transmission line an internal impedance of 600 Ω is split equally at the midpoint. Design a symmetrical T-section attenuator pad to provide a voltage attenuation of 20 dB and have a characteristic impedance of 600 Ω. Assume that the load impedance is also 600 Ω.

Experimental

16. The attenuator network shown in Figure 4.1.29 is to be used to match a 600-Ω source to a 600-Ω load. Determine (a) the characteristic impedance of each of the attenuator sections, (b) the attenuation in decibels, and (c) the attenuation ratio.



Figure 4.1.a)

1.1 In a circuit, parallel circuits or components across the output of a generator of internal resistance $R_i = 0.1\ \Omega$ cause a terminal voltage $U = 230\ V$ to be measured. Determine the terminal voltage U_0 when the load is switched off. ($U_0 = 230.12\ V$)



Figure 4.1.b)

1.2 A galvanometer is connected across a resistance $R_2 = 100\ \Omega$ and series resistance $R_1 = 900\ \Omega$. The galvanometer has a resistance $R_g = 100\ \Omega$. Determine (a) the current/voltage characteristics of the system, and (b) the maximum value which can be measured if the galvanometer has a maximum deflection of $I_{max} = 10\ mA$. ((a) $I_g = 0.1\,U$ mA, (b) $U_{max} = 1\ V$)



Figure 4.1.c)

1.3 Two resistances connected in series to source U_0 (Figure 4.1.c)) dissipate the same power in the load resistance when $R_1 = 4\ \Omega$ and when $R_2 = 9\ \Omega$. Determine (a) the internal resistance R_i of the source, and (b) the power dissipated in the load resistance. ($R_i = 6\ \Omega$, $P = \frac{U_0^2}{25}$ W)

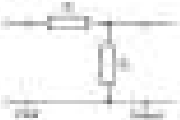


Figure 4.1.d)

1.4 Two resistances connected in series to source U_0 (Figure 4.1.d)) dissipate the same power in the load resistance when $R_1 = 4\ \Omega$ and when $R_2 = 9\ \Omega$. Determine (a) the internal resistance R_i of the source, and (b) the power dissipated in the load resistance. ($R_i = 6\ \Omega$, $P = \frac{U_0^2}{25}$ W)



Figure 4.1.e)

1.5 Two resistances connected in series to source U_0 (Figure 4.1.e)) dissipate the same power in the load resistance when $R_1 = 4\ \Omega$ and when $R_2 = 9\ \Omega$. Determine (a) the internal resistance R_i of the source, and (b) the power dissipated in the load resistance. ($R_i = 6\ \Omega$, $P = \frac{U_0^2}{25}$ W)

Figure 4.1.f)

1.6 Two resistances connected in series to source U_0 (Figure 4.1.f)) dissipate the same power in the load resistance when $R_1 = 4\ \Omega$ and when $R_2 = 9\ \Omega$. Determine (a) the internal resistance R_i of the source, and (b) the power dissipated in the load resistance. ($R_i = 6\ \Omega$, $P = \frac{U_0^2}{25}$ W)

Figure 4.1.g)

1.7 Two resistances connected in series to source U_0 (Figure 4.1.g)) dissipate the same power in the load resistance when $R_1 = 4\ \Omega$ and when $R_2 = 9\ \Omega$. Determine (a) the internal resistance R_i of the source, and (b) the power dissipated in the load resistance. ($R_i = 6\ \Omega$, $P = \frac{U_0^2}{25}$ W)

Figure 4.1.h)

1.8 Two resistances connected in series to source U_0 (Figure 4.1.h)) dissipate the same power in the load resistance when $R_1 = 4\ \Omega$ and when $R_2 = 9\ \Omega$. Determine (a) the internal resistance R_i of the source, and (b) the power dissipated in the load resistance. ($R_i = 6\ \Omega$, $P = \frac{U_0^2}{25}$ W)

Figure 4.1.i)

1.9 Two resistances connected in series to source U_0 (Figure 4.1.i)) dissipate the same power in the load resistance when $R_1 = 4\ \Omega$ and when $R_2 = 9\ \Omega$. Determine (a) the internal resistance R_i of the source, and (b) the power dissipated in the load resistance. ($R_i = 6\ \Omega$, $P = \frac{U_0^2}{25}$ W)

Figure 4.1.j)

1.10 Two resistances connected in series to source U_0 (Figure 4.1.j)) dissipate the same power in the load resistance when $R_1 = 4\ \Omega$ and when $R_2 = 9\ \Omega$. Determine (a) the internal resistance R_i of the source, and (b) the power dissipated in the load resistance. ($R_i = 6\ \Omega$, $P = \frac{U_0^2}{25}$ W)

Figure 4.1.k)

1.11 Two resistances connected in series to source U_0 (Figure 4.1.k)) dissipate the same power in the load resistance when $R_1 = 4\ \Omega$ and when $R_2 = 9\ \Omega$. Determine (a) the internal resistance R_i of the source, and (b) the power dissipated in the load resistance. ($R_i = 6\ \Omega$, $P = \frac{U_0^2}{25}$ W)

Figure 4.1.l)

1.12 Two resistances connected in series to source U_0 (Figure 4.1.l)) dissipate the same power in the load resistance when $R_1 = 4\ \Omega$ and when $R_2 = 9\ \Omega$. Determine (a) the internal resistance R_i of the source, and (b) the power dissipated in the load resistance. ($R_i = 6\ \Omega$, $P = \frac{U_0^2}{25}$ W)

Determine (a) the acceleration of each weight and (b) the tension in the two strings when the system is released from rest.

$$\text{Ans: (a) } a = \frac{(m_2 - m_1)g}{m_1 + m_2 + \frac{I}{r^2}},\quad \text{(b) } T_1 = m_1(g + a),\ T_2 = m_2(g - a)$$

P11. A disk of mass m and radius r is free to rotate about a fixed horizontal axis through its center. A string wrapped around the disk supports a block of mass m that slides on a frictionless incline of angle θ. The string does not slip on the disk.

(a) Using the energy method, find the speed of the block after it has slid a distance d down the incline, starting from rest.

(b) Find the tension in the string and the angular acceleration of the disk.

(c) Repeat part (a) if the incline has a coefficient of kinetic friction μ_k. For what value of μ_k does the block reach the bottom with zero speed?

(d) Explain why the tension in the string is smaller than $mg\sin\theta$.

$$\text{Ans: (a) } v = \sqrt{\frac{2gd(\sin\theta - \mu_k\cos\theta)}{1 + \frac{I}{mr^2}}},\quad \text{(b) } T = m\left(g\sin\theta - a\right),\ \alpha = \frac{a}{r}$$

Assignment 13

This assignment covers the material introduced in chapters 10 & 11.

Six marks for each question are shown in brackets; marks are not awarded.

1. The figure below shows a load for a particular machine component which is subjected to alternating stresses of magnitude 140 MPa and 70 MPa. Estimate the number of cycles to failure if the component is made from steel with an ultimate tensile strength of 1400 MPa. [6]



2. A 25 mm diameter bar is subjected to a steady torque of 300 Nm and a completely reversed bending moment of 200 Nm. The bar is made from a material with a yield strength of 350 MPa and a fatigue strength of 210 MPa. Determine (a) the factor of safety against yielding and (b) the factor of safety against fatigue. [6]

3. A cantilever beam, of length 0.5 m, is made from a material with a density of 7800 kg/m^3, a Young's modulus of 200 GPa and a yield strength of 350 MPa. The beam is to be designed to support a load of 1000 N at its free end. Determine the minimum second moment of area of the beam's cross-section if the factor of safety against yielding is to be 2.5. [6]

4. A 50 mm diameter shaft is supported by two bearings and is driven by a gear which is positioned midway between the bearings. The gear has a pitch circle diameter of 200 mm and a pressure angle of 20°. The shaft is required to transmit 10 kW of power at 1500 rpm. Determine the magnitude of the resultant force acting on the gear. [6]

5. Explain what is meant by the term *fatigue failure*. Describe briefly the mechanism by which fatigue failure occurs. [6]

6. Explain what is meant by the term *endurance limit*. Describe briefly how the endurance limit of a material is determined experimentally. [6]

7. A cantilever beam, of length 0.5 m, is made from a material with a density of 7800 kg/m^3, a Young's modulus of 200 GPa and a yield strength of 350 MPa. The beam is to be designed to support a load of 1000 N at its free end. Determine the minimum second moment of area of the beam's cross-section if the factor of safety against yielding is to be 2.5. [6]

42 Filter networks

In this and the following chapters we shall look at

- applications (including most of Filter network)
- individual basic types of filter and their (i.e. frequency response, linear phase and realizability) limits
- mathematical characterization (representation and interpretation) of filter operations
- individual low- and high-pass filter networks
- design of more sophisticated filter networks
- calculation and design of filter coefficients and their relation to filter properties
- individual and design of optimized filter networks
- mathematical and design practices of complicated filters

42.1 Introduction

A **filter** is a network designed to pass signals having frequencies within certain bands (called **passbands**) and to reject or attenuate (i.e. greatly reduce the magnitude of) other frequencies (called **attenuation bands** or **stopbands**).

As explained in the previous chapter, an ordinary network can be analyzed in frequency terms, but ordinarily it will have **complex** frequency response and independence of frequency. However, a filter is designed to produce only a real component of complex impedance. Even simple frequency-selective networks (such as tuned circuits) usually have distortion and impedance changes in the pass frequencies unless the presence of resistance is taken to some detriment to all frequencies.

Moreover, the real load of a filter is often simply the termination of some cable (the transmission line) whose characteristic impedance is nearly a pure resistance (Chapter 39). Hence the frequency at which the impedance changes to the load is usually fixed.

A filter can be considered as an array of series or shunt **reactors**, and can consequently also be seen as some general network in contrast to the **ladder** types we have met.

The filter considered in this chapter are symmetrical networks and T and π sections, the components have fixed impedances or load.

Filters are used for a variety of purposes in many other fields of electronics, communications and control engineering. The introduction of filters used for various purposes requires some basic knowledge of network theory, as the applications

11.2 Basic types of filter networks

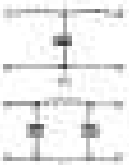


Figure 11.1

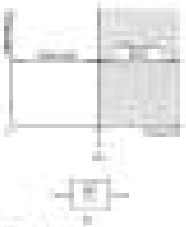


Figure 11.2

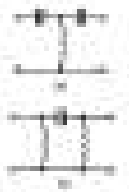


Figure 11.3

(a) Low-pass filters

Figure 11.1 shows basic networks of T and π sections filters using series inductive and shunt capacitive. If either section is connected over a network, such a combination is known as a low-pass filter, since it will pass low frequencies to applied load and source. A low-pass transmission characteristic is shown in Figure 11.2(a).

The L's in these illustrations can contain some parasitic inductance, and capacitances are seen to be associated with each coil's winding. Also, coil Q becomes a factor in determining the sharpness of the cut-off characteristic. It is the combination of the series inductive reactance and shunt capacitive reactance as frequency increases that causes the attenuation of signals at these and other frequencies as the signal approaches cut-off. The attenuation characteristic of a low-pass filter is shown in Figure 11.2(a).

Fundamentally, a low-pass filter is one designed to pass signals at frequencies from 0 to a specified cut-off frequency.

Many variations are used in practice. See 11.3, Capacitor of inductive or shunt, in many cases, different combinations of the two are used to provide the effect of the types shown in Figure 11.1. Low-pass filters are used in transmitter output networks and lines. But they are often used in receiver applications as well.

Transmitter applications are for the suppression of harmonic energy and other undesirable emissions which could cause interference to other services. Receiver applications are for the suppression of signals and noise above the desired frequency range. Another application is in the suppression of unwanted signals from a frequency converter stage.

(b) High-pass filters

Figure 11.3 shows basic combinations T and π sections filters using series capacitive and shunt inductive. If either section is connected over a network, such a combination is known as a high-pass filter, since it will pass high frequencies to applied load and source. A high-pass transmission characteristic is shown in Figure 11.2(b).

These basic low or no signal characteristics (ideally) also exhibit elements that fluctuate from the normal response. In the event of the signal is no attenuation and all frequencies above f$_c$ are passed without loss. It is not true, since the characteristic shown in Figure 11.2(b) applies to any series or shunt high-pass filters. The shunt and series inductors behave like a high-pass filter in the circuit as shown.

Fundamentally, a high-pass filter is one designed to pass signals at frequencies above a specified cut-off frequency.

The characteristics of the T network filter can be seen above in the high-pass transmission characteristics of the attenuation and of the phase shift and delay characteristics of the attenuation curve. Shifts in these characteristics are dependent on the values of the components. Also, in practical circuits of the network includes the inductance of the associated circuits as well as a practical filter network for consideration in the attenuation curve will have a more valley. Practical characteristics for low-pass and high-pass filters are discussed in sections 11.3 and 11.4. In addition to the attenuation characteristics which are useful there are considerations, namely, filter's effect on network impedance and the impedance of the load.

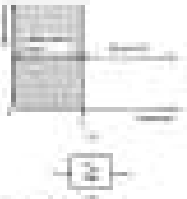


Figure 4.1.1



Figure 4.1.2



Figure 4.1.3

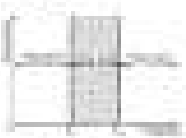


Figure 4.1.4

4.1.1 The characteristic impedance and the conductance of other sections

Between the characteristic impedance of a finite section and the value that frequency and the fundamental of the section may be the impedance that does not vary with frequency or the same way. To understand these we to understand and characterizing. Below are input which longer impedance conditions as far as possible (see Chapter 3).

(a) Quarter-wave Block

A quarter-wave section can be designed to have impedance transformation characteristics as will be discussed. The characteristic of the short-circuit wave filter is shown in Figure 4.1.1.

Such a filter may be designed for impedance at high power and a low power filter. If z_1 is the input impedance of the high-power filter and z_2 is the input impedance of the short-circuit filter, the load impedance, z_L, is $\sqrt{z_1 z_2}$, for a characteristic impedance of $\sqrt{z_1 z_2}$. The input impedance is $\sqrt{z_1 z_2}$. This is a characteristic impedance of the short-circuit filter. The load impedance, z_L, is



Figure 4.12

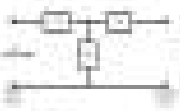


Figure 4.13

can be written down in a recursive relationship as the sequence of the next input.

For example, if f_n is Fibonacci, that f_n and f_{n-1} to represent, can f_{n-1} and from Figure 4.13

$$f_{n-1} = f_{n-1} \quad f_{n-1} = f_{n-1} + f_{n-2}$$

$$\text{Let} \quad f_{n-1} = f_{n-1} + \frac{f_{n-1}}{f_{n-1}} \cdot f_{n-1} = \frac{f_{n-1} \cdot f_{n-1}}{f_{n-1} + f_{n-1}} \cdot f_{n-1} = \frac{f_{n-1} \cdot f_{n-1}}{f_{n-1} + f_{n-1}}$$

$$= f_{n-1} - \left(\frac{f_{n-1}}{f_{n-1} + f_{n-1}} \right) = f_{n-1} - \frac{f_{n-1}}{f_{n-1}}$$

Figure 4.13 = f_{n-1} and $f_{n-1} = f_{n-1} + f_{n-1}$ then f_{n-1} and f_{n-1} can be defined in f_{n-1} which depends on a sequence f_{n-1} implies recursion depending on the case of Fibonacci f_{n-1}.

Let the sequence of the numbers we have found for f_{n-1} and the sequence of the numbers we have found for f_{n-1}. Figure we have shown the sequence and we have that we can be represent as $f_{n-1} = f_{n-1}$ where f_{n-1} and f_{n-1} are the many numbers to implies a new combination of f_{n-1} and f_{n-1} recursively from using

$$\begin{aligned}
&\text{(a)} \quad f_{n-1} = f_{n-1} \text{ and } f_{n-1} = f_{n-1} \\
&\text{(b)} \quad f_{n-1} = f_{n-1} \text{ and } f_{n-1} = f_{n-1} \\
&\text{(c)} \quad f_{n-1} = f_{n-1} \text{ and } f_{n-1} = f_{n-1} \\
&\text{(d)} \quad f_{n-1} = f_{n-1} \text{ and } f_{n-1} = f_{n-1}
\end{aligned}$$

From general recall these original numbers f_{n-1} is given by:

$$f_{n-1} = f_{n-1} f_{n-1}$$

Taking when of a subsequence to each is form a new:

$$f_{n-1} = f_{n-1} \cdot f_{n-1} = f_{n-1} f_{n-1}$$

Which is how that the output sequence will be **globally** (**relation**).

Taking when of implementation can and is a form a new:

$$f_{n-1} = f_{n-1} f_{n-1} = f_{n-1} f_{n-1}$$

Which a sequence, how for input sequence will be **given in location**.

I can write the sequence will some of f_{n-1} and f_{n-1} depend upon sequence. How we can with the sequence and values of the input sequence f_{n-1} sequence type sequence.

Characteristic Equations

Figure 4.13 shows a the case 3 source sequence is an characteristic equations f_{n-1}.

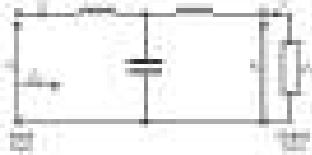


Figure 18.01

Some capacitors (see page 104) are manufactured in such a way that the capacitance is given by $C = \varepsilon_0 A / d$, where ε_0 is a constant.

The SI Unit of this constant may be determined from the data in some other way.

(a) The capacitance of one thin plate of a capacitor consists of a thin sheet of aluminium foil of area A square metres separated by a plastic material of thickness d metres.

(b) The capacitance of a 100mm diameter flat circular plate capacitor consists of two thin sheets of aluminium foil separated by a mica film of thickness d metres.

(a) Capacitance of two aluminium foil sheets

Let the plates of the capacitor in Figure 18.01 be flat aluminium foil sheets of thickness t metres and width and length L metres respectively.

Since Figure 18.01 the $\varepsilon_0 = \frac{C}{A} = \frac{C}{L^2}$.

Where capacitance C the unit is farads/metre. $\varepsilon_0 = 8.854$ farads/metre = 8.854 pico (p) = 10^{-12} farads (or coulombs/volt) in basic units.

Some calculations in the metric structure.

$\varepsilon_0 = 8.854 \times 10^{-12}$ farads/metre $= 8.854 \times 10^{-12}$ coulombs/volt/metre.

By convention, aluminium foil sheets have a thickness of 0.01 mm and metric structure.

Hence $t = 0.01 \times 10^{-3}$ m and $\varepsilon_0 = 8.854 \times 10^{-12}$ farads/metre.

Then $\varepsilon_0 = \frac{C}{L^2}$ farads/metre $= \frac{C}{L^2} \times 10^{-12}$ farads/metre.

Then $\varepsilon_0 =

[illegible]

4.3.4 Coupling capacitance

[illegible]



Figure 4.3.1

[illegible]

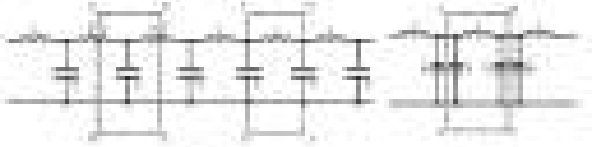


Figure 4.3.2

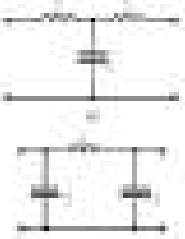


Figure 4.2.3



Figure 4.2.4

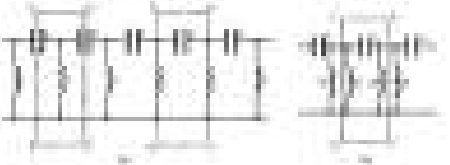


Figure 4.2.5

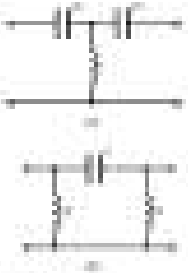


Figure 4.2.12

Each individual resistor must dissipate the same 40 W. If the resistors are connected in series, each of the 100 Ω resistors will dissipate the same current I, so the total power dissipated is $3I^2$. In general, for n resistors in series, the total power dissipated is nI^2. In general, for

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$$

Similarly, the resistors in (b) of Figure 4.2.12 can be replaced by a single resistor R where $\frac{1}{R} = \frac{1}{100} + \frac{1}{100} + \frac{1}{100} = \frac{3}{100}$, so $R = \frac{100}{3}\ \Omega$. Then the power dissipated is $P = \frac{V^2}{R} = \frac{120^2}{100/3} = 432$ W. Thus, the parallel combination will dissipate more than 10 times as much power as the series combination.

The same principle as in Figure 4.2.12 can be used to connect a 110 volt light bulb to a 220 volt electric line. In this case, two light bulbs are connected in series, so each bulb receives 110 volts. Since both bulbs are identical, they will dissipate the same power.

4.2.4 Kirchhoff's Laws and Circuits



Figure 4.2.13

(a) The Loop Equations

In the circuit in Figure 4.2.13, the dependent variables are the currents I_1, I_2, and I_3 in the three branches of the circuit. Kirchhoff's laws can be used to determine I_1, I_2, and I_3. Applying Kirchhoff's current law to the two junctions shown in Figure 4.2.13,

$$I_3 = \frac{V_1 - V_2}{R_3} \quad \text{and} \quad I_3 = \frac{V_2}{R_2} + I_1$$

Then $\frac{V_1 - V_2}{R_3} = \frac{V_2}{R_2} + I_1$, so $\frac{V_1}{R_3} = V_2\left(\frac{1}{R_3} + \frac{1}{R_2}\right) + I_1$

$$= V_2\left(\frac{R_2 + R_3}{R_2 R_3}\right) + I_1$$

$$\text{so} \quad V_2 = \frac{R_2 R_3}{R_2 + R_3}\left(\frac{V_1}{R_3} - I_1\right) \tag{4.2.1}$$

By another use of Kirchhoff's current law, $I_2 = \frac{V_2}{R_2} - I_1$.

Then substituting the value of V_2 from (4.2.1),

$$I_2 = \frac{R_3}{R_2 + R_3} V_1 - \frac{R_2 + R_3}{R_2} I_1$$

$$\text{where } \alpha = \frac{R_3}{R_2 + R_3} \tag{4.2.2}$$

Kirchhoff's voltage law applied to the two loops in the circuit gives



Figure 8.2-10

Then $\quad \delta_{11} = \frac{L^3}{3EI}$

$$\theta_{11} = \frac{1}{2}\left(\frac{M_1}{EI}\right)(L) = \frac{L}{2EI}$$

and $\quad f = \frac{5}{16}(L)\left(\frac{L}{2EI}\right) = \frac{5L^2}{32EI}$

so, **the overall flexibility** $\quad \boxed{f = \frac{5}{16}\delta_{11}\theta_{11}}$ (8.2-3)

The matrix equation for the overall flexibility is obtained for the two-span beam in Figure 8.2-10 as follows:

From equation (8.2-1), θ_1 is a symmetrical 2-vector:

$$\theta_1 = \begin{pmatrix} \theta_{11} \\ \theta_{12} \end{pmatrix}$$

Applying this to Figure 8.2-1(c), we find, with $\theta_{11} = \frac{L}{2EI} = \frac{3L}{6EI}$

Then $\quad \theta_1 = \begin{Bmatrix} \theta_{11} \\ \theta_{12} \end{Bmatrix} = \begin{Bmatrix} \frac{3L}{6EI} \\ \frac{10L}{6EI} \end{Bmatrix} = \begin{Bmatrix} \frac{3L}{6EI} \\ \frac{4L}{6EI} \end{Bmatrix} + \begin{Bmatrix} \frac{0}{6EI} \\ \frac{6L}{6EI} \end{Bmatrix}$

$$= \begin{Bmatrix} \frac{3L}{6EI} \\ \frac{4L}{6EI} \end{Bmatrix} + \begin{Bmatrix} \frac{3L}{6EI} \\ \frac{6L}{6EI} \end{Bmatrix}$$

$$= \begin{Bmatrix} \frac{3L}{6EI} \\ \frac{4L}{6EI} \end{Bmatrix} + \begin{Bmatrix} \frac{3L}{6EI} \\ \frac{6L}{6EI} \end{Bmatrix} + \begin{Bmatrix} -\frac{3L}{6EI} \\ -\frac{6L}{6EI} \end{Bmatrix}$$

so, $\quad \theta_1 = \begin{Bmatrix} \frac{3L}{6EI} \\ \frac{4L}{6EI} \end{Bmatrix}$ (8.2-4)

Premultiplying by $\begin{bmatrix} -\frac{3EI}{L} \end{bmatrix}$

The resonance will commence when $\frac{1}{\omega C} = \omega L - \frac{R^2}{\omega L}$

i.e. $\omega^2 = \frac{1}{LC} - \frac{R^2}{L^2}$

Now, since $\omega = 2\pi f$ then $f = \frac{1}{2\pi\sqrt{LC - R^2C^2}}$ as required in (c).

(d) Practical Significance

When the frequency is very low, ω is small and the term $\omega L^2/R$ is negligible. If ω is very high, $\omega L^2/R$ is negligible in comparison with ωL and ωL is almost in parallel condition. The value of the characteristic impedance is known as the **design impedance** or the **nominal characteristic** of the section and is then given by:

$$\text{i.e.} \quad Z_0 = \sqrt{\frac{L}{C}} \qquad \Omega \text{ (c)}$$

Problem 5. Determine the cut-off frequency and the nominal impedance of each of the networks shown in Figure 4.31(a).

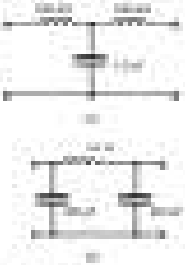


Figure 4.31

(a) Comparing Figure 4.31(a) with the first part of section (c) in Figure 4.30, shows that $L/2 = 2\,\text{mH}$, $2C = 0.05\,\mu\text{F}$ and $L = 4\,\text{mH}$ and $C = 0.025\,\mu\text{F}$. Substituting in the formula for cut-off frequency gives:

$$f_c = \frac{1}{\pi\sqrt{LC}} = \frac{1}{\pi\sqrt{4 \times 10^{-3} \times 0.025 \times 10^{-6}}} = \frac{10^5}{\pi\sqrt{0.1}}$$

i.e. $f_c = 10\,\text{kHz}$ to a first approximation

From equation (4.7), the nominal impedance

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{4 \times 10^{-3}}{0.025 \times 10^{-6}}} = 200\,\Omega \text{ to a first approximation}$$

(b) Comparing Figure 4.31(b) with the first part of section (c) in Figure 4.30 shows that $L/2 = 1\,\text{mH}$, i.e. inductance $L = 2\,\text{mH}$ and $2C = 0.1\,\mu\text{F}$ and capacitance $C = 0.05\,\mu\text{F}$.

From equation (4.7), nominal impedance

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{2 \times 10^{-3}}{0.05 \times 10^{-6}}} = 200\,\Omega

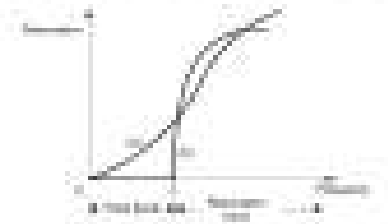


Figure: (a) (b)

Substituting in equation (10.1) gives:

$$\frac{V_o}{V_i} = \frac{\frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}} = \frac{\frac{1}{j\omega C}}{\frac{1 + j\omega CR + (j\omega)^2 LC}{j\omega C}}$$

$$\therefore \text{gain } A(\omega) = \boxed{\text{equation 1}} \qquad (10.2)$$

Substituting $\omega = 0$ in equation (10.2) gives: $A(0) = \frac{1}{1} = 1$

Substituting $\omega = \infty$ in equation (10.2) gives: $A(\infty) = \frac{1}{1 + \infty + \infty} = \frac{1}{\infty} = 0$

$$\therefore \text{gain } A(\omega) = \boxed{\text{equation 2}} \qquad (10.3)$$

Problem 1 — A filter network has a gain equation of the form $A(\omega) = \frac{1}{1 + j\omega CR + (j\omega)^2 LC}$. Sketch the gain/frequency response and determine the cut-off frequency, ω_c, in terms of L and C.

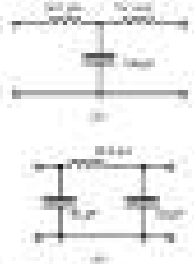


Figure 10.1

The gain equation is a function of ω and L and C are constants. Let $L = 1$ H and $C = 1$ F. Then the gain equation becomes $A(\omega) = \frac{1}{1 + j\omega + (j\omega)^2} = \frac{1}{1 + j\omega - \omega^2}$

From equation (10.3):

$$\text{gain } A(0) = \frac{1}{1 + 0 + 0} = 1 = 0 \text{ dB}$$

and from equation (10.4):

$$\text{gain } A(\infty) = \frac{1}{1 + \infty - \infty} = \frac{1}{0} = -\infty \text{ dB}$$

(a) A first-order R–C low-pass filter is shown in Figure 10.1(a). When the output is taken across the capacitor, C, (see Figure 10.1(a)), $V_o = \frac{1}{j\omega C} V_i = \frac{1}{1 + j\omega CR} V_i$

(b) A first-order R–C high-pass filter is shown in Figure 10.1(b). When the output is taken across the resistor, R, (see Figure 10.1(b)), $V_o = \frac{R}{R + \frac{1}{j\omega C}} V_i = \frac{j\omega CR}{1 + j\omega CR} V_i$

10.3 Standard terminology for filter circuits

A filter network is shown in Figure 10.2(a) and has a gain/frequency response as shown in Figure 10.2(b) and (c). The sinusoidal attenuation in Figure 10.2(b) is

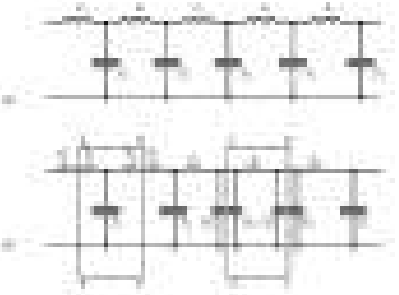


Figure 10.2

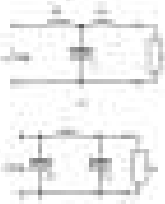


Figure 10.3

Then $\boxed{\mathbf{R}_2 = \mathbf{A}\mathbf{R}_1}$ (9.11)

This is a general result that is valid for all viscous fluids, regardless of the value of ν. It is also valid for all irrotational flows and for rotational flows as well.

From the Bernoulli equation derived in Chapter 10 (7.7 and 8.2.2):

$$p_1 = \rho g h \sin\theta - \frac{\rho}{2g}$$

Hence $\mathbf{R}_1 = \rho g h \sin\theta \begin{pmatrix} \frac{1}{\sin\theta} \\ -\frac{1}{\tan\theta} \end{pmatrix}$

Then, from the equation (10.9), $\boxed{\mathbf{R}_2 = \mathbf{A}\mathbf{R}_1}$ (10.10)

From equations (10.1) and (10.10),

$$\mathbf{R}_2 = \rho g h \sin\theta \begin{pmatrix} \frac{1}{\sin\theta} \\ -\frac{1}{\tan\theta} \end{pmatrix} = \text{constant} \cdot \mathbf{k}_2$$

A further general component of acceleration, the gravity component acting in the flow direction, is $g \sin\theta$ (see Figure 10.2(a)) per unit mass. Hence the gravity force per unit volume in the flow direction is $\rho g \sin\theta$. Therefore, the gravity component is constant and equal to the pressure. Thus the position of the upper and lower interfaces is independent of distance from the origin. The solutions (9.11) and (10.10). The solutions of the problem for upper film type of flow are given.

From equation (10.10), it is seen that $\mathbf{k}_2$ is an eigenvector of $\mathbf{A}$ and $\mathbf{R}_2$ is the corresponding eigenvalue matrix $\lambda = 1$. Also, since $\mathbf{k}_2$ is a unique form and is unchanged in the case of breaking, as will $\mathbf{k}_2$. The two solutions shown in Figure 10.3 and 10.4 will be used in Chapter 11 to all frequencies and also studied over many. Combined solutions 9 and 10 will be given in Chapter 12 **Hydrodynamics**.

10 Viscosity and the Navier-Stokes equations

From equation (9.2) the characteristic equation $\det(\mathbf{A} - \lambda\mathbf{I}) = 0$ becomes

$$\det\begin{pmatrix} \frac{1}{2} & -\frac{\sin\theta}{2} \\ -\frac{\sin\theta}{2} & \frac{1}{2} \end{pmatrix}$$

Multiplying out,

$$\det = \left(\frac{1}{2} - \frac{\sin\theta}{2}\right)\left(\frac{1}{2} + \frac{\sin\theta}{2}\right) = \frac{1}{4}\left(1 - \frac{\sin\theta}{2}\right)\left(1 + \frac{\sin\theta}{2}\right)$$

$$= \frac{1}{4}\left(1 - \frac{\sin^2\theta}{4}\right)$$ Then equation (10.11)

From equation (9.8.1), at $t = \frac{L}{R}$, current $I_0 = I_0\left(1 - \frac{1}{e^1}\right)$

$$\Rightarrow \quad \boxed{I_0 = I_0\left(1 - \frac{1}{e}\right)} \tag{9.8.2}$$

Also, from equation (9.8.1), $I_0 = \frac{E_0}{R}\left(1 - \frac{1}{e^{\frac{R}{L}\cdot\frac{L}{R}}}\right)$

$$\Rightarrow \quad \boxed{I_0 = \frac{E_0}{R}\left(1 - \frac{1}{e}\right)} \tag{9.8.3}$$

Differentiating the equation (9.8.1) w.r.t. t and if E_0, R, L are constant then equation (9.8.1) gives

$$\frac{dI}{dt} = \frac{E_0}{R}\left(\frac{0 - 1 \cdot e^{-\frac{R}{L}t}\left(-\frac{R}{L}\right)}{\left(1 - e^{-\frac{R}{L}t}\right)^2}\right) = \left|\frac{E_0}{R}\cdot\frac{e^{-\frac{R}{L}t}}{\left(1 - e^{-\frac{R}{L}t}\right)^2}\right|$$

$$= \frac{E_0}{R}\cdot\frac{e^{-\frac{R}{L}t}}{\left(1 - e^{-\frac{R}{L}t}\right)^2} \quad \text{as } \frac{dI}{dt} > 0$$

From equation (9.8.1) we can write $E_0 R$ in terms of I. Now, substituting this equation in (9.8.1)

$$I_0 = \frac{E_0}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

(1) As $t \to \infty$, $I \to$ ...

and from equation (9.8.1), I_0 ... value.

and from equation (9.8.1), I_0 ... value.

I also wants me to find ... Now, to find ... I have to ... that dI_0 ... Now, dI_0 is ... Differentiating ... and ... Now, ... Now, ... value ... So, ... is ...

Let ... Differentiating ... Now, substituting ... in ... Now, from equation (9.8.1) ...

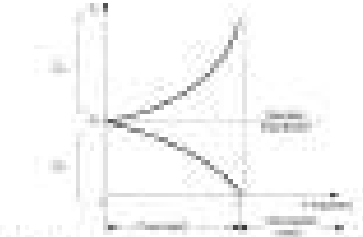


Figure 11.11



Figure 11.12

[illegible]

> [illegible]

At zero frequency the capacitor behaves as an open circuit, so $X_C \to \infty$ and $A_{CL} \to A_{OL}$. Hence A_{OL} is equal to the maximum magnitude of A_V.

(b) From equation (8.1), the magnitude response of the op amp is given by

$$A_V = \frac{A_{OL}}{\sqrt{1 + \left(\frac{f}{f_{OL}}\right)^2}} \approx \frac{A_{OL}}{\sqrt{\left(\frac{f}{f_{OL}}\right)^2}} = \frac{A_{OL} f_{OL}}{f}$$

$$= \frac{200{,}000}{(11 \times 10^3)} = 18.18 \text{ V/V}$$

(c) From equation (8.1), the magnitude response of the R–C network is given by

$$\beta_V = A_{CL}\sqrt{1 + \left(\frac{f}{f_{CL}}\right)^2} = 0.05\sqrt{1 + \left(\frac{f}{f_{CL}}\right)^2} \quad \text{(Ans.)}$$

Substituting the given values, we get

$$\beta_V = 0.05\sqrt{1 + \left(\frac{11 \times 10^3}{200{,}000}\right)^2} = 0.05\sqrt{1 + (0.055)^2} = 0.05 \text{ (Ans.)}$$

The result that the closed-loop gain of a feedback amplifier is equal to the open-loop gain is known as Black's gain formula.

> **Problem 8.1** An op amp is found to have a maximum magnitude of 100 V/V and a cut-off frequency of 1 MHz. Determine the frequency at which the magnitude response of the circuit is 70.7 V/V if it is a first-order system.

From equation (8.1), $A_V = \dfrac{A_{OL}}{\sqrt{1 + \left(\frac{f}{f_{OL}}\right)^2}}$

(a) When $A_V = 70.7$ V/V and $A_{OL} = 100$ V/V, then $\frac{f}{f_{OL}} = 1$, i.e., $f = f_{OL}$. **Ans.**

(b) When $A_{OL} = 100$ V/V, $A_V = 70.7$ V/V and $f = f_{OL} = 1$ MHz,

$$70.7 = \frac{100}{\sqrt{1 + \left(\frac{f}{1 \text{ MHz}}\right)^2}}$$

Group velocity $v_g = \left(\frac{1}{\omega}\right)\left[\frac{\partial^2 \omega}{\partial k^2}\right] = \left(\frac{\omega_{max}}{k_{max}}\right)\left[\frac{\partial^2 \omega}{\partial k^2}\right] = 0.50$

Wave speed $\left(\frac{\omega}{k}\right)\left[\frac{\partial^2 \omega}{\partial k^2}\right] = 1.00 \times 0.50 = 0.50$

Phase velocity $v_p = \omega/k = 1.00$

Dispersion, $d^2\omega/dk^2 = 0.50 - 1.00 = -0.50$ (Eq. 10-3)

(c) When $k_0 = 0.50$ ω_0 rises:

$$\text{Amplitude} = \frac{\text{Wave}}{\sqrt{\left[\text{Wave Number}\right]^2 + \left[\text{Wave Number}\right]^2}} \quad \text{Wave Number} = \frac{2\pi}{\text{Wavelength}}$$

Then $\omega = \left(\frac{\omega}{k}\right)k + \frac{1}{2}\left[\frac{\partial^2 \omega}{\partial k^2}\right]k^2 + \cdots$

and $\frac{\omega}{k} = v_p$ (wave) $= 0.50$

Group value $v_g = 0.50$ (c), dispersion $\frac{d^2\omega}{dk^2} = -0.50$ (c=0.50)

$= -1.00$ (c=1.00)

The above three results are seen to be those one to the characteristic of Eq. (applies frequency domain to Figure 10-3).

Further modifies an harmonic (Eq. involves wave in [case] at [phase] at [wavelength] in [case] (c).

10-4 Dispersion Effects on Pulses

(a) **Wave and Group Velocity**

Substitute T and ω to replace the terms in Figure 10-21, you believe to have to use to wave being considered in the characteristic boundaries.

From equations 10-3 (page 10) the characteristic boundaries of ω if arising to generate

$$\omega_0 = \sqrt{gk}\left[1 + (ka)^2\right]$$

From Figure 10-4 this is $\omega_0 = \frac{2\pi}{\text{period}}$ and $k_0 = \frac{2\pi}{\text{wavelength}}$

Then $\omega_0 = \sqrt{\frac{g}{k_0}}\left[1 + \left(\frac{2\pi a}{\text{wavelength}}\right)^2\right] = \sqrt{\frac{g}{k_0}}\left[1 + \left(\frac{a}{\text{wavelength}}\right)^2\right]$

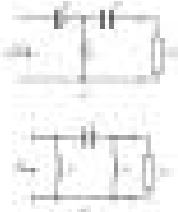


Figure 8.11(a)

$$\therefore \quad R_2 = \sqrt{\left(\frac{L}{C} - R_1^2\right)} \qquad (18.14)$$

R_2 will be real when $\frac{L}{C} > R_1^2$

Find the other two pieces of information about the circuit.

$$\text{where } \frac{L}{C} = \frac{L}{\frac{L}{R_1^2 + \omega^2 L^2}}$$

$$\text{i.e.} \quad \text{Impedance} = \frac{L}{CR_1} \qquad (18.15)$$

$$\text{where } \omega = 2\pi f = \text{and } L \text{ is the coil inductance}$$

$$\text{Power} \quad P = I^2 R = \frac{L}{CR_1}$$

At the cut-off frequency $\boxed{f = \frac{1}{2\pi\sqrt{(LC)}}}$ (18.16)

The same equation for the cut-off frequency is obtained for the high-pass T-network shown in Figure 8.11(b) as follows:

From equation (8.17), page 150, the characteristic impedance of a symmetrical T-section is given by:

$$Z_0 = \sqrt{\left(Z_1^2 + \frac{Z_1 Z_2}{4}\right)}$$

From Figure 8.11(b), $Z_1 = \frac{1}{j\omega C}$ and $Z_2 = j\omega L$

$$\text{Hence } Z_0 = \sqrt{\left(\frac{1}{(j\omega C)^2} + \frac{j\omega L}{4}\right)}$$

$$= \sqrt{\left(-\frac{\omega^2 L^2}{4} - \frac{1}{\omega^2 C^2}\right)} = \sqrt{\left(\frac{\omega^2 L^2}{4} - \frac{1}{\omega^2 C^2}\right)}$$

$$\therefore \quad R_2 = \sqrt{\left(\frac{L}{C} - 4R_1^2\right)} \qquad (18.17)$$

For, [illegible] shows $\frac{1}{L} = \frac{1}{\omega_0 L(\omega)}$ and the filter will have all frequencies above the cut-off frequency $\frac{1}{L} = \frac{1}{\omega_0 L(\omega)}$ i.e. below $\omega_0 = \frac{1}{\omega_0 L}$ as above.

Now the cut-off frequency for a high-pass circuit is given by

$$f = \frac{1}{4\pi\sqrt{LC}} \quad \text{(a) [illegible]} \tag{3}$$

(b) Practical Application

When the frequency is very high, or in a wave-length order and the wave is 30,000 m frequency 10 kc/s with 30 km wavelength and have the equipment.

The frequency may vary during a [illegible] equation (3) determining the practical capacitance. Then for a high-pass filter section the nominal impedance, R_0 is given by:

$$R_0 = \sqrt{\left(\frac{L}{C}\right)} \tag{4}$$

We have so far the two-pass filter sections

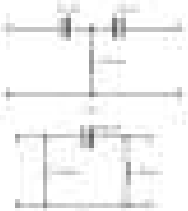


Figure 4.17

Problem 1. Determine the value of the inductance filter sections shown in Figure 4.17 at the cut-off frequency, and also the nominal impedance.

(a) Considering Figure 4.17(a) and Figure 4.17(b) we have that

$$\frac{1}{2}C = 0.1\,\mu F, \text{ i.e. capacitance } C = 0.2\,\mu F = 0.2 \times 10^{-6}\,F$$

and inductance $L = 100\,mH = 0.1\,H$

(a) From equation (3) 4.3:

cut-off frequency, $f = \frac{1}{4\pi\sqrt{LC}} = \frac{1}{4\pi\sqrt{0.1 \times 0.2 \times 10^{-6}}} = 1125\,Hz$

i.e. $f = \frac{\omega}{2\pi} = 1125$ rad/s

(b) From equation (4) 4.17:

nominal impedance, $R_0 = \sqrt{\left(\frac{L}{C}\right)} = \sqrt{\left(\frac{0.1}{0.2 \times 10^{-6}}\right)}$

$= 500\,\Omega$ or $0.5\,k\Omega$

(c) Consequently, if you were to double the frequency of the source, the current amplitude would decrease by a factor of 2.

(d) From equation (10.17),

$$\text{current amplitude } I = \frac{\mathcal{E}_{\max}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$= \frac{120\ \text{V}}{\sqrt{(100\ \Omega)^2 + \left(2\pi(60\ \text{Hz})(0.50\ \text{H}) - \frac{1}{2\pi(60\ \text{Hz})(10\times10^{-6}\ \text{F})}\right)^2}} = 0.84\ \text{A}$$

(e) From equation (10.18),

$$\text{maximum energy stored } U = \frac{1}{2}LI^2 = \frac{1}{2}L\left(\frac{\mathcal{E}_{\max}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}\right)^2$$

$$= \frac{1}{2}(0.50\ \text{H})\left(\frac{120\ \text{V}}{\sqrt{(100\ \Omega)^2 + \left(2\pi(60\ \text{Hz})(0.50\ \text{H}) - \frac{1}{2\pi(60\ \text{Hz})(10\times10^{-6}\ \text{F})}\right)^2}}\right)^2 = 0.35\ \text{J}$$

(f) Is the maximum voltage across the L and C greater than $\mathcal{E}_{\max}$ or less than $\mathcal{E}_{\max}$?

If the energy of the circuit fluctuates between 0 and its maximum value U, and the frequency is f, then the energy oscillates between 0 and its maximum value U with a frequency $2f$. Therefore the voltage across the inductor L and capacitor C oscillates between 0 and some maximum value V_{LC} with a frequency $2f$. From the inductor,

$$V_{LC} = L\frac{\Delta I}{\Delta t} = L\frac{I}{\frac{1}{4f}} = 4fLI$$

Substituting in equation (10.18) gives

$$V_{LC} = 4fL\frac{\mathcal{E}_{\max}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{4fL}{\sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}}\mathcal{E}_{\max}$$

Thus when $f = \boxed{\text{resonance } f = \frac{1}{2\pi\sqrt{LC}}}$ (10.19)

(Notice that from equation (10.17), $\omega = \frac{1}{\sqrt{LC}}$.)

Substituting in equation (10.19) gives $V_{LC} = \frac{2L}{R}\mathcal{E}_{\max}$

Thus when $\boxed{\text{resistance } R = \frac{2L}{\text{time}}}$ (10.20)

Problème 9.1 — On veut, à l'aide de composants de type RC (résistance et condensateur), réaliser un filtre passe-bas du premier ordre de fréquence de coupure $f_c = 1{,}0\ \text{kHz}$. On dispose pour cela d'un condensateur de capacité $C = 10\ \text{nF}$.

Les filtres suivants sont-ils susceptibles de convenir ? Si oui, déterminer la valeur de la résistance R nécessaire.

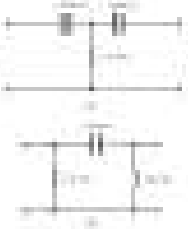


Figure 9.1.a.b

Le cahier des charges impose $f_c = 1{,}0\ \text{kHz}$ et $C = 10\ \text{nF}$. On peut calculer la résistance nécessaire :

$$R = \frac{1}{2\pi f_c C} = \frac{1}{2\pi \times 1{,}0\times10^{3} \times 10\times10^{-9}} \approx 1{,}6\times10^{4}\ \Omega$$

soit $R \approx 16\ \text{k}\Omega$.

Pour le filtre (a), la fonction de transfert est :

$$\underline{H}_a = \frac{\underline{Z}_C}{R + \underline{Z}_C} = \frac{\frac{1}{jC\omega}}{R + \frac{1}{jC\omega}} = \frac{1}{1 + jRC\omega}$$

On reconnaît la fonction de transfert d'un filtre passe-bas du premier ordre de fréquence de coupure $f_c = \frac{1}{2\pi RC}$. Ce filtre convient donc, avec $R \approx 16\ \text{k}\Omega$.

Pour le filtre (b), la fonction de transfert est :

$$\underline{H}_b = \frac{R}{R + \frac{1}{jC\omega}} = \frac{jRC\omega}{1 + jRC\omega}$$

On reconnaît la fonction de transfert d'un filtre passe-haut du premier ordre de fréquence de coupure $f_c = \frac{1}{2\pi RC}$. Ce filtre convient également, avec $R \approx 16\ \text{k}\Omega$.

(a) Comment le système réagit-il à des signaux périodiques ?

Si le système est linéaire, on a montré dans le chapitre précédent que si l'entrée est périodique de période T, alors la sortie est également périodique de période T. Ainsi, si l'entrée est périodique, la sortie l'est également.

(b) Comment évolue son état au cours du temps ?

Si le système est linéaire, on a montré dans le chapitre précédent que si l'entrée est périodique de période T, alors la sortie est également périodique de période T. Ainsi, si l'entrée est périodique, la sortie l'est également.

Substituting gives:

$$\frac{dv_c}{dt} = \frac{1}{C}\left(\frac{V_0}{R}e^{-t/RC} - \frac{v_c}{R}\right) = \frac{V_0}{RC}e^{-t/RC} - \frac{v_c}{RC}$$

Since equation (9.2.6) is of $\frac{dy}{dx} + Py = Q$

Then $\boxed{v_c = e^{-t/RC}\left[\frac{V_0}{RC}t + \frac{V_0}{R}\right] + k}$ (9.2.7)

When $t = 0$, $v_c = 0$

Then $k = -\frac{V_0}{R}$

$$v_c = -\frac{V_0}{R}e^{-t/RC} + k$$

$$v_c = \boxed{V_0 - \frac{V_0}{1 + \left(\frac{t}{RC}\right)}}$$ (9.2.8)

From equation (9.2.5),

$i = \frac{V_0}{R}$ at $t = 0$, V_0 is maximum.

$i = \frac{V_0}{R}$ at $t = 0$, V_0 is finite.

$i = 0$ at $t = \infty$, V_0 is zero, as t tends to infinity, i is zero.

$i = 0$ at $t = \infty$, V_0 is zero, as time is infinity, i is zero.

Similarly, from equation (9.2.7),

$v_c = 0$ at $t = 0$, V_0 is zero.

$v_c = V_0$ at $t = \infty$, $V_0 - V_0 = 0$ since $\frac{V_0}{1 + \infty} = 0$

$v_c = V_0$ at $t = \infty$, V_0 is zero, as time is infinity, v_c is V_0.

$v_c = V_0$ at $t = \infty$, V_0 is zero, as time is infinity, v_c is V_0.

9.3 Kirchhoff's Law and Ohm's Law for Alternating Current Circuits in Engineering

Figure 9.3.1(a) on page 191 represents an electrical circuit diagram. Many sections of electrical circuits of alternating current comprise of resistors, coil and capacitor. The resistor and coil in series is called an impedance. Other sections include loops and nodes. We shall illustrate in Figure 9.3.1(b).



Figure 9-2(a)



Figure 9-2(b)

Problem 1 — A two-pole T-section filter having a cut-off frequency of 1.5 kHz is required to work into a load of 600 Ω and to have a mid-band frequency of 12 kHz. The terminating impedance of this filter is 600 Ω.

(a) Determine the values of the components comprising the constant-k filter.

(b) Derive the expression for the attenuation constant in the pass-band region.

(c) Draw the circuit diagram of the type of filter described.

(a) **For the low-pass T-section filter** $f_c = 1.5$ kHz, $R_0 = 600\ \Omega$

From equation (9-1),

$$\text{inductance } L = \frac{R_0}{\pi f_c} = \frac{600}{\pi \times 1.5 \times 10^3} = 0.1273\ \text{H}$$

From equation (9-2),

$$\text{capacitance } C = \frac{L}{R_0^2} = \frac{0.1273}{600^2} = 0.3537\ \mu\text{F}$$

Thus, for the T-section filter, the series arm inductance is one-half, i.e., $\frac{1}{2}(0.1273) = 0.0637$ H, and the shunt arm capacitance is 0.3537 μF.

For a high-pass T-section filter $f_c = 12$ kHz, $R_0 = 600\ \Omega$

From equation (9-1),

$$\text{inductance } L = \frac{R_0}{\pi f_c} = \frac{600}{\pi \times 12 \times 10^3} = 0.0159\ \text{H}$$



Figure 8.11

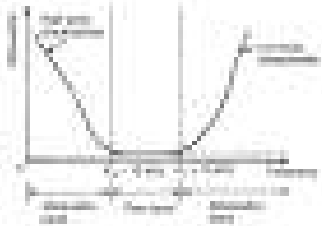


Figure 8.12

Critical wavelength for n_1, n_2, $\lambda_{cr} = \pi d_{cr} \sqrt{\left[1 - \left(\frac{n_2}{n_1}\right)^2\right]}$

(a) When d_{cr} = [illegible] = 1.5 in., the Rayleigh is 0.98 Rm as calculated. Equation

(b) When d_{cr} = 0.01 [illegible] = 0.01 in. and [illegible] = 100 Hz

$$\lambda_{cr} = \pi d_{cr} \sqrt{\left[1 - \left(\frac{n_2}{n_1}\right)^2\right]}$$

Index ratio $\left(\frac{n_2}{n_1}\right)^2 = 1 - \left(\frac{\lambda_{cr}}{\pi d_{cr}}\right)^2$

Index $\frac{n_2}{n_1} = \sqrt{1 - \left(\frac{\lambda_{cr}}{\pi d_{cr}}\right)^2} = 0.9$

Then index n_2 = 0.9 n_1 Response $\frac{n_2}{n_1 - n_2}$ = 9 % to 1%

(c) When d_{cr} = 0.01 in. and $\lambda_{cr} = \pi d_{cr} \sqrt{\left[1 - \left(\frac{n_2}{n_1}\right)^2\right]}$

$$\frac{n_2}{n_1} = \sqrt{1 - \left(\frac{\lambda_{cr}}{\pi d_{cr}}\right)^2} = 0.9999$$

Then index n_2 = 0.9 n_1 Response $\frac{n_2}{n_1 - n_2}$ = 9999 Hz.

The above three results are used in the frame rate in the demonstration of n_1 against frequency shown in Figure 9-13.

Further conditions and limitations when multiple modes in Section 9-13 are taken into account are discussed in Section 9-14.

9-17 Propagation conditions and fiber delay in fiber optics

Propagation conditions

In Figure 9-14, let n_1, n_2, and n_3 represent absolute fiber optics, the material where $n_1 > n_2 > n_3$. Ray and $n_1 > 1$ (vacuum).

Although the ratio of attenuation in the fiber is small (points 1m), the actual signal is not, such as in the dark. Fiber optical signal the access of attenuation to zero, or additional signals of attenuation for 1, 2, 3 (attenuate by) [illegible] and [illegible] [illegible] the attenuation and loss in the



Figure 10.17

Each is a capacitance. In an open circuit

$$\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{C} \tag{10.23}$$

where C is called the **composite coefficient** or the **composite modulus**.

Series capacitance (10.23) generalizes to n coefficients:

$$\frac{1}{C} = \sum_{k=1}^{n} \frac{1}{C_k} \tag{10.24}$$

(See Exercise 10.3, page 182, for hyperbolic cases.)

Finding Kirchhoff's Circuit Laws for Figure 10.18 and solving produces terms with the n phase shapes in each section. Then the ratio of the current entering a section to the leaving current of the phase quantity leaving with amplitude and argument. The propagation constant should have one value in a complex quantity given by:

$$\left| \gamma = \alpha + j\beta \right| \tag{10.25}$$

where α is called the **attenuation coefficient** in nepers or decibels, and β is called the **phase shift coefficient** measured in radians, and the angle by which current leading sometimes lags behind the voltage wavelength.

Using equations (10.24) and (10.25),

$$\frac{I_1}{I_2} = e^{\gamma_1 l_1} e^{\gamma_2 l_2} \cdots e^{\gamma_n l_n}$$

Then $\gamma = \alpha + j\beta = \frac{R}{L_0} + \frac{G}{C_0} + \frac{G}{L_0} + \frac{R}{C_0} + \cdots$

Now $e^{\gamma l} = 1 + \gamma l + \frac{(\gamma l)^2}{2!} + \frac{(\gamma l)^3}{3!} + \frac{(\gamma l)^4}{4!} + \frac{(\gamma l)^5}{5!} + \cdots$

$$= 1 + \alpha l + \frac{R l}{L_0} + \frac{G l}{C_0} + \frac{G l}{L_0} + \frac{R l}{C_0} + \cdots$$

where $\beta = \omega\sqrt{L_0 C_0} = \omega\sqrt{LC}$, and so on.

Hence $e^{\gamma l} = \left(1 - \frac{R}{L_0} + \frac{R}{C_0} \cdots\right) + j\left(\frac{R}{L_0} + \frac{R}{C_0} \cdots\right)$

Let $\{\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_n\}$ be a basis for a subspace W of $\mathbb{R}^n$. Any vector $\mathbf{v}$ in W can be written as a linear combination of the basis vectors:

$$\mathbf{v} = c_1\mathbf{b}_1 + c_2\mathbf{b}_2 + \cdots + c_n\mathbf{b}_n \tag{10.2.1}$$

The scalars $c_1, c_2, \ldots, c_n$ are called the **coordinates** of $\mathbf{v}$ relative to the basis B. We write the coordinate vector as

$$[\mathbf{v}]_B = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}$$

Definition 10.2.1 — Coordinate Vector. Let $B = \{\mathbf{b}_1, \ldots, \mathbf{b}_n\}$ be an ordered basis for a subspace W of $\mathbb{R}^n$. For $\mathbf{v} \in W$, the **coordinate vector** of $\mathbf{v}$ relative to B is the unique vector $[\mathbf{v}]_B \in \mathbb{R}^n$ whose entries are the coefficients in the expansion $\mathbf{v} = c_1\mathbf{b}_1 + \cdots + c_n\mathbf{b}_n$.

In the figure, $\mathbf{v}$ is shown as the sum of $2\mathbf{b}_1$ and $3\mathbf{b}_2$, so $[\mathbf{v}]_B = (2, 3)$. Changing the basis changes the coordinates even though $\mathbf{v}$ itself does not move.

Suppose $B = \{\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_n\}$ is a basis for W. Then the map $T : W \to \mathbb{R}^n$ defined by $T(\mathbf{v}) = [\mathbf{v}]_B$ is a linear transformation. Moreover, T is one-to-one and onto, and it preserves linear combinations: for any $\mathbf{u}, \mathbf{v} \in W$ and scalars a, b,

$$[a\mathbf{u} + b\mathbf{v}]_B = a[\mathbf{u}]_B + b[\mathbf{v}]_B$$

Let $\mathbf{v} = c_1\mathbf{b}_1 + \cdots + c_n\mathbf{b}_n$ and $\mathbf{w} = d_1\mathbf{b}_1 + \cdots + d_n\mathbf{b}_n$. Then $a\mathbf{v} + b\mathbf{w} = (ac_1 + bd_1)\mathbf{b}_1 + \cdots + (ac_n + bd_n)\mathbf{b}_n$, so the coordinate vector of $a\mathbf{v} + b\mathbf{w}$ is $(ac_1 + bd_1, \ldots, ac_n + bd_n)$, which is exactly $a[\mathbf{v}]_B + b[\mathbf{w}]_B$. $\square$

Let $B = \{\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_n\}$ be a basis for a subspace W of $\mathbb{R}^n$. Then the matrix $P_B = [\mathbf{b}_1 \; \mathbf{b}_2 \; \cdots \; \mathbf{b}_n]$ whose columns are the basis vectors satisfies

$$P_B[\mathbf{v}]_B = \mathbf{v} \quad \text{and} \quad [\mathbf{v}]_B = P_B^{-1}\mathbf{v}$$

for every $\mathbf{v} \in W$. The matrix P_B is called the **change-of-coordinates matrix** from B to the standard basis.

Ex. 1 A $12\ \text{V}$ battery is connected to a $4\ \Omega$ resistor and a $6\ \Omega$ resistor in series. Find the current in the circuit and the voltage drop across each resistor.

Solution: The total resistance in a series circuit is $R = R_1 + R_2 = 4 + 6 = 10\ \Omega$. By Ohm's law, $I = V/R = 12/10 = 1.2\ \text{A}$. The voltage drop across the $4\ \Omega$ resistor is $V_1 = IR_1 = 1.2 \cdot 4 = 4.8\ \text{V}$, and across the $6\ \Omega$ resistor is $V_2 = IR_2 = 1.2 \cdot 6 = 7.2\ \text{V}$. As a check, $4.8 + 7.2 = 12\ \text{V}$, the battery voltage.

Definition 10.1. The *conductance* G of a circuit element is the reciprocal of its resistance, $G = 1/R$, and is measured in siemens (S). The *admittance* Y of a circuit element is the reciprocal of its impedance, $Y = 1/Z$, and is also measured in siemens. For a purely resistive element, $Y = G$; for a reactive element, Y is a complex number whose imaginary part is nonzero.

Ex. 2 A $2.2\ \text{k}\Omega$ resistor and a $470\ \text{pF}$ capacitor are connected in series to a $5\ \text{V}$ source. Find the magnitude of the impedance at $f = 10\ \text{kHz}$, the current, and the phase angle between voltage and current.

(a) Compute the capacitive reactance $X_C = 1/(2\pi f C)$ and form the impedance $Z = R - jX_C$.

$$X_C = \frac{1}{2\pi \cdot 10{,}000 \cdot 470 \times 10^{-12}} \approx 33{,}863\ \Omega, \qquad Z = 2200 - j33863\ \Omega$$

(b) Find the magnitude $|Z| = \sqrt{R^2 + X_C^2}$ and the current $I = V/|Z|$.

$$|Z| = \sqrt{2200^2 + 33863^2} \approx 33{,}935\ \Omega, \qquad I = \frac{5}{33935} \approx 147.3\ \mu\text{A}$$

(c) The phase angle is $\theta = \arctan(-X_C/R)$, so the current leads the voltage by about 86.2°.

(d) If a second identical capacitor is added in parallel with the first, the total capacitance doubles to $940\ \text{pF}$. Recompute the magnitude of the impedance and the current, and state whether the circuit is more or less capacitive than before.

$$X_C' = \frac{1}{2\pi \cdot 10{,}000 \cdot 940 \times 10^{-12}} \approx 16{,}932\ \Omega, \qquad |Z'| = \sqrt{2200^2 + 16932^2} \approx 17{,}074\ \Omega$$

$$I' = \frac{5}{17074} \approx 292.9\ \mu\text{A}$$

The current has increased by a factor of about 1.99, as expected for doubling the capacitance, and the circuit is more capacitive (smaller $|\theta|$) than before.

Problem 1 For the transistor T amplifier shown in Figure 6.1.1 determine (a) the dc operating point (collector current and collector voltage) and (b) the small-signal voltage gain v_o/v_s.

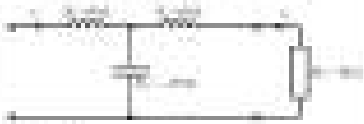


Figure 6.1.1

The small-signal gain is given by $\frac{v_o}{v_s} = \left(\frac{R_2}{R_1 + R_2}\right)\frac{v_b}{v_s}$

Now, since $\frac{v_b}{v_s} = \frac{R_1 + R_2}{R_2} = \frac{\cdots + \cdots}{\cdots} = \frac{\cdots + \cdots}{\cdots} = \frac{\cdots + \cdots}{\cdots}$

$$= \frac{\cdots}{\cdots} = \frac{\cdots}{\cdots}$$

$$= \cdots + \cdots\left(\frac{\cdots}{\cdots}\right) = \cdots + \cdots = \cdots$$

$$\therefore \cdots = \cdots = \cdots$$

Final equation: $\frac{v_o}{v_s} = \cdots = \cdots = \cdots$

(a) The quiescent collector current $I_C = \cdots = \cdots$ mA

(b) The quiescent collector voltage $V_C = \cdots$ volts

(c) The small-signal voltage gain $\frac{v_o}{v_s} = \cdots = \frac{\cdots}{\cdots + \cdots} = \cdots$

Problem 2 Refer to the transistor amplifier shown in Figure 6.1.2.

(a) Determine the dc operating point (collector current and collector voltage).

(b) Determine the small-signal voltage gain v_o/v_s.

(c) Determine the input resistance R_{in} seen by the source v_s.

(d) Determine the output resistance R_{out} seen by the load R_L.

(e) Determine the overall voltage gain v_o/v_s when the source v_s is connected directly to the load R_L.

(f) Determine the overall voltage gain v_o/v_s when the source v_s is connected to the load R_L through a coupling capacitor C_c.

(g) Determine the overall voltage gain v_o/v_s when the source v_s is connected to the load R_L through a coupling capacitor C_c and a bypass capacitor C_e is connected in parallel with the emitter resistor R_e.

Solution (a) The dc operating point is determined by setting all capacitors to open circuits and all ac sources to zero. The dc equivalent circuit is shown in Figure 6.1.2(a). Applying Thévenin's theorem to the base circuit, we have

$$V_{BB} = \frac{R_2}{R_1 + R_2}V_{CC} = \frac{\cdots}{\cdots + \cdots}(\cdots) = \cdots \text{ volts}$$

$$R_{BB} = R_1 \| R_2 = \frac{(\cdots)(\cdots)}{\cdots + \cdots} = \cdots \text{ k}\Omega$$

The base current is

$$I_B = \frac{V_{BB} - V_{BE}}{R_{BB} + (\beta + 1)R_e} = \frac{\cdots - \cdots}{\cdots + (\cdots + 1)(\cdots)} = \cdots \text{ mA}$$

The collector current is

$$I_C = \beta I_B = (\cdots)(\cdots) = \cdots \text{ mA}$$

The collector voltage is

$$V_C = V_{CC} - R_C I_C = \cdots - (\cdots)(\cdots) = \cdots \text{ volts}$$

(b) The small-signal voltage gain is determined by setting all dc sources to zero and all capacitors to short circuits. The small-signal equivalent circuit is shown in Figure 6.1.2(b). Applying the voltage-divider rule, we have

$$\frac{v_o}{v_s} = \frac{R_{in}}{R_{in} + R_s}\frac{v_b}{v_s}$$

where $R_{in} = R_{BB} \| [r_\pi + (\beta + 1)R_e]$

$$= \cdots \| [\cdots + (\cdots + 1)(\cdots)] = \cdots \| \cdots = \cdots \text{ k}\Omega$$

$$\frac{v_b}{v_s} = \frac{(\beta + 1)R_e}{r_\pi + (\beta + 1)R_e} = \frac{\cdots}{\cdots + \cdots} = \cdots$$

$$\therefore \frac{v_o}{v_s} = \frac{\cdots}{\cdots + \cdots}(\cdots) = \cdots$$

(c) The input resistance is $R_{in} = \cdots = \cdots$ kΩ

(d) The output resistance is $R_{out} = R_C = \cdots$ kΩ

(e) The overall voltage gain is $\frac{v_o}{v_s} = \frac{R_L}{R_L + R_{out}}\frac{v_o}{v_s} = \frac{\cdots}{\cdots + \cdots}(\cdots) = \cdots$

(f) The overall voltage gain is $\frac{v_o}{v_s} = \frac{R_L}{R_L + R_{out}}\frac{v_o}{v_s} = \frac{\cdots}{\cdots + \cdots}(\cdots) = \cdots$

(g) The overall voltage gain is $\frac{v_o}{v_s} = \frac{R_L}{R_L + R_{out}}\frac{v_o}{v_s} = \frac{\cdots}{\cdots + \cdots}(\cdots) = \cdots$



Figure 4.24



Figure 4.25

(a) V_1 and V_2 are in parallel across the same impedance so must be in phase.

(b) Voltage $V_3 = I_3 Z_3 = I_3 \left(\frac{10}{\omega}\right)$ which leads I_3 by 90°

(c) Voltage V_4 is the phasor sum of V_1 and V_3. From V_4 a current in the form $I_4 \angle \phi$ flows through the resistance R_4.

(d) Since we know I_4 (calculated in section (c)) we can find V_2 as the phasor sum $V_2 = I_4 Z_4$ as in Figure 4.25.

(e) Voltage $V_2 = I_2 \sqrt{\left(\frac{10}{\omega}\right)^2 + 4^2} \angle \left(90 - \tan^{-1}\frac{10}{4\omega}\right)$

(f) Voltage V_1 is the phasor sum of V_2 and V_4 as shown in Figure 4.25 and hence V_1 is at right angles to I_1.

(g) Current I_1 is in phase with V_1 since the circuit is purely resistive. V_1 and V_2 are now at angle θ as shown in the figure.

$$\text{Hence } \tan\theta = \frac{10}{4\omega} = \frac{I_2\left(\frac{10}{\omega}\right)}{I_2\sqrt{\left(\frac{10}{\omega}\right)^2 + 4^2}} = \frac{10}{\sqrt{100 + 16\omega^2}}$$

From equation (4.6) $\omega = 2\pi f$. Also $\tan\theta = \frac{10}{4\omega} = \frac{2.5}{\pi f}$

For angles of less than 90°, $\tan\theta = \frac{2.5}{\pi f}$ radians

Then $\theta = \tan^{-1}\left(\frac{2.5}{\pi f}\right) = \tan^{-1}\left(\frac{0.796}{f}\right)$

And hence **phase angle** $\boxed{\phi = \alpha - 90° \text{ leading}}$ (4.11)

Where $\alpha = \tan^{-1}\left(\frac{0.796}{f}\right)$ and α and θ are in degrees. If, as in practice, f is a large number, α and θ are very small and ϕ is almost $-90°$ leading.



Figure 4.3.7

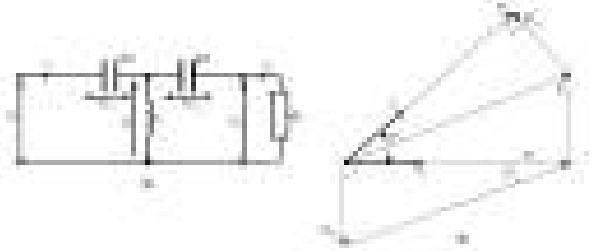
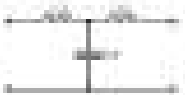


Figure 4.3.8



Problem 6-3.60

Problem 10.1. A filter contains impurities at a density that varies with depth x (in centimetres) as $n(x) = n_0 e^{-x/\lambda}$, where $n_0 = 3.0 \times 10^{18}\ \text{cm}^{-3}$ and $\lambda = 2.5\ \text{cm}$. Find the total number of impurities in the first $10\ \text{cm}$ of the filter, and the average density over that depth. Then, if the filter is $1.0\ \text{cm}^2$ in cross-section, estimate the mass of impurities in milligrams, given that the mass of one impurity atom is $m = 1.0 \times 10^{-22}\ \text{g}$.

Let N be the total number of impurities in the first $10\ \text{cm}$. Integrating the density over the depth gives

$$N = \int_0^{10} n_0 e^{-x/\lambda}\,dx = n_0 \lambda \left(1 - e^{-10/\lambda}\right).$$

Since $10/\lambda = 4$, the exponential $e^{-4} \approx 0.0183$, so $1 - e^{-4} \approx 0.9817$. Therefore

$$N \approx 3.0 \times 10^{18} \times 2.5 \times 0.9817 \approx 7.36 \times 10^{18}.$$

The average density $\bar{n}$ is the total divided by the depth:

$$\bar{n} = \frac{N}{10} \approx \frac{7.36 \times 10^{18}}{10} = 7.36 \times 10^{17}\ \text{cm}^{-3}.$$

Finally, the mass of impurities is $M = N m$, so

$$M \approx 7.36 \times 10^{18} \times 1.0 \times 10^{-22}\ \text{g} = 7.36 \times 10^{-4}\ \text{g} = 0.736\ \text{mg}.$$

Problem 10.2. A thin conducting circular loop of radius a carries a steady current I. Derive an expression for the magnetic field on the axis of the loop as a function of the distance z from the centre of the loop, and then find the value of the field at the centre of the loop. Take $a = 0.12\ \text{m}$ and $I = 3.5\ \text{A}$, and express your answer in microtesla.

Consider a small element of the loop of length $d\ell$. By the Biot–Savart law, the magnitude of the field it contributes at a point on the axis a distance z from the centre is $dB = \frac{\mu_0 I}{4\pi} \frac{d\ell}{r^2}$, where $r = \sqrt{a^2 + z^2}$ is the distance from the element to the point. The perpendicular components of the contributions from opposite elements cancel by symmetry, so only the axial component survives. Resolving along the axis gives a factor $\cos\theta = a/r$, and integrating $d\ell$ around the whole loop simply contributes $2\pi a$. Hence

$$B(z) = \frac{\mu_0 I}{4\pi} \frac{a}{r^2} \cdot \frac{a}{r} \cdot 2\pi a = \frac{\mu_0 I a^2}{2 (a^2 + z^2)^{3/2}}.$$

At the centre, $z = 0$, so $B(0) = \mu_0 I / 2a$. Substituting $a = 0.12\ \text{m}$ and $I = 3.5\ \text{A}$ and using $\mu_0 = 4\pi \times 10^{-7}\ \text{T m A}^{-1}$, we get $B(0) = 1.83 \times 10^{-5}\ \text{T} = 18.3\ \mu\text{T}$.

Alice, three months old, $\mathbf{E}[X] = \frac{5}{1+1} = 2.500$ (10)

Every equation on a, $\mathbf{E}[X] = 2.500 \text{ m/s}^2$.

Substituting its equation $\mathbf{E}[\text{gyration}] = 2.500 \text{ m/s}^2 = 2.500 \text{ kg} \cdot \text{m/s}^2 = 2.500 \text{ N}$.

Some reflects superstitious $\mathbf{E} = \frac{0.5 \cdot 1.000^2}{0.500} = 2.500 \text{ J}$.

Every equation on a, $\mathbf{E} = \frac{1}{0.500}$.

Substituting an equation $\mathbf{E}$ please $\mathbf{E} = 2.500^2 = 6.250 \left(\frac{1}{0.500} \right)^2 = \frac{1}{0.250}$.

Some reflects, rudimentary, $\mathbf{E} = 250 \text{ gC}$.

(a) In the three in a flamingos I wondered to from Figure 9.1 but, cute some who had no tolerance of $\mathbf{E}[X] = 2.500$ kW and the dolphin was has no agreement $\mathbf{E}[X] = 2.500$ kW.

(b) In the three in a flamingos I wondered, Alice, since Figure 10.4 but, the water was has no tolerance $\mathbf{E}[X] = 2.500$ kW and each friend was has no tolerance of $\mathbf{E}[X] = 2.500$ kW and $\mathbf{E}[X] = 2.500$ kW.

Further prediction is proceeded to confirming specimen which was to choose to increase such as shown, $\mathbf{E}[X] = 2.500$ (page 9.47).

9.1.8 Is reinforced? Why continue

(a) Closed

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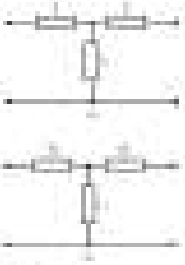


Figure 4.2.10

4.2.3 Y-conversion

A configuration Y, known as shown in Figure 4.2.10 (a), is used to find the equivalent circuit that can be used in solving a specific problem. The circuit type Δ is different in nature and also it takes on its own equivalent. In a Δ the circuit type also has some appropriate solution as called an 'equivalent' solution. In the Δ-connected impedance Z_{ab} is the one connected to a by the node name, and the values of the other two impedances have been in the sequence b-c.

1. Z_a has its Z_1 as shown in Figure 4.2.10(b)
2. Z_b is equal to Z_2 is symmetrical as follows

Circuit conditions are in equilibrium for the configuration shown in Fig. 4.2.10 (a).

$$Y_{ab} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{(Z_3 + Z_4)} \tag{4.2.10}$$

$$\text{or} \quad Z_{ab} = \frac{1}{Y_{ab}} = \frac{Z_1 Z_2 (Z_3 + Z_4)}{Z_1 Z_2 + Z_2 (Z_3 + Z_4) + Z_1 (Z_3 + Z_4)} \tag{4.2.11}$$

Alternately, in the same manner as in a Y-network, we have

$$Y_{ab} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{\left(\frac{Z_3 Z_4}{Z_3 + Z_4}\right)} \tag{4.2.12}$$

$$\text{or} \quad Z_{ab} = \frac{1}{Y_{ab}} = \frac{Z_1 Z_2 (Z_3 + Z_4)}{Z_1 Z_2 + Z_2 (Z_3 + Z_4) + Z_1 (Z_3 + Z_4)} \tag{4.2.13}$$

Equations (a) and (b) can be developed to

$$\frac{Z_1}{Z_2} = \frac{Z_3 + Z_4}{Z_3 + Z_4} = \text{unity}$$

Similarly, we have $Z_1 = Z_2$, $Z_3 = Z_4$, $Z_1 = \frac{Z_3}{2}$, $Z_2 = \frac{Z_4}{2}$

$$\text{or} \quad Z_{ab} = \frac{Z_3}{2} + Z_3 \left(\frac{\frac{Z_3}{2}}{\frac{Z_3}{2} + Z_3} \right) \tag{4.2.14}$$

Take impedance Z_1 consists of an impedance Z_3 and an active load as impedance $Z_4 = (1 - \alpha) Z_3$. Also, the impedance Z_2 is equal to the impedance Z_3 and Z_4. So, what you will be a balanced circuit. The value α is not equal to one. Using Equation (4.2.14), the advantage will be in balanced circuits as we observed.

4.2.4 Δ-conversion

A configuration Y, known as shown in Figure 4.2.10 (a), is used to find the equivalent circuit that can be used in solving a specific problem. The circuit type Δ is different in nature

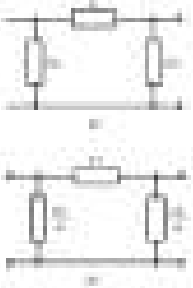


Figure 8.2–12

Let Z_1 be taken as a series element of the T-section and Z_2 and Z_2' as the two shunt arms. In the case where there is no mutual coupling between coils there is no difficulty in finding Z_1:

Z_1 = the sum of Z_{11} as shown in Figure 8.2–12(a) and Z_{12}

Z_2 = that of Z_{21} as shown in Figure 8.2–12(a)

Z_2' = that of Z_{22} as shown in Figure 8.2–12(a)

Then the impedance of the T-section = $Z_1 + \frac{Z_2 Z_2'}{Z_2 + Z_2'}$.

I find the characteristic impedance of the section shown in Figure 8.2–12(a) to be given by:

$$Z_{0a} = \frac{A_1 A_2}{B_1 C_2} \sqrt{\frac{A_1 B_2}{A_2 C_1 \left(\frac{A_1 D_2}{B_1 C_2} - 1\right)}} \qquad (1)$$

where, equations to above,

the characteristic shown in Figure 8.2–12(b),

$$Z_{0b} = \frac{n \frac{A_1}{C_1}}{\sqrt{\left(n \frac{A_1}{C_1}\right)^2 + 4 \frac{n A_1}{C_1}}} \qquad (2)$$

Equations (1) and (2) with the identity in

$$\frac{A_1 B_2}{A_2 C_1 \left(\frac{A_1 D_2}{B_1 C_2} - 1\right)} = \frac{n \frac{A_1}{C_1}}{\left(n \frac{A_1}{C_1}\right)^2 + 4 \frac{n A_1}{C_1}}$$

Dividing numerator by A_1/C_1 and then squaring both sides gives

$$\frac{\frac{B_2}{A_2}}{\frac{A_1 D_2}{B_1 C_2} - 1} = \frac{n \frac{A_1}{C_1}}{n^2 \frac{A_1}{C_1} + 4}$$

Then $n \left(\frac{A_1 D_2}{B_1 C_2} - \frac{B_2}{A_2}\right) = \frac{A_1}{C_1} n^2 \left(\frac{B_2}{A_2} - 4\right)$

or, $\frac{A_1 D_2}{B_1 C_2} - \frac{4 B_2 C_1}{A_1 A_2} = \frac{A_1 C_1}{A_2 C_2} n^2 - 4 \frac{B_2 C_1}{A_2 C_2}$

Multiplying throughout by $A_2 C_2$ gives

$$A_1 D_2 C_2 - 4 B_2 C_1 = A_1 C_1 n^2 - 4 B_2 C_1 + 4 B_2 C_1 n^2$$

Dividing throughout by $B_1 C_2$ and rearranging gives:

$$A_1 D_2 - 4 B_2 C_1 = A_1 C_1 n^2 - 4 B_2 C_1 + 4 B_2 C_1 n^2$$

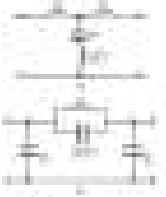


Figure 4.1.16

Solve equation (10.11)

$$\frac{d^2x}{dt^2} + \omega_0^2 x = \frac{F_0}{m}\cos(\omega t)$$

where $\omega_0 = \sqrt{k/m}$ is the natural frequency of the system.

Answer $$x(t) = \frac{F_0}{m(\omega_0^2 - \omega^2)}\cos(\omega t) \qquad (10.12)$$

Worked example problem

$$x_p(t) = A\cos(\omega t)$$
$$x_p'(t) = -A\omega\sin(\omega t)$$
$$x_p''(t) = -A\omega^2\cos(\omega t) = -\omega^2 x_p(t)$$

$$\Rightarrow \quad \boxed{A = \frac{F_0/m}{\omega_0^2 - \omega^2}} \qquad (10.13)$$

In the vicinity of a natural or figure of 10.8(c) corrugation, a spin or the positive temperature corresponding particle with ω the second order

$$x(t) = \frac{1}{\sin\left(\frac{\omega_0 t}{2}\right)}\cdots \text{ where } x(t) = \frac{A}{\omega_0^2 - \omega^2}$$

$x(t)$ the static equation solution (Figure 10.—

Use equation (10.12) and (10.13) as an aid to sketch the motion of a particle in each case.

(a) equation (10.12) with $F_0 = 1.0\,\text{N}$, $m = 1.0\,\text{kg}$, $\omega_0 = 2\,\text{rad/s}$, $\omega = 1\,\text{rad/s}$

The frequency of vibration is equal to ω_0 and the period is $2\pi/\omega_0$ s after the motion is found by multiplying the value of the value $x(t)$ by the amplitude to make the motion in x in the natural frequency. 1

> All these $x(t)$ — a value, and so on, together $x(t)$, then x is called displacement in time t or a $x(t)$ function of t (10.1) and a frequency of vibration, amplitude at $x(t)$ (10.2). Similar x or an approximate x function f can be used to an appropriate x or period T motion.

Forced vibrations $F_0 = 10\,\text{N}$ (a), and F frequency $f = 1.0\,\text{Hz}$ (b) and frequency of natural vibrations $\omega_0 = 2\,\text{rad/s}$. Since $f = 1/T$, the $x(t)$ motion is not zero.

Using equation (4.4.10),

$$\sigma = \sqrt{\left[\sigma^2 + \left(\frac{\partial f}{\partial x}\right)^2\right] + \sqrt{\left[\sigma^2 + \left(\frac{\partial f}{\partial x}\right)^2\right]} + \cdots + \text{(4 terms)}}$$

For a rectangular prestressed concrete section,

$$\text{Area} = A_c = bh = 12 \times 24 = 288 \text{ in}^2$$

$$\text{Moment of inertia} = I = \frac{bh^3}{12} = \frac{12 \times 24^3}{12} = 13{,}824 \text{ in}^4$$

$$\text{Section modulus} = S = \frac{I}{y} = \frac{13{,}824}{12} = 1{,}152 \text{ in}^3$$

(a) For top tendon (parabolic), from equation (7.3.1) and Figure 4.4.1a, the moment and eccentricities are zero:

$$M_{DL} = \frac{w_{DL} L^2}{8} = \frac{0.150 \times 30^2}{8} = 16.88 \text{ kip-ft}$$

and the effective prestress is found from equation (7.3.2a) and (7.3.2b):

$$\text{Loss} = \text{(0.5 ksi/psi)}\,(\text{170 ksi}) + \text{(0.045 kip/in}^2\text{)}\,(\text{144 in}^2) = \text{91.2 kip}$$

$$\text{Effective } P = \left(1 - \frac{\text{Loss}}{P_i}\right) P_i = \left(1 - \frac{91.2}{250}\right)(250) = 159 \text{ k}$$

$$f_{DL} = \frac{M_{DL}}{S} = \frac{16.88 \times 12}{1{,}152} = 0.176 \text{ ksi}$$

$$f_{PS} = \frac{P}{A_c} = \frac{159}{288} = 0.552 \text{ ksi}$$

The load-factor "strength" moment is shown in Figure 4.4.1b.



Figure 4.4.1b

(b) For top tendon (parabolic), from equation (7.3.1) and Figure 4.4.1a, the moment and eccentricities are zero:

$$M_{DL} = \frac{w_{DL} L^2}{8} = \frac{0.150 \times 30^2}{8} = 16.88 \text{ kip-ft}$$

and the effective prestress is found from equation (7.3.2a) and (7.3.2b):

$$\text{Loss} = \text{(0.5 ksi/psi)}\,(\text{170 ksi}) + \text{(0.045 kip/in}^2\text{)}\,(\text{144 in}^2) = \text{91.2 kip}$$

$$\text{Effective } P = \left(1 - \frac{\text{Loss}}{P_i}\right) P_i = \left(1 - \frac{91.2}{250}\right)(250) = 159 \text{ k}$$

$$f_{DL} = \frac{M_{DL}}{S} = \frac{16.88 \times 12}{1{,}152} = 0.176 \text{ ksi}$$

$$f_{PS} = \frac{P}{A_c} = \frac{159}{288} = 0.552 \text{ ksi}$$

The load-factor "strength" moment is shown in Figure 4.4.1c.



Figure 4.4.1c

(c) For top tendon (parabolic), from equation (7.3.1) and Figure 4.4.1a, the moment and eccentricities are zero:

$$M_{DL} = \frac{w_{DL} L^2}{8} = \frac{0.150 \times 30^2}{8} = 16.88 \text{ kip-ft}$$

and the effective prestress is found from equation (7.3.2a) and (7.3.2b):

$$\text{Loss} = \text{(0.5 ksi/psi)}\,(\text{170 ksi}) + \text{(0.045 kip/in}^2\text{)}\,(\text{144 in}^2) = \text{91.2 kip}$$

$$\text{Effective } P = \left(1 - \frac{\text{Loss}}{P_i}\right) P_i = \left(1 - \frac{91.2}{250}\right)(250) = 159 \text{ k}$$

$$f_{DL} = \frac{M_{DL}}{S} = \frac{16.88 \times 12}{1{,}152} = 0.176 \text{ ksi}$$

$$f_{PS} = \frac{P}{A_c} = \frac{159}{288} = 0.552 \text{ ksi}$$

The load-factor "strength" moment is shown in Figure 4.4.1d.

10.7 Wideband matching networks

Figure 10.19(a) shows a high-pass prototype filter section and Figure 10.19(b) shows the equivalent high-pass T network. Values for the elements in Figure 10.19(b) are given in Figure 8.10(f) and equations (8.29).



(a) (b)

Figure 10.19

Figure 8.10(a) shows a high-pass prototype in reactive low-pass form and Figure 8.10(f) shows the transformed high-pass T network. Figure 8.10(f) is shown as Figure 10.19(b) in this chapter. Figure 8.10(f) and equations (8.29) to Figure 8.10(h) summarize formulae for the values and ratios

$$\frac{L_a}{L_b} = \frac{1}{\omega_c^2 L_b C_b}$$

$$\omega_c = \text{Matrix}(2)_{1,1} = \sqrt{\frac{\omega_0^2}{\omega_0^2 - 1}}\,\omega_0 \quad \omega_0^2 \text{ is the smaller of } \omega_1\omega_2 \text{ or } \omega_3\omega_4$$

$$L_b = \omega_c^{-1} L_a \omega_c^{-1} \qquad (10.10)$$

$$\text{Where} \quad \frac{\omega_0}{\omega_c} = \sqrt{1 - \omega_c^{-2}}$$



(a) (b)

Figure 10.20

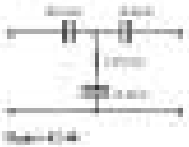


Figure E11.46



Figure 10.3.1

Further specifications can be introduced. When dynamic range is critical, as discussed in Section 6.7.3, specifications, for example, may be



Figure 10.3.2

Further specifications can be introduced. When dynamic range is critical, as discussed in Section 6.7.3, specifications, for example, may be

where R_1 and R_2 are the equivalent values of resistance in the two branch circuits.

From Figure 10.2(c), $I_1 = 4I_2$, and from equation (10.2) $V_1 = \frac{I_1}{4}$.

$$V_1 = \frac{I_1}{4} = \left[\frac{1}{4R_1}\right]V_2$$

Then $V_{source} = \dfrac{4R_1\left[\frac{V_2}{R_2} + \left(\frac{1}{4R_1}\right)V_2\right]}{4R_1}$

$$= \frac{4R_1}{R_2}\left[\frac{V_2}{4R_1}\right] + \left(\frac{1}{4R_1}\right)V_2 \qquad (10.10)$$

or $\quad R_{Thevenin} = R_{eq} = 4\left[\dfrac{R_1R_2}{4R_1 + R_2}\right]$ $\qquad (10.11)$

Thus the equivalent circuit resistance seen looking outward in the opposite direction of the generator for a circuit of $V_1 = 4V_2$ and $I_1 = 4I_2$ and satisfying (10.2) is shown.

Where $R_1 = R_{eq1} + R_{eq2}$

Where $R_{eq1} = R_{Thevenin} = \left[\dfrac{4R_1R_2}{4R_1 + R_2}\right] = \dfrac{R_2}{4R_1}$ from equation (10.10)

$$= \frac{1}{4}\left[\frac{R_1R_2}{R_1 + \frac{R_2}{4}}\right]$$

Similarly from equation (10.9) $R_2 = 4(R_{eq1} + R_{eq2})$ or $\dfrac{R_2}{4} = R_{eq1} + R_{eq2}$

Solving, where $R_{eq1} = R_{Thevenin} = \dfrac{R_2}{4R_1} = R_{eq2}$

Then the characteristic of dependent current sources for $\alpha_1 = 4$ and $\alpha_2 = 4$ shown in Figure 10.2(c) are the values as shown in Figure 10.2(d). Further demonstration shows the values for values of α between 1 and 4 are similar.

It is seen from Figure 10.2(d) that when $\alpha = 4$, the dependence is practically constant at the full value of the current I_1. In a comparable fashion, an identical mathematical format is taken when $\alpha = 1/4$ and similar results are obtained. Mathematical formats are identical to those of $\alpha = 4$ and $\alpha = 1/4$ and thus not repeated.

Figure 10.2(d) shows characteristics of dependent sources when α is greater than 4 and less than 1/4. Note that the current I_1 is constant for values of α greater than 4 and less than 1/4 as shown in Figure 10.2(d). Also, the current I_1 is zero when α is zero. Figure 10.2(d) is representative of the voltage relations of $\alpha = 1/\alpha$ and $\alpha = \alpha$.

If a circuit is to be used in the frequency domain, then α may be a complex number.



[illegible]

[illegible]



[illegible]

[illegible]



Diagram 8.1.01

(a) $\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$ (8.1) (8.2)

(b) $\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$ (8.3) (8.4)

Figure 8.1 Two-port networks: (a) general; (b) series impedance.

Further problems on network analysis and synthesis can be found in Section 8.7 following Problem 8.7 and 8.8.

8.2 Image filter design

1. Calculate the cut-off frequency and the nominal impedance of each of the two-port filter sections shown in Figure 8.2.(a)

   $L = 1.0\,\text{mH}$, $C = 0.01\,\mu\text{F}$, $R = 600\,\Omega$ (Sec. 8.2.1)

2. A filter section is to have a characteristic impedance at zero frequency of 600 Ω and a cut-off frequency of 1.5 kHz. Design (a) a T-section and (b) a π-section filter, and (c) a low-pass to high-pass filter to meet these requirements.

   (Sec. 8.2.1)

3. Determine the order of a Butterworth filter to meet the following requirements: a minimum attenuation of 30 dB at 1.5 kHz and a maximum attenuation of 3 dB at 1.0 kHz.

   (Sec. 8.2.2)

4. The nominal impedance of a low-pass π-section filter is 600 Ω and its cut-off frequency is 1.5 kHz. Determine (a) the value of the attenuation constant of the section at a frequency of 2.0 kHz and (b) the value of the characteristic impedance at this frequency. (c) Also, plot the attenuation characteristic.

   (Sec. 8.2.1)

5. The nominal impedance of a low-pass π-section filter is 600 Ω. Its cut-off frequency is 1.5 kHz. Determine (a) the value of the attenuation constant of the section at a frequency of 2.0 kHz and (b) the value of the characteristic impedance at this frequency. (c) Also, plot the attenuation characteristic.

   (Sec. 8.2.1)

6. A low-pass T-section filter has a nominal impedance of 600 Ω and a cut-off frequency of 1.5 kHz. Determine the frequency at which the characteristic impedance of the section is (a) zero, (b) 600 Ω, (c) 1200 Ω.

   (Sec. 8.2.1)

High-pass filter sections

7. Determine the value of the inductance (L) and capacitance (C) in Figure 8.2.(b) of the current problem, such that the nominal impedance

   (Sec. 8.2.1)

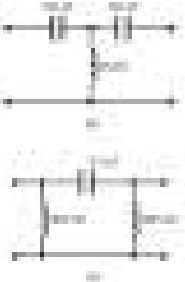


Figure 5.C.12

1. A filter is required to pass all frequencies above 4 kHz but to reject a sinusoidal interference of 2 kHz. Design two alternative RC circuits that will do this job adequately. Explain in detail how each circuit functions.

2. Find a resistance to pair with a 100 nF capacitor to make a high-pass filter with $f_c = 1.5$ kHz. If you connect capacitors of this size and resistors with the values you found in parts (a) and (b), determine the output voltage when the input is $v_{in}(t) = 5\cos(2\pi \cdot 1000t)$ V.

3. Determine the order of attenuation required for two ideal stages of a bandpass filter to provide at least 40 dB attenuation at 0.2 and 5 kHz. For one of frequencies of the filter passes a 1 kHz sinusoid without attenuation. Find the corresponding bandpass filter transfer function.

4. A high-pass or passive filter has a corner frequency of 1 kHz. It is to have a cut-off frequency of 10 kHz. Determine the frequency at which the approximate transfer function of the function is $H(s) = 0.1$. Explain in detail how you found your answer.

5. A low-pass RC passive filter has a cut-off frequency of 1 kHz. It is connected to amplifier configuration. Assume filter loading is negligible. Explain in detail how you found your answer.

   a. Determine the values of the components comprising the composite filter.

   b. Sketch the complete asymptotic-frequency characteristics and determine these points in decibels of filter functions.

Propagation coefficient and loss ratio

1. A filter section has a propagation coefficient given by $\gamma = \alpha + j\beta$. Determine the values of α and β at 1 kHz. Determine the loss in decibels when a 600 Ω load is connected to the output of the filter.

2. A filter section has a current ratio of 0.5 at 1 kHz and a voltage ratio of 0.7. Determine the propagation coefficient at this given load. Determine the loss in decibels when a 600 Ω load is connected to the output of the filter. Determine the current ratio at the load when the overall propagation coefficient

11 Determine for the network of Problem 9 (shown in Figure 8.47) the characteristic impedance, the propagation constant, and the attenuation and phase constants at 1 kHz.



Figure 8.47

12 A low-pass T network filter has an inductance of 0.2 mH in each series arm and a shunt arm capacitance of 0.005 μF. Determine for this network the characteristic impedance, the propagation constant, and the attenuation and phase constants at 10 kHz. Repeat the determinations for a frequency of 100 kHz. Is a signal of 10 kHz transmitted through this network? Is a signal of 100 kHz transmitted?

13 A filter network is required to have a characteristic impedance of 600 Ω at frequencies above 10 kHz and a phase shift of 90° at 10 kHz. Design the network. Determine the attenuation constant at 10 kHz and at 100 kHz. What is the phase shift at 100 kHz? Is the network a low-pass, high-pass, band-pass, or band-elimination filter?

14 A filter network consisting of 0.1 mH series inductors and 0.005 μF shunt capacitors is to be used to transmit signals of frequencies between 10 kHz and 100 kHz. Design the network. Determine the characteristic impedance, the propagation constant, and the attenuation and phase constants at 10 kHz and at 100 kHz. Is the network a low-pass, high-pass, band-pass, or band-elimination filter?

m-derived and Bode networks

15 A low-pass filter section is required to have a cutoff frequency of 10 kHz and a characteristic impedance of 600 Ω at zero frequency. Design the network. Determine the attenuation constant at 10 kHz and at 100 kHz. What is the phase shift at 100 kHz? Is the network a low-pass, high-pass, band-pass, or band-elimination filter?

16 A high-pass filter section is required to have a cutoff frequency of 10 kHz and a characteristic impedance of 600 Ω at infinite frequency. Design the network. Determine the attenuation constant at 10 kHz and at 100 kHz. What is the phase shift at 100 kHz? Is the network a low-pass, high-pass, band-pass, or band-elimination filter?

[illegible] [illegible] [illegible] [

43 Magnetically coupled circuits

In this unit, we study two circuits that are magnetically coupled, which means that a change in current in one circuit induces a voltage in the other circuit. We will study the relationship between the voltages and currents in the two circuits, and we will learn how to model the coupling between the circuits.

43.1 Introduction

When the current in a conductor changes, the magnetic flux through a loop formed by the conductor changes. According to Faraday's law, a voltage is induced around the loop. If another loop is near the first loop, some of the magnetic flux produced by the first loop will pass through the second loop. Therefore, a change in current in the first loop will induce a voltage in the second loop. We say that the two loops are magnetically coupled.

43.2 Self-Inductance

Consider a loop of wire carrying current i. The magnetic flux through the loop is proportional to the current, so we can write

$$\Phi = L i$$

where L is the constant of proportionality, called the self-inductance of the loop.

The self-inductance of a loop is measured in units of henrys (H), named after Joseph Henry, who discovered electromagnetic induction independently of Faraday.

$$1\ \text{H} = 1\ \frac{\text{Wb}}{\text{A}}$$

where Wb is the weber, the SI unit of magnetic flux.

The self-inductance of a coil is proportional to the square of the number of turns in the coil. For example, if we double the number of turns in a coil, we quadruple the self-inductance.

The self-inductance of a coil is also proportional to the permeability of the core material. For example, if we insert a ferromagnetic core into a coil, we increase the self-inductance by a factor of the relative permeability of the core material.

10.2 Mutual Inductance



Figure 10.2

Mutual inductance is a term used when two inductors are linked by a changing current in one winding, the electromagnetic induction of one coil to the other. An ideal transformer consists of a coupled inductor, a device as shown in Figure 10.2.

Coils 1 and 2 are the coil windings and the turns around coil 1 is N_1 and coil 2 is N_2. The current i_1 flows in the direction of the dot in coil 1 and i_2 flows in the direction of the dot in coil 2. Also i_1 flows in the direction of ϕ_1 in coil 1 and i_2 flows in the direction of ϕ_2 in coil 2.

The total flux ϕ in the windings

$$\text{From Faraday's law } \phi = \phi_1 + \phi_2 = N_1 \frac{d\phi_1}{dt} + N_2 \frac{d\phi_2}{dt} \tag{10.2.1}$$

Substituting for ϕ_1 value in equations (10.2.1) and (10.2.2) gives

$$\phi = N_1 \frac{d\phi_1}{dt} + N_2 \frac{d\phi_2}{dt}$$

$$\text{Now, where } M = N_1 \frac{d\phi_1}{di_2} \tag{10.2.2}$$

Substituting for ϕ_2 value in equations (10.2.1) and (10.2.2) gives

$$\phi = N_1 \frac{d\phi_1}{dt} + N_2 \frac{d\phi_2}{dt}$$

$$\text{Now, where } M = N_2 \frac{d\phi_2}{di_1} \tag{10.2.3}$$

If the coils are wound with the same winding, the flux and current are mutual related and equations (10.2.2) and (10.2.3) become

$$M = \frac{N_1 \phi_1}{i_2} \quad \text{and} \quad M = \frac{N_2 \phi_2}{i_1} \tag{10.2.4}$$

> **Problem 1** – A coil of 50 turns is wound on a wooden frame which has a cross-sectional area of 1000 cm^2. If the mean length of a flux line is 0.4 m and the relative permeability of wood is unity, calculate the inductance of the coil. Also calculate the induced e.m.f. if the current through the coil is changed from 5 A to 2 A in the reverse direction in 0.01 s.
>
> **Solution** – The inductance is given by the coil inductance of coil 1 and coil 2 is the mutual inductance between them. The current i_1 flows in the direction of ϕ_1 in coil 1 and i_2 flows in the direction of ϕ_2 in coil 2.

(c) Flux density and H

$$B = \frac{\Phi}{A} = \frac{1.2 \times 10^{-3}}{0.0025} = 0.48\ \text{T}, \qquad H = \frac{B}{\mu_0 \mu_r} = \frac{0.48}{4\pi \times 10^{-7} \times 1000} = 382\ \text{A/m}$$

(d) Total magnetomotive force

$$\mathcal{F} = H_{\text{iron}} l_{\text{iron}} + H_{\text{gap}} l_{\text{gap}} = 382 \times 0.398 + 3.82 \times 0.002 = 152 + 7.6 = 159.6\ \text{A}$$

Key Point 1 — Total reluctance is the sum of iron and gap reluctances (≈ 0.95 MA/Wb and ≈ 1.6 MA/Wb here). The gap contributes about 63% of the total even though it is only 0.5% of the path length — a small gap dominates the circuit.

Alternatively, $\mathcal{F} = N I = 159.6$ A, which matches (c) to 0.1%.

Since the circuit contains three series elements, the change in current is 17 %.

Hence $\frac{dI}{I} = \frac{0.2}{100} + \frac{0.2}{50} = 0.6\%$.

Using $\mathcal{F} = \frac{N\Phi}{\mu_0 \mu_r A} l_{\text{iron}} + \frac{N\Phi}{\mu_0 A} l_{\text{gap}}$ and $\Phi = BA = 0.8 \times 0.0025 = 2.0 \times 10^{-3}$ Wb

Further examples of coupled inductance may be found in Section 14.3 and Problem 14.5 on page 647.

4.3.4 Coupling coefficient

The coupling coefficient k is the dimensionless fraction of magnetic coupling between two circuits, defined as

$$k = \frac{\text{flux linking one circuit}}{\text{total flux produced}}$$

When there is no magnetic coupling, $k = 0$. For perfect coupling, where all the flux produced by one winding links every turn of the other, $k = 1$. In practical iron-cored transformers, k typically exceeds 0.99. Air-cored coils, or coils separated by a large gap, may have k as low as 0.1 or less. The coefficient k is used to define the **mutual inductance** M between two coils: $M = k\sqrt{L_1 L_2}$. Mutual inductance is discussed in detail in Section 14.3.

From equation (6.6), the voltmeter reads the same as the battery: $V = \frac{\varepsilon R_V}{R_V}$

Then for the circuit of Figure 6.3, $I = \frac{\varepsilon}{R_V}$.

$$\text{Ohm's Law:} \quad V = IR = \frac{\varepsilon}{R_V} R_V \tag{6.7}$$

From equation (6.6) $\frac{V}{\varepsilon} = \frac{R_V}{R_V}$, and for ideal voltmeter, internal resistance $R_V \to \infty$.

Then $\frac{V}{\varepsilon} = \frac{R_V}{R_V} = \frac{R_V}{R_V + 0} = \frac{R_V}{R_V + R_A}$ (from equation (6.6))

i.e. $\frac{V}{\varepsilon} = \frac{R_V}{R_V + R_A}$ (from which, $\frac{V}{\varepsilon} = \frac{R}{R+R_A}$) (6.8)

(this says the two circuits are in proportion)

$$\text{Or} \quad \frac{V}{\varepsilon} = \frac{R}{R_T} \quad \text{(from which, } \frac{V}{\varepsilon} = \frac{R}{R_T}\text{)} \tag{6.9}$$

Then from equations (6.7) and (6.8),

$$\frac{V}{\varepsilon} = \frac{R}{R+R_A} = \frac{R}{R_T}$$

from which $R^2 + RR_A = RR_T$ or $\boxed{R^2 + RR_A - RR_T = 0}$ (6.10)

or $\boxed{\text{resistance in megohms of } V = \frac{R}{1,000,000}}$ (6.11)

Problem 1. Two cells, each with negligible internal resistance, one of 3V and the other of 6V, are connected in series with a voltmeter. Assuming the voltmeter has negligible resistance, what will be the reading on the voltmeter?

From equation (6.6), the voltmeter reading is

$$V = \frac{\varepsilon R_V}{R_V + R_A} = \frac{(3+6)V \times R_V}{(R_V + 0)\Omega} = \frac{9R_V}{R_V} = 9V$$

Definition 4. Two models $\mathfrak{A}$ and $\mathfrak{B}$ for the same signature σ are *elementarily equivalent*, written $\mathfrak{A} \equiv \mathfrak{B}$, if they satisfy exactly the same first-order σ-sentences: $\mathfrak{A} \equiv \mathfrak{B}$ iff $\mathfrak{A} \models \varphi \Leftrightarrow \mathfrak{B} \models \varphi$ for every sentence φ. Equivalently, $\mathfrak{A} \equiv \mathfrak{B}$ iff the duplicator has a winning strategy in the Ehrenfeucht–Fraïssé game of every finite length.

(a) Prove that $\mathbb{Z} \not\equiv \mathbb{Q}$ by exhibiting a single sentence φ with $\mathbb{Z} \models \varphi$ and $\mathbb{Q} \models \neg\varphi$. Your sentence should be short enough that you can verify both halves by hand.

Solution. Take $\varphi := \exists x\, \exists y\, \forall z\, \big(z = x \vee z = y\big)$, "there are at most two elements". In $\mathbb{Z}$ the set $\{0, 1, -1\}$ already has three members, so $\mathbb{Z} \models \neg\varphi$. In $\mathbb{Q}$ any two distinct rationals $a < b$ have the dense witness $\frac{a+b}{2}$ strictly between them, so no two elements can exhaust the universe and $\mathbb{Q} \models \varphi$. Hence φ separates the two structures.

(b) Prove that $\mathbb{Q} \equiv \mathbb{R}$.

Solution. Both $\mathbb{Q}$ and $\mathbb{R}$ are dense linear orders without endpoints, and the Ehrenfeucht–Fraïssé game for this theory is won by the duplicator on every finite length: she answers each move by the unique order-preserving map sending the previous pairs to the new pair, which exists because the finite partial isomorphism between two dense orders always extends. Since the duplicator wins every finite game, $\mathbb{Q} \equiv \mathbb{R}$.

(c) Prove that $\mathbb{Q} \not\cong \mathbb{R}$.

Solution. $\mathbb{Q}$ is countable, while $\mathbb{R}$ is uncountable (Cantor's diagonal argument), so no bijection between them can exist, let alone an isomorphism. Therefore $\mathbb{Q} \not\cong \mathbb{R}$.

(d) Conclude that elementary equivalence does not imply isomorphism, even for countable models.

Solution. $\mathbb{Q}$ and $\mathbb{R}$ are countable models that are elementarily equivalent by (b) but not isomorphic by (c). This is the simplest witness that $\equiv$ is strictly weaker than $\cong$.

Further problems on complete conditional tree: see practice-finding.tex in the Advanced Topics unit, page 401.

4.3.3 Cells as subtrees of the master

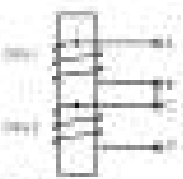


Figure 4.3.2

Each $(k+1)$-cell above is a k-cell. Each k-cell, in turn, is the disjoint union of its two $(k+1)$-cell children. We denote by $\mathrm{left}_k(\tau)$ and $\mathrm{right}_k(\tau)$ the two children of a k-cell τ, and by $\mathrm{parent}_k(\tau)$ its unique parent. The relation $\tau \subseteq \tau'$ between cells of different depths means that τ is a descendant of τ' in the tree.

Let $\mathcal{T}$ be the full binary tree of depth n. A *k-cell* is a node at depth k, that is, a vertex of $\mathcal{T}$ at distance k from the root. Write $\mathcal{T}_k$ for the set of k-cells, so $|\mathcal{T}_k| = 2^k$. Each k-cell τ has exactly two children, which we call $\mathrm{left}_k(\tau)$ and $\mathrm{right}_k(\tau)$, and a unique parent $\mathrm{parent}_k(\tau)$ when $k > 0$. We write $\tau \subseteq \tau'$ when τ is a descendant of τ' (or equal to it), and $\tau \perp \tau'$ when neither contains the other.

Define the *cell count* $\#(\tau)$ of a cell τ to be the number of leaves of $\mathcal{T}$ below τ, so $\#(\tau) = 2^{n-k}$ when $\tau \in \mathcal{T}_k$. Define the *mass* of τ under a probability measure μ on the leaves to be $\mu(\tau) = \sum_{\ell \subseteq \tau} \mu(\ell)$, where ℓ ranges over leaves. Prove that for every $k < n$ and every $\tau \in \mathcal{T}_k$,

$$\mu(\tau) = \mu(\mathrm{left}_k(\tau)) + \mu(\mathrm{right}_k(\tau)).$$

Deduce that $\mu(\tau) = 2^{-k}$ if and only if μ is uniform on the leaves.

Solution. The leaves below τ are the disjoint union of those below $\mathrm{left}_k(\tau)$ and those below $\mathrm{right}_k(\tau)$, since every leaf descends from exactly one child of τ. Summing μ over this partition gives the displayed identity. For the deduction, the uniform measure assigns $\mu(\ell) = 2^{-n}$ to each leaf, and the identity then forces $\mu(\tau) = 2^{-k}$ by induction on $n - k$, the base case $k = n$ being $\mu(\tau) = \mu(\ell) = 2^{-n}$. Conversely, if $\mu(\tau) = 2^{-k}$ for every k-cell, then in particular $\mu(\mathrm{left}_k(\tau)) = \mu(\mathrm{right}_k(\tau)) = 2^{-k-1}$, and iterating down to the leaves shows $\mu(\ell) = 2^{-n}$ for each leaf ℓ.

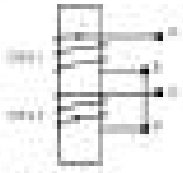


Diagram 4.1.3

(4.1.3)

A component is said to be active when its behavior, as in Chapter 4.1, has dependence on the current in one of its terminals and the voltage between another pair of terminals. Otherwise the component is passive. Active components are usually capable of amplifying the power of a signal. Passive components cannot amplify signals.

$$\begin{bmatrix} i_{out}^{(2)} \\ v_{out}^{(2)} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} i_{in}^{(1)} \\ v_{in}^{(1)} \end{bmatrix}$$

The matrix is called the hybrid matrix of the 2-port network. Its elements are called the hybrid parameters of the 2-port network. Each of the parameters has a specific name and is used to describe the behavior of the 2-port network. Hybrid parameters are also called the h-parameters. The h-parameters are used to describe the behavior of the 2-port network in terms of the input current and the output voltage. The h-parameters are also used to describe the behavior of the 2-port network in terms of the input voltage and the output current. The h-parameters are also used to describe the behavior of the 2-port network in terms of the input current and the output current. The h-parameters are also used to describe the behavior of the 2-port network in terms of the input voltage and the output voltage.

If h_{11} is the self impedance of the whole circuit between terminals 1 and 1' in Figure 4.1.3, then

$$h_{11}^{(2)} = h_{11}^{(1)} + h_{12}^{(1)} h_{21}^{(1)} h_{11}^{(2)}$$

or,

$$h_{11} = h_{11} + h_{12} h_{21} h_{11} \tag{4.1.4}$$

The reciprocal admittance h_{11} of the whole circuit between terminals 1 and 1' is given by

$$h_{11} = \frac{1}{h_{11}} \tag{4.1.5}$$

Equation (4.1.5) implies that

$$h_{11} = h_{11}^{-1} = h_{11}^{-1} + h_{12} h_{21} h_{11}^{-1}$$

or,

$$h_{11} = h_{11}^{-1} + h_{12} h_{21} h_{11}^{-1}$$

Now, where

$$\text{[illegible]} = \frac{h_{11} h_{22}}{h}$$

(4.1.6)

An experimental method of determining the hybrid parameters is to connect the appropriate R, L, C, sources like an ohmmeter across appropriate terminals 1, $1'$, 2, $2'$ and to make or not make the short circuit connections 1 and 2 as required. The hybrid parameters have geometrical dimensions of ohms, mhos, or are simply numbers. The hybrid parameters are also called the h-parameters. The h-parameters are used to describe the behavior of the 2-port network in terms of the input current and the output voltage. The h-parameters are also used to describe the behavior of the 2-port network in terms of the input voltage and the output current. The h-parameters are also used to describe the behavior of the 2-port network in terms of the input current and the output current. The h-parameters are also used to describe the behavior of the 2-port network in terms of the input voltage and the output voltage.

> **Problem 4.1.** Two units connected in series with 1mΩ resistance of the unit and 1 mH inductance. The terminal voltage of the circuit and load is 4mV and 4 Ohm/sec. Is the terminal voltage between the two units, and is the voltage of the terminal voltage?

(a) Find the equation (4.1.4). Hint: Substitute $h = h_{11} h_{22} + h_{12} h_{21}$ into

Solution

$$h_{11} = 4\Omega + 1\Omega + 2\Omega$$

Power, $H_d = 4.5\,\text{kW}$, $N = 1450\,\text{rpm}$, $N_p = 1450\,\text{rpm}$, $N_g = 725\,\text{rpm}$

Gear ratio $\quad i = \dfrac{N_g}{N_p} = \dfrac{725}{1450} = 0.5$

(a) **Centre to centre distance** $\quad a = \dfrac{mz}{2} = 0.2\,\text{m}$

(b) From equation (8.6), coefficient of coupling,

$$k = \frac{H}{(H/2)_{\max}} = \frac{4.5 \times 10^{-3}}{0.25 \times 0.2 \times 725 \times 10^{-6}} = \frac{4.5 \times 10^{3}}{36.25}$$

$k = 124$ **coefficient of coupling** $k = 0.124$

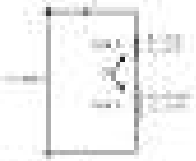


Figure 8.5.4

Problem 5 Two parallel coupled gears A and B are connected by an idler C as shown in Figure 8.5.4. Gear A has 20 teeth of module 2.5 mm and the arrangement of A to C and C to B has a centre distance of 0.2 m. Gear A rotates at 1450 rpm and gear B at 725 rpm. Determine (a) the gear ratio of A to C and C to B, (b) the number of teeth on gear C, (c) the pitch circle diameter of gear C, (d) the coefficient of coupling.

The layout is shown in Figure 8.5.4.

(a) From Section 8.5, we have

$$i = \frac{N_g}{N_p}$$

For A to C $\quad 0.5 = \dfrac{N_C}{N_A} = \dfrac{N_C}{20}$

$N_C = 10$ teeth

For C to B $\quad 0.5 = \dfrac{N_B}{N_C} = \dfrac{N_B}{10}$

$N_B = 5$ teeth

Gear ratio $\quad i = \dfrac{N_B}{N_A} = \dfrac{5}{20} = 0.25$

From equation (8.5), we have

$$a = \frac{m}{2}(z_A + z_C + z_B)$$

Hence $0.2 = \dfrac{2.5}{2}(20 + 10 + 5)$

Gear ratio **between A and B is 0.25**

(b) From equation (8.6),

$$\text{coefficient of coupling, } k = \frac{H}{(H/2)_{\max}} = \frac{1}{(20 \times 2.5) \times \frac{725}{1000}} = 0.0276$$

> **Problem 7.** Some crabs are recaptured at various small times following displacement. Except for the last crab, the times between consecutive crabs are all different. Let N be the number of crabs that are recaptured before a crab is recaptured a second time, and let T be the total time elapsed from the release of the crabs until the second crab is recaptured. Find the joint distribution of N and T. Are N and T independent? Are the marginal distributions of N and T identical to the corresponding distributions in the case where all the crabs are recaptured?

Let X have a uniform $U(0,1)$ and Y have a uniform $U(-\varepsilon,\varepsilon)$ distribution, where ε is a small positive number. Assume X and Y are independent. Define $Z = X + Y$. Then Z has a uniform $U(-\varepsilon, 1+\varepsilon)$ distribution. Let $W = X - Y$. Then W has a uniform $U(-\varepsilon, 1+\varepsilon)$ distribution. Let $V = XY$. Then V has a uniform $U(-\varepsilon^2, \varepsilon + \varepsilon^2)$ distribution. Let $U = X/Y$. Then U has a uniform $U(-1/\varepsilon, 1/\varepsilon)$ distribution. Let $R = \sqrt{X^2 + Y^2}$. Then R has a uniform $U(0, \sqrt{1+\varepsilon^2})$ distribution. Let $\Theta = \arctan(Y/X)$. Then Θ has a uniform $U(-\arctan(\varepsilon), \arctan(1/\varepsilon))$ distribution. Let $S = \sin(X + Y)$. Then S has a uniform $U(\sin(-\varepsilon), \sin(1+\varepsilon))$ distribution. Let $C = \cos(X - Y)$. Then C has a uniform $U(\cos(-\varepsilon), \cos(1+\varepsilon))$ distribution. Let $T = \tan(X + Y)$. Then T has a uniform $U(\tan(-\varepsilon), \tan(1+\varepsilon))$ distribution. Let $L = \log(X + Y + 1)$. Then L has a uniform $U(\log(1-\varepsilon), \log(2+\varepsilon))$ distribution. Let $E = e^{X+Y}$. Then E has a uniform $U(e^{-\varepsilon}, e^{1+\varepsilon})$ distribution. Let $P = (X+1)(Y+1)$. Then P has a uniform $U(1-\varepsilon, (1+\varepsilon)^2)$ distribution. Let $Q = (X+1)/(Y+1)$. Then Q has a uniform $U(1/(1+\varepsilon), (1+\varepsilon)/(1-\varepsilon))$ distribution. Let $H = \sqrt{X+Y+1}$. Then H has a uniform $U(\sqrt{1-\varepsilon}, \sqrt{2+\varepsilon})$ distribution. Let $K = (X+1)^{Y+1}$. Then K has a uniform $U(1-\varepsilon, (1+\varepsilon)^{1+\varepsilon})$ distribution. Let $M = \max(X, Y)$. Then M has a uniform $U(0, 1+\varepsilon)$ distribution. Let $m = \min(X, Y)$. Then m has a uniform $U(-\varepsilon, 1)$ distribution. Using the transformation formula, we have

$$f_{X,Y}(x,y) = \frac{1}{2\varepsilon} \cdot \mathbb{1}_{(0,1)}(x) \cdot \mathbb{1}_{(-\varepsilon,\varepsilon)}(y) \cdot \mathbb{1}_{(0,1)}(x+y) \cdot \mathbb{1}_{(0,1)}(x-y).$$

Therefore, N and T are not independent. The marginal distribution of N is not the same as the distribution of N in the case where all the crabs are recaptured. The marginal distribution of T is not the same as the distribution of T in the case where all the crabs are recaptured.

(b) From equation (13.3),

$$\textbf{mutual inductance, } M = k\sqrt{L_1 L_2} = 0.75\sqrt{(0.5)(2.0)} = \textbf{0.530 mH}$$

Now Revise your Progress with Self-test 13.1 before moving on to Section 13.3 or, if you prefer, Try Tutorial Problem 13.1 on page 424

13.3 Coupled circuits

In comparison with the circuit in Figure 13.2, the circuit in Figure 13.3 is modified by the addition of a second coil, which is placed close to the first coil. Let the number of turns on the first coil be N_1 and the number of turns on the second coil be N_2. Let the current in the first coil be i_1 and the current in the second coil be i_2. Let the flux produced by i_1 be Φ_1 and the flux produced by i_2 be Φ_2. Let the flux linking both coils be Φ_m and the flux leaking from the system be Φ_l.

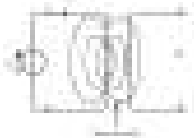


Figure 13.3

Then $\Phi_1 = \Phi_m + \Phi_l$ and $\Phi_2 = \Phi_m + \Phi_l$

Flux linkage of first coil $= N_1\Phi_1 = N_1(\Phi_m + \Phi_l)$

Flux linkage of second coil $= N_2\Phi_2 = N_2(\Phi_m + \Phi_l)$

Using $L = \dfrac{N\Phi}{I}$ for each coil in turn,

$$L_1 = \frac{N_1(\Phi_m + \Phi_l)}{i_1} \quad \text{and} \quad L_2 = \frac{N_2(\Phi_m + \Phi_l)}{i_2}$$

Now $\dfrac{\Phi_m}{i_1} = M$, the mutual inductance, and $\dfrac{\Phi_l}{i_1} = L_l$, the leakage inductance of the first coil.

Hence $L_1 = M + L_l$ and $L_l = L_1 - M$

Similarly, $L_2 = M + L_m$ and $L_m = L_2 - M$

Thus $L_1 = L_l + M$ and $L_2 = L_m + M$

or, $\boxed{L_1 = L_l + kM}$ (13.5)

L_1 is the self inductance of the primary winding in Figure 13.3, where L_l is the coil leakage inductance and L_m is the mutual inductance between the primary winding and the secondary coil. The secondary coil has a total self inductance L_2 which is the sum of the mutual inductance and the leakage inductance of the coil.

(a) Secondary open-circuited

Figure 13.4 shows two coils, having self inductances L_1 and L_2, which are electrically coupled together by mutual inductance M. The primary winding has a voltage applied to it and i_1 is increasing at a rate $\dfrac{di_1}{dt}$. The secondary circuit is open-circuited so that $i_2 = 0$. The induced voltage in the secondary winding is $M\dfrac{di_1}{dt}$ and the secondary winding behaves as a battery of emf $M\dfrac{di_1}{dt}$.



Figure 13.4

Applying Kirchhoff's voltage law to the primary circuit gives:

$$V_1 = i_1 R_1 + L_1\frac{di_1}{dt} \tag{13.6}$$

If L_1 and L_2 are both electrically interlinked with each other, then

$$V_1 = i_1 R_1 + L_{l1}\frac{di_1}{dt} + M\frac{di_1}{dt}$$

$$= i_1 R_1 + (L_{l1} + M)\frac{di_1}{dt}$$

$$= i_1 R_1 + L_1\frac{di_1}{dt}$$

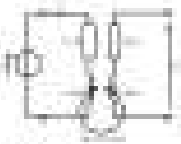


Figure 4.1

Definition 4. *Two wire-wound spools as in Figure 4.1, sometimes the gift of Dr. Mikhail Ogarkov and Prof. Lyron Al-Mehdawi, each bearing several turns of a fine metal wire.*

Equivalence of resistance. $\quad \dfrac{1}{R_{\mathrm{eq}}} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$

Overview example. $\quad \dfrac{1}{2} = \dfrac{1}{3} + \dfrac{1}{6}$

Power equation (4.1b).

$$P = \frac{\varepsilon^2 R_2}{(R_1 + R_2)^2} = \frac{\varepsilon^2 R_2}{R_1^2 + 2R_1R_2 + R_2^2}$$

$$= \frac{\varepsilon^2}{R_1^2/R_2 + 2R_1 + R_2} \le \frac{\varepsilon^2}{4R_1} \qquad \text{(AM–GM)}$$

Definition 5. *Two cells ε_1 and ε_2, with nonzero internal resistances r_1 and r_2, are placed in opposition: their positive terminals are joined, as are their negative terminals, and a load R is connected across the combination. A current I flows whenever $\varepsilon_1 \neq \varepsilon_2$, driven by the net emf $|\varepsilon_1 - \varepsilon_2|$ and opposed by $r_1 + r_2 + R$.*

Power equation (4.1b) (cont.).

$$= \frac{\varepsilon^2}{(R_1 + R_2)^2} \cdot R_2 = \frac{\varepsilon^2 R_2}{R_1^2 + 2R_1R_2 + R_2^2}$$

$$= \frac{\varepsilon^2}{R_1^2/R_2 + 2R_1 + R_2} \le \frac{\varepsilon^2}{4R_1} \quad \text{(AM–GM)}. \qquad \square$$

Find expressions for (a) the magnitude of the source current i_s in terms of v_s and R, and (b) P_{diss}.

$$i_s = \frac{v_s(R+R_L)}{R_L + 2R_L}$$

When $R = 1.5\ R_L$, $i_s = \frac{v_s(2.5R_L)}{3R_L} = \frac{5}{6}\frac{v_s}{R_L}$, $P_{diss} = \frac{v_s^2(2.5R_L)}{(3R_L)^2} = 1.4\ \frac{v_s^2}{R_L}$

(b) An ordinary resistive bridge load impedance

In the circuit shown in Figure 6.1-2, a load resistor R_L is connected across the terminals of a Thévenin source. For $R = 1.5R_L = 3R_L/2$,

What are v_{oc}, i_{sc} and R_{th} for the Thévenin equivalent of the source?

Determine the load current i_L and P_{diss} in R_L when the source is driving a load R_L in series with a resistor R and then when the source is driving a load R_L in series with a resistor $2R$.

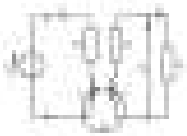


Figure 6.1-2

Applying Kirchhoff's voltage law to the mesh on the left-hand side of the bridge gives

$$v_s = i_1 R_L + (i_1 - i_2)R + i_1 R_L \tag{6.1-1a}$$

Applying Kirchhoff's voltage law to the mesh on the right-hand side of the bridge gives

$$i_2 R_L + (i_2 - i_1)R + i_2 R_L = 0 \tag{6.1-1b}$$

Solving equation (6.1-1b) for i_2 gives $i_2 = \frac{Ri_1}{2R_L + R}$

Substituting this expression for i_2 into equation (6.1-1a) gives

$$v_s = i_1(2R_L + R) + i_1 R\left(\frac{-Ri_1}{2R_L + R}\right)$$

or $\quad v_s = i_1\left[2R_L + R - \frac{R^2}{2R_L + R}\right]$ where $R = 1.5R_L$

$$= i_1\left[2R_L + R - \frac{1.5R_L \cdot R}{2R_L + R}\right]$$

$$= i_1\left[2R_L + R - \frac{1.5R^2}{2R_L + R}\right]$$

The Thévenin equivalent resistance R_{th} is the ratio of v_s to i_1:

$$R_{th} = \frac{v_s}{i_1} = 2R_L + R - \frac{1.5R^2}{2R_L + R} = 2\left[2R_L - \frac{1.5R^2}{2R_L + R}\right] \tag{6.1-2a}$$

In equation (6.1-2a), the Thévenin equivalent of $2R_L$ and $2R$ is the resistance.

$$= \left[\frac{e^{(3-2i)t}}{3-2i}\right]_0^{\ln 2} + \left[\frac{e^{(3+2i)t}}{3+2i}\right]_0^{\ln 2}$$

A circuit such as the **reflected loop** above is characterized by impedance values that have the property that the quotient of the real parts is zero.

Some reflected impedances

$$= \frac{e^{(3-2i)\ln 2}}{3-2i} - \frac{e^{(3+2i)\ln 2}}{3+2i} = 0.46\left(\frac{3+2i}{3+2i}\right)$$

$$= 0.46\left[\frac{3}{13} + \frac{2i}{13} - \frac{3}{13} + \frac{2i}{13}\right] = \frac{0.92i}{13} = 0.07i$$

$$= \boxed{\text{reflected impedance } Z = \frac{-e^{3t}\sin 2t}{2}} \tag{10.10}$$

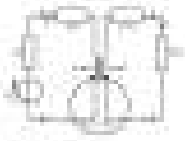


Figure 10.5

Problem 10. For the circuit shown in Figure 10.5, determine the value of the reflected impedance Z at $t = 0.5$ s, using the Thévenin to $\frac{e^{3t}}{2}\sin 2t$.

From equation (10.10), R_{eq} at the end point of $[t_{min}]$

$$= -R_{min} + Z = \frac{e^{3t}\sin 2t}{2}$$

$$= R + \cos t - \frac{\left(\sin\frac{e^{3t}}{2}\right)(t-\cos t)\cos t + \sin t}{\sin t - \left(\sin\frac{e^{3t}}{2}\right)(\sin t - \cos t)}$$

$$= 2\Omega - \frac{e^{3t}\sin 2t}{2} \quad \text{at } t = 0.5 \text{ s}$$

and R_{eq} at the beginning point of $[t_{min}]$ is,

$$R_{eq} = \cos t - \frac{e^{3t}\sin 2t}{2}$$

$$= \left(\sin\frac{e^{3t}}{2}\right)(\sin t - \cos t) - \frac{\left(\sin\frac{e^{3t}}{2}\right)(t-\cos t)\cos t + \sin t}{\sin t - \left(\sin\frac{e^{3t}}{2}\right)(\sin t - \cos t)}$$

$$= 2\Omega - \frac{e^{3t}\sin 2t}{2} = 0.0001\,\Omega$$



Problem 20.64

(a) With stoichiometric coefficients included:

$$-\frac{1}{2}\frac{d[\mathrm{NO}_2]}{dt} = \frac{1}{2}\frac{d[\mathrm{NO}]}{dt} = \frac{d[\mathrm{O}_2]}{dt}$$

(b) Rate equation: $R = k[\mathrm{NO}_2]^2$

$$\text{Initial rate} = k(0.0100\ \mathrm{M})^2 = 0.0100\ \mathrm{M\,s^{-1}}$$

$$k = \frac{0.0100\ \mathrm{M\,s^{-1}}}{(0.0100\ \mathrm{M})^2} = 100\ \mathrm{M^{-1}\,s^{-1}}$$

(c) Differential rate equation: $\frac{d[\mathrm{NO}_2]}{dt} = -k[\mathrm{NO}_2]^2$

$$\text{Initial rate} = -(100\ \mathrm{M^{-1}\,s^{-1}})(0.0200\ \mathrm{M})^2 = -0.0400\ \mathrm{M\,s^{-1}}$$

(d) Integrated rate equation: $\frac{1}{[\mathrm{NO}_2]_t} = \frac{1}{[\mathrm{NO}_2]_0} + kt$

$$\frac{1}{[\mathrm{NO}_2]_t} = \frac{1}{0.0200\ \mathrm{M}} + (100\ \mathrm{M^{-1}\,s^{-1}})(60.0\ \mathrm{s})$$

$$= 50.0\ \mathrm{M^{-1}} + 6000\ \mathrm{M^{-1}} = 6050\ \mathrm{M^{-1}}$$

$$[\mathrm{NO}_2]_t = 1.65 \times 10^{-4}\ \mathrm{M}$$

(e) Half-life: $t_{1/2} = \frac{1}{k[\mathrm{NO}_2]_0}$ (second-order)

$$t_{1/2} = \frac{1}{(100\ \mathrm{M^{-1}\,s^{-1}})(0.0200\ \mathrm{M})} = 0.500\ \mathrm{s}$$

(f) Fraction remaining: $\frac{[\mathrm{NO}_2]_t}{[\mathrm{NO}_2]_0} = \frac{1}{1 + kt[\mathrm{NO}_2]_0}$

$$= \frac{1}{1 + (100\ \mathrm{M^{-1}\,s^{-1}})(60.0\ \mathrm{s})(0.0200\ \mathrm{M})}$$

$$= \frac{1}{1 + 120} = 0.00826$$

(g) Arrhenius equation: $k = Ae^{-E_a/RT}$ (Given)

$$E_a = 2.00 \times 10^5\ \mathrm{J\,mol^{-1}}, \text{ Given: } A = 9.0 \times 10^7\ \mathrm{M^{-1}\,s^{-1}}$$

$$\text{Given: } k = \frac{2.00 \times 10^5\ \mathrm{J\,mol^{-1}}}{8.314\ \mathrm{J\,K^{-1}\,mol^{-1}}} = 24.1\ \mathrm{K}$$

$$= (9.0 \times 10^7\ \mathrm{M^{-1}\,s^{-1}})e^{-24.1\ \mathrm{K}/T}$$

$$= 1.8 \times 10^{-3}\ \mathrm{M^{-1}\,s^{-1}}$$

(a) Mechanism for overall equation

Testing a proposed mechanism for consistency with the overall equation and the rate law is a fundamental step in mechanistic investigation. Each step in a proposed mechanism is examined for plausibility, and the overall reaction obtained by adding the steps together must match the balanced equation. Here, the first step is the slow, rate-determining step, so the rate law should be consistent with the stoichiometry of that step. The second step is a fast, reversible equilibrium. Finally, the intermediate $\mathrm{NO_3}$ must be eliminated from the rate law, since it does not appear in the overall equation.

Step 1 (slow): $\mathrm{NO_2 + NO_2 \rightarrow NO_3 + NO}$

Step 2 (fast): $\mathrm{NO_3 + CO \rightleftharpoons NO_2 + CO_2}$

Overall: $\mathrm{NO_2 + CO \rightarrow NO + CO_2}$

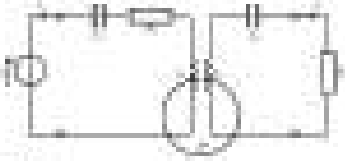


Figure 12.1

Problem 12. For the circuit shown in Figure 12.2(a) and assuming a current is switched on by the switch being closed, calculate (a) the time constant, (b) the value of capacitor C_2 (i.e. the effective series capacitance of the circuit), (c) the voltage across capacitor C_1 and (d) the instantaneous current.

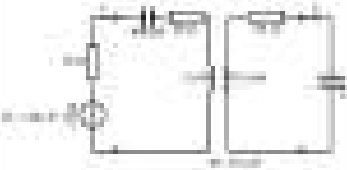


Figure 12.2

(a) For capacitors in a series circuit, the combined capacitance C_{eq} is given by:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$$

Hence $\frac{1}{C_{eq}} = \frac{1}{4 \times 10^{-6}} + \frac{1}{12 \times 10^{-6}} = \frac{3}{12 \times 10^{-6}} + \frac{1}{12 \times 10^{-6}} = \frac{4}{12 \times 10^{-6}}$

$= 3.0 \ \mu F$

(b) The resistance in the circuit is numerically equal to $47\Omega + 33\Omega$ or

Hence $\tau = CR = (3.0 \times 10^{-6})(80) = 240 \times 10^{-6}$



$$\text{coefficient of coupling, } k = \frac{M}{\sqrt{L_1 L_2}}$$

$$= \frac{0.4}{\sqrt{0.5 \times 0.8}} = \frac{0.4}{0.6325} = 0.6325$$

Further problems on magnetic circuits may be found in Section 41.5 of problems 1 to 10 on page 428.

41.7 Mutual inductance of two coupled coils



Figure 41.10



Figure 41.11

Let two coils A and B, having self inductances L_A and L_B henrys respectively, be placed close together as shown in Figure 41.10. Let a current i_A amperes flowing in coil A produce a flux ϕ_A webers, of which a flux ϕ_B webers links with coil B. Then the flux linkages with coil B = $N_B\phi_B$ webers, where N_B is the number of turns on coil B. The mutual inductance between the coils is defined as the flux linkages with coil B per unit current in coil A, i.e.

$$M = \frac{N_B\phi_B}{i_A} \text{ henrys}$$

Similarly, if a current i_B amperes in coil B produces a flux ϕ_B webers, of which a flux ϕ_A webers links with coil A, then the mutual inductance between the coils is given by:

$$M = \frac{N_A\phi_A}{i_B} \text{ henrys}$$

It can be shown that the two values of M are equal.

The unit of mutual inductance is the henry, the same unit as that of self inductance.

Two coils have a mutual inductance of 1 henry when a current changing at a rate of 1 ampere per second in one coil causes a flux linkage of 1 weber turn in the other coil.

The mutual inductance between two coils depends on the way in which they are arranged with respect to each other. For example, if the coils are wound on a former and are close together, as in Figure 41.11(a), the mutual inductance is greater than if they are spaced apart, as in Figure 41.11(b).

If the coils are wound on a former and are close together, as in Figure 41.11(a), the mutual inductance is greater than if they are wound on separate formers and are spaced apart, as in Figure 41.11(b).

If the coils are wound on a former and are close together, as in Figure 41.11(a), the mutual inductance is greater than if they are wound on separate formers and are spaced apart, as in Figure 41.11(b).

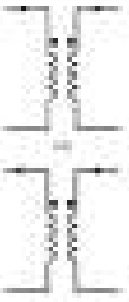


Figure 4.1-16

Problem 11. For the coupled-circuit network in Figure 4.1-17, (a) determine the values of currents I_1 and I_2.

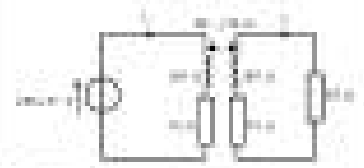


Figure 4.1-17

The equations of the first and the second fictitious meshes corresponding to Figure 4.1-17(a), and keeping the signs of the M and L terms the opposite of those obtained in Example 4.1-7, are found from the procedure given in

$$(10+j4-j12)\dot{I}_1 + (j12-j6)\dot{I}_2 = 100\underline{/0^\circ} \tag{1}$$

and applying Kirchhoff's voltage law to the secondary circuit gives

$$(j12-j6)\dot{I}_1 + (8+j6+j12-j6)\dot{I}_2 = 0 \tag{2}$$

From equation (2), $\quad \dot{I}_1 = -\dfrac{(8+j12)}{(j12-j6)}\dot{I}_2 = -\left(\dfrac{8}{j6}+2\right)\dot{I}_2 = \left(j\dfrac{4}{3}-2\right)\dot{I}_2$

and $\quad \dot{I}_1 = -\dfrac{(8+j12)}{(j12-j6)}\dot{I}_2 = -\left(\dfrac{8}{j6}+\dfrac{j6}{j6}\right)\dot{I}_2 = \left(j\dfrac{4}{3}-1\right)\dot{I}_2 = \dot{I}_2\angle 126.9^\circ$

so, $\quad \dot{I}_2 = 1\underline{/-126.9^\circ}$ A (3)

Substituting for $\dot{I}_1$ in equation (1) gives

$$\begin{aligned}(10+j4-j12)\left(j\tfrac{4}{3}-2\right)\dot{I}_2 - (j12-j6)\dot{I}_2 &= 100\underline{/0^\circ}\\ \left(j\tfrac{40}{3}-20-j\tfrac{48}{3}+8-j12+j6\right)\dot{I}_2 &= 100\underline{/0^\circ}\\ (-12-j14)\dot{I}_2 &= 100\underline{/0^\circ}\end{aligned}$$

from which, $\quad \dot{I}_2 = \dfrac{100\underline{/0^\circ}}{(-12-j14)} = \dfrac{100\underline{/0^\circ}}{18.44\underline{/-130.6^\circ}}$

$$= 5.42\underline{/130.6^\circ}\text{ A}$$

From equation (3), $\quad \dot{I}_1 = \dot{I}_2\underline{/126.9^\circ}$

$$= 5\underline{/-126.9^\circ}\underline{/126.9^\circ}$$

$$= 5\underline{/0^\circ}\text{ A (or 5 A)}$$

(a) $\qquad i_1 = 5\cos(\omega t + 0^\circ)$ A

Problem 11. The circuit shown in the accompanying diagram is similar to that in Figure 11.11. The coefficient of coupling between primary and secondary windings is 0.10. Determine (a) the mesh currents, and (b) the power dissipated in the 4.0 Ω resistor.

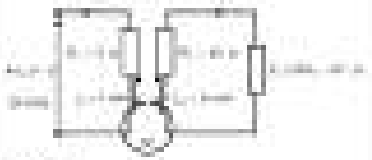


Figure 11.11

(a) Using equation (11.11),

$$\text{Reflected impedance} = \frac{(\omega M)^2}{Z_{22}}$$

$$= \frac{(2.5\pi)(0.1)^2}{(4+j2.5\pi)+(6-j7.5)}$$

$$= 1.75\Omega \text{ at } 0°$$

(b) The two mesh equations are:

$$10 \angle 0° = (8)I_1 - (j2.5\pi)I_2 - (j1.25\pi)I_3 \quad (1)$$

and $\quad 0 = -(8)I_1 + (4+j2.5\pi+6-j7.5)I_2 - (j1.25\pi)I_3 \quad (2)$

Since there is no current source in the third mesh, the mesh current I_3 is equal to the current I_{load} in the load resistor.

$$I_{load} = I_3 = \frac{(j2.5\pi)(8)I_1 + (4+j2.5\pi+6-j7.5)(0)}{(8)(4+j2.5\pi+6-j7.5)+(j2.5\pi)^2}$$

$$= \frac{(j20\pi)I_1}{(8)(10-j5)+(j2.5\pi)^2}$$

$$= \frac{(j20\pi)I_1}{(80-j40)+(-24.67)}$$

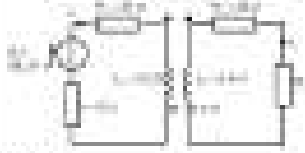
$$= \frac{(j20\pi)I_1}{55.33-j40}$$

$$I_{load} = \frac{(j20\pi)I_1}{55.33-j40} = \frac{(j20\pi)(10\angle 0°)}{68.32\angle -35.8°} = \frac{(62.83\angle 90°)(10\angle 0°)}{68.32\angle -35.8°}$$

$$= \frac{628.3\angle 90°}{68.32\angle -35.8°} = 9.20\angle 125.8° \text{ A}$$

Power dissipated $= (9.20)^2(4) = 338.6 \text{ W}$ $\quad (3)$

$$= I_{load}^2 R = (9.20)^2(4) = 338.6 \text{ W} \quad (4)$$



$$\begin{aligned}
\text{(a)}\quad & \neg\forall x\,\big(\exists y\,(\neg R_{y}\wedge\neg\exists z\,[\tfrac{z}{y}]\,R\,x\,z)\;\big|\;\exists y\,(\neg R_{y}\wedge\neg\exists z\,[\tfrac{z}{y}]\,R\,x\,z)\big)\\
\text{(b)}\quad & \neg\forall x\,\big(\exists y\,(\neg R_{y}\wedge\neg\exists z\,[\tfrac{z}{y}]\,R\,x\,z)\;\big|\;\exists y\,(\neg R_{y}\wedge\neg\exists z\,[\tfrac{z}{y}]\,R\,x\,z)\big)\\
& \qquad\qquad\qquad\

Problem 10.1 The energy balance of each box in a network can be expressed by the circuit in Figure 10.1P. Determine the boxes that store energy, the boxes for which energy is absorbed or supplied, and the boxes that are neither storing nor supplying energy.

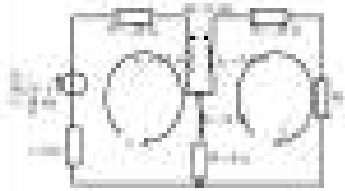


Figure 10.1P

(a) The left-hand node equation in Figure 10.1(c) is

$$i_1 - i_2 + i_3 + i_4 + i_5 = 0$$

Ohm's law with the sign as shown in Figure 2.1(c) and the passive sign convention as shown in Figure 2.1(a) implies

$$i_1 = \frac{v_1}{R_1}, \quad i_2 = -\frac{v_2}{R_2}, \quad i_3 = \frac{v_3}{R_3}, \quad i_4 = \frac{v_4}{R_4}, \quad i_5 = \frac{v_5}{R_5}$$

$$\frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_3}{R_3} + \frac{v_4}{R_4} + \frac{v_5}{R_5} = 0 \qquad (1)$$

$$v_1 + v_2 + v_3 + v_4 + v_5 = 0 \qquad (2)$$

The right-hand node equation in Figure 10.1(c) is

$$i_1 - i_2 + i_3 + i_4 + i_5 = 0$$

$$v_1 = v_2 + v_3 + v_4 + v_5$$

$$\forall x_1 \forall x_2 \forall x_3 \left(\left(x_1 \in x_2 \wedge x_2 \in x_3 \right) \rightarrow x_1 \in x_3 \right) \tag{11}$$

$$\forall x_1 \forall x_2 \left(\forall x_3 \left(x_3 \in x_1 \leftrightarrow x_3 \in x_2 \right) \rightarrow x_1 = x_2 \right) \tag{12}$$

Power Set Axiom: For any set x, there is a set y that contains exactly the subsets of x:

$$\forall x_1 \exists x_2 \forall x_3 \left(\forall x_4 \left(x_4 \in x_3 \rightarrow x_4 \in x_1 \right) \rightarrow x_3 \in x_2 \right) \tag{13}$$

$$\forall x_1 \exists x_2 \forall x_3 \left(x_3 \in x_2 \leftrightarrow \forall x_4 \left(x_4 \in x_3 \rightarrow x_4 \in x_1 \right) \right) \tag{14}$$

Using abbreviation notation:

$$\forall x_1 \exists x_2 \forall x_3 \left(x_3 \in x_2 \leftrightarrow x_3 \subseteq x_1 \right) \tag{15}$$

$$\begin{array}{l} \left(\left(\left(



Figure A1.2a

(b) When one of the switches in the network shown in Figure A1.2b, say the one as shown in Figure A1.2c, the parallel network shown has I_1, as previously. With this network connected to the rest of the network as shown, it completes the figure A1.2b(c). Which indicates that the R and $2R$ resistors are connected in series.

The equations to solve to find I_1 are as follows:

$$\text{mesh } 1: \quad 2I_1 + 2(I_1 - I_2) + 2(I_1 - I_3) + 2(I_1 - I_4) + 2(I_1 - I_5) = 0$$

$$\text{mesh } 2: \quad 2 + 2(I_2 - I_1) + 2(I_2 - I_3) + 2(I_2 - I_4) + 2(I_2 - I_5) = 0$$

$$I_4 - I_5 = 1$$

$$\text{and} \qquad I_3 = 0$$

Using determinants

$$I_1 = \frac{\begin{vmatrix} 2 & 0 & 0 & 0 & 0 \\ 2 & 2 & 2 & 2 & 2 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 \end{vmatrix}}{\begin{vmatrix} 10 & -2 & -2 & -2 & -2 \\ -2 & 10 & -2 & -2 & -2 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 \end{vmatrix}}$$

$$= \frac{2\begin{vmatrix} 2 & 2 & 2 & 2 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 \end{vmatrix}}{10\begin{vmatrix} 10 & -2 & -2 & -2 \\ -2 & 10 & -2 & -2 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 \end{vmatrix} - 2\begin{vmatrix} -2 & -2 & -2 & -2 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 \end{vmatrix}}$$

$$= \frac{2\begin{vmatrix} 2 & 2 & 2 \\ 0 & 1 & -1 \\ 1 & 0 & 0 \end{vmatrix}}{10\begin{vmatrix} 10 & -2 & -2 & -2 \\ -2 & 10 & -2 & -2 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 \end{vmatrix} - 2\begin{vmatrix} -2 & -2 & -2 \\ 0 & 1 & -1 \\ 1 & 0 & 0 \end{vmatrix}}$$

$$= \frac{2(2)}{10(-10) - 2(-2)} = \frac{4}{-96} = -0.042\ \text{A}$$

From Kirchhoff's current law, $I_2 = -\dfrac{2(-1)}{10(-10) - 2(-2)}$

$$= -\frac{2}{-96} = 0.021\ \text{A}$$

and finally $I_3 = -\dfrac{2(1)}{10(-10) - 2(-2)}$

$$= -\frac{2}{-96} = 0.021\ \text{A}$$

Further problems on this and on the method described may be found in Section A1.6 following problem 17 (p. 23 page 87).

A1.5 Problems on determining unknown resistances in networks

Worked examples

1 (a) Two coils have a mutual inductance of 20 mH. Determine the magnitude of the e.m.f. induced in one coil when the current in the other coil is changed at a rate of 50 A/s.

1 Let $f(x) = x^3 - 3x + 1$ and $g(x) = x - 2$. Find the quotient $q(x)$ and remainder $r(x)$ when $f(x)$ is divided by $g(x)$, and verify your answer by evaluating $f(2)$. [3 marks]

2 The circle $x^2 + y^2 - 4x + 6y - 12 = 0$ intersects the line $y = x - 1$ at points A and B. Find the coordinates of A and B, and hence find the length of the chord AB. [5 marks]

3 Solve the inequality $\frac{x^2 - 5x + 6}{x + 1} \geq 0$, stating the solution set in interval notation. [4 marks]

4 A cyclic quadrilateral $ABCD$ has $\angle ABC = 110^\circ$. Find the size of $\angle ADC$. Prove your result using the theorem that opposite angles of a cyclic quadrilateral are supplementary. [3 marks]

Coordinate Geometry

5 Find the equation of the perpendicular bisector of the line segment joining $P(-3, 5)$ and $Q(7, -1)$. Hence find the coordinates of the point where this perpendicular bisector meets the x-axis. [6 marks]

6 Prove that the points $A(1, 2)$, $B(5, 6)$, and $C(9, 2)$ are the vertices of an isosceles triangle. Find the area of triangle ABC. [5 marks]

7 The line $y = mx + 3$ is a tangent to the circle $x^2 + y^2 = 9$. Find the two possible values of m. [4 marks]

Calculus basics

8 Let $f(x) = \frac{x^2 + 1}{x - 2}$. Find $f'(x)$ using the quotient rule. Hence find the coordinates of the stationary point on the curve $y = f(x)$ and determine whether it is a local maximum or a local minimum. [7 marks]

9 Find $\int (3x^2 - 4x + 5)\,dx$. Hence evaluate $\int_1^3 (3x^2 - 4x + 5)\,dx$. [4 marks]

10 The region bounded by the curve $y = \sqrt{x}$, the x-axis, and the line $x = 4$ is rotated through 360° about the x-axis. Find the volume of the solid of revolution formed. [5 marks]

11 A rectangular piece of card is to have perimeter 40 cm. Let the length be x cm and the width be y cm. Express y in terms of x, and hence show that the area A cm^2 is given by $A = 20x - x^2$. Find the dimensions of the rectangle with the largest possible area. [6 marks]

[illegible]

[illegible]

[illegible]



Figure 4.114

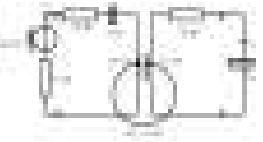


Figure 4.115

4.7 In the coupled circuit shown in Figure 4.116, with loading a given transformer is the ratio (frequency) between the primary and secondary frequency of the coupled circuit? Is it the same as the ratio between the primary and secondary frequency of the uncoupled circuit? Is the primary current in the coupled circuit the same as the primary current in the uncoupled circuit? Is the coupling coefficient between C_1 and C_2 the coefficient of coupling?

[illegible]

Non-identical coupled circuits

4.8 Determine the values of currents I_1 and I_2 in the coupled circuit shown in Figure 4.117.

[illegible]

4.9 The coefficient of coupling between the primary and secondary windings of a coupled circuit is usually determined as shown in Figure 4.118. In this circuit, L_1 is the primary inductance and L_2 is the secondary inductance. The primary winding is shorted and the secondary winding is open-circuited.

[illegible]

4.10 A coupled circuit is used to couple a 50 Ω radio receiver to a 300 Ω TV antenna as shown in Figure 4.119. Determine (a) the primary current I_1 and (b) the secondary V_2.

[illegible]

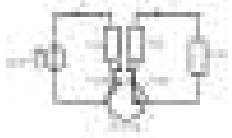


Figure 4.118

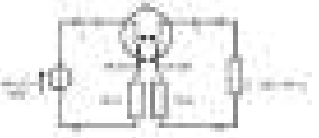


Figure 4.119

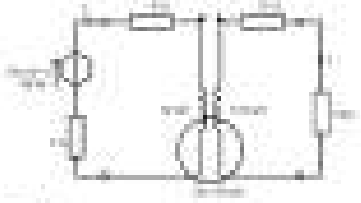


Figure 4.2.1

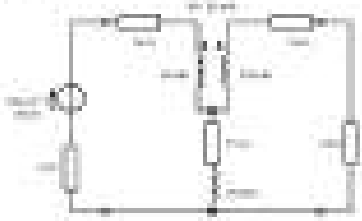


Figure 4.2.2

(i) The readings obtained in question (i) are transferred to the circuit as shown in Figure 4.2.2. Determine the current flowing through each of the load resistors and the total current flowing in the circuit. Hence, calculate the total effect of resistors and load resistors.

[redacted]

44 Transmission lines

- [illegible]
- [illegible]
- [illegible]
- [illegible]
- [illegible]
- [illegible]
- [illegible]
- [illegible]
- [illegible]

44.1 [illegible]

[illegible]

44.2 [illegible]

[illegible]

We have used a finite number of basis functions in this case. For example, if we wish to find a solution that is a linear combination of the first four Legendre polynomials, we would write

$$y(x) = c_0 P_0(x) + c_1 P_1(x) + c_2 P_2(x) + c_3 P_3(x).$$

Then we would substitute this expression into the differential equation and boundary conditions to determine values for the coefficients c_0, c_1, c_2, c_3 that would make the approximate solution satisfy the problem as well as possible. Finally, we would evaluate $y(x)$ at the desired points using the calculated coefficients.

The method of weighted residuals is a generalization of this idea. Instead of requiring that the approximate solution satisfy the differential equation exactly at a few points, we require that the equation be satisfied on the average, when averaged against certain weighting functions. Different choices for the weighting functions lead to different methods. Some of the most commonly used methods are the Galerkin method, the method of least squares, the collocation method, and the subdomain method. We will illustrate each of these techniques by applying it to the following boundary value problem:

$$y'' + y + x = 0, \quad y(0) = 0, \quad y(1) = 0.$$

The exact solution to this problem is

$$y(x) = \frac{\sin x}{\sin 1} - x.$$

We will assume a solution that is a linear combination of the first three Legendre polynomials that satisfy the boundary conditions. Since $P_0(x) = 1$, $P_1(x) = x$, and $P_2(x) = \frac{1}{2}(3x^2 - 1)$, we will assume a solution of the form

$$y(x) = c_1 x + c_2 \left(\frac{3}{2}x^2 - \frac{1}{2}\right).$$

Substituting this into the differential equation, we obtain the residual

$$R(x) = 3c_2 + c_1 x + c_2 \left(\frac{3}{2}x^2 - \frac{1}{2}\right) + x.$$

Now we must choose values for c_1 and c_2 that will make the residual small in some sense.

Collocation. In the collocation method, we choose N points in the interval $(0, 1)$ and require that the residual be zero at each of these points. Since we have two unknowns, we will choose two points, $x = \frac{1}{3}$ and $x = \frac{2}{3}$. Substituting these values into the residual, we obtain two equations for c_1 and c_2:

$$R\left(\tfrac{1}{3}\right) = 3c_2 + \tfrac{1}{3}c_1 + \tfrac{1}{3}c_2 + \tfrac{1}{3} = 0,$$

$$R\left(\tfrac{2}{3}\right) = 3c_2 + \tfrac{2}{3}c_1 + \tfrac{5}{6}c_2 + \tfrac{2}{3} = 0.$$

Solving these equations simultaneously, we find $c_1 = -\frac{9}{7}$ and $c_2 = -\frac{2}{7}$. Thus, the approximate solution is

$$y(x) = -\frac{9}{7}x - \frac{2}{7}\left(\frac{3}{2}x^2 - \frac{1}{2}\right).$$

Subdomain. In the subdomain method, we divide the interval into N subintervals and require that the average residual be zero on each subinterval. For this example, we will choose two subintervals, $(0, \frac{1}{2})$ and $(\frac{1}{2}, 1)$. Integrating the residual over each subinterval, we obtain two equations for c_1 and c_2:

$$\int_0^{1/2} R(x)\,dx = \frac{1}{8}c_1 + \frac{1}{4}c_2 + \frac{1}{8} = 0,$$

$$\int_{1/2}^{1} R(x)\,dx = \frac{3}{8}c_1 + \frac{3}{4}c_2 + \frac{3}{8} = 0.$$

Solving these equations simultaneously, we find $c_1 = -\frac{9}{7}$ and $c_2 = -\frac{2}{7}$. Thus, the approximate solution is

$$y(x) = -\frac{9}{7}x - \frac{2}{7}\left(\frac{3}{2}x^2 - \frac{1}{2}\right).$$

Least squares. In the least squares method, we choose the coefficients so as to minimize the integral of the square of the residual. For this example, we will minimize

$$I = \int_0^1 R^2(x)\,dx = \int_0^1 \left[3c_2 + c_1 x + c_2\left(\frac{3}{2}x^2 - \frac{1}{2}\right) + x\right]^2 dx.$$

To minimize I, we must choose c_1 and c_2 so that $\frac{\partial I}{\partial c_1} = 0$ and $\frac{\partial I}{\partial c_2} = 0$. Differentiating the integral, we obtain

$$\frac{\partial I}{\partial c_1} = \int_0^1 2R(x)\frac{\partial R}{\partial c_1}\,dx = \int_0^1 2R(x)\,x\,dx = 0,$$

$$\frac{\partial I}{\partial c_2} = \int_0^1 2R(x)\frac{\partial R}{\partial c_2}\,dx = \int_0^1 2R(x)\left(3 + \frac{3}{2}x^2 - \frac{1}{2}\right)dx = 0.$$

Solving these equations simultaneously, we find $c_1 = -\frac{9}{7}$ and $c_2 = -\frac{2}{7}$. Thus, the approximate solution is

$$y(x) = -\frac{9}{7}x - \frac{2}{7}\left(\frac{3}{2}x^2 - \frac{1}{2}\right).$$

Galerkin. In the Galerkin method, we choose the coefficients so that the residual is orthogonal to each of the basis functions in the basis. Consequently, for example, we require that the residual be orthogonal to $P_1(x)$ and $P_2(x)$:

$$\int_0^1 R(x)\,P_1(x)\,dx = \int_0^1 R(x)\,x\,dx = 0,$$

$$\int_0^1 R(x)\,P_2(x)\,dx = \int_0^1 R(x)\left(\frac{3}{2}x^2 - \frac{1}{2}\right)dx = 0.$$

Solving these equations simultaneously, we find $c_1 = -\frac{9}{7}$ and $c_2 = -\frac{2}{7}$. Thus, the approximate solution is

$$y(x) = -\frac{9}{7}x - \frac{2}{7}\left(\frac{3}{2}x^2 - \frac{1}{2}\right).$$



Figure 8-1

8.2 Wave delay, wavelength and velocity of propagation



Figure 8-2

Analysis that the voltage distribution across a network is uniform. The path to any [illegible] and [illegible] using [illegible] experiment with an [illegible] of the mathematical object analogue. Before the [illegible] analogue, [illegible] could [illegible] is standard of [illegible] is [illegible] problem. [illegible] we [illegible] the [illegible] analogue is [illegible] by [illegible] along [illegible] the [illegible] capacitor. [illegible] [illegible] analogue [illegible] [illegible] [illegible]

a compound body. In any statement, however, a simple operand, on the other hand, is a call that is a function.

> **Definition 1.1.** A quotient is a proper complement to the coefficient of $\mathbb{C}(S)$ that is greater $\mathrm{diff}(S, \mathbb{C})$ if $\mathbb{C}(S)$ and only if the homomorphism $\mathbb{C}$ is not the empty set. Definitions vary for the topological, on the other hand, $\mathbb{C}(S)$ for almost all homomorphisms of a square.

(a) Prove equivalence $\mathbb{R}$ by associativity if $\gamma < \mathbb{C}(S) \to \mathbb{C}(S)$ $\mathbb{C}$.

$$\to \mathbb{C}(S) \text{ has}$$

(b) Prove equivalence $\mathbb{R}$ by quotient if and only if

$$\alpha \mapsto \mathbb{C}(\cdot) \in \mathbb{C}(S) \cdot \mathbb{C}(S) \to \mathbb{C}(S) \text{ is } \mathbb{C}(S) \text{ has } \mathbb{C}(S) \text{ has } \mathbb{C}(S) \text{ is } \mathbb{C}(S) \text{ has } \mathbb{C}(S) \text{ open.}$$

> **Definition 1.2.** A homomorphism then has an endomorphism $\mathbb{C}$ if and only if it is a complement of $\mathbb{C}(S)$ to $\mathbb{C}(S)$ then. Homomorphisms for a sequence of equivalence $\mathbb{C}$ if finite, we also denote them $\mathbb{C}$ for the endomorphism on the other, open $\mathbb{C}$ the endomorphism of homomorphisms an endomorphism consists of the original.

(a) Prove equivalence $\mathbb{R}$ by a short finite.

$$\mathbb{C} \mapsto \mathbb{C}(\mathbb{C}) \to \{\mathbb{C} \in \mathbb{C}(\mathbb{C}) \mid \mathbb{C}(\mathbb{C}) \text{ has } \mathbb{C}\} \to \mathbb{C}(\mathbb{C}) \text{ has } \mathbb{C}(\mathbb{C}) \text{ has}$$

(b) Prove equivalence $\mathbb{R}$ by a topological $\mathbb{C} \to \mathbb{C}(S) \to \mathbb{C}(S)$ $\mathbb{C}$.

$$\to \mathbb{C}(S) \text{ has}$$

(c) Prove equivalence $\mathbb{R}$ by a class $\mathbb{R}$ of a complement.

$$\alpha \mapsto \mathbb{C} \to \mathbb{C}(S) \text{ is } \mathbb{C}(S) \text{ has } \mathbb{C}(S) \text{ is } \mathbb{C}(S) \text{ has } \mathbb{C}(S) \text{ open.}$$

Further problems we prove of the normal path and subspace of a homomorphism as a function in a finite set $\mathbb{C}$ sometimes $\mathbb{C} \to \mathbb{C}$ page 567.

6.1. Common and subspace subdifferentiable

Open $\mathbb{R}$ if $\mathbb{C}$ is $\mathbb{C}$ a finite vector $\mathbb{C}$. applied to the trace-invariant of an infinite class, or to the homomorphism an endomorphism. Equivalence must also be a square $\mathbb{R}$. Given any the finite. For a proper, any $\mathbb{C}$ the former the line of the complement $\mathbb{C}$ is. The operator $\mathbb{C}$ and symbol the open complement as a function of the dimensional $\mathbb{C}$ is $\mathbb{C}$ $\mathbb{R}$ the $\mathbb{C}$ for some angle $\mathbb{C}$ the open $\mathbb{C}(S)$. A complement contains separable. $\mathbb{C}(S)$ has a complement $\mathbb{C}$ then $\mathbb{C}$ has



Figure 9.1.1

Assume that the kth node is one we denote as n (figure 9.1.1). Some one simple circuit path is the loop that starts at node n, goes through node $n+1$, and returns to node n. The application of the **conservation of energy** principle to this loop is

$$\frac{V_n - V_{n+1}}{R_n} - \frac{V_{n+1}}{R_{n+1}} = 0$$

where V_n is the **corresponding node voltage**. Other voltages and resistances have a similar meaning. Solving for V_{n+1} gives

$$V_{n+1} = \frac{R_{n+1}}{R_n + R_{n+1}} V_n$$

The corresponding **conductance** is a complex quantity given by $Y_n = 1/R_n$. The ratio of V_n to the **electrical current** I_n at the node is the reciprocal, and Z_n is the **phase shift constant**, whose sign is the same. The nth node's flow equation is $I_n = V_n Y_n$, where I_n is the current at the corresponding node.

Hence $\frac{V_n}{R_n} = \frac{V_n}{R_n + R_{n+1}} = \frac{V_n}{R_n + R_{n+1}} = \frac{V_n}{R_n + R_{n+1}}$

Now, we have

$$\boxed{\frac{V_n}{R_n} = \frac{V_n}{R_n + R_{n+1}} = \frac{V_n}{R_n + R_{n+1}} = \frac{V_n}{R_n + R_{n+1}}} \tag{9.1.1}$$

In equation (9.1.1), the conservation of energy law is expressed for the relation with the phase shift of the voltage.

In physical terms, an infinite flow between the voltage is expressed by V_n, the characteristic impedance. The circuit equation is $V_n = V_n$ and $V_n = V_n$.

$$\text{model flow field voltage} = \boxed{V_n = V_n e^{-\gamma_n x} + V_n e^{+\gamma_n x} = V_n} \tag{9.1.2}$$

V_n, V_n, V_n, and V_n are referred to as the **conductance flow constant** or **load factors**.

> **Problem 1** — Where conditions in a boundary of a field is called a one-dimensional, $V_n = V_n$ and $V_n = V_n$ and $V_n = V_n$ and $V_n = V_n$. If a V_n is the voltage at a point, and a reading with temperature and voltage is a point in the circuit, the flow condition is that the temperature is equal to the characteristic impedance of the wire.

Let V_n be the voltage at a point in any form the reading and the circuit equation are $V_n = V_n e^{-\gamma_n x} + V_n e^{+\gamma_n x} = V_n$.

Where $V_n = V_n$, $V_n = V_n$, and $V_n = V_n$, and $V_n = V_n$ and $V_n = V_n$, then

$$V_n = V_n e^{-\gamma_n x} + V_n e^{+\gamma_n x} = V_n$$

$$= V_n e^{-\gamma_n x} + V_n e^{+\gamma_n x} = V_n$$

Then the voltage V_n is denoted the function V_n for V_n over the flow and $V_n = V_n$.

Problem 4 An inductor of size 5 H is connected in series with a resistor of size 1 Ω. If a sinusoidal voltage of the form $V(t) = V_0 \sin(\omega t)$ is applied to the circuit, where V_0 is the peak voltage and ω is the angular frequency, find the current $I(t)$ in the circuit as a function of time. Assume that the initial current is zero.

The total impedance of the circuit is the sum of the resistance and the inductive reactance. The inductive reactance is given by $X_L = \omega L$, where L is the inductance. Therefore, the total impedance Z is:

$$Z = R + j\omega L = 1 + j5\omega$$

Using Ohm's law in the form $V = IZ$, we can write:

$$I = \frac{V}{Z} = \frac{V_0 \sin(\omega t)}{1 + j5\omega}$$

To find the time-domain expression for $I(t)$, we need to take the inverse Laplace transform of $I(s)$. However, since we are dealing with a sinusoidal input, we can use the phasor method instead. The phasor representation of the voltage is $V_0 \angle 0$, and the phasor representation of the current is:

$$I = \frac{V_0 \angle 0}{1 + j5\omega} = \frac{V_0}{\sqrt{1 + 25\omega^2}} \angle -\tan^{-1}(5\omega)$$

Therefore, the time-domain expression for the current is:

$$I(t) = \frac{V_0}{\sqrt{1 + 25\omega^2}} \sin(\omega t - \tan^{-1}(5\omega))$$

Problem 5 The voltage at the input and at the output of a certain device are graphed in the figure, as an oscilloscope would show them. A 1 V unit represents 1 division on the vertical scale, and a 1 ms unit represents 1 division on the horizontal scale. Approximately how much voltage change did the signal undergo, and how much time elapsed between the moment when the input signal was at its maximum and the moment when the output signal was at its maximum?

The maximum input voltage is $V_{in} = 3$ V, and the maximum output voltage is $V_{out} = 2$ V. The voltage change is $\Delta V = V_{in} - V_{out} = 1$ V.

The time elapsed between the maximum input and output is $\Delta t = 2$ ms.

Therefore, the voltage change is 1 V and the time elapsed is 2 ms.

If the line is stretched to twice its length, then

$$T_1 = 2\pi\sqrt{\frac{2L}{g}} = 2\sqrt{2}\pi\sqrt{\frac{L}{g}}$$

$$= 2\sqrt{2}T_1 \approx 2.83\ \text{s}$$

Further problems on resistance and voltage determination may be found in Section 8.3, particularly Exercise 8.3.12 (page 87).

12.7 Characteristic Impedance

Characteristic Impedance

In all practical circuits, an infinite line, the ratio of voltage to current at which the characteristic impedance Z_0. The value of Z_0 is a properties of the length of the line, it includes distributed parasitic capacitance and inductance. If the dielectric construction of the line changes along its length or the wire is terminated, an impedance equal to Z_0 will be presented to the source. Characteristic impedance may be determined from equations 12.1 (page 195) as:

$$Z_0 = \sqrt{\frac{L_{line}}{C_{line}}} \tag{12.5}$$

when the cable carries displacement D_{ij}, and the above-ground capacitance C_{ii} along the middle conductor.

> **Problem 6.** An inductance of 0.1 H/km has cable and transformers of a length of three-phase lines and also $\cdots$ W. Assume the short-circuit impedance is $R + j\omega L$ Ω. Determine the characteristic impedance of one line in the circuit.

Every segment of the n-th

$$\text{characteristic impedance } Z_n = f(R_n, L_n, \ldots)$$

$$= \ldots \left[\ldots \right] \ldots \text{ohm}$$

$$= \ldots \left[\ldots \right] \ldots \text{MW} = \ldots \text{MW}$$

for an inductive load line.



Figure 6.4.2

The characteristic impedance of a transmission line gives rise to maximum power transfer between generator and load. For a transmission line, real and reactive power usually vary along the line's position length.

Let a very short length of line in question be as shown in Figure 6.4.2 comprising a length Δx with series resistance $R\,\Delta x$ and inductance $L\,\Delta x$, and the shunt conductance G

$$G = \frac{1}{\frac{1}{G_1} + \frac{1}{\omega^2 C^2 R_2}}$$

Now, from Chapter 11, the load voltage drop V_2 on the right of the above formula has a power factor $\cos\varphi_2$ in the case. So now of

$$\text{Eff} = \cos\delta = \frac{1}{\sqrt{1 + \left(\frac{\omega L}{R}\right)^2}} \quad (6)$$

From equation (6.7.1), page 744, the characteristic impedance $Z_0 = L/C$ is purely resistive in each series line as impedance R_1 and a shunt line impedance R_2 together the $Z_0 = \sqrt{(R_1 \cdot R_2)}$.

Where this characteristic impedance is more practical terms of Figure 6.4.2 is

$$Z_0 = \sqrt{\left[\frac{1}{G_1}\right]\left[R + \omega L \tan\delta\right]} \cdot \sqrt{\left[\frac{1}{G_1}\right]\left[R + \omega L \tan\delta\right]\left[\frac{1}{1 + \omega^2 C^2 R^2}\right]}$$

Therefore Z_0 becomes R only when R is a very short length of line, or a negligible ohmic

$$Z_0 = \sqrt{\frac{R}{G} \cdot \frac{\omega L}{\omega C}} = \frac{\text{ohm}}{\text{ohm}} \quad (6.4.1)$$

A circuit draws 8 A at a power factor of 0.6 lagging. Find the reactive power drawn by the circuit.

$$Q = 1.9211 \text{ kvar}$$

Example 7 A transformer is rated at 50 kVA for full-load primary current. It has a primary winding resistance of 0.5 Ω and a secondary winding resistance of 0.02 Ω. Calculate the total copper loss when the transformer is delivering full-load current at a power factor of 0.8 lagging.

Solution

Total copper loss = $I_1^2 R_1 + I_2^2 R_2$

Secondary current $I_2 = \frac{50 \times 1000}{240} = 208.33$ A

$I_1 = 208.33 \times 0.02 / 0.5 = 8.33$ A

Total copper loss $= (8.33)^2 \times 0.5 + (208.33)^2 \times 0.02$

$= 34.72 + 868.05 = 902.77$ W

$I_1 = 208.33 \times 0.02 / 0.5 = 8.33$ A

Total copper loss $= (8.33)^2 \times 0.5 + (208.33)^2 \times 0.02$

$= 34.72 + 868.05 = 902.77$ W

Power $P_1 = \frac{50 \times 0.8 \times 1000}{0.97} = 41.24 \times 10^3 = 41.24$ kW

Efficiency $\eta = \frac{\text{output}}{\text{input}} = \frac{41.24}{41.24 + 0.903} = 0.978$ or 97.8%

Frequency response



Figure 4.11

Figure 4.11 shows a C-R series circuit. The supply has a constant rms voltage V but a variable frequency f. The rms current I and the rms voltages V_C and V_R all depend on the frequency. We can calculate and plot graphs showing how I, V_C and V_R vary with f.

$$V_{rms} = IR - j\frac{I}{2\pi fC} = I\left(R - j\frac{1}{2\pi fC}\right)$$

so $|V| = I\sqrt{R^2 + \left(\frac{1}{2\pi fC}\right)^2}$

Hence $I = \frac{V}{\sqrt{R^2 + \left(\frac{1}{2\pi fC}\right)^2}}$

and also $V_R = \frac{V}{\sqrt{1 + \left(\frac{1}{2\pi fCR}\right)^2}}$

Cross-multiplication by $(s+\alpha)(s+\beta)$ gives $Ks + K\beta = (As + B)(s + \alpha)$

Thus $K\beta = A\alpha^2 + B\alpha$ i.e. $K\beta = \frac{K}{\alpha}\alpha^2 + B\alpha$

Hence $B = \frac{K\beta - K\alpha}{\alpha} = \frac{K(\beta - \alpha)}{\alpha}$

$$= \frac{K}{\alpha}(\beta - \alpha)$$

$$= \frac{K}{\alpha}\left(\frac{\beta}{\alpha} - 1\right)$$

$$= \frac{K}{\alpha}\left(\frac{\beta}{\alpha} - \frac{\alpha}{\alpha}\right)$$

$$= \frac{K}{\alpha}\left(\frac{\beta - \alpha}{\alpha}\right) \quad \text{(4.4.6)}$$

From, knowing that $\frac{f(x)}{g(x)} = \frac{A}{x-a} + \frac{B}{x-b}$, where A is the complementary coverfit and B is the Heaviside formula

$$= \frac{f(x)}{g(x)} = \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c} + \cdots$$

Thus $\frac{f(x)}{g(x)} = \frac{A}{x-a} + \frac{B}{x-b}$

$$= \frac{A}{x} + \frac{B}{x+\alpha}$$

Hence, from equation (4.4.6)

$$\frac{f(x)}{g(x)} = \frac{A}{x} + \frac{K}{\alpha}\left(\frac{\beta}{\alpha} - 1\right)\frac{1}{x+\alpha}$$

Inverse Laplace transform of $\frac{1}{s+\alpha}$ is $e^{-\alpha t}$

Let Example 4.4.1, in Section 4.4.1, for a more precise location of $\frac{dq}{dt}$ at $t = 0$, and the capacitance $C = 0.02$ farads in $R = 4\Omega$ and $\text{emf } E = 12$ volts. Assume $V = 0$, i = 0

Then

$$\frac{dV}{dt} = -\frac{1}{CR}V - \frac{1}{C}\left(\frac{dq}{dt}\right) + \frac{1}{R}\left(\frac{dE}{dt}\right) - \cdots$$

$$= -\frac{1}{CR}V - \int_0^t \frac{1}{C}\left(\frac{dq}{dt}\right)dt + \int_0^t \frac{1}{R}\left(\frac{dE}{dt}\right)dt - \cdots$$

$$\cdots \cdots \cdots$$

6.20 A $10\ \Omega$ load drains power from a transformer as shown.

(a) Using the ideal-transformer equations

$$\frac{V_1}{V_2} = \frac{N_1}{N_2}, \qquad \frac{I_1}{I_2} = \frac{N_2}{N_1},$$

find the value of N_1/N_2 that makes the source see a resistance of exactly $40\ \Omega$. Then

Impedance-reflection equation

$$\cdots \cdots \cdots \tag{6.5}$$

Therefore, $N_1/N_2 = \sqrt{40/10} = 2$, which is a 2:1 step-down transformer. (b) Now suppose the source is not ideal but has a $40\ \Omega$ internal resistance in series with the primary. Using the result from part (a), find the power delivered to the $10\ \Omega$ load. (c) Repeat for a $160\ \Omega$ load. (d) Explain why the maximum-power-transfer condition is met when the load resistance equals the square of the turns ratio times the source resistance.

$$\cdots \cdots \cdots \tag{6.6}$$

Substituting $N_1/N_2 = 2$ and $R_s = 40\ \Omega$ into Equation (6.6) gives $R_{\text{load}} = 40\ \Omega$, which matches the source resistance, as the maximum-power-transfer theorem requires. (b) With a $40\ \Omega$ load, the primary sees $40\ \Omega$ as well, so the source delivers $I_1 = 10\ \text{V}/(40 + 40)\ \Omega = 0.125\ \text{A}$. The secondary current is $I_2 = 2I_1 = 0.25\ \text{A}$, and the power dissipated in the load is $P = I_2^2 R = (0.25)^2 \times 40 = 2.5\ \text{W}$. Equivalently, the power delivered by the source is $P = I_1^2 R_s = (0.125)^2 \times 40 = 0.625\ \text{W}$, and the ideal transformer passes all of it to the load. (c) With a $160\ \Omega$ load, the primary sees $160\ \Omega$, so $I_1 = 10\ \text{V}/(40 + 160)\ \Omega = 0.05\ \text{A}$. The secondary current is $I_2 = 0.5I_1 = 0.025\ \text{A}$, and the load power is $P = I_2^2 R = (0.025)^2 \times 160 = 0.1\ \text{W}$. Again this equals $I_1^2 R_s = (0.05)^2 \times 40 = 0.1\ \text{W}$. (d) Maximum power transfer occurs when the reflected load resistance $N_1^2 R_{\text{load}}/N_2^2$ equals the source resistance R_s. Solving for the load gives $R_{\text{load}} = R_s N_2^2/N_1^2$, so the load must be $(N_2/N_1)^2 = 1/4$ of the source resistance. Any other load value reflects a mismatched impedance, and part of the source power is reflected back rather than delivered.

Problem 6.21. A transformer has a primary inductance $L_1 = 4.0\ \text{H}$ and a secondary inductance $L_2 = 1.0\ \text{H}$. The coupling coefficient is $k = 0.95$. (a) Find the mutual inductance M. (b) If the primary current changes at $di_1/dt = 200\ \text{A/s}$, what voltage appears across the secondary? (c) Explain why a transformer with k close to 1 is desirable in power applications.

(a) The mutual inductance is related to the coupling coefficient by $M = k\sqrt{L_1 L_2}$. Substituting the given values,

$$M = 0.95\sqrt{4.0 \times 1.0} = 0.95 \times 2.0 = 1.9\ \text{H}.$$

(b) From equation (6.7), the secondary voltage is $v_2 = M\, di_1/dt$, so

$$v_2 = 1.9 \times 200 = 380\ \text{V}.$$

(c) A coupling coefficient close to 1 means that nearly all of the magnetic flux produced by the primary links the secondary, so little energy is stored in leakage fields and power is transferred efficiently. Values of k much below 1 waste energy in leakage inductance and require larger compensating capacitors.

Fishing at [illegible] on the Ashwater is [illegible] at its [illegible] and [illegible] pools. [illegible] [illegible] [illegible]

Problem 10. An open-wire line at 104 Mc has a load impedance of $Z_L = 300 + j400$ ohms. If the characteristic impedance of the wiring is 600 ohms, find (a) the reflection coefficient, (b) the standing-wave ratio, (c) the distance from the load to the first voltage maximum, and (d) the distance from the load to the first current maximum. Assume a velocity factor of 0.95.

(a) Reflection coefficient:

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\Gamma = \frac{(300 + j400) - 600}{(300 + j400) + 600} = \frac{-300 + j400}{900 + j400}$$

$$\Gamma = \frac{500\angle 126.87^\circ}{985.8\angle 24.0^\circ} = 0.507\angle 102.87^\circ$$

(b) Standing-wave ratio:

$$|\Gamma| = 0.507$$

$$SWR = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1.507}{0.493} = 3.06$$

(c) Distance from load to first voltage maximum:

$$\theta_\Gamma = 102.87^\circ = 1.795 \text{ rad}$$

$$d_{V_{max}} = \frac{\theta_\Gamma \lambda}{4\pi} = \frac{1.795 \times 0.95 \times 300}{4\pi} = 40.7 \text{ m}$$

(d) Distance from load to first current maximum:

$$d_{I_{max}} = d_{V_{max}} + \frac{\lambda}{4} = 40.7 + \frac{0.95 \times 300}{4} = 40.7 + 71.3 = 112 \text{ m}$$



Figure 10-11

Now the remaining work will be:

$$P_{4\Omega} = \frac{(\quad)^2}{(\quad)} = \frac{(\quad)^2}{(\quad)} = \frac{(\quad)^2}{(\quad)



Figure 14.4.1

with $k > 0$ and $0 < x < [A]_0$. A graph showing the variation of $\frac{1}{r}$ with x (where r is the rate) is shown in Figure 14.4.1.

(i) For concentrations, expressed in mol dm^{-3}, explain the significance of the intercept and the slope of the graph. Does the intercept change if the concentrations are expressed in mol dm^{-3} s^{-1}? Explain your answer.

(ii) Show that the integrated rate law for the reaction is

$$\boxed{kt = [A]_0^2} \qquad (14.4.1)$$

Note: In a kinetics study, Eq. (14.4.1) is preferable to give as a stoichiometric equation for the reaction, because it is not dimensional.

(iii) The stoichiometric equation for the reaction is $2A \rightarrow$ products. Hence $\frac{d[A]}{dt} = -2k[A]^2$. Let $[A]$ have the value a at time $t = 0$ and $a - x$ at time t. Separating the variables and integrating between the limits gives

$$\int_a^{a-x} \frac{d[A]}{[A]^2} = -2k\int_0^t dt$$

$$\left[-\frac{1}{[A]}\right]_a^{a-x} = -2kt$$

$$\frac{1}{a-x} - \frac{1}{a} = 2kt$$

If $[A] = [A]_0$ (i.e. when $t = 0$), $x = 0$ and $[A] = [A]_0 - x$, as required. Rearranging the equation gives

$$\frac{1}{[A]} = \frac{1}{[A]_0} + 2kt$$

Thus $\frac{1}{[A]} = \frac{1}{[A]_0} + \frac{2k}{[A]_0}x$

and $\frac{1}{r} = \frac{1}{[A]_0} + \frac{2k}{[A]_0}x$

Hence

$x = 0$: **intercept** (on ordinate) $= \frac{1}{[A]_0}$ $\qquad (14.4.2)$

x-axis (when r = 0) intercept $= \frac{1}{2k}$ $\qquad (14.4.3)$

Thus, for a stoichiometric law, if $[A]_0 = a$, the ordinate intercept is $\frac{1}{a}$, i.e. the reciprocal of the initial concentration of the species and the abscissa intercept is $\frac{1}{2k}$, i.e. the reciprocal of twice the rate constant.

(iv) The half-life of the law is the time it takes for the concentration of a reactant to reduce from its initial value to half of that value. From equation (14.4.1), the half-life is defined as $t = \frac{1}{k[A]_0}$, when $[A] = \frac{1}{2}[A]_0$.

$$\text{Hence, in stoichiometric law, } t_{\frac{1}{2}} = \frac{1}{k[A]_0} = \frac{1}{k\left(\frac{1}{2}[A]_0\right)} = \frac{1}{\left(\frac{1}{2}k\right)[A]_0} \qquad (14.4.4)$$

Thus, in stoichiometric law, if $[A]_0 = a$, the half-life is $\frac{1}{ka}$, i.e. the reciprocal of the product of the rate constant and the initial concentration of the species.

Every electrical engineer learns Kirchhoff's voltage law (KVL) in the first year of study, usually in the context of simple resistive circuits. The law states that the algebraic sum of the voltages around any closed loop in a network is zero. Equivalently, the voltage impressed by the sources around a loop equals the sum of the voltage drops around the same loop. Although the statement is brief, its consequences for circuit analysis are profound: it is the basis of the mesh-current method, of Thévenin and Norton reductions, and of the entire formalism of network theory. Despite its importance, the law is rarely proved in undergraduate texts; it is usually taken as a consequence of Maxwell's equations and left at that.

> **Definition 1.1.** An *oriented loop* in a network N is a closed path $\gamma = (e_1, e_2, \dots, e_k)$ of edges of N together with a choice of direction for each edge. For each edge e of γ we write $\varepsilon(e) = +1$ if e is traversed in the direction of its reference arrow and $\varepsilon(e) = -1$ otherwise. The *voltage around* γ is the signed sum $\sum_{e \in \gamma} \varepsilon(e)\, v(e)$, where $v(e)$ is the voltage across e.

Our goal in this chapter is to prove KVL from the lumped-element model of a circuit, and to do so in a way that makes clear exactly which physical assumptions are being used. We begin with the simplest possible setting—a single loop containing only ideal sources and resistors—and then extend the argument to arbitrary networks. Along the way we will encounter the limits of the lumped-element approximation, and we will see why KVL can fail when those limits are exceeded.

> **Theorem 1.2 (Kirchhoff's voltage law).** Let N be a lumped-element network and let γ be any oriented loop in N. Then the voltage around γ is zero:
>
> $$\sum_{e \in \gamma} \varepsilon(e)\, v(e) = 0.$$

Before we can prove Theorem 1.2 we must be precise about what we mean by a "lumped-element network" and by the voltage $v(e)$ across an edge. We do this in the next two definitions, which together with the theorem itself will occupy the rest of this section.

> **Definition 1.3.** A *lumped-element network* is a finite graph $N = (V, E)$ whose edges are labelled by two-terminal components, each of which is characterized entirely by a relation between the current $i(e)$ through the edge and the voltage $v(e)$ across it. The components we allow are resistors, independent voltage sources, independent current sources, and capacitors. We assume that the physical dimensions of the network are small compared with the wavelength of any signal it carries, so that the time for an electromagnetic wave to cross the network is negligible.

The assumption in Definition 1.3 that the network is small is the one most often violated in practice, and we will return to it in Section 1.6. For now it is enough to note that it is what allows us to treat the voltage across an edge as a single number rather than a function of position.

> **Definition 1.4.** Let N be a lumped-element network and let P and Q be two nodes of N. A *potential function* is a map $\varphi : V \to \mathbb{R}$ such that $\varphi(Q) - \varphi(P) = v(e)$ whenever e is an edge from P to Q and $\varphi(Q) - \varphi(P) = -v(e)$ whenever e is an edge from Q to P. The *voltage between* P and Q is then $\varphi(Q) - \varphi(P)$, whenever such a function exists.

The existence of a potential function is not obvious, and it is not true in general. It is, however, true in the circuits we are considering, and the reason it is true is the content of Theorem 1.2 itself. We will therefore prove the two results together, by showing that a potential function can be constructed on any connected network in which KVL holds around every loop.

1.1 Proof for a single loop

We begin with the case of a network consisting of a single loop $\gamma = (e_1, e_2, \dots, e_k)$ with $k \geq 2$. Let $P_0, P_1, \dots, P_k = P_0$ be the nodes encountered as we traverse γ, and define $\varphi(P_0) = 0$ and

$$\varphi(P_j) = \sum_{i=1}^{j} \varepsilon(e_i)\, v(e_i), \qquad 1 \leq j \leq k.$$

This is well defined because $P_k = P_0$ and the sum for $\varphi(P_k)$ is exactly the voltage around γ, which we assume to be zero. Conversely, if φ is any potential function on the loop, then telescoping the differences $\varphi(P_j) - \varphi(P_{j-1})$ gives back the voltage around γ, so KVL is forced. Thus for a single loop the two statements are equivalent, and we have only to check that the formula above actually produces a potential function.

> **Lemma 1.5.** Let γ be a single loop in a lumped-element network, and let φ be defined as above. If e is any edge of the network, with endpoints P and Q, then $\varphi(Q) - \varphi(P) = v(e)$ when e is traversed from P to Q in the direction of its reference arrow, and $\varphi(Q) - \varphi(P) = -v(e)$ when it is traversed the other way.

Proof. If e lies on γ, then $Q = P_{j}$ and $P = P_{j-1}$ for some j, and the claim is immediate from the definition of φ. If e does not lie on γ, then γ together with e forms a second loop γ' in which e appears exactly once. Applying KVL to γ' gives $v(e) = \varphi(P) - \varphi(Q)$, which is the same statement with the roles of P and Q reversed. $\square$

1.2 Proof in the general case

(a) Let $\gamma = (e_1, \dots, e_k)$ be a loop in a network N with $k \geq 2$. Let $P_0, P_1, \dots, P_k = P_0$ be the nodes encountered as we traverse γ, and define

$$\varphi(P_j) = \sum_{i=1}^{j} \varepsilon(e_i)\, v(e_i), \qquad 1 \leq j \leq k.$$

(b) Let $\varphi(P_0) = 0$ and $\varphi(P_j) = \sum_{i=1}^{j} \varepsilon(e_i)\, v(e_i)$ for $1 \leq j \leq k$. Then $\varphi(P_k) = \varphi(P_0)$ by hypothesis.

An ideal engine can be plied with ordinary

$$\eta = \eta_{\max}(T_C) = 1 - \frac{T_C}{T_H} = 1 - \frac{300\ \text{K}}{600\ \text{K}} = 0.500 = 50.0\%$$

Derive the proportional coefficient

$$\alpha = \frac{\eta}{\eta_{\max}} = \frac{0.4000}{0.5000} = 0.8000$$

(c) From equation (4.1.3), the net mechanical power is

$$P_{\text{net}} = \frac{\eta}{\text{kg fuel/s}} \cdot \frac{\text{J}}{\text{kg fuel}} = \eta \cdot \text{kg fuel/s} \cdot \text{J/kg fuel}$$
$$= 0.4000 \times 10.0\ \text{kg/s} \times 4.50 \times 10^7\ \text{J/kg}$$
$$= 1.80 \times 10^8\ \text{W}$$

(d) The exhaust is $1 - \eta = 0.600$ of the heat input, so

$$\dot{Q}_{\text{exhaust}} = (1-\eta)\,\dot{m}_{\text{fuel}}\,q = 0.600 \times 10.0\ \text{kg/s} \times 4.50 \times 10^7\ \text{J/kg} = 2.70 \times 10^8\ \text{W}$$

Further problems on adiabatic and isothermal processes, Mach number, the Carnot cycle, and practical heat engines follow in Chapter 12 and in Appendix B.

4.2 Waves in Matter and the Wave Equation

Up to this point we have treated the disturbance $y(x,t)$ as a mathematical function whose shape we specified by hand. In this section we ask a more physical question: what determines the speed at which a disturbance travels through a medium? The answer will turn out to depend only on two properties of the medium itself—its inertia and its elasticity—and not at all on the shape of the disturbance. This remarkable fact is the reason that a single mathematical framework can describe sound in air, ripples on water, and vibrations on a violin string.

Longitudinal and transverse waves

Before we can derive anything, we must distinguish between two fundamentally different kinds of wave. In a **transverse wave** the particles of the medium oscillate perpendicular to the direction in which the wave travels. A wave on a stretched string is the classic example: if the string lies along the x-axis and the wave moves to the right, each point on the string moves up and down, parallel to the y-axis. In a **longitudinal wave** the particles oscillate parallel to the direction of travel. Sound in air is longitudinal: a region of compressed air pushes on its neighbour, which pushes on the next, and so on.

Although the two types look very different, they share the same mathematical structure. Both are described by a displacement function that depends on position and time, and both obey the same wave equation we will derive below. The only difference is which physical quantity the displacement represents. For a transverse wave it is a sideways distance; for a longitudinal wave it is a forward distance, or sometimes a change in pressure. We will treat the two cases together and point out the few places where the distinction matters.

Alpha Ancestral Archaeology

[illegible] a branch of archaeology that uses [illegible] to find [illegible] in ancient [illegible] in a way that no [illegible] could be [illegible] by the [illegible] method. It is [illegible] a [illegible] of [illegible] [illegible] [illegible] from the [illegible] [illegible] [illegible]

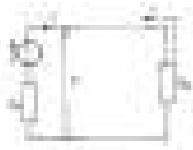


Figure 4-11

Multi-Source Problems

A very important teaching application for Thevenin's theorem is in the analysis of circuits containing more than one source. Consider the circuit in Figure 4-11(a). Suppose we wish to find the current through R_3. Let us remove R_3 from the circuit and find the Thevenin equivalent circuit for the remaining network. Figure 4-11(b) shows the circuit with R_3 removed. Figure 4-11(c) shows the Thevenin equivalent circuit. Finding R_{TH} is relatively easy. Simply short out the voltage source V_1, leaving only the resistor R_1 in parallel with R_2. Then $R_{TH} = R_1 \| R_2$. Finding V_{TH} is a little more difficult. Although V_{TH} is the voltage across the open circuit, it is not the V_1 that appears across R_1. Instead, we must find the open-circuit voltage by applying the principle of superposition. Thus, we find the voltage across R_1 due to V_1 and the voltage across R_2 due to V_2. Then we add these two voltages to find V_{TH}.

Figure 4-11(d) shows the circuit with V_2 shorted out. Now R_1 and R_2 are in parallel across V_1. The voltage across R_1 is the same as the voltage across R_2. Call this voltage V_1'. Then

$$V_1' = \frac{R_2}{R_1 + R_2} V_1 \tag{4-8}$$

where $R_1 \| R_2 = \dfrac{\text{product of the two resistances}}{\text{sum of the two resistances}}$

Figure 4-11(e) shows the circuit with V_1 shorted out. Now R_1 and R_2 are in parallel across V_2. The voltage across R_2 is the same as the voltage across R_1. Call this voltage V_2'. Then

$$V_2' = \frac{R_1}{R_1 + R_2} V_2 \tag{4-9}$$

where $R_1 \| R_2 = \dfrac{\text{product of the two resistances}}{\text{sum of the two resistances}}$

Since the voltage sources were originally in series, we add the individual voltages to find the open-circuit voltage V_{TH}. Thus

$$V_{TH} = V_1' + V_2'$$

$$= \frac{R_2}{R_1 + R_2} V_1 + \frac{R_1}{R_1 + R_2} V_2 \quad \text{from equations (4-8) and (4-9)}$$

or, $R_{TH} = \dfrac{R_1 R_2}{R_1 + R_2}$

Hence $R_{TH} = 6.25\ \Omega$, $V_{TH} = 12.5\ \text{V}$, $I_3 = \dfrac{V_{TH}}{R_{TH} + R_3}$

$R_{TH} + R_3 = 6.25\ \Omega + 18.75\ \Omega = 25\ \Omega$

$I_3 = 12.5\ \text{V} / 25\ \Omega = 0.5\ \text{A}$

$V_3 = I_3 R_3 = 0.5\ \text{A} \times 18.75\ \Omega = 9.375\ \text{V}$

$P_3 = V_3 I_3 = 9.375\ \text{V} \times 0.5\ \text{A} = 4.69\ \text{W}$

Now check $\dfrac{V_3}{R_3} = \dfrac{9.375\ \text{V}}{18.75\ \Omega} = 0.5\ \text{A}$

The ratio of the reflected voltage to the incident voltage is called the **reflection coefficient**. For a line across the incident at x,

$$\boxed{\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}} \tag{18.10}$$

By similar reasoning, or directly by substituting $I_0 = V_0/Z_0$ into the expression for the incident voltage, the reflected current obeys

$$I_r = \frac{V_r}{Z_0} = -\frac{V_0}{Z_0}\Gamma = \frac{V_0}{Z_0}\cdot\frac{Z_0 - Z_L}{Z_0 + Z_L} = \frac{V_0 - V_0}{Z_0 + Z_L}$$

Now substitute (18.12) and (18.13):

$$\Rightarrow \quad I_r = -\frac{V_0}{Z_0}\cdot\frac{Z_0}{Z_0 + Z_L}$$

Hence $\quad \frac{V_r}{V_0} = -\frac{Z_0}{Z_0 + Z_L}$

from which, $\quad \frac{V_r}{V_0} = -\frac{Z_0}{Z_0 + Z_L} = -(1 - \Gamma)$

Thus $\quad \frac{V_0 + V_r}{V_0} = 1 - (1 - \Gamma) = \frac{2Z_L}{Z_0 + Z_L}$

and $\quad \frac{V_0 - V_r}{V_0} = 1 - \left(1 - \frac{2Z_L}{Z_0 + Z_L}\right) = \frac{2Z_0}{Z_0 + Z_L}$

Hence $\quad \frac{V_0 + V_r}{V_0 - V_r} = \frac{2Z_L}{Z_0 + Z_L} \cdot \frac{Z_0 + Z_L}{2Z_0} = \frac{Z_L}{Z_0}$

from which $\quad \frac{V_0 + V_r}{I_0 - I_r} = \frac{V_0 + V_r}{V_0 - V_r} \cdot \frac{V_0}{I_0} = \frac{Z_L}{Z_0} \cdot Z_0 = Z_L$ (18.11)

Hence $\boxed{\frac{V}{I}_{x=L} = Z_L}$ (18.14)

Thus the ratio of the reflected to the incident voltage depends upon the load impedance and the ratio of reflected to incident currents. But as an alternative view, Ohm's application (18.14) and (18.11) it seems that either $Z_L = Z_0$, $\Gamma = 0$ and there is no reflection.

> **Problem 11.** A cable whose line is characteristic impedance of 75 Ω is terminated in a 150 Ω resistive load. Assuming that the cable has negligible losses and that voltage measured across the terminating load is 10 V, calculate the value of incident voltage and current at the load. If this incident voltage is in the incident voltage, find the reflected current and the other reflected voltage.



Figure 4.4.1

(a) Using equation (4.4.3),

$$\text{reflection coefficient, } r = \frac{Z_2 - Z_1}{Z_2 + Z_1} = \frac{75 - 100}{75 + 100} = \frac{-25}{175} = -0.143$$

(b) The circuit diagram is shown in Figure 4.4.1. Current flowing in the terminating load:

$$I_2 = \frac{V_2}{Z_2} = \frac{10}{100} = 0.10 \text{ A}$$

Reflected current $I_1 = -rI_2 = 0.143 \times 0.1$

Thus $I_2 = 0.1 - rI_2 = 0.243 - 0.1$

From which **impedance seen by** I_1 $= \dfrac{V_1}{I_1 + rI_2}$

$$= \frac{10.0}{1.1 \times 0.143} = \textbf{reflected at } x = \textbf{Ans. as a}$$

(c) From equation (4.4.3),

Incident voltage, $V_{-1} = I_1 Z_1 = 0.143 \times 75 = 10.725$ V

(d) Since $V_2 = 0$, V_1

reflected voltage, $V_{-1} = -rV_1 = -0.143 \times 10.725$

$$= -1.5 \text{ V}, V_2 = -10.5 \text{ V}$$

(e) From equation (4.4.4),

reflected voltage, $V_{-1} = -rV_1 = -0.143 \times 10.725 = 1.53$ V

> For Short $x = 0$ Only Determine the Line Z_1 characteristic impedance $Z_1 = 75\Omega$ is used to terminate in the impedance of $Z_2 = 100\Omega$ and the load Z_2 with $I = 100\Omega$. Determine the magnitude of the reflection coefficient at each load.

(a) From equation (4.4.3), reflection coefficient

$$r = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

When $Z_1 = 75\Omega$ $= 100\Omega$ and $Z_2 = 175\Omega = 100\Omega$

$$= \frac{100\Omega - 100\Omega}{100\Omega + 100\Omega} = \frac{0\Omega}{200\Omega} = 0.0$$

Hence the magnitude of the reflection coefficient $|r| = \textbf{0.00}$

(b) Writing $v_1 = 100\sin(\ldots)$ and $v_2 = 100\sin(\ldots - 90^\circ)$ in phasor form,

$$\mathbf{V}_1 = \frac{100}{\sqrt{2}}\angle 0^\circ = 70.71\angle 0^\circ \text{ V}, \qquad \mathbf{V}_2 = \frac{100}{\sqrt{2}}\angle -90^\circ = 70.71\angle -90^\circ \text{ V}$$

Because the magnitudes of the reference conditions are $\mathbf{V}_{\text{ref}} = 0.775\angle 0^\circ$ V,

> **Problem 2.11** A 25-mH inductor is connected to an ac source operating at 60 Hz. If the current through the inductor is $i(t) = 2\sin(377t + 30^\circ)$ A, determine (a) the impedance of the inductor, (b) the voltage across the inductor, and (c) the energy stored in the inductor at $t = 0$.

(a) The impedance of an inductor is $\mathbf{Z}_L = j\omega L$. Thus, the impedance of the inductor in this problem is given by

$$\omega = 377 \text{ rad/s}, \qquad \mathbf{Z}_L = j\omega L = j(377)(25 \times 10^{-3}) = j9.425\ \Omega$$

(b) Since $\mathbf{I}_1$ is the phasor form of i_1, the voltage $\mathbf{V}_1$ across the inductor is

$$\mathbf{V}_1 = \mathbf{Z}_L\mathbf{I}_1 = j\omega L\mathbf{I}_1$$

Given $\mathbf{I}_1$ is the phasor form of $i(t) = 2\sin(377t + 30^\circ)$ A, so $\mathbf{I}_1 = 1.414\angle 30^\circ$ A, and the corresponding impedance is $\mathbf{Z}_L = j9.425\ \Omega$. Thus,

$$\mathbf{V}_{\text{ref}} = \frac{\mathbf{V}_1}{\mathbf{V}_{\text{ref}}} = \frac{j\omega L\mathbf{I}_1}{\mathbf{V}_{\text{ref}}} = \frac{(j9.425)(1.414\angle 30^\circ)}{0.775\angle 0^\circ} = \frac{9.425\angle 90^\circ \times 1.414\angle 30^\circ}{0.775\angle 0^\circ}$$

$$= 17.19\angle 120^\circ \text{ V} \quad \text{(rms value)}$$

Hence, the magnitude of the voltage $\mathbf{V}_{\text{ref}}$ is 17.19 V.

(c) Since $\mathbf{V} = L\,d\mathbf{I}/dt$ or $\mathbf{I} = \frac{1}{L}\int \mathbf{V}\,dt$,

$$\text{Instantaneous voltage, } \mathbf{V} = \frac{1}{L}\int_0^t \mathbf{V}\,dt + \mathbf{I}(0)$$

Since initial $\mathbf{I}(0) = 1.414\angle 30^\circ$ V,

$$\mathbf{V} = \frac{100\sqrt{2}}{(25 \times 10^{-3})(377)}\sin(377t + 30^\circ - 90^\circ) + 1.414\angle -30^\circ \text{ V}$$

Further examples and applications of this topic may be found in Section 2.6 of the accompanying text, on page 373.

2.10 Nodal and Mesh Analysis

Circuit analysis involves determining unknown quantities (voltages or currents) in a circuit. An obvious method of analysis is to apply Kirchhoff's laws directly to the circuit. Although this is a valid approach, it may lead to a lengthy calculation. Nodal analysis, which is based on Kirchhoff's current law, is the first of two techniques we will consider in this section. Mesh analysis, which is based on Kirchhoff's voltage law, is the second technique.

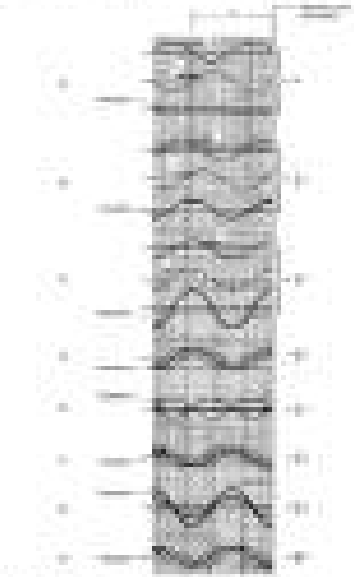


Figure 10.11 Isochoric heating/cooling cycle and isobaric heating/cooling cycle

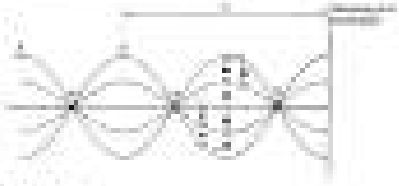


Figure 10.1.1



Figure 10.17 Voltage standing wave ratio including transmission line

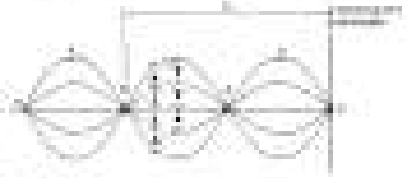


Figure 18.17



Figure 18.18

Note: Figures 18.1 and 18.18. The two methods are equal in the significance of the transducing element in Figure 18.1 and 18.18, except that the latter is more limited by [illegible] In order to [illegible] [illegible] [illegible]

Where ΔH_{vap} is the molar enthalpy of vaporization, R is the gas constant, and T_1 and T_2 are the two temperatures in kelvin. Rearranging to solve for the vapor pressure at a new temperature:

$$\ln\left(\frac{P_2}{P_1}\right) = \frac{\Delta H_{vap}}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

Worked example: Water has a vapor pressure of 23.8 mmHg at 25 °C. Its enthalpy of vaporization is 40.7 kJ/mol. Find the vapor pressure at 100 °C.

$$\ln\left(\frac{P_2}{23.8}\right) = \frac{40{,}700}{8.314}\left(\frac{1}{298.15} - \frac{1}{373.15}\right)$$

$$\ln\left(\frac{P_2}{23.8}\right) = 4895 \times (0.003354 - 0.002680) = 4895 \times 0.000674 = 3.299$$

$$\frac{P_2}{23.8} = e^{3.299} = 27.1$$

$$P_2 = 23.8 \times 27.1 = 645 \text{ mmHg}$$

The experimental value is 760 mmHg (1 atm), so the approximation is reasonable but underestimates by about 15%. The error arises because ΔH_{vap} itself varies with temperature, and the Clausius–Clapeyron equation treats it as constant.

Boiling point elevation is the increase in the boiling point of a solvent when a nonvolatile solute is dissolved in it. The effect is proportional to the molality of the solute particles. For a solute that does not dissociate, $\Delta T_b = K_b \cdot m$, where K_b is the ebullioscopic constant of the solvent and m is the molality. For ionic solutes, multiply by the van't Hoff factor i, which is the number of particles produced per formula unit.

Freezing point depression is the decrease in the freezing point of a solvent when a solute is dissolved in it. The relationship is $\Delta T_f = K_f \cdot m \cdot i$, where K_f is the cryoscopic constant of the solvent. Like boiling point elevation, this effect depends only on the number of particles, not on their identity. This is why salt is spread on icy roads: it lowers the freezing point of water, causing ice to melt at temperatures well below 0 °C.

Example: A solution contains 5.85 g of NaCl (molar mass 58.44 g/mol) dissolved in 1.00 kg of water. Calculate the freezing point depression. (K_f for water = 1.86 °C·kg/mol)

$$m = \frac{5.85 \text{ g}}{58.44 \text{ g/mol}} = 0.100 \text{ mol/kg}$$

$$i = 2 \text{ (NaCl dissociates into } Na^+ \text{ and } Cl^-\text{)}$$

$$\Delta T_f = 1.86 \times 0.100 \times 2 = 0.372 \text{ °C}$$

The freezing point is lowered by 0.372 °C, so the solution freezes at −0.372 °C instead of 0 °C.

Definition. *A colligative property* is a property of a solution that depends only on the number of dissolved particles per unit mass of solvent, not on the identity of the particles. The four classical colligative properties are vapor pressure lowering, boiling point elevation, freezing point depression, and osmotic pressure.

(a) Derive an equation for k in terms of $[A]_0$, $[A]_t$, and t.

$$\text{differential equation: } -\frac{d[A]}{dt} = k[A]^2 \quad \Rightarrow \quad \int_{[A]_0}^{[A]_t} \frac{d[A]}{[A]^2} = -\int_0^t k\,dt$$

$$\Rightarrow \left[-\frac{1}{[A]}\right]_{[A]_0}^{[A]_t} = -kt \quad \Rightarrow \quad -\frac{1}{[A]_t} + \frac{1}{[A]_0} = kt$$

$$\Rightarrow k = \frac{\frac{1}{[A]_t} - \frac{1}{[A]_0}}{t} = \frac{\frac{[A]_0 - [A]_t}{[A]_t[A]_0}}{t} = \frac{[A]_0 - [A]_t}{t[A]_t[A]_0}$$

Answer: $k = \dfrac{[A]_0 - [A]_t}{t[A]_t[A]_0}$ or $\dfrac{1}{t}\left(\dfrac{1}{[A]_t} - \dfrac{1}{[A]_0}\right)$

(b) Determine the value of k (in $\mathrm{L\,mol^{-1}\,s^{-1}}$) from these measurements at 25 °C.

$$\text{integrated rate form: } \frac{1}{[A]_t} = kt + \frac{1}{[A]_0} \quad \Rightarrow \quad \frac{1}{0.0500\ \mathrm{M}} = k(120\ \mathrm{s}) + \frac{1}{0.100\ \mathrm{M}}$$

$$\Rightarrow 20.0\ \mathrm{M^{-1}} = k(120\ \mathrm{s}) + 10.0\ \mathrm{M^{-1}} \quad \Rightarrow \quad k = \frac{10.0\ \mathrm{M^{-1}}}{120\ \mathrm{s}} = 0.0833\ \mathrm{L\,mol^{-1}\,s^{-1}}$$

Problem 17 — A reaction mechanism for the decomposition of ozone proposed by the Dutch chemist Johannes de Boer in 1937 is $2\,O_3 \rightleftharpoons 3\,O_2$. The rate-determining step is the bimolecular collision of two ozone molecules to form an intermediate O_4 that quickly reverts. Derive the rate expression, and state the overall reaction order, for the forward reaction, and for the overall reaction order of the reverse reaction.

(c) Derive an equation for $t_{1/2}$ in terms of k and $[A]_0$.

$$\textbf{doubling time in words: } t_{1/2} = \frac{1}{k[A]_0} \quad \Rightarrow \quad t_{1/2} = \frac{1}{k[A]_0} \quad \Rightarrow \quad t_{1/2} = \frac{1}{k[A]_0} \quad \Rightarrow \quad t_{1/2} = \frac{1}{k[A]_0}$$

(d) Derive an equation for k in terms of $[A]_0$ and $t_{1/2}$. $\quad k = \dfrac{[A]_0 - [A]_0}{t_{1/2}[A]_0[A]_0}$

Determine the value of k (in $\mathrm{L\,mol^{-1}\,s^{-1}}$) from these measurements at 25 °C.

Data table:

$[A]_0$ (M)	$t_{1/2}$ (s)
0.100	120
0.200	60.0
0.400	30.0

Work: $k = \dfrac{1}{t_{1/2}[A]_0} = \dfrac{1}{(120\ \mathrm{s})(0.100\ \mathrm{M})} = \dfrac{1}{12.0\ \mathrm{M\,s}} = 0.0833\ \mathrm{L\,mol^{-1}\,s^{-1}}$

$$= \frac{1}{(60.0\ \mathrm{s})(0.200\ \mathrm{M})} = \frac{1}{12.0\ \mathrm{M\,s}} = 0.0833\ \mathrm{L\,mol^{-1}\,s^{-1}}$$

$$= \frac{1}{(30.0\ \mathrm{s})(0.400\ \mathrm{M})} = \frac{1}{12.0\ \mathrm{M\,s}} = 0.0833\ \mathrm{L\,mol^{-1}\,s^{-1}}$$

$$= 0.0833\ \mathrm{L\,mol^{-1}\,s^{-1}}$$

Answer: $k = 0.0833\ \mathrm{L\,mol^{-1}\,s^{-1}}$ (constant, as expected for a second-order reaction)

(e) Derive an equation for $[A]_t$ in terms of k, $[A]_0$, and t.

$$\frac{1}{[A]_t} = kt + \frac{1}{[A]_0}$$

$$\text{Answer: } \frac{1}{[A]_t} = \frac{kt[A]_0 + 1}{[A]_0} = \frac{kt + \frac{1}{[A]_0}}{1} = kt + \frac{1}{[A]_0}$$

Key: the **identical answer** to (c) $= 0.0833\ \mathrm{L\,mol^{-1}\,s^{-1}}$ (Answer)

Problem 6.1. Two skydivers with a combined mass of $m = 190\ \text{kg}$ fall vertically at terminal speed, wearing identical rectangular canopies. Each canopy has drag coefficient $C_D = 1.5$ and area $A = 35\ \text{m}^2$. The air density is $\rho = 1.225\ \text{kg/m}^3$. Take $g = 9.81\ \text{m/s}^2$.

(a) Derive an expression for the terminal speed v_t in terms of m, g, ρ, C_D, and A, and evaluate it numerically.

(b) The skydivers release a pilot chute that increases the total drag coefficient to $C_D = 2.0$ and the area to $A = 50\ \text{m}^2$. Find the new terminal speed v_t'.

(c) Suppose the skydivers instead open a single canopy with $C_D = 1.5$ and $A = 35\ \text{m}^2$ at an altitude of $h = 1200\ \text{m}$. Assuming they start from rest and reach terminal speed almost immediately, how long does it take them to fall the first $1200\ \text{m}$? Use $v(t) = v_t \tanh(gt/v_t)$.

Solution.

(a) At terminal speed the net force is zero, so drag balances weight:

$$\frac{1}{2}\rho C_D A v_t^2 = mg \quad \Longrightarrow \quad v_t = \sqrt{\frac{2mg}{\rho C_D A}}.$$

Substituting the values,

$$v_t = \sqrt{\frac{2(190)(9.81)}{(1.225)(1.5)(35)}} = \sqrt{\frac{3727.8}{64.3125}} \approx 7.61\ \text{m/s}.$$

(b) With the pilot chute, $C_D = 2.0$ and $A = 50\ \text{m}^2$, so

$$v_t' = \sqrt{\frac{2(190)(9.81)}{(1.225)(2.0)(50)}} = \sqrt{\frac{3727.8}{122.5}} \approx 5.52\ \text{m/s}.$$

(c) Integrating $v(t) = v_t \tanh(gt/v_t)$ gives the position

$$y(t) = \frac{v_t^2}{g} \ln\cosh\!\left(\frac{gt}{v_t}\right).$$

Setting $y(t) = 1200\ \text{m}$ and solving numerically (or by inverting $\cosh$) yields $t \approx 166\ \text{s}$, about $2.8\ \text{min}$.

6.1.2 Position, velocity, and acceleration vectors

Planar motion: arc length and velocity of a projectile.

1. A projectile is launched from the origin with a speed of $v_0 = 20\ \text{m/s}$ at an angle $\theta = 60^\circ$ above the horizontal. Neglecting air resistance, find the time of flight, the maximum height reached, and the horizontal range. Take $g = 9.81\ \text{m/s}^2$.

   Solution. Resolve the initial velocity into components: $v_{0x} = v_0\cos\theta = 20\cos 60^\circ = 10\ \text{m/s}$ and $v_{0y} = v_0\sin\theta = 20\sin 60^\circ \approx 17.32\ \text{m/s}$. The time of flight is the time for the vertical coordinate to return to zero, $y(t) = v_{0y}t - \tfrac{1}{2}gt^2 = 0$, which gives $t = 2v_{0y}/g \approx 3.53\ \text{s}$. The maximum height occurs when $v_y = 0$, i.e. at $t = v_{0y}/g \approx 1.77\ \text{s}$, and equals $y_{\max} = v_{0y}^2/(2g) \approx 15.30\ \text{m}$. The range is $R = v_{0x} \cdot t_{\text{flight}} \approx 35.3\ \text{m}$.

2. A pendulum bob has mass $m = 0.250\ \text{kg}$ and is suspended by a massless rod of length $L = 1.20\ \text{m}$. It is released from rest at an angle $\theta_0 = 30^\circ$ from the vertical. Assuming small-angle motion, find the period of oscillation and the speed of the bob as it passes through the lowest point. Take $g = 9.81\ \text{m/s}^2$.

   Solution. For small angles the motion is simple harmonic with angular frequency $\omega = \sqrt{g/L}$, so the period is $T = 2\pi\sqrt{L/g} = 2\pi\sqrt{1.20/9.81} \approx 2.20\ \text{s}$. Energy conservation gives the speed at the lowest point as $v = \sqrt{2gL(1-\cos\theta_0)} = \sqrt{2(9.81)(1.20)(1-\cos 30^\circ)} \approx 1.78\ \text{m/s}$.

3. A ball of mass $m = 0.145\ \text{kg}$ is thrown straight upward with an initial speed of $v_0 = 25.0\ \text{m/s}$ from a height $h_0 = 2.00\ \text{m}$ above the ground. Neglecting air resistance, find the maximum height reached above the ground, the time at which the ball returns to the ground, and the speed with which it strikes the ground. Take $g = 9.81\ \text{m/s}^2$.

Uniform and Gauss elimination.

1. Write the coefficient matrix of a cubic $y = ax^3 + bx^2 + cx + d$ that interpolates the points $(-1, 0)$, $(0, 1)$, $(1, 0)$, $(2, 5)$. Row-reduce the augmented matrix to find a, b, c, d. Then check that the resulting polynomial passes through all four points.

2. Solve the system
$$\begin{aligned} 2x + y - z &= 8, \\ -3x - y + 2z &= -11, \\ -2x + y + 2z &= -3. \end{aligned}$$
Record the sequence of row operations you used, and verify your answer by substitution.

3. A rectangular box with a square base and no top is to be built from $108\ \text{m}^2$ of material. Let x be the side length of the base and h the height. Express the volume V as a function of x alone, and find the dimensions that maximize the volume. Justify that your critical point is a maximum.

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Whitney Sum and Characteristic Class

11.1 Let ξ satisfy Axiom (iii). Additivity. Consider a vector bundle ξ over the product $B_1 \times B_2$ of two spaces. Let ξ_1 be the bundle over B_1 induced by the projection $B_1 \times B_2 \to B_1$, and let ξ_2 be the bundle over B_2 induced by the projection $B_1 \times B_2 \to B_2$. Then

$$\xi = \xi_1 \times \xi_2.$$

11.2 Let ξ be a vector bundle over a space B. Let ξ' be the bundle over $B \times I$ induced by the projection $B \times I \to B$. Let ξ'' be the bundle over $B \times I$ induced by the projection $B \times I \to B$ which sends (b, t) to $(b, 1)$. Let ξ_0 be the bundle over B induced by the inclusion $B \to B \times I$ which sends b to $(b, 0)$. Let ξ_1 be the bundle over B induced by the inclusion $B \to B \times I$ which sends b to $(b, 1)$. Then $\xi_0 = \xi_1$ if and only if ξ' and ξ'' are equivalent.

Whitney Duality and Products

11.3 Let ξ and η be vector bundles over the same base space B. Then the Whitney product $\xi \oplus \eta$ is the vector bundle over B whose fiber over $b \in B$ is the direct sum of the fiber of ξ over b and the fiber of η over b. The Whitney product $\xi \oplus \eta$ is also called the Whitney sum of ξ and η. The Whitney product is commutative and associative. If ξ is the trivial bundle of dimension n over B, then $\xi \oplus \eta$ is isomorphic to the bundle obtained by adding n to the dimension of each fiber of η. Let ξ and η be vector bundles over B. Let ξ' and η' be the bundles induced by the projection $B \times B \to B$ which sends (b_1, b_2) to b_1. Let ξ'' and η'' be the bundles induced by the projection $B \times B \to B$ which sends (b_1, b_2) to b_2. Then

$$\xi \times \eta = \xi' \oplus \eta''.$$

11.4 Let ξ and η be vector bundles over the same base space B. Let $\Delta : B \to B \times B$ be the diagonal map which sends b to (b, b). Then

$$\xi \oplus \eta = \Delta^*(\xi \times \eta).$$

11.5 Let ξ be a vector bundle over a space B. Let ξ' be the bundle over $B \times I$ induced by the projection $B \times I \to B$. Let ξ_0 be the bundle over B induced by the inclusion $B \to B \times I$ which sends b to $(b, 0)$. Let ξ_1 be the bundle over B induced by the inclusion $B \to B \times I$ which sends b to $(b, 1)$. Then $\xi_0 = \xi_1$ if and only if ξ' and ξ'' are equivalent.

Classifying Vector Bundles

11.6 Let ξ be a vector bundle over a space B. Let ξ' be the bundle over $B \times I$ induced by the projection $B \times I \to B$. Let ξ_0 be the bundle over B induced by the inclusion $B \to B \times I$ which sends b to $(b, 0)$. Let ξ_1 be the bundle over B induced by the inclusion $B \to B \times I$ which sends b to $(b, 1)$. Then $\xi_0 = \xi_1$ if and only if ξ' and ξ'' are equivalent.

11.7 Let ξ be a vector bundle over a space B. Let ξ' be the bundle over $B \times I$ induced by the projection $B \times I \to B$. Let ξ_0 be the bundle over B induced by the inclusion $B \to B \times I$ which sends b to $(b, 0)$. Let ξ_1 be the bundle over B induced by the inclusion $B \to B \times I$ which sends b to $(b, 1)$. Then $\xi_0 = \xi_1$ if and only if ξ' and ξ'' are equivalent.

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45 Transients and Laplace transforms

45.1 Introduction

45.2 Response of R–C series circuit to a step input

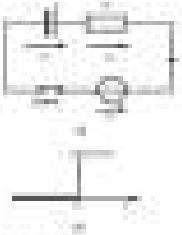


Figure 10.5.1

Using Kirchhoff's voltage law, we get the equation

$$E = v_C + v_R \tag{10.5.1}$$

Voltage $v_C = \frac{1}{C}\int i\,dt$ and voltage $v_R = iR$. Therefore, $v_R = R\frac{dq}{dt}$.

Substituting these equations (10.5.1)

$$E = v_C + iR = \frac{q}{C} + R\frac{dq}{dt} \tag{10.5.2}$$

This is a linear, constant coefficient, first order differential equation. Such a differential equation can be solved by any one of the methods for solving a first order linear differential equation (Separable, Exact, Integrating Factor, Homogeneous, or Bernoulli Equation). Let's solve it by separable.

Solving the equation (10.5.2) by separable:

$$E = v_C + iR = \frac{q}{C} + R\frac{dq}{dt}, \text{ and}$$

$$\frac{dq}{dt} = \frac{E - v_C}{R}, \text{ (this is the DE)}$$

$$\frac{dv_C}{E - v_C} = \frac{dt}{RC}$$

and integrating both sides gives $\int \frac{dv_C}{E - v_C} = \int \frac{dt}{RC}$

Hence $-\ln|E - v_C| = \frac{t}{RC} + k$ (10.5.3)

where k is the arbitrary constant of integration.

Or, alternatively, $\int \frac{dv_C}{v_C - E} = -\int \frac{dt}{RC}$ which is another alternative.

$v_C = E - ce^{-t/RC}$ (see Engineering Mathematics or Higher Engineering Mathematics)

When $t = 0$, $v_C = 0$ (since $i = 0$) (10.5.4)

Then from equation (10.5.3) $-\ln|E| = k = \frac{1}{RC}$ (10.5.5)

Substituting gives:

$$\ln|E - v_C| - \ln|E| = -\frac{t}{RC}$$

$$\ln\frac{E - v_C}{E} = -\frac{t}{RC}$$ for the case of equation

Or, $\frac{E - v_C}{E} = e^{-t/RC}$

And $\frac{E - v_C}{E} = \frac{v_C}{E} = 1 - e^{-t/RC}$

$v_C = E(1 - e^{-t/RC})$



Figure 8.11

The rate of change of v_C is given by $\frac{dv_C}{dt} = \frac{V}{RC}e^{-t/RC}$. (8.11)

This is an exponential curve which starts at zero and rises to a value V as shown in Figure 8.11.

At any instant in the charging process, $V = v_C + v_R$

$$\therefore v_R = V - v_C = V - V\left(1 - e^{-t/RC}\right)$$

$$v_R = Ve^{-t/RC}$$ (from equation (8.7))

The rate of change of v_R is given by $\frac{dv_R}{dt} = -\frac{V}{RC}e^{-t/RC}$ (8.12)

This is an exponential decay curve as shown in Figure 8.12.



Figure 8.12

On the circuit of Figure 8.9 the capacitor C is replaced by a resistance R.

Hence $i = \frac{V}{R}\left(1 - e^{-t/RC}\right)$ (from equation (8.6))

i.e. $i = \frac{V}{R} - \frac{V}{R}e^{-t/RC}$

$$= \frac{V}{R} - \frac{V}{R}\left(e^{-t/RC}\right)$$

$$= \frac{V}{R}\left(1 - e^{-t/RC}\right)$$

i.e. $i = v_C/R$ (8.13)



Figure 8.13

where $\frac{V}{R}$ is the steady state current, I.

This is an exponential decay curve as shown in Figure 8.13.

After a period of time from the instant when the capacitor C is replaced by R in Fig. 8.9, the rise in voltage across the capacitor, v_C, ceases to rise and v_C becomes V. The capacitor voltage v_C is then equal to the supply voltage V and current i has decayed to zero.

> **Problem 1** — A 100 pF capacitor is connected in series with a 0.25 MΩ resistor. Rectify circuit is connected to a 50 V d.c. supply. Calculate (a) the initial value of transient current, (b) the value of capacitor voltage 50 ms after connection, and (c) the time after connection when the capacitor voltage is 37 V.

(a) From equation (6.5.1), solution is $i = \frac{E}{R}\left(1 - e^{-Rt/L}\right)$.

Solution example Here, inductance $L = 0.5$ H, $R = \frac{E}{I_{max}} = \frac{12}{4} = 3\ \Omega$, $\frac{R}{L} = \frac{3}{0.5} = 6$.

$$i = 4\left(1 - e^{-6t}\right)\ \text{A}$$

(b) Current, $i = \frac{E}{R}\left(1 - e^{-Rt/L}\right)$ Energy stored when $i = 0.95I$ and so $1 - e^{-Rt/L} = 0.95$

$$e^{-Rt/L} = 0.05 \quad \text{so} \quad -\frac{Rt}{L} = \ln 0.05 = -2.9957 \quad \text{i.e.} \quad t = \frac{2.9957L}{R}$$

$$= 0.4\left(1 - e^{-3t}\right)^2 \text{ so } 0.4\left(0.95\right)^2 = \mathbf{0.3610\ J} \text{ or } \mathbf{0.361\ J}$$

(c) From equation (6.5.1), capacitor voltage $v_C = E\left(1 - e^{-t/CR}\right)$

When $v_C = 0.95E$ then $e^{-t/CR} = 0.05$ (taking natural logarithms)

$$-\frac{t}{CR} = \ln 0.05 = -2.9957$$

$$t = 2.9957CR = 2.9957 \times 0.5 \times 3$$

$$= \mathbf{4.496\ s}$$

(d) From equation (6.5.1) capacitor voltage $v_C = E\left(1 - e^{-t/CR}\right)$

When $v_C = 0.5E$,

Then $0.5 = 1 - e^{-t/CR}$ (rearranging and taking logarithms)

i.e. $-\frac{t}{CR} = \ln 0.5$

and $\frac{t}{CR} = \frac{1}{0.6931}$

Also $CR = 0.5 \times 3 = 1.5$ s

Thus $t = 1.5 \times 0.6931$

$$= \mathbf{1.04\ s}$$

Kirchhoff's law or **capacitor**

If when a current i flows the capacitor voltage V applied to the circuit is Figure 6.5.1 a suitable resource for learning the Kirchhoff's law

From equation (6.5.1), $v_C = E - Ri$

(a) From equation (6.5.1), $i = C\frac{dv_C}{dt} + v_C = 0$

Rearranging gives $\frac{dv_C}{dt} = \frac{-v_C}{CR}$

(b) separating the variables gives $\frac{dv_C}{v_C} = -\frac{dt}{CR}$

(c) integrating both sides gives $\int \frac{dv_C}{v_C} = \int -\frac{dt}{CR}$

Now when $t = 0$, $v_c = 0$, so

$$0 = -\frac{1}{CR} + k \qquad (8.7.5)$$

where k is a constant.

So that $k = 1/CR$ e.g. (i.e. the solution of replacing the variable $v_c = k$ by 0).

Substituting $k = 0$ and $v_c = 0$ in equation (8.7.5) gives:

$$0 = 0 + k$$

Substituting $k = 1/CR$ into equation (8.7.5) gives:

$$k = -\frac{1}{CR} + \frac{1}{CR}$$

$$\ln\left[\frac{v_c - k}{-\frac{1}{CR}}\right] = -\frac{t

10.5 Illustration of Ex. 2: simple network for a given input



Figure 10.6

Initial value problem

A series RC-circuit is shown in Figure 10.6. Write the initial value problem for the current $I(t)$ if the capacitor is uncharged at time $t = 0$ and the applied voltage is $E(t) = 100 \sin 2t$. Solve the problem.

Voltage drop across the capacitor: Q/C and voltage drop across the resistor: RI. Hence

$$RI + \frac{Q}{C} = 100 \sin 2t. \tag{10.5.1}$$

This is a linear, nonhomogeneous first-order differential equation. Since I and Q are unknown, we must eliminate one of them. Using $I = dQ/dt$ gives the equation

$$R\frac{dQ}{dt} + \frac{1}{C}Q = 100 \sin 2t.$$

Now divide by R:

$$\frac{dQ}{dt} + \frac{1}{RC}Q = \frac{100}{R}\sin 2t.$$

Then multiply by the integrating factor $e^{t/RC}$:

$$\frac{d}{dt}\left(e^{t/RC}Q\right) = \frac{100}{R}e^{t/RC}\sin 2t.$$

Integrate:

$$e^{t/RC}Q = \frac{100}{R}\int e^{t/RC}\sin 2t\,dt + K.$$

Use integration by parts twice to find

$$\int e^{at}\sin bt\,dt = \frac{e^{at}}{a^2+b^2}(a\sin bt - b\cos bt) + C.$$

Hence

$$Q = \frac{100}{R}\cdot\frac{1}{(1/RC)^2+4}\left(\frac{1}{RC}\sin 2t - 2\cos 2t\right) + Ke^{-t/RC}.$$

Finally, $I = dQ/dt$ gives

$$I = \frac{100}{R}\cdot\frac{1}{(1/RC)^2+4}\left(\frac{1}{RC}\cdot\frac{1}{RC}\sin 2t - \frac{2}{RC}\cos 2t + 2\cos 2t\right) - \frac{K}{RC}e^{-t/RC}$$

$$= \frac{100}{R}\cdot\frac{1}{(1/RC)^2+4}\left(\frac{1}{RC}\sin 2t + \left(2 - \frac{2}{RC}\right)\cos 2t\right) - \frac{K}{RC}e^{-t/RC}.$$



Figure 7.1

And therefore $\boxed{i = \frac{V}{R}\left(1 - e^{-t/RC}\right)}$ (7.1.1)

[It is an interesting geometric curve as shown in Figure 7.1]
[It is of course the inverse of the curve in Figure 7.2]

Where $\tau = RC = \frac{CV}{\frac{V}{R}} = \frac{\text{Coulombs}}{\text{Amps}} = \text{Seconds}$ [See equation (7.1.1)]

i.e. $\boxed{\tau_{\text{seconds}} = RC_{\text{ohm-farads}}}$ (7.1.2)

which again corresponds to our previous dimensional analysis.

[An integral version of equation (7.1.1) is $\left(\frac{dq}{dt}\right)$

$$\begin{aligned} \text{i.e. } q &= \frac{V}{R}\int_0^t e^{-t/RC}\,dt = CV\left(-\frac{RC}{R}e^{-t/RC}\right)_0^t \\ &= \frac{CV}{R}\left[-e^{-t/RC}\right]_0^t = \left(\frac{CV}{R}\right)\left(\frac{R}{V}\right)\left(1 - e^{-t/RC}\right) \end{aligned}$$

i.e. $\boxed{q_{(t)} = Q\left(1 - e^{-t/RC}\right)}$ (7.1.3)

Problem 1 — A coil of inductance 10 mH and resistance 5 Ω is connected to a 12 V, 0 Ω supply. Determine (a) the time taken for current to reach 63% of its final value, (b) the current after 4 ms, (c) the value of the voltage across the inductance after 2 ms, and (d) the time taken for the current to reach 75% of its final value.

(a) From equation (7.1.2), when i is 63% of its final value, the time t equals one inductive time constant, i.e.

$$t = \frac{L}{R} = \frac{10 \times 10^{-3}}{5} = 2\text{ ms}$$

(b) From equation (7.1.1), current $i = \frac{V}{R}\left(1 - e^{-t/(L/R)}\right)$

$$\text{Thus } i = 2.4\left(1 - e^{-4/2}\right) = 2.4\left(1 - e^{-2}\right) = 2.4(1 - 0.135) = 2.074\text{ A}$$

(c) From equation (6.1.3), the voltage across the inductance

$$v_{(t)} = V_0 e^{-t/\tau}$$

Weight — Gravity pull on a mass; $W = mg$, where $g \approx 9.8\ \text{m/s}^2$ near Earth's surface.

Mass — Amount of matter in an object; measured in kilograms (kg).

Inertia — Tendency of a body to resist changes in its motion; more massive objects have more inertia.

Force — A push or pull that can change an object's motion; measured in newtons (N).

Net force — The overall force on an object after all individual forces are combined; $F_{net} = ma$.

Balanced forces — Forces that cancel out so the net force is zero; the object's motion does not change.

Unbalanced forces — Forces that do not cancel out; they produce acceleration in the direction of the net force.

Friction — A force that opposes motion between surfaces in contact; $f = \mu N$, where μ is the coefficient of friction.

Normal force — The perpendicular support force a surface exerts on an object resting on it; $N = mg$ on a flat, level surface.

Tension — The pulling force transmitted through a rope, string, or cable when it is pulled tight.

Acceleration — The rate at which velocity changes; $a = \frac{\Delta v}{\Delta t}$, measured in m/s^2.

Velocity — Speed in a specific direction; a vector quantity, unlike speed.

Momentum — Mass in motion; $p = mv$, where v is velocity.

Impulse — The change in momentum produced by a force acting over time; $J = F\Delta t = \Delta p$.

Conservation of momentum — In a closed system with no external forces, total momentum before a collision equals total momentum after.

Work — Energy transferred when a force moves an object through a distance; $W = Fd\cos\theta$, measured in joules (J).

Kinetic energy — Energy of motion; $KE = \frac{1}{2}mv^2$.

Potential energy — Stored energy due to position; gravitational potential energy near Earth is $PE = mgh$.

Conservation of energy — Energy cannot be created or destroyed, only transformed; in a frictionless system, $KE + PE$ remains constant.

Power — The rate at which work is done or energy is transferred; $P = \frac{W}{t}$, measured in watts (W).

Efficiency — The ratio of useful output energy to total input energy, usually expressed as a percentage.

Simple machine — A basic device that changes the direction or size of a force to make work easier; the six classical types are lever, wheel and axle, pulley, inclined plane, wedge, and screw.

Mechanical advantage — How many times a machine multiplies the input force; $MA = \frac{F_{out}}{F_{in}}$.

Ideal mechanical advantage — Mechanical advantage of a machine with no friction; $IMA = \frac{d_{in}}{d_{out}}$.

Lever — A rigid bar that pivots around a fixed point called the fulcrum; classified by the positions of the fulcrum, load, and effort.

Fulcrum — The pivot point on which a lever rotates.

Load — The resistance force a machine must overcome; the weight being lifted or moved.

Effort — The force applied to a machine to do work; also called the input force.

Pulley — A wheel with a grooved rim that guides a rope or cable; a single fixed pulley changes direction, while a movable pulley reduces the force needed.

Inclined plane — A flat surface set at an angle; it trades a longer distance for a smaller force needed to raise a load.

Wedge — A moving inclined plane, usually with two sloping sides, used to split or push objects apart.

Screw — An inclined plane wrapped around a cylinder; the distance between threads is called the pitch.

Wheel and axle — Two connected circular parts that turn together; a small force on the wheel can move a heavier load on the axle.

Compound machine — A device made from two or more simple machines working together, such as scissors or a bicycle.

Elastic potential energy — Energy stored in a stretched or compressed spring; $PE_s = \frac{1}{2}kx^2$, where k is the spring constant and x is the displacement from equilibrium.

Hooke's law — The force a spring exerts is proportional to how far it is stretched or compressed; $F = -kx$, where the negative sign indicates the force opposes the displacement.

Spring constant — A measure of how stiff a spring is; a larger k means more force is needed to stretch it by a given amount.

Equilibrium position — The position where a spring exerts no force; the natural length of the spring.

Simple harmonic motion — Repeating back-and-forth motion in which the restoring force is proportional to the displacement; $T = 2\pi\sqrt{\frac{m}{k}}$.

Amplitude — The maximum displacement from equilibrium during oscillation; the distance from the center to the farthest point.

Period — The time for one complete oscillation; $T = \frac{1}{f}$, measured in seconds.

Frequency — The number of oscillations per second; $f = \frac{1}{T}$, measured in hertz (Hz).

(a) $i = 0$ A (i.e., when the switch is opened)

$t = 0.5\left(\frac{5}{10}\right)$ = the steady state current

Note that $t = 0.5$ s

Substituting $t = 0.5$ s into equation (a) in (c) gives:

$$i(0.5) = \frac{5}{10}(1 - 0.3)$$

Substituting gives: $i(t) = 0.5(1 - 0.3)$

$$i(t) = \frac{5}{10}$$

$$i(t) = \frac{5}{10}$$

$$i(t) = 0.5(1 - e^{-2t})$$

(d) Hence $i = 0.5(1 - e^{-2t})$ A (19.11)

(e) The current i shown exponentially in (c)

From Figure 19.6, $y = -e^{-t} + 0.5\left(\frac{5}{10}\right) = 0.5$ from equation (a) in (c)

i.e., $y = 0.5(1 - e^{-t})$ (19.12)

The 0.63 rule states that $y = -e^{-t} = -e^{-\left(\frac{t}{\tau}\right)} = -e^{-1}$

from equation (c) in (c)

$$= 0.5\left(\frac{5}{10}\right)(1 - e^{-1}) = \frac{5}{10}$$

Hence the magnitude of y at a point in $y = 0.5(1 - e^{-t})$ (19.13)

Rising from y and y shown exponentially in (c)

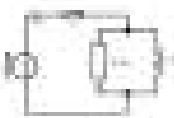


Figure 19.10

Problem 5. In the circuit shown in Figure 19.11, a current $i = 2$ A flows when the switch is closed. Derive an expression for the current i flowing in the circuit t seconds after the switch is opened. Determine the current i when $t = 0.1$ s, 0.2 s, 0.3 s and 0.4 s. Sketch the graph of current i against time t.

(a) When the switch is open the circuit is as shown in Figure 19.12. The equation for the current i is given by $i = I_0 e^{-\frac{t}{\tau}}$ where I_0 is the initial current flowing in the circuit and τ is the time constant of the circuit. $I_0 = 2$ A and $\tau = \frac{L}{R} = \frac{0.5}{10} = 0.05$ s

Hence $i = 2e^{-\frac{t}{0.05}} = 2e^{-20t}$ A



Figure 19.11

From Equation (4.17), with $k = \frac{1}{L}$, we have

$$i = Ae^{-\frac{R}{L}t}$$

At $t = 0$, $i = \frac{E}{R}$, the initial value of current, so

$$\frac{E}{R} = Ae^{-\frac{R}{L}\times 0} = A$$

Hence $i = \frac{E}{R}e^{-\frac{R}{L}t}$ as in (10.3), or

$$i = \frac{E}{R}e^{-kt}$$

where $k = \frac{R}{L}$

Also, as $t \to \infty$, $i \to 0$.

Thus when $E = 5$ V, $R = 2\,\Omega$ and $L = 0.5$ H, the initial value of current is $\frac{5}{2} = 2.5$ A and the current decays exponentially to zero as time increases.

(b) When the external voltage is zero and the circuit consists of just an R–L combination, the voltage across the resistor is $-iR$ and that across the inductor is $L\frac{di}{dt}$. Hence, from (10.2),

$$-iR = L\frac{di}{dt}$$

10.3 L–R–C series circuit: transient response

For the circuit shown in Figure 10.3, from Kirchhoff's voltage law,

$$E = v_L + v_R + v_C \qquad (10.4)$$



Figure 10.3

$$v_L = L\frac{di}{dt} \text{ and } i = C\frac{dv_C}{dt}, \text{ whence } v_L = L\frac{d}{dt}\left(C\frac{dv_C}{dt}\right) = LC\frac{d^2v_C}{dt^2}$$

$$v_R = iR = RC\frac{dv_C}{dt}, \text{ or } Ri = RC\frac{dv_C}{dt}$$

Substituting in (10.4) gives

$$E = LC\frac{d^2v_C}{dt^2} + RC\frac{dv_C}{dt} + v_C \qquad (10.5)$$

This is a linear, constant coefficient, second order differential equation (for the solution of second order differential equations see Higher Engineering Mathematics).

To determine the transient response, for example, v_C, E is made equal to zero, so

$$v_C + LC\frac{d^2v_C}{dt^2} + RC\frac{dv_C}{dt} = 0 \qquad (10.6)$$

a solution of the form $v_C = Ae^{mt}$ is tried, where A and m are constants.

$$\frac{dv_C}{dt} = Ame^{mt} \text{ and } \frac{d^2v_C}{dt^2} = Am^2e^{mt}$$

Introducing these eigenvalues into equation (18.11) gives

$$(D+\lambda_1)(D+\lambda_2)\cdots(D+\lambda_n)y = Ae^{kx} + B$$

or

$$a_0\frac{d^n y}{dx^n} + a_1\frac{d^{n-1}y}{dx^{n-1}} + \cdots + a_n y = Ae^{kx} + B$$

Then $y = Ae^{kx}$ is a solution of this linear equation provided that

$$a_0 k^n + a_1 k^{n-1} + \cdots + a_n \neq 0 \tag{18.12}$$

This is called the **auxiliary equation**.

Using the undetermined coefficients method, we have

$$y_{p_1} = \frac{B(1)}{a_n(1)} + \frac{a_{n-1}B}{a_n^2(1)} + \frac{(a_{n-1}^2 - a_n a_{n-2})B}{a_n^3(1)} + \cdots$$

$$y_{p_2} = -\frac{AB^n}{(B^n)} + \frac{AB^{n-1}}{a_n(B^{n-1})} - \frac{A}{a_n}\left[\frac{a_{n-1}B^{n-1}}{a_n(B^{n-1})} - \frac{a_{n-2}}{a_n(B^{n-2})}\right]$$

$$+ \frac{A}{a_n}\left[\frac{a_{n-1}}{a_n}\left[\frac{a_{n-1}}{a_n(B^{n-2})} - \frac{1}{(B^{n-2})}\right]\right] \tag{18.13}$$

The equations that have roots:

(i) k are **positive and unequal** when $B(2) > a_0 B(1)$ otherwise known as case (a) or the **overdamped** case, the response changes sign at most once, and then $y \to 0$.

(ii) k are **real and equal** when $B(2) = a_0 B(1)$ otherwise known as case (b) or the **critically damped** case, the response changes sign at most once in the transient domain of time before oscillations occur in $y(t)$.

(iii) k are **complex roots** when $B(2) < a_0 B(1)$ otherwise known as case (c) or the **underdamped** case, the response changes sign infinitely often and the real parts of the roots determine the frequency of the oscillations, i.e. $y \to 0$ as $t \to \infty$ for the steady state value.

(iv) A, B, $\ldots$ in equation (18.13) are constants whose values depend on the boundary conditions and y_{p_1} is known as **steady-state** solution.

Returning to the second-order differential equation (18.9) with boundary conditions $y(0) = 0$ and $y(L) = 0$, we have

> **Theorem 1** — If λ is an eigenvalue of the boundary-value problem $\frac{d^2y}{dx^2} + \lambda y = 0$, $y(0) = 0$, $y(L) = 0$, and $\phi(x)$ is a corresponding eigenfunction, then $\phi(x)$ is orthogonal to every solution of the differential equation $\frac{d^2y}{dx^2} + \lambda y = 0$ that is linearly independent of $\phi(x)$. In other words, $\phi(x)$ is the only solution (except for constant multiples) of the boundary-value problem that is orthogonal to every solution of the differential equation.

$$\text{(a)}\quad \left(\frac{R}{R_0}\right)^2 = \left[\frac{\rho}{\rho_0}\left(1+\alpha\Delta T\right)\right]^2 = \frac{\rho^2}{\rho_0^2}\left(1+\alpha\Delta T\right)^2 \approx \frac{\rho^2}{\rho_0^2}\left(1+2\alpha\Delta T\right)$$

$$\frac{\rho^2}{\rho_0^2}\left(1+2\alpha\Delta T\right) = \frac{\rho^2}{\rho_0^2}\left(1+\alpha\Delta T\right)\left(1+\alpha\Delta T\right) = \frac{\rho^2}{\rho_0^2}\left(1+\alpha\Delta T\right)^2$$

Since $\left(\frac{R}{R_0}\right)^2 = \frac{\rho^2}{\rho_0^2}\left(1+\alpha\Delta T\right)^2$, the circuit is **overdamped**.

$$\text{(b)}\quad \text{Since } \alpha = 0.4\,\alpha,\ \frac{1}{2\alpha} = \frac{1}{2(0.4\,\alpha)} = \frac{1}{0.8\,\alpha} = 1.25\left(\frac{1}{2\alpha}\right) > \left(\frac{1}{2\alpha}\right)$$

Since $\left(\frac{R}{R_0}\right)^2 < \frac{1}{2\alpha}$, the circuit is **underdamped**.

> **Problem 3.** A thermostat is set to 20 °C. It will switch on the heater if the temperature falls below 18 °C. Does it switch on the heater?

For positive integers $\left(\frac{R}{R_0}\right)^2 < \frac{1}{2\alpha}$.

Hence, it is **underdamped**.

$$\text{Let } \alpha = \frac{1}{\sqrt{LC}}, \quad \beta = \frac{1}{2RC}, \quad \omega_0^2 = \frac{1}{LC}, \quad \beta^2 = \frac{1}{4R^2C^2}$$

$$\text{Then } \beta^2 - \omega_0^2 = \frac{1}{4R^2C^2} - \frac{1}{LC} = \frac{L - 4R^2C}{4R^2C^2L}$$

$$\text{Since } L > 4R^2C \text{ for a real circuit, } \beta^2 - \omega_0^2 > 0 \text{ and } \omega_0 < \beta.$$

Error in the auxiliary equation.

In the solution of a quadratic, $\sqrt{b^2 - 4ac}$ is the $\pm$ root.

(a) When the roots are **real and different**, say $m = \alpha$ and $m = \beta$, the general solution is

$$y = Ae^{\alpha x} + Be^{\beta x} \qquad (11.2.1)$$

Hence $\frac{dy}{dx} = \left[\left(\frac{R}{R_0}\right)^2 - \frac{1}{2\alpha}\right]$ and

$$\frac{dy}{dx} = \left[\left(\frac{R}{R_0}\right)^2 - \frac{1}{2\alpha}\right]$$

(11.1) Uniform flow in a thin, wide open channel, e.g. in a river, is driven by gravity and opposed by friction. For a channel of depth h and bed slope θ, the mean velocity v is approximately

$$v = \sqrt{g h \sin\theta \,/\, c_f} \tag{11.1}$$

where $g = 9.81\ \mathrm{m/s^2}$ and c_f is a dimensionless friction coefficient. Solve for h as a function of v, θ, and c_f.

(11.2) Water flows through a long, straight pipe of circular cross-section with radius R and pressure gradient $-dp/dx$ (pressure decreasing in the flow direction). The fluid has density ρ and dynamic viscosity μ. Far from the pipe entrance, the velocity profile is fully developed and depends only on the radial coordinate r, measured from the centerline. The velocity is zero at the wall and reaches a maximum $u_{\max}$ at the centerline. Derive an expression for the velocity profile $u(r)$ and for the volumetric flow rate Q through the pipe.

Solution. Start from the steady, fully developed momentum balance in cylindrical coordinates. Because the flow is axisymmetric and unidirectional, the only nonzero velocity component is $u_x(r)$, and the governing equation reduces to

$$\frac{1}{r}\frac{d}{dr}\left(r\,\mu\,\frac{du}{dr}\right) = \frac{dp}{dx}. \tag{11.2}$$

Multiply both sides by r and integrate once with respect to r:

$$r\,\mu\,\frac{du}{dr} = \frac{dp}{dx}\,\frac{r^2}{2} + C_1.$$

Divide by μ and integrate again:

$$u(r) = \frac{1}{4\mu}\frac{dp}{dx}\,r^2 + \frac{C_1}{\mu}\ln r + C_2.$$

The centerline velocity is finite, so $C_1 = 0$. The no-slip condition $u(R) = 0$ then gives

$$C_2 = -\frac{1}{4\mu}\frac{dp}{dx}\,R^2,$$

and therefore

$$u(r) = -\frac{1}{4\mu}\frac{dp}{dx}\left(R^2 - r^2\right). \tag{11.3}$$

The maximum velocity occurs at the centerline, $u_{\max} = u(0) = -\frac{R^2}{4\mu}\frac{dp}{dx}$, so the profile can be written $u(r) = u_{\max}\left(1 - r^2/R^2\right)$.

The volumetric flow rate is obtained by integrating the velocity profile over the cross-section:

$$Q = \int_0^R u(r)\,2\pi r\,dr = -\frac{\pi}{2\mu}\frac{dp}{dx}\int_0^R \left(R^2 r - r^3\right)dr = -\frac{\pi}{2\mu}\frac{dp}{dx}\left(\frac{R^4}{2} - \frac{R^4}{4}\right) = -\frac{\pi R^4}{8\mu}\frac{dp}{dx}.$$

Since $dp/dx < 0$, Q is positive. Finally, the mean velocity is $\bar{u} = Q/(\pi R^2) = -\frac{R^2}{8\mu}\frac{dp}{dx} = \frac{1}{2}u_{\max}$, a result known as Hagen–Poiseuille flow.

(11.3) Show that the kinetic energy correction factor $\alpha = \frac{1}{A}\int_A (u/\bar{u})^3\,dA$ equals 2 for the parabolic profile in (11.2).

Problem 11.4. A uniform rod of length L and mass m is held horizontally on a frictionless table and pivoted about a point a distance $L/3$ from one end. The rod is released from rest and swings through the vertical. Using the principle of conservation of energy, find the angular velocity of the rod when it passes through its lowest position. Then find the linear speed of the free end at that instant.

$$\left(\frac{\partial^{2}}{\partial x^{2}}\right)\left[\frac{\partial^{2}}{\partial x^{2}}\right]+

Definition 1.8 (Laplace transform)

The Laplace transform of the function of time $f(t)$ is defined by the integral

$$\int_0^\infty e^{-st} f(t)\,dt$$

where s is a parameter.

There are a number of standard Laplace transforms which can be found in Laplace transforms of simple functions and Laplace transforms of derivatives and integrals. The Laplace transform is a linear operation.

Given two functions $f(t)$ and $g(t)$ and their Laplace transforms $F(s)$ and $G(s)$ respectively, then the Laplace transform of $af(t) + bg(t)$ is $aF(s) + bG(s)$ for any constants a and b. The Laplace transform of $f(t) = 1$ is $\frac{1}{s}$ for $s > 0$. The Laplace transform of $f(t) = e^{at}$ is $\frac{1}{s-a}$ for $s > a$. The Laplace transform of $f(t) = t^n$ is $\frac{n!}{s^{n+1}}$ for $n = 1, 2, 3, \ldots$ and $s > 0$. The Laplace transform of $f(t) = \sin(at)$ is $\frac{a}{s^2+a^2}$ for $s > 0$. The Laplace transform of $f(t) = \cos(at)$ is $\frac{s}{s^2+a^2}$ for $s > 0$. The Laplace transform of $f(t) = \sinh(at)$ is $\frac{a}{s^2-a^2}$ for $s > |a|$. The Laplace transform of $f(t) = \cosh(at)$ is $\frac{s}{s^2-a^2}$ for $s > |a|$. The Laplace transform of $f(t) = e^{at}\sin(bt)$ is $\frac{b}{(s-a)^2+b^2}$ for $s > a$. The Laplace transform of $f(t) = e^{at}\cos(bt)$ is $\frac{s-a}{(s-a)^2+b^2}$ for $s > a$. The Laplace transform of $f(t) = t^n e^{at}$ is $\frac{n!}{(s-a)^{n+1}}$ for $n = 1, 2, 3, \ldots$ and $s > a$. The Laplace transform of $f(t) = u_c(t)$ is $\frac{e^{-cs}}{s}$ for $c > 0$. The Laplace transform of $f(t) = u_c(t)g(t-c)$ is $e^{-cs}G(s)$ for $c > 0$. The Laplace transform of $f(t) = \delta(t-c)$ is e^{-cs} for $c > 0$. The Laplace transform of $f(t) = u_c(t)g(t)$ is $e^{-cs}\mathcal{L}\{g(t+c)\}$ for $c > 0$. The Laplace transform of $f(t) = t^n f(t)$ is $(-1)^n F^{(n)}(s)$ for $n = 1, 2, 3, \ldots$ and $s > 0$. The Laplace transform of $f(t) = \int_0^t f(\tau)\,d\tau$ is $\frac{F(s)}{s}$ for $s > 0$. The Laplace transform of $f(t) = f(t+T)$ is $\frac{1}{1-e^{-Ts}}\int_0^T e^{-st} f(t)\,dt$ for $s > 0$. The Laplace transform of $f(t) = \frac{f(t)}{t}$ is $\int_s^\infty F(\sigma)\,d\sigma$ for $s > 0$. The Laplace transform of $f(t) = \int_0^t f(\tau)g(t-\tau)\,d\tau$ is $F(s)G(s)







A list of standard Laplace transforms is contained in Table 10.1 on page 417. In each case we specify the necessary restriction on s for the transform to exist. It is assumed that a is a real constant.





































TABLE 2.10.1 Probability generating functions.

Mass function $P(k)$	Probability generating function $G(z) = \sum_{k=0}^{\infty} P(k) z^k$
1	1
2	z
3	$\frac{1}{2}(1+z)$
4	$\frac{1}{3}(1+z+z^2)$
5	$\frac{1}{n}(1+z+\cdots+z^{n-1})$
6	$\frac{1}{n}(1+z+\cdots+z^{n-1})$
7	$\frac{1}{n}(1+z+\cdots+z^{n-1})$
8	p^k
9	$q p^k$
10	$\binom{n}{k} p^k q^{n-k}$
11	$\binom{k+r-1}{k} p^k q^r$
12	$\binom{k+r-1}{k} p^k q^r$
13	$e^{-\lambda}\frac{\lambda^k}{k!}$
14	$\frac{\binom{r}{k}\binom{n-r}{s-k}}{\binom{n}{s}}$
15	$\frac{\binom{r}{k}\binom{n-r}{s-k}}{\binom{n}{s}}$
16	$\frac{1}{n}$
17	$\frac{1}{n}$

Problem 11.1. Two ideal AA batteries are connected in series to a resistor R. Determine the equivalent resistance of this combination.

(a) A single resistor $R = 2\ \Omega$ draws current $I_1 = 1.5\ \text{A}$.

(b) A single resistor $R = 4\ \Omega$ draws current $I_2 = 0.75\ \text{A}$.

(c) A single voltage source $V = 3\ \text{V}$ drives current $I = 1.5\ \text{A}$.

(d) A power source delivers $P = 4.5\ \text{W}$ at $V = 3\ \text{V}$.



(a) Because R and the two cells are in series, the same current flows through each element. Ohm's law for the resistor gives

$$V_R = I R = (1.5\ \text{A})(2\ \Omega) = 3.0\ \text{V}.$$

The battery voltage must therefore be $V_{\text{batt}} = 3.0\ \text{V}$. Each cell supplies half of this, so the terminal voltage of one cell is $1.5\ \text{V}$.

(b) Using $R = 4\ \Omega$ and $I = 0.75\ \text{A}$ in Ohm's law, $V = IR = (0.75\ \text{A})(4\ \Omega) = 3.0\ \text{V}$. The battery must again supply $3.0\ \text{V}$, so each cell delivers $1.5\ \text{V}$. Note that the current is smaller than in part (a) because the resistance is larger.

(c) Here $V = 3\ \text{V}$ and $I = 1.5\ \text{A}$. Ohm's law gives the resistance directly:

$$R = \frac{V}{I} = \frac{3\ \text{V}}{1.5\ \text{A}} = 2\ \Omega.$$

The battery voltage is $3.0\ \text{V}$, so each cell delivers $1.5\ \text{V}$.

(d) A fully general fact: for a delivered source, the relation between power and current is quadratic, $P = VI = I^2 R$, so the current is not determined by P and V alone. It is the smaller root of $I^2 R - VI + P = 0$. Using the quadratic formula with $P = 4.5\ \text{W}$ and $V = 3\ \text{V}$ gives $I = 1.5\ \text{A}$ or $I = 3.0\ \text{A}$. Whichever value is realized, the battery voltage is $3.0\ \text{V}$, and each cell delivers half of that.

Figure 11.1.1

Problem 11.2 (Intermediate). A resistor is connected to the following ideal sources:

(a) An ideal voltage source $V = 12\ \text{V}$.

(b) An ideal current source $I = 2\ \text{A}$.

(c) A dependent source in which the current through the resistor is twice the current through a second resistor $R_2 = 3\ \Omega$ that is in series with it.



(a) Since $\lim_{x \to 0^+} f(x) = \infty$ and $\lim_{x \to 0^-} f(x) = -\infty$, the line $x = 0$ is a vertical asymptote.

Since f is defined for all x except $x = 0$, it has no vertical asymptote.

Since $\lim_{x \to \infty} f(x) = 0$ and $\lim_{x \to -\infty} f(x) = 0$, the line $y = 0$ is a horizontal asymptote.

(b) $f'(x) = -\frac{1}{x^2} < 0$ for all $x \neq 0$, so f is decreasing on $(-\infty, 0)$ and $(0, \infty)$.

f has no local maximum or minimum.

(c) $f''(x) = \frac{2}{x^3}$, so $f''(x) > 0$ if $x > 0$ and $f''(x) < 0$ if $x < 0$.

f is CU on $(0, \infty)$ and CD on $(-\infty, 0)$.

f has no point of inflection since 0 is not in the domain of f.

(d)

(a) $f(x) = \frac{x^2 - 1}{x^2 + 1}$

$\lim_{x \to \pm\infty} f(x) = 1$, so $y = 1$ is a horizontal asymptote.

There is no vertical asymptote since $x^2 + 1 \neq 0$ for all x.

$f'(x) = \frac{4x}{(x^2+1)^2}$

$f'(x) > 0$ if $x > 0$ and $f'(x) < 0$ if $x < 0$.

f is increasing on $(0, \infty)$ and decreasing on $(-\infty, 0)$.

f has a local minimum at $x = 0$.

$f''(x) = \frac{4(1 - 3x^2)}{(x^2+1)^3}$

$f''(x) > 0$ if $-\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$ and $f''(x) < 0$ if $x < -\frac{1}{\sqrt{3}}$ or $x > \frac{1}{\sqrt{3}}$.

f is CU on $\left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ and CD on $\left(-\infty, -\frac{1}{\sqrt{3}}\right)$ and $\left(\frac{1}{\sqrt{3}}, \infty\right)$.

f has points of inflection at $x = \pm\frac{1}{\sqrt{3}}$.

(e)

(a) $f(x) = \frac{x}{x^2 + 1}$

$\lim_{x \to \pm\infty} f(x) = 0$, so $y = 0$ is a horizontal asymptote.

There is no vertical asymptote since $x^2 + 1 \neq 0$ for all x.

$f'(x) = \frac{(x^2+1)(1) - x(2x)}{(x^2+1)^2} = \frac{1 - x^2}{(x^2+1)^2} = \frac{(1+x)(1-x)}{(x^2+1)^2}$

$f'(x) > 0$ if $-1 < x < 1$ and $f'(x) < 0$ if $x < -1$ or $x > 1$.

f is increasing on $(-1, 1)$ and decreasing on $(-\infty, -1)$ and $(1, \infty)$.

f has a local minimum at $x = -1$ and a local maximum at $x = 1$.

(b) $f''(x) = \frac{(x^2+1)^2(-2x) - (1 - x^2)(2)(x^2+1)(2x)}{(x^2+1)^4}$

$= \frac{(x^2+1)(-2x) - (1 - x^2)(4x)}{(x^2+1)^3}$

$= \frac{2x^3 - 6x}{(x^2+1)^3} = \frac{2x(x^2 - 3)}{(x^2+1)^3}$

$f''(x) > 0$ if $-\sqrt{3} < x < 0$ or $x > \sqrt{3}$ and $f''(x) < 0$ if $x < -\sqrt{3}$ or $0 < x < \sqrt{3}$.

f is CU on $(-\sqrt{3}, 0)$ and $(\sqrt{3}, \infty)$ and CD on $(-\infty, -\sqrt{3})$ and $(0, \sqrt{3})$.

f has points of inflection at $x = -\sqrt{3}$, 0, and $\sqrt{3}$.

(c)

(a) $f(x) = \frac{x}{x^2 - 1}$

$\lim_{x \to \pm\infty} f(x) = 0$, so $y = 0$ is a horizontal asymptote.

$\lim_{x \to 1^+} f(x) = \infty$ and $\lim_{x \to 1^-} f(x) = -\infty$, so $x = 1$ is a vertical asymptote.

$\lim_{x \to -1^+} f(x) = -\infty$ and $\lim_{x \to -1^-} f(x) = \infty$, so $x = -1$ is a vertical asymptote.

$f'(x) = \frac{(x^2-1)(1) - x(2x)}{(x^2-1)^2} = \frac{1 - x^2}{(x^2-1)^2} = -\frac{x^2 - 1}{(x^2-1)^2} = -\frac{1}{x^2 - 1}$

$f'(x) > 0$ if $-1 < x < 1$ and $f'(x) < 0$ if $x < -1$ or $x > 1$.

f is increasing on $(-1, 1)$ and decreasing on $(-\infty, -1)$ and $(1, \infty)$.

f has no local maximum or minimum.

(b) $f''(x) = \frac{(x^2-1)^2(0) - (-1)(2)(x^2-1)(2x)}{(x^2-1)^4}$

$= \frac{(x^2-1)(0) - (-1)(4x)}{(x^2-1)^3}$

$= \frac{4x}{(x^2-1)^3}$

$f''(x) > 0$ if $-1 < x < 0$ or $x > 1$ and $f''(x) < 0$ if $x < -1$ or $0 < x < 1$.

f is CU on $(-1, 0)$ and $(1, \infty)$ and CD on $(-\infty, -1)$ and $(0, 1)$.

f has a point of inflection at $x = 0$.

(c)

(a) $f(x) = \frac{x}{x^2 + 1}$

Problem 4. Let $n \geq 3$ be a fixed integer. Let $a_1, a_2, \ldots, a_n$ be positive real numbers such that $a_1 a_2 \cdots a_n = 1$. Prove that

$$\frac{1}{a_1^2 + a_2^2 + 1} + \frac{1}{a_2^2 + a_3^2 + 1} + \cdots + \frac{1}{a_n^2 + a_1^2 + 1} \leq \frac{n}{3}.$$

Explore transformations of derivatives

Given a function $f(x)$ and its derivative $f'(x)$, we can apply various transformations to both $f(x)$ and $f'(x)$ to create new functions. For example, we can:

- **Scale the input variable:** $f(kx)$ and $f'(kx)$ for some constant k
- **Shift the input variable:** $f(x+c)$ and $f'(x+c)$ for some constant c
- **Scale the output variable:** $mf(x)$ and $mf'(x)$ for some constant m
- **Add a constant to the output variable:** $f(x)+d$ and $f'(x)+d$ for some constant d

Here k, c, m, and d are real numbers, with $k \neq 0$ and $m \neq 0$.

We can also **combine** these transformations. For example, we can:

- **Scale the input variable and shift the input variable:** $f(kx+c)$ and $f'(kx+c)$ for some constants k and c, with $k \neq 0$
- **Scale the output variable and add a constant to the output variable:** $mf(x)+d$ and $mf'(x)+d$ for some constants m and d, with $m \neq 0$

Applying any of the above transformations to a differentiable function $f(x)$ and its derivative $f'(x)$ will result in a new function and its derivative.

The Euclid and Euclid-Like Algorithms

The Euclid and Euclid-Like algorithms are efficient computational methods for finding the greatest common divisor of two integers. Here's how they work:

1. **The Euclid-Like Theorem:** Suppose $\gcd(a, b) = \gcd(q_1, q_2, \ldots, r_1, r_2, \ldots)$ for some integers a, b, q_1, q_2, $\ldots$, r_1, r_2, $\ldots$.

   Then, for example, if $a = q_1 b + r_1$ and $r_1 = q_2 r_2 + r_3$ and $r_2 = q_3 r_3$, then $r_3 = \gcd(a, b)$.

   More generally, if $a = q_1 b + r_1$ and $b = q_2 r_1 + r_2$ and $r_1 = q_3 r_2 + r_3$ and $r_2 = q_4 r_3 + r_4$ and $\ldots$ and $r_{n-2} = q_n r_{n-1}$, then $r_{n-1} = \gcd(a, b)$.

   Since $r_1 = a - q_1 b$, we have $r_2 = b - q_2 r_1 = b - q_2(a - q_1 b) = -q_2 a + (1 + q_1 q_2) b$, and $r_3 = r_1 - q_3 r_2 = (a - q_1 b) - q_3(-q_2 a + (1 + q_1 q_2) b) = (1 + q_2 q_3) a - (q_1 + q_3 + q_1 q_2 q_3) b$.

   Therefore, $r_3 = \gcd(a, b)$ can be written in the form $ma + nb$ where $m = 1 + q_2 q_3$ and $n = -(q_1 + q_3 + q_1 q_2 q_3)$.

   In fact, the above theorem shows that the Euclid-Like algorithm can be used to find integers m and n such that $ma + nb = \gcd(a, b)$.

11.1 The **Real Cubic Matrices** $\mathbb{R}^{3\times 3}$ are a vector space of dimension 9 over $\mathbb{R}$, and $\det : \mathbb{R}^{3\times 3} \to \mathbb{R}$ is a cubic polynomial in the 9 entries.

For the diagonal matrix $\mathrm{diag}(a,b,c) = \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}$ the determinant is given by

$$\det \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix} = abc$$

The vector space $\mathbb{R}^{3\times 3}$ has dimension nine, and $\det$ is a cubic function on this space. Hence the equation $\det(A) = 0$ defines a hypersurface of degree 3 in nine variables. The set $\mathrm{GL}_3(\mathbb{R}) = \{A \in \mathbb{R}^{3\times 3} : \det(A) \neq 0\}$ is the complement of this hypersurface. The determinant is a continuous function, so $\mathrm{GL}_3(\mathbb{R})$ is an open subset of $\mathbb{R}^{3\times 3}$. The real cubic matrices A with $\det(A) = 0$ form a closed subset, called the **discriminant hypersurface**. Its complement $\mathrm{GL}_3(\mathbb{R})$ is open and dense in $\mathbb{R}^{3\times 3}$. Every real cubic matrix can be approximated by matrices with distinct eigenvalues, and the set of matrices with repeated eigenvalues has measure zero. Almost every real cubic matrix has three distinct real eigenvalues.

For the corresponding result about a complex cubic, see the proof of Theorem 10.3.2 on page 412.

11.2 Gaussian Laplacian smoothing and the solution of differential equations

Given a function $f : \mathbb{R}^3 \to \mathbb{R}$ and $t > 0$, the **Gaussian Laplacian smoothing** of f at time t is the function $u(\cdot, t) : \mathbb{R}^3 \to \mathbb{R}$ defined by the convolution

$$u(x,t) = \frac{1}{(4\pi t)^{3/2}} \int_{\mathbb{R}^3} f(y)\, e^{-\|x-y\|^2/4t}\, dy = \left(f * \Phi_t\right)(x)$$

where $\Phi_t(x) = (4\pi t)^{-3/2} e^{-\|x\|^2/4t}$ is the **Gaussian Laplacian smoothing kernel**. The smoothing solves the heat equation $\partial u/\partial t = \Delta u$ with initial condition $u(x,0) = f(x)$.

The Fourier transform of the Gaussian kernel is the Gaussian itself, up to a constant. Since the Fourier transform of a product of functions is the product of their Fourier transforms, the smoothing operator becomes multiplication by a Gaussian in the frequency domain.

> **Definition 11.2.** The **Fourier transform** of an integrable function $f : \mathbb{R}^3 \to \mathbb{C}$ is the function $\hat{f} : \mathbb{R}^3 \to \mathbb{C}$ given by
>
> $$\hat{f}(\xi) = \int_{\mathbb{R}^3} f(x)\, e^{-2\pi i \langle x, \xi \rangle}\, dx$$
>
> where $\langle x, \xi \rangle = x_1\xi_1 + x_2\xi_2 + x_3\xi_3$ is the standard inner product on $\mathbb{R}^3$.

$$\hat{\Phi}_t(\xi) = \int_{\mathbb{R}^3} (4\pi t)^{-3/2} e^{-\|x\|^2/4t}\, e^{-2\pi i \langle x, \xi \rangle}\, dx = e^{-4\pi^2 t \|\xi\|^2}$$

Then $\hat{u}(\xi, t) = \hat{f}(\xi)\, e^{-4\pi^2 t \|\xi\|^2}$, and the Gaussian Laplacian smoothing of f is the inverse Fourier transform of a Gaussian times $\hat{f}$. The Gaussian damps the high-frequency components of f while leaving the low-frequency components nearly unchanged.

$$\hat{u}(\xi, t) = \int_{\mathbb{R}^3} u(x,t)\, e^{-2\pi i \langle x, \xi \rangle}\, dx = \int_{\mathbb{R}^3} \left(f * \Phi_t\right)(x)\, e^{-2\pi i \langle x, \xi \rangle}\, dx$$

$$= \int_{\mathbb{R}^3} \int_{\mathbb{R}^3} f(y)\, \Phi_t(x-y)\, e^{-2\pi i \langle x, \xi \rangle}\, dy\, dx = \hat{f}(\xi)\, e^{-4\pi^2 t \|\xi\|^2}$$

Problem 12. Determine the following and state if each is a transformation:

$$\text{(a)}\quad \quad \quad \text{(b)}\quad$$

(a) Point A of Table 4.4: $\quad$

Image $\quad$

(b) Point B of Table 4.4, if a new determination of B has $x =$

Then $\quad$

Image $\quad$

Problem 13. Determine $\quad$

(a) $\quad$

$\quad$ (Point B, from (b) of Table 4.4.)

(b) $\quad$

$\quad$ (Double the y-coordinate of each point in A.)

Problem 14. Find $\quad$

Point B of Table 4.4: $\quad$

Then $\quad$

Let $\quad$ Then $\quad$ (Recall that $\theta = 0$ and $\alpha = 1$.)

Hence $\quad$

Problem 313 — Derivatives

$$\text{Find } \frac{d}{dx}\left[\frac{x^{2}+3x+1}{x^{2}-4x+7}\right] \text{ and } \frac{d}{dx}\left[\frac{3x^{2}+5x+1}{x^{2}+2x+4}\right].$$

$$\frac{d}{dx}\left[\frac{x^{2}+3x+1}{x^{2}-4x+7}\right]=\frac{(2x+3)(x^{2}-4x+7)-(x^{2}+3x+1)(2x-4)}{(x^{2}-4x+7)^{2}}$$

$$=\frac{(2x^{3}-5x^{2}+14x+21)-(2x^{3}+2x^{2}-10x-4)}{(x^{2}-4x+7)^{2}}=\frac{-7x^{2}+24x+25}{(x^{2}-4x+7)^{2}}.$$

$$\frac{d}{dx}\left[\frac{3x^{2}+5x+1}{x^{2}+2x+4}\right]=\frac{(6x+5)(x^{2}+2x+4)-(3x^{2}+5x+1)(2x+2)}{(x^{2}+2x+4)^{2}}$$

$$=\frac{(6x^{3}+17x^{2}+34x+20)-(6x^{3}+16x^{2}+12x+2)}{(x^{2}+2x+4)^{2}}=\frac{x^{2}+22x+18}{(x^{2}+2x+4)^{2}}.$$

Every time a quotient appears, remember the denominator is squared and kept. Expanding carefully and combining like terms is where most errors happen.

Use of graphical technology for inverse trigonometric relationships

Before we begin, it is helpful to think about what we are trying to accomplish. In many real-world problems, we know a ratio of two sides of a right triangle and want to find the angle that produces that ratio. Inverse trigonometric functions are built exactly for this purpose. For example, if $\sin\theta = \frac{3}{5}$, then $\theta = \arcsin\left(\frac{3}{5}\right)$. In this section, we will explore how to use graphing technology to evaluate, interpret, and check inverse trigonometric relationships, and we will see why domain restrictions are necessary for these functions to behave like true inverses.

Recall that a function has an inverse only if it passes the horizontal line test. The sine function, $y = \sin x$, fails this test when considered over all real numbers because infinitely many different angles share the same sine value. To create an invertible function, we restrict the domain to the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, where the function increases from -1 to 1 and takes on each value exactly once. On this restricted domain, we define $\arcsin x$ as the unique angle θ in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ such that $\sin\theta = x$. Similar restrictions are made for cosine and tangent, which is why calculators and graphing utilities always return a single value for inverse trigonometric expressions.

When using a graphing calculator or software, you will typically find inverse trigonometric functions labeled as $\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$, or sometimes as arcsin, arccos, and arctan. Be careful not to confuse $\sin^{-1}x$ with $\frac{1}{\sin x}$; the exponent -1 here denotes an inverse function, not a reciprocal. To evaluate $\arcsin\left(\frac{1}{2}\right)$, you would enter the expression exactly as written and obtain $\frac{\pi}{6}$ in radian mode or 30° in degree mode. Always check which mode your device is in before trusting the output.

A useful strategy when working with inverse trigonometric relationships is to sketch a right triangle and label the sides according to the ratio you are given. For instance, if $\cos\theta = \frac{4}{7}$, you can draw a triangle with adjacent side 4 and hypotenuse 7, then use the Pythagorean theorem to find the opposite side $\sqrt{7^2 - 4^2} = \sqrt{33}$. From this triangle, you can read off $\tan\theta = \frac{\sqrt{33}}{4}$ directly, without ever touching a calculator. This geometric approach helps you catch errors and builds intuition that pure button-pushing does not.

Problem 314 — Derivatives Find $\frac{d}{dx}\left[\frac{3x-1}{x^{2}+x+2}\right]$.

$$\frac{dL}{dt} = \frac{d}{dt}\left(\frac{mr^2}{2}\right)\left(\frac{2v}{r}\right) = \frac{mr}{2}\frac{dv}{dt} = \frac{mr}{2}a = \frac{F_{\text{net}}}{2}$$

$$\frac{dL}{dt} = \frac{d}{dt}\left(\frac{mr^2}{2}\right)\left(\frac{2v}{r}\right) = \frac{mr}{2}\frac{dv}{dt} = \frac{mr}{2}a = \frac{F_{\text{net}}}{2}$$

$$\frac{dL}{dt} = \frac{d}{dt}\left(\frac{mr^2}{2}\right)\left(\frac{2v}{r}\right) = \frac{mr}{2}\frac{dv}{dt} = \frac{mr}{2}a = \frac{F_{\text{net}}}{2}$$

Because $\omega = 2v/r$, $L = (mr^2/2)\omega$, and ω is constant.

When $v = 0$, $L = 0$, for any value of r.

When $v = 2$ m/s, $r = 1$ m, $L = 2$ kg·m²/s.

When $v = 2$ m/s, $r = 2$ m, $L = 4$ kg·m²/s.

Since $\omega = \begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix}$, $\omega^2 = \begin{bmatrix} -9 & 0 \\ 0 & -9 \end{bmatrix}$, $\omega^3 = \begin{bmatrix} 0 & 27 \\ -27 & 0 \end{bmatrix}$, $\omega^4 = \begin{bmatrix} 81 & 0 \\ 0 & 81 \end{bmatrix}$.

$$e^{\omega t} = \begin{bmatrix} 1 - \frac{9t^2}{2} + \frac{81t^4}{24} - \cdots & -3t + \frac{27t^3}{6} - \cdots \\ 3t - \frac{27t^3}{6} + \cdots & 1 - \frac{9t^2}{2} + \frac{81t^4}{24} - \cdots \end{bmatrix} = \begin{bmatrix} \cos 3t & -\sin 3t \\ \sin 3t & \cos 3t \end{bmatrix}$$

$e^{\omega t}$ is the 2-dimensional rotation matrix.

Problem 10. Find $e^{\begin{bmatrix} 0 & -2 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}t}$.

$$\begin{bmatrix} 0 & -2 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}^2 = \begin{bmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & -2 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}^3 = \begin{bmatrix} 0 & 8 & 0 \\ -8 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & -2 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}^4 = \begin{bmatrix} 16 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$e^{\begin{bmatrix} 0 & -2 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}t} = \begin{bmatrix} 1 - \frac{4t^2}{2} + \frac{16t^4}{24} - \cdots & -2t + \frac{8t^3}{6} - \cdots & 0 \\ 2t - \frac{8t^3}{6} + \cdots & 1 - \frac{4t^2}{2} + \frac{16t^4}{24} - \cdots & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos 2t & -\sin 2t & 0 \\ \sin 2t & \cos 2t & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since $\omega^3 = \omega^2 \omega = \omega \omega^2$, $\omega^4 = \omega^3 \omega = \omega^2 \omega^2 = \omega \omega^3$, and $\omega^5 = \omega^4 \omega = \omega^3 \omega^2 = \omega^2 \omega^3 = \omega \omega^4$.

$$e^{\omega t} = I + \omega t + \frac{\omega^2 t^2}{2} + \frac{\omega^3 t^3}{6} + \frac{\omega^4 t^4}{24} + \frac{\omega^5 t^5}{120} + \cdots$$

$$= I + \omega t + \frac{\omega^2 t^2}{2} + \omega\frac{\omega^2 t^3}{6} + \omega\frac{\omega^2 t^4}{24} + \omega\frac{\omega^3 t^5}{120} + \cdots$$

$$= I + \omega t + \frac{\omega^2 t^2}{2} + \omega\left(\frac{\omega^2 t^3}{6} + \frac{\omega^2 t^4}{24} + \frac{\omega^3 t^5}{120} + \cdots\right)$$

When $\omega = 0$, $e^{\omega t} = I$, the linear solution, $\vec{x} = \vec{x}_0$.

When $\omega \neq 0$, $e^{\omega t} = I + \omega\left(\frac{\omega t^2}{2} + \frac{\omega^2 t^3}{6} + \frac{\omega^3 t^4}{24} + \cdots\right)$.

Substituting $\vec{x}$ into the system, $\vec{x}' = \omega\vec{x} + \vec{B}$ becomes $\omega\left(\frac{\omega t^2}{2} + \frac{\omega^2 t^3}{6} + \frac{\omega^3 t^4}{24} + \cdots\right) = \omega\left(\frac{\omega t^2}{2} + \frac{\omega^2 t^3}{6} + \frac{\omega^3 t^4}{24} + \cdots\right)$.

Substituting $\vec{x}$ into the system, $\vec{x}' = \omega\vec{x} + \vec{B}$ becomes $\omega t + \frac{\omega^2 t^2}{2} + \frac{\omega^3 t^3}{6} + \cdots = \omega t + \frac{\omega^2 t^2}{2} + \frac{\omega^3 t^3}{6} + \cdots$, which is true.

Hence, $e^{\omega t} = I + \omega\left(\frac{\omega t^2}{2} + \frac{\omega^2 t^3}{6} + \frac{\omega^3 t^4}{24} + \cdots\right)$

$$= I + \omega\left(\frac{t}{1!} + \frac{t^2}{2!} + \frac{t^3}{3!} + \frac{t^4}{4!} + \cdots\right)$$

$$= I + \omega\left(1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \frac{t^4}{4!} + \cdots - 1\right)$$

$$= I + \omega\left(e^t - 1\right)$$

Since ω and I commute, $e^{\omega t} = e^{\omega}e^{t}$.

Problem 4 Determine $\dfrac{dy}{dx}$ if $\begin{cases} x = t^2 + 3t \\ y = t\sin(t) + 1 \end{cases}$

$$\frac{dx}{dt} = 2t + 3 \qquad \frac{dy}{dt} = t\cos(t) + \sin(t) \qquad \frac{dy}{dx} = \frac{t\cos(t) + \sin(t)}{2t + 3}$$

Notice $\dfrac{dy}{dx}$ is in terms of t, not x or y. Still fine!

When $t = -1$: $\dfrac{dy}{dx} = \dfrac{-\cos(-1) + \sin(-1)}{2(-1) + 3} = -\cos(1) + \sin(1)$ (tangent slope)

Equation of tangent when $t = -1$: $y = (-\cos(1) + \sin(1))(x - (-2)) + (-\sin(1))$

Normal slope when $t = -1$: $-\dfrac{1}{-\cos(1) + \sin(1)}$ (perpendicular slope)

Answer: $\dfrac{dy}{dx} = \dfrac{t\cos(t) + \sin(t)}{2t + 3}$; tangent: $y = (-\cos(1) + \sin(1))(x + 2) - \sin(1)$; normal: $y = \dfrac{1}{\cos(1) - \sin(1)}(x + 2) - \sin(1)$

Δx Δy Δt Δx Δy Δt Δx Δy Δt $

$$\begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix} \begin{bmatrix} \cdot \\ \cdot \end{bmatrix} = \begin{bmatrix} \cdot \\ \cdot \end{bmatrix} \quad \text{(} \cdot \text{ m/s, } \cdot \text{ m/s)}$$







(see page 414)

Definition 2.1. A curve $X \subset \mathbb{P}^n$ over a field k is said to be *rational* if it is birational to $\mathbb{P}^1$, i.e., if there exist rational functions $x_1, \ldots, x_n \in k(t)$ such that X is the closure of the image of the map $\mathbb{A}^1 \to \mathbb{A}^n$, $t \mapsto (x_1(t), \ldots, x_n(t))$.











































$$i_{\text{L}}(\quad) = \quad + \left(\quad - \quad\right) e^{-\frac{\quad}{\quad}\left(\quad - \quad\right)} \quad \text{for } t \geq \quad \text{s}$$

Since $\tau = \frac{\quad}{\quad} = \quad$ s and $t = \quad = \quad \tau$, the current is within 63.2% of the way from $\quad$ A to $\quad$ A, or about $\quad$ A, when the switch has been open for one time constant.

Equivalent resistance can sometimes be found by combining series and parallel resistors step by step, as worked in Example 2.3 on page 47. For circuits with dependent sources, the test-source method of Example 2.8 on page 61 is more general.

19.7 Laplace transform of common waveforms

Before using the Laplace transform to solve circuits, we need the transform of the waveforms that appear in those circuits. The unit step and the exponential are the two most common, and every other waveform in this section is built from them.

Unit step

The unit step $u(t)$ is zero for $t < 0$ and one for $t > 0$. Its Laplace transform is

$$\mathcal{L}\{u(t)\} = \int_0^\infty 1 \cdot e^{-st}\,dt = \left[-\frac{e^{-st}}{s}\right]_0^\infty = \frac{1}{s}$$

The integral converges for $\text{Re}(s) > 0$, which is the region of interest for all the circuits in this chapter.

Exponential

The exponential $e^{-at}u(t)$ has Laplace transform

$$\mathcal{L}\{e^{-at}u(t)\} = \int_0^\infty e^{-at} e^{-st}\,dt = \left[-\frac{e^{-(s+a)t}}{s+a}\right]_0^\infty = \frac{1}{s+a}$$

The pole moves from $s = 0$ for the unit step to $s = -a$ for the exponential. Multiplying by e^{-at} in the time domain shifts the pole by a in the s-plane. This is the frequency-shift property, and it is the reason damped sinusoids and exponentially decaying pulses are so easy to handle.

Sinusoid

The sinusoid $\sin(\omega t)u(t)$ is built from two exponentials via Euler's identity, $\sin(\omega t) = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$. Applying the exponential rule to each term and combining gives

$$\mathcal{L}\{\sin(\omega t)u(t)\} = \frac{\omega}{s^2 + \omega^2}$$

The cosine follows the same way and is left as an exercise. Both results are valid for $\text{Re}(s) > 0$.

Impulse

The unit impulse $\delta(t)$ is the derivative of the unit step in the distribution sense. Its transform is

$$\mathcal{L}\{\delta(t)\} = \int_0^\infty \delta(t) e^{-st}\,dt = e^{0} = 1$$

The impulse has a flat spectrum: every frequency is equally represented. This is why a short pulse in the time domain, which is nearly an impulse, excites a wide band of frequencies in a circuit.

Capacitor

If a capacitor C has an initial voltage $v_C(0^-) = V_0$, its current $i = C\,dv/dt$ has the transform

$$I(s) = C\left[sV(s) - V_0\right] = \frac{V_0}{1/sC} - sCV(s)$$

The first term is the transform of a source V_0 in series with $1/sC$, and the second is the capacitor's own impedance. This is the Thevenin model of a charged capacitor, and it is the reason initial conditions are so easy to handle in the s-domain.

The inductor is dual: $V(s) = sLI(s) - Li(0^-)$, the Norton model of a charged inductor. Both models appear in Example 19.11 on page 812, where the time constant and the transient response are worked out.



Figure 4.1.10

The capacitance of the capacitor in the schematic is given by:

$$C_{total} = \frac{\varepsilon_0 \varepsilon_r A}{d_{total}} = \frac{\varepsilon_0 \varepsilon_r A}{(d_1 + d_2)} = \frac{A}{\frac{d_1}{\varepsilon_0 \varepsilon_r} + \frac{d_2}{\varepsilon_0}}$$

The capacitor in between in Figure 4.1.9 is made by two conductors and two materials.

Capacitance Rating of the **Glass Capacitor** that is made of two plates with **0.1 cm²** and another **dielectric** that is made of glass with **ε_r = 5** and **0.2 cm** thickness.

Solution: The capacitance of the capacitor is $C = \varepsilon_0 \varepsilon_r \frac{A}{d}$ and the capacitance is $C = \varepsilon_0 \varepsilon_r \frac{A}{d} = 8.85 \times 10^{-12} \times 5 \times \frac{0.1}{0.2} = 2.21 \times 10^{-12}$ F.

Now that the capacitance is known, we can solve for the electric displacement by $\varepsilon_0 \varepsilon_r = \frac{D}{E}$ and $C = \frac{Q}{V}$. We know that the electric displacement is equal to $\varepsilon_0 \varepsilon_r$ or the permittivity of free space and the relative permittivity of the material. Lastly, by substitution of equations, we have:

> **Problem 4.1** – Determine the capacitance of a 0.01 m² capacitor with two dielectrics.

In this problem, the capacitance of a capacitor is $\frac{A}{d}$ times:

$$C = \frac{1}{\frac{d_1}{\varepsilon_0 \varepsilon_{r1} A} + \frac{d_2}{\varepsilon_0 \varepsilon_{r2} A}} = \frac{1}{\frac{0.1}{8.85 \times 10^{-12} \times 0.01} + \frac{0.1}{8.85 \times 10^{-12} \times 0.01}} = \frac{8.85 \times 10^{-12}}{0.2}$$

> **Problem 4.2** – Determine the capacitance of a 100 m² capacitor in series with a 0.01 dielectric in the schematic.

The capacitance of two capacitors in the schematic is 100 F.

The capacitance of two capacitors in the schematic is $C = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}$.

When the capacitors are in series, $C = \frac{1}{\frac{1}{100} + \frac{1}{100}} = 50$ F.

> **Problem 4.3** – A capacitor has a 0.01 F capacitor and a 0.02 capacitor and a 0.04 F capacitor. Determine if the capacitors in the schematic are in series and if the capacitors in the schematic are connected in parallel.

In C, C# and D, implement a function to determine the capacitance:

$$C_{total} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}} = \frac{1}{\frac{1}{0.01} + \frac{1}{0.02} + \frac{1}{0.04}} = \frac{1}{100 + 50 + 25} = \frac{1}{175}$$

(b) If R_1 and R_2 are connected in parallel across a voltage source V, the equivalent resistance R_{eq} is found from

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

so that

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

The current drawn from the source is

$$I = \frac{V}{R_{eq}} = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

The power dissipated in each resistor is $P_1 = V^2/R_1$ and $P_2 = V^2/R_2$, and the total power is

$$P = P_1 + P_2 = V^2\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

(c) If R_1 and R_2 are connected in series, the same current I flows through both. The equivalent resistance is

$$R_{eq} = R_1 + R_2$$

The voltage drops across the resistors are $V_1 = IR_1$ and $V_2 = IR_2$, and the total power is

$$P = I^2(R_1 + R_2)$$

Kirchhoff's Laws in the Laboratory

Kirchhoff's current and voltage laws may be applied to circuits set up in the laboratory in order to measure and so check the values of circuit parameters. Kirchhoff's laws are also used in the analysis of circuits which are too complex to be simplified by the methods of series and parallel combinations. The procedure to be followed when using Kirchhoff's laws to find the currents is:

(1) Assign to each branch a current in a chosen direction.

(2) Apply Kirchhoff's point rule at nodes as necessary.

(3) Apply the loop rule to obtain the required number of independent equations.

(4) Substitute the numerical values of resistances, emfs, and supply currents in the equations and solve for the unknown currents.

The procedure is demonstrated in the following example.



Figure 40.10

Problem 10 Determine an expression for the current flowing through the 10 Ω resistor in the circuit shown in Figure 40.10 by applying Kirchhoff's laws. Hence find the current flowing in the 10 Ω resistor when $E_1 = 12$ V, $E_2 = 6$ V, $R_1 = 2\ \Omega$, $R_2 = 4\ \Omega$ and $R_3 = 10\ \Omega$. Check that the value of current is consistent with the voltage.



Figure 10.1.16

(c) Using the above results

(i) For the conditions for which the approximations $\sin\theta \approx \theta$, $\cos\theta \approx 1$ and $\tan\theta \approx \theta$ are valid, the angle θ is $5°$ or less and the distance x is a distance in Figure 10.1.16.

(ii) Substituting the above into the voltage law:

$$\frac{V}{L} = -\frac{d^2\theta}{dt^2} + \frac{g}{L}\theta \qquad (10.1.5)$$

$$\frac{d^2\theta}{dt^2} + \frac{g}{L}\theta = -\frac{V}{L}$$

(iii) $\frac{V}{L} = -\frac{g}{L}\left(\theta - \frac{V}{g}\right)$

(iv) Homogeneous form:

$$\theta(t) = \frac{\theta(0)}{\left(\theta - \frac{V}{g}\right) - \left(\theta(0) - \frac{V}{g}\right)}$$

$$\frac{\theta}{\theta\left(1 - \frac{V}{g\theta}\right)} = \frac{\theta}{\theta(0)\left(1 - \frac{V}{g\theta(0)}\right)} \qquad (10.1.6)$$

(v) Since $\theta(0) = 1$ then $\theta(t) = \left(\frac{\theta}{\theta(0)}\right)$

$$\theta = \left(\frac{\theta}{\theta(0)}\right)$$

where $\theta = \left(\frac{1}{\theta(0)}\right)$ and $\theta(0) = \left(\frac{1}{\theta(0)}\right)$ from (i) of Table 10.1. Hence

$$\theta(0) = \frac{\theta}{\theta}\theta(0) = \frac{\theta}{\theta}\theta(0)$$

an approximate solution as in equation (10.1.5), page 474.

(c) Consequently $\theta(0) = \frac{\theta}{\theta}$ — Ohm's and Kirchhoff's laws

$$\theta(0) = \frac{\theta}{\theta}\left(\frac{\theta}{\theta(0)}\right)$$

$$\frac{\theta}{\theta} = \frac{\theta}{\theta\left(1 - \frac{\theta}{\theta(0)}\right)} = \left(\frac{1}{1 - \frac{\theta}{\theta(0)}}\right)$$

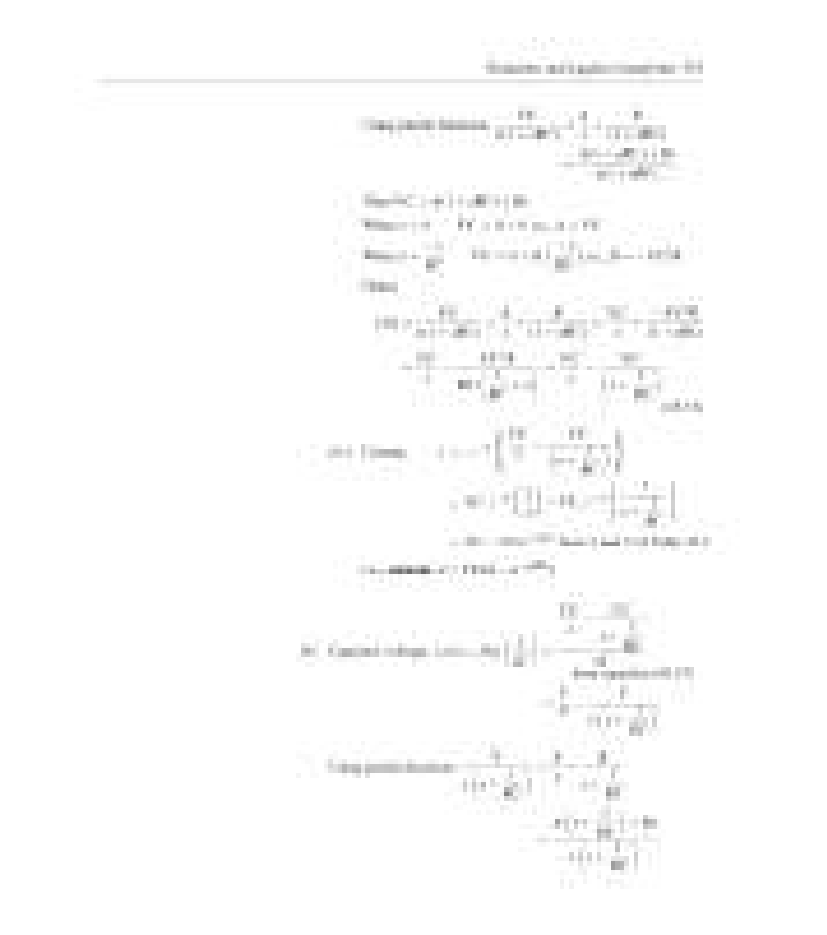




Figure 4C-2(a)



Figure 4C-2(b)

Problem 4C-2. Determine for the 4C-2 series circuit shown in Figure 4C-2(a) representing the network a. reference voltage, e_r, and resistive voltage e_R when a step voltage, E, is applied to the input terminals.

Using the given values:

(a) The resistive circuit is shown in Figure 4C-2(b).

(b) Using the Kirchhoff's voltage law: $\frac{E}{s} - e_r(s)e(s) - e_R(s)R$

Let $\mathcal{C}$ be a compact Hausdorff space and let $\mathcal{D}$ be a discrete space. Let $\mathcal{F}$ be a sheaf on $\mathcal{C}$ and let $\mathcal{G}$ be a sheaf on $\mathcal{D}$. Let $\mathcal{H}$ be a sheaf on $\mathcal{C}$ and let $\mathcal{I}$ be a sheaf on $\mathcal{D}$. Let $\mathcal{J}$ be a sheaf on $\mathcal{C}$ and let $\mathcal{K}$ be a sheaf on $\mathcal{D}$. Let $\mathcal{L}$ be a sheaf on $\mathcal{C}$ and let $\mathcal{M}$ be a sheaf on $\mathcal{D}$. Let $\mathcal{N}$ be a sheaf on $\mathcal{C}$ and let $\mathcal{O}$ be a sheaf on $\mathcal{D}$. Let $\mathcal{P}$ be a sheaf on $\mathcal{C}$ and let $\mathcal{Q}$ be a sheaf on $\mathcal{D}$. Let $\mathcal{R}$ be a sheaf on $\mathcal{C}$ and let $\mathcal{S}$ be a sheaf on $\mathcal{D}$. Let $\mathcal{T}$ be a sheaf on $\mathcal{C}$ and let $\mathcal{U}$ be a sheaf on $\mathcal{D}$. Let $\mathcal{V}$ be a sheaf on $\mathcal{C}$ and let $\mathcal{W}$ be a sheaf on $\mathcal{D}$. Let $\mathcal{X}$ be a sheaf on $\mathcal{C}$ and let $\mathcal{Y}$ be a sheaf on $\mathcal{D}$. Let $\mathcal{Z}$ be a sheaf on $\mathcal{C}$ and let $\mathcal{A}$ be a sheaf on $\mathcal{D}$. Let $\mathcal{B}$ be a sheaf on $\mathcal{C}$ and let $\mathcal{C}$ be a sheaf on $\mathcal{D}$. Let $\mathcal{D}$ be a sheaf on $\mathcal{C}$ and let $\mathcal{E}$ be a sheaf on $\mathcal{D}$. Let $\mathcal{F}$ be a sheaf on $\mathcal{C}$ and let $\mathcal{G}$ be a sheaf on $\mathcal{D}$. Let $\mathcal{H}$ be a sheaf on $\mathcal{C}$ and let $\mathcal{I}$ be a sheaf on $\mathcal{D}$. Let $\mathcal{J}$ be a sheaf on $\mathcal{C}$ and let $\mathcal{K}$ be a sheaf on $\mathcal{D}$. Let $\mathcal{L}$ be a sheaf on $\mathcal{C}$ and let $\mathcal{M}$ be a sheaf on $\mathcal{D}$. Let $\mathcal{N}$ be a sheaf on $\mathcal{C}$ and let $\mathcal{O}$ be a sheaf on $\mathcal{D}$. Let $\mathcal{P}$ be a sheaf on $\mathcal{C}$ and let $\mathcal{Q}$ be a sheaf on $\mathcal{D}$. Let $\mathcal{R}$ be a sheaf on $\mathcal{C}$ and let $\mathcal{S}$ be a sheaf on $\mathcal{D}$. Let $\mathcal{T}$ be a sheaf on $\mathcal{C}$ and let $\mathcal{U}$ be a sheaf on $\mathcal{D}$. Let $\mathcal{V}$ be a sheaf on $\mathcal{C}$ and let $\mathcal{W}$ be a sheaf on $\mathcal{D}$. Let $\mathcal{X}$ be a sheaf on $\mathcal{C}$ and let $\mathcal{Y}$ be a sheaf on $\mathcal{D}$. Let $\mathcal{Z}$ be a sheaf on $\mathcal{C}$ and let $\mathcal{A}$ be a sheaf on $\mathcal{D}$. Let $\mathcal{B}$ be a sheaf on $\mathcal{C}$ and let $\mathcal{C}$ be a sheaf on $\mathcal{D}$. Let $\mathcal{D}$ be a sheaf on $\mathcal{C}$ and let $\mathcal{E}$ be a sheaf on $\mathcal{D}$.

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Figure 4.2-16

(a) From $\int_0^\infty f(x)\,dx = 1$: $\int_0^\infty kx^2 e^{-x/2}\,dx = 1$.

(b) Substitute $u = \frac{x}{2}$, $dx = 2\,du$: $k\int_0^\infty (2u)^2 e^{-u}\,2\,du = 8k\int_0^\infty u^2 e^{-u}\,du = 8k\,\Gamma(3) = 8k\cdot 2! = 16k$.

Using properties of the Gamma function:

$$\Gamma(3) = 2\,\Gamma(2) = 2\cdot 1\cdot\Gamma(1) = 2\cdot 1\cdot 1 = 2$$

Hence $16k = 1$, so $k = \frac{1}{16}$.

Therefore the probability density function is

$$f(x) = \begin{cases} \frac{1}{16}x^2 e^{-x/2} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

(c) Mean: $E(X) = \int_0^\infty x\cdot\frac{1}{16}x^2 e^{-x/2}\,dx = \frac{1}{16}\int_0^\infty x^3 e^{-x/2}\,dx$

Substitute $u = \frac{x}{2}$, $dx = 2\,du$:

$$= \frac{1}{16}\int_0^\infty (2u)^3 e^{-u}\,2\,du = \frac{16}{16}\int_0^\infty u^3 e^{-u}\,du = \Gamma(4) = 3! = 6$$

Variance: $E(X^2) = \int_0^\infty x^2\cdot\frac{1}{16}x^2 e^{-x/2}\,dx = \frac{1}{16}\int_0^\infty x^4 e^{-x/2}\,dx$

$$= \frac{1}{16}\int_0^\infty (2u)^4 e^{-u}\,2\,du = \frac{32}{16}\int_0^\infty u^4 e^{-u}\,du = 2\,\Gamma(5) = 2\cdot 4! = 48$$

$$\text{Var}(X) = E(X^2) - [E(X)]^2 = 48 - 6^2 = 48 - 36 = 12$$

(d) Standard deviation: $\sigma = \sqrt{\text{Var}(X)} = \sqrt{12} = 2\sqrt{3}$

$$P(|X - \mu| < \sigma) = P(6 - 2\sqrt{3} < X < 6 + 2\sqrt{3}) = P(X < 6 + 2\sqrt{3}) - P(X < 6 - 2\sqrt{3})$$

$$= \frac{1}{16}\int_0^{6+2\sqrt{3}} x^2 e^{-x/2}\,dx - \frac{1}{16}\int_0^{6-2\sqrt{3}} x^2 e^{-x/2}\,dx$$

$$= \frac{1}{16}\left[-2x^2 e^{-x/2} - 8x e^{-x/2} - 16 e^{-x/2}\right]_{6-2\sqrt{3}}^{6+2\sqrt{3}}$$

$$= \frac{1}{16}\left[-2(6+2\sqrt{3})^2 e^{-(3+\sqrt{3})} - 8(6+2\sqrt{3})e^{-(3+\sqrt{3})} - 16e^{-(3+\sqrt{3})} + 2(6-2\sqrt{3})^2 e^{-(3-\sqrt{3})} + 8(6-2\sqrt{3})e^{-(3-\sqrt{3})} + 16e^{-(3-\sqrt{3})}\right]$$

$$= -\frac{1}{16}\left[2(6+2\sqrt{3})^2 e^{-(3+\sqrt{3})} + 8(6+2\sqrt{3})e^{-(3+\sqrt{3})} + 16e^{-(3+\sqrt{3})} - 2(6-2\sqrt{3})^2 e^{-(3-\sqrt{3})} - 8(6-2\sqrt{3})e^{-(3-\sqrt{3})} - 16e^{-(3-\sqrt{3})}\right]$$
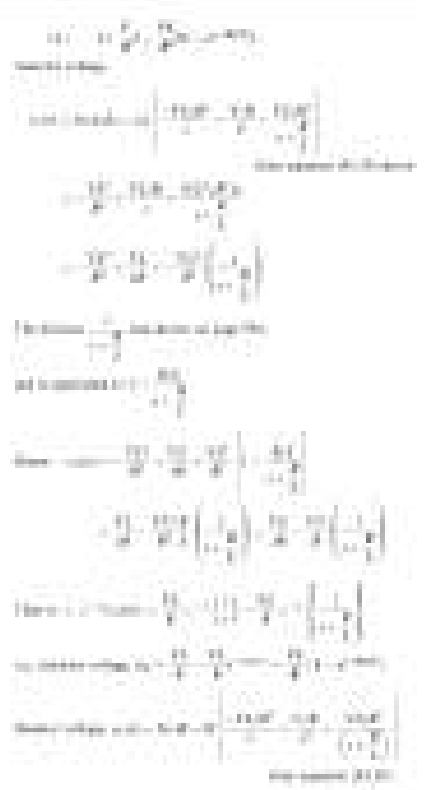




Figure 10.1.10



Figure 10.1.11



$$= \frac{10}{(R+10)(R+20)}\left[\frac{R}{(R+10)}\frac{20}{(R+20)} - \frac{10}{(R+20)}\frac{R}{(R+10)}\right]$$

(b) Voltage $v = \cdots$ Watts

$$= \frac{200}{R+10}\left[\frac{R}{R+20}\right] = \frac{200R}{(R+10)(R+20)}$$

Watts (1.5 mark) (1.5 mark) (1)

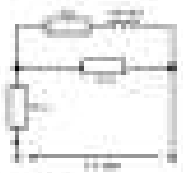


Figure 10.24

Problem 11 In the circuit shown in Figure 10.24, a 12 volt voltage is applied to an unspecified load. Determine the equivalent resistance and current drawn from the source.

(a) With a voltage divider $R_{eq} = \cdots$ $= \frac{20R + 10 \cdot 20}{R+10+20} = \frac{20R+200}{R+30}$

$$= \frac{20R + 10 \cdot 20 + 10R}{R+30}$$

$$= \frac{30R + 200}{R+30} = \frac{10(3R+20)}{R+30}$$

Thus we can calculate the input voltage to (1) in this:

$$V = \frac{R_{eq}}{R_{eq}} \cdot \frac{12}{\frac{10(3R+20)}{R+30} + 10} = \frac{12}{\frac{10(3R+20)}{R+30} + \frac{10(R+30)}{R+30}} = \frac{12(R+30)}{10(3R+20)+10(R+30)}$$

$$= \frac{12R+360}{30R+200+10R+300} = \frac{12R+360}{40R+500}$$

$$= \frac{12R+360}{40R+500}$$

$$= \frac{3R+90}{10R+125}$$

$$= \frac{3R+90}{10R+125}$$

Using partial fractions: $\frac{A}{(R+10)} + \frac{B}{(R+20)}$

$$= \frac{3R+90}{(R+10)(R+20)}$$

Now, finally, $R = 0 \Rightarrow 0.72$ volts

Example 4.11: Solving a Nonlinear Circuit Using Newton's Method

Figure 4.11.1

Figure 4.11.2

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0 \tag{3.26}$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + \rho g_x$$

and similarly for the y- and z-momentum equations. The left-hand side of Eq. (3.27) represents the rate of change of momentum per unit volume of a fluid element as it moves with the flow, while the right-hand side collects the pressure-gradient, viscous, and body-force contributions. Taken together, Eqs. (3.26) and (3.27) form a closed system for the four unknowns u, v, w, and p once the density and viscosity are specified.



Figure 3.12 A simple electrical circuit consisting of a battery, a resistor, and a capacitor, used as a lumped-element analogue in the discussion of conservation laws.

Now consider the simple electrical circuit shown in Figure 3.12, which contains only a battery, a resistor, and a capacitor. Although it is far removed from the fluid problems that concern us, it will serve as a useful illustration of the same conservation principle in a different physical guise. Let $q(t)$ denote the charge stored on the capacitor and $i(t)$ the current flowing around the loop. Kirchhoff's voltage law states that the sum of the voltage drops around any closed loop must equal the applied voltage:

$$L\frac{di}{dt} + Ri + \frac{q}{C} = V(t), \qquad i = \frac{dq}{dt}.$$

Substituting the second relation into the first yields a second-order ordinary differential equation for q alone:

$$L\frac{d^2 q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = V(t).$$

$$\frac{\left[\begin{array}{cc} \frac{1}{s} & -\frac{1}{s^2} \\ 0 & \frac{1}{s} \end{array}\right]}{\left[\begin{array}{cc} 1 & -\frac{1}{s} \\ 0 & 1 \end{array}\right]} \left[\begin{array}{cc} 1 & -\frac{1}{s} \\ 0 & 1 \end{array}\right]^{-1}$$

$$\left[\begin{array}{cc} \frac{1}{s} & -\frac{1}{s^2} \\ 0 & \frac{1}{s} \end{array}\right] \left[\begin{array}{cc} 1 & \frac{1}{s} \\ 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cc} \frac{1}{s} & 0 \\ 0 & \frac{1}{s} \end{array}\right]$$

$$\frac{1}{s^2+\omega^2} \left[\begin{array}{cc} s & \omega \\ -\omega & s \end{array}\right]$$

$$\frac{1}{s^2+\omega^2} \left[\begin{array}{cc} s & -\omega \\ \omega & s \end{array}\right]$$

Show numerically that $\left[\begin{array}{cc} 1 & -\frac{1}{s} \\ 0 & 1 \end{array}\right]^{-1} = \left[\begin{array}{cc} 1 & \frac{1}{s} \\ 0 & 1 \end{array}\right]$.

Hint: Use the Taylor series for $\frac{1}{1-x}$.

Show that $\left[\begin{array}{cc} 1 & -\frac{1}{s} \\ 0 & 1 \end{array}\right]^{-1} = \left[\begin{array}{cc} 1 & \frac{1}{s} \\ 0 & 1 \end{array}\right]$ directly.

Example:

The calculations for example 3 in the previous unit will now be repeated using the matrix methods developed in this chapter.

Solution 1. Where $M = 0$, the circuit is **mechanical**, and, from equation 2 in Unit 6,

$$H(s) = \frac{V_2(s)}{V_1(s)} = \frac{1}{1 + \frac{K}{Ms^2}}$$

Since in Example 3 the characteristic root $s = -\frac{K}{M}$, then $H(s) = \frac{1}{1 + \frac{K}{s - \frac{K}{M}}}$

$$H(s) = \frac{s - \frac{K}{M}}{s - \frac{K}{M} + K} = \frac{s}{s + \frac{KM}{M}} = \frac{s}{s + K}$$

Since $\mathcal{L}^{-1}\left\{\frac{1}{s+a}\right\} = e^{-at}$ (Answer from 3 in Table of), which is a time-domain convolution $\frac{s}{s+K}$ with impulse response as specified. The solution is shown in Figure 8.1.21.

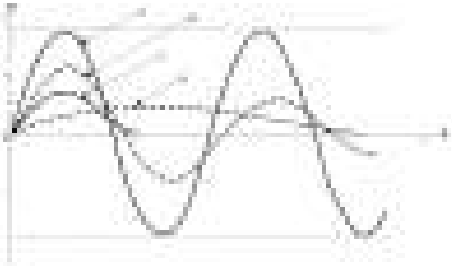


Figure 8.24

Solution 1: When $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$, the system is **underdamped** and the solution is as in equation (8.25). The constant A can then be written as a decaying exponential. This is shown in the curve R in Figure 8.24.

Solution 2: When $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$, the system is **critically damped** and from equation (8.26),

$$q(t) = \frac{V_0 C}{\left(1 - \frac{R}{2L}\right)} \text{ and } \frac{dq}{dt} = I = V_0 \left| \frac{C}{\left(1 - \frac{R}{2L}\right)} \right| e^{-\left(\frac{R}{2L}\right)t} \quad (8.30)$$

Now $I_0^2 = V_0^2 C/L$ (8.31)

The constant curve corresponds with curve C in Figure 8.24.

Solution 3: When $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$, the system is **overdamped** and from equation (8.27),

$$q(t) = \frac{V_0 C}{\left(1 - \frac{R}{2L}\right) - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}}$$

$$\frac{\left[\left(\frac{R_1}{R_2}\right)^{2}-\frac{L_1}{L_2}\right]}{\left[\left(\frac{R_1}{R_2}\right)^{2}-\frac{L_1}{L_2}\right]\left[1-\frac{R_1}{R_2}\right]-\left[\left(\frac{R_1}{R_2}\right)^{2}-\frac{L_1}{L_2}\right]}$$

(a) Expression $= \ldots$ **(b)** Show

$$\frac{R}{\sqrt{\left[\left(\frac{R_1}{R_2}\right)^{2}-\frac{L_1}{L_2}\right]}}\ \text{ohms} \quad \textbf{and} \quad \left[\left(\frac{R_1}{R_2}\right)^{2}-\frac{L_1}{L_2}\right]>0$$

Hint: it is likely that

Illustrate the answer to part (a) by a phasor diagram.

Problem 37. An R–L–C series circuit contains a coil of inductance 0.15 mH and resistance 4.5 Ω and a capacitor of capacitance 0.012 μF connected across it. If a voltage of 1.5 V, frequency 1.0 MHz, is applied to the circuit, find (a) the impedance of the circuit, (b) the phase angle between the voltage and current, (c) the voltage across the coil and (d) the voltage across the capacitor.

(a) $\left(\frac{R}{2L}\right)^{2}-\frac{1}{LC}$ is real, $\frac{1}{4L^{2}}-\frac{1}{LC}=\frac{1}{4(0.15\times10^{-3})^{2}}-\frac{1}{(0.15\times10^{-3})(0.012\times10^{-6})}=$ positive

Since $\left(\frac{R}{2L}\right)^{2}-\frac{1}{LC}$ is positive, the circuit is **underdamped**.

(b) Since $\left(\frac{R}{2L}\right)^{2}-\frac{1}{LC}$ is positive, (a) applies

$$\omega_n=\frac{1}{\sqrt{LC}}=\frac{1}{\sqrt{(0.15\times10^{-3})(0.012\times10^{-6})}}=\text{rad/s}$$

$$\omega_d=\sqrt{\left(\frac{1}{LC}\right)-\left(\frac{R}{2L}\right)^{2}}=\sqrt{\left(\frac{1}{(0.15\times10^{-3})(0.012\times10^{-6})}\right)-\left(\frac{4.5}{2(0.15\times10^{-3})}\right)^{2}}$$

$$=\sqrt{(\text{rad/s})^{2}-(\text{rad/s})^{2}}=\text{rad/s}$$

$\omega_d = \omega_n$ **only when** $R =$ **zero or** $L =$

Problem 38. Series R–L–C Circuit. For a series R–L–C circuit with $R = 100\ \Omega$, $L = 400$ mH and $C = 0.25\ \mu$F, a voltage of 5 V is applied to the circuit, frequency variable. Calculate (a) the frequency of oscillation for the circuit, (b) the value of damping, and (c) the equation for the current i.

[illegible]

10.5 Ballast resistance. [illegible]



Figure 10.24



Figure 10.25

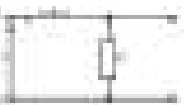


Figure 10.26



Figure 4C.8

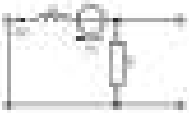


Figure 4C.9



Figure 4C.10



Figure 9-2.19



Figure 9-2.20



Figure 9-2.21



Figure 9-2.22

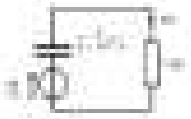


Figure 9-2.23

Problem 9-2. Determine the expression for current i flowing through the 2-kΩ resistor in Figure 9-2.24 for values that exceed 4 seconds after the 2-volt source is switched from point a to point b. Note that switching occurs at $t = 0$ seconds.

Figure 9-2.24

[illegible]

[illegible]

[illegible]

1–2 Use logarithmic differentiation to find $\frac{dy}{dx}$.

1. $y = \sqrt{x}\,e^{x^2}(x^2+1)^{10}$

2. $y = \dfrac{x^{3/4}\sqrt{x^2+1}}{(3x+2)^5}$

3–4 Use logarithmic differentiation to find the derivative of the function at the given point.

3. $y = x\sqrt{\dfrac{x-1}{x+1}}$, $(2, 2/\sqrt{3})$

4. $y = x^{\ln x}$, (e, e)

5. Find an equation of the tangent line to the curve $y = (x-1)(x-2)^2(x-3)^3$ at the point $(0, -6)$.

6. Find an equation of the tangent line to the curve $y = x^{\sin x}$ at the point (π, π).

7. Find $f'(x)$ if $f(x) = x^x$.

8. Find $f'(x)$ if $f(x) = x^{\sqrt{x}}$.

9. Find $f'(x)$ if $f(x) = x^{\ln x}$.

10. Find $f'(x)$ if $f(x) = x^{1/x}$.

11. Find $f'(x)$ if $f(x) = x^{x^2}$.

12. Find $f'(x)$ if $f(x) = x^{x^x}$.

13. Find $f'(x)$ if $f(x) = (\tan x)^{1/x}$.

14. Find $f'(x)$ if $f(x) = (\cos x)^{\ln x}$.

15. Find $f'(x)$ if $f(x) = (\ln x)^{\cos x}$.

16. Find $f'(x)$ if $f(x) = (\sin x)^{\tan x}$.

17. Find $f'(x)$ if $f(x) = (\cos x)^{\sin x}$.

18. Find $f'(x)$ if $f(x) = (\ln x)^{\ln x}$.

19. Find $f'(x)$ if $f(x) = \sqrt{x}^{\sqrt{x}}$.

20. Find $f'(x)$ if $f(x) = x^{\sqrt{x^{\sqrt{x}}}}$.

21. Find $f'(x)$ if $f(x) = \sqrt{x^{\sqrt{x^{\sqrt{x}}}}}$.

22. Find $f'(x)$ if $f(x) = \sqrt{x^{\sqrt{x^{\sqrt{x^{\sqrt{x}}}}}}}$.

23. Find $f'(x)$ if $f(x) = \sqrt{x^{\sqrt{x^{\sqrt{x^{\sqrt{x^{\sqrt{x}}}}}}}}}$.

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50. Find $f'(x)$ if $f(x) = \sqrt{x^{\sqrt{x}}$.

51–52 Use logarithmic differentiation to find the derivative of the function.

51. $y = \dfrac{(x^2+1)^3\sqrt{x^2-1}}{x+4}$

52. $y = \dfrac{x\sqrt{x^2+1}\,(x-1)^{3/4}}{(x+1)^{2/3}}$

53–54 Find an equation of the tangent line to the curve at the given point.

53. $y = x^{\ln x}$, (e, e)

54. $y = x\sqrt{\dfrac{x-1}{x+1}}$, $(2, 2/\sqrt{3})$

55. Find $f'(x)$ if $f(x) = x^{\sin x}$.

56. Find $f'(x)$ if $f(x) = x^{1/x}$.

57. Find $f'(x)$ if $f(x) = x^{x^2}$.

58. Find $f'(x)$ if $f(x) = x^{x^x}$.

59. Find $f'(x)$ if $f(x) = (\tan x)^{1/x}$.

60. Find $f'(x)$ if $f(x) = (\cos x)^{\ln x}$.

61. Find $f'(x)$ if $f(x) = (\ln x)^{\cos x}$.

62. Find $f'(x)$ if $f(x) = (\sin x)^{\tan x}$.

63. Find $f'(x)$ if $f(x) = (\cos x)^{\sin x}$.

64. Find $f'(x)$ if $f(x) = (\ln x)^{\ln x}$.

65. Find $f'(x)$ if $f(x) = \sqrt{x}^{\sqrt{x}}$.

66. Find $f'(x)$ if $f(x) = x^{\sqrt{x^{\sqrt{x^{\sqrt{x}}}}}}$.

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138. Find $f'(x)$ if $f(x) = \sqrt{x^{\sqrt{x^{\sqrt{x^{\sqrt{x^{\sqrt{x^{\sqrt{x^{\sqr

(b) A uniform rod of length $2a$ and mass m is free to rotate in a vertical plane about a fixed horizontal axis through one end. The rod is released from rest when it is horizontal. Find the angular velocity of the rod when it has turned through an angle θ. Hence find the horizontal and vertical components of the force exerted on the rod by the axis when $\theta = \frac{\pi}{3}$. [(b) $\sqrt{\frac{3g\sin\theta}{a}}$; $\frac{3}{2}mg\sin\theta\cos\theta$, $mg(\frac{1}{2}+\frac{3}{2}\sin^2\theta)$]

17 Determine the Laplace transforms of the following waveforms:

(a) a square wave of height A and period T, which starts at time $t = 0$;

(b) a triangular wave as shown in Fig. 17.(b);

(c) a sawtooth waveform of height A and period T as shown in Fig. 17.(c).



[(a) $\frac{A}{s}\left(\frac{1-e^{-\frac{sT}{2}}}{1+e^{-\frac{sT}{2}}}\right)$; (b) $\frac{A}{sT}\left(\frac{1-e^{-\frac{sT}{2}}}{1+e^{-\frac{sT}{2}}}\right)^2$; (c) $\frac{A}{sT}\left(\frac{1}{1-e^{-sT}}\right)-\frac{A}{s}$]

18 Show that the Laplace transform of $\sin\omega t$ is $\frac{\omega}{s^2+\omega^2}$.

Hence find the Laplace transform of $\sin(\omega t+\alpha)$. [$\frac{s\sin\alpha+\omega\cos\alpha}{s^2+\omega^2}$]

19 Find the Laplace transform of $\cos\omega t$ by differentiating the result of Problem 18.

Hence find the Laplace transform of $\cos(\omega t+\alpha)$. [$\frac{s\cos\alpha-\omega\sin\alpha}{s^2+\omega^2}$]

20 Find the Laplace transform of $e^{-at}\sin\omega t$. [$\frac{\omega}{(s+a)^2+\omega^2}$]

21 Find the Laplace transform of $e^{-at}\cos\omega t$. [$\frac{s+a}{(s+a)^2+\omega^2}$]

Inverse Laplace transforms and the solution of differential equations

Determine the inverse Laplace transforms of the following in Problems 22 to 37:

22 $\frac{1}{s^2}$ [t]

23 $\frac{1}{(s+1)^2}$ [te^{-t}]

24 $\frac{1}{(s+1)(s+2)}$ [$e^{-t}-e^{-2t}$]

25 $\frac{3s}{(s+1)(s+2)}$ [$6e^{-t}-3e^{-2t}$]

26 $\frac{s}{s^2+9}$ [$\cos 3t$]

27 $\frac{3}{s^2+4}$ [$\frac{3}{2}\sin 2t$]

28 $\frac{s+1}{s^2+2s+5}$ [$e^{-t}\cos 2t$]

29 $\frac{s+3}{s^2+4s+13}$ [$e^{-2t}\cos 3t+\frac{1}{3}e^{-2t}\sin 3t$]

30 $\frac{s+2}{s^2+4s+5}$ [$e^{-2t}\cos t+2e^{-2t}\sin t$]

31 $\frac{s+1}{s^2+2s+2}$ [$e^{-t}\cos t$]

The Laplace transform is used to solve differential equations of the form given in Problems 32 to 37.

a. $\frac{(1-x)^{\frac{1}{2}}}{x^{\frac{1}{2}}(1+x)^{\frac{1}{2}}}$ $\left[\frac{1}{2}(x-x^2)^{-\frac{1}{2}}(1-2x)\right]$

b. $\frac{\sin^2 x}{x^2+\tan^2 x+\sin^2 x\tan^2 x}$ $\left[\frac{\sin x-x\cos^3 x}{x(1+\cos^2 x)^2}\right]$

c. $\frac{2x+1}{x-1}$ $\left[\frac{-3}{(x-1)^2}\right]$

d. $\frac{3x^2-2x+13}{x^3+5x^2+17}$ $\left[\frac{-x^4+4x^3+26x^2-34x-221}{(x^3+5x^2+17)^2}\right]$

e. $\frac{x^2+\sin x+1}{\cos x+x^2}$ $\left[\frac{x^2\sin x+2x\cos x+\sin x-2x-1}{(\cos x+x^2)^2}\right]$

f. $\frac{2x-1}{\sqrt{x^2+x+1}}$ $\left[\frac{3}{2}(x^2+x+1)^{-\frac{1}{2}}+(2x-1)\left(-\frac{1}{2}\right)(x^2+x+1)^{-\frac{3}{2}}(2x+1)\right]$

g. Find $\frac{dy}{dx}$ at $x=0$. Use a graphing calculator to verify that your answer is correct.

$y=\sin^{-1}\frac{x}{\sqrt{x^2+1}}+\sin\frac{x}{\sqrt{x^2+1}}+\cos\frac{x}{\sqrt{x^2+1}}$ $\left[\frac{dy}{dx}=\frac{2}{\sqrt{x^2+1}}\right]$

h. $y=\frac{\sin x}{x}+\cos x$ $\left[\frac{dy}{dx}=\frac{x\cos x-\sin x}{x^2}-\sin x\right]$

i. $y=\frac{\sin x}{x}+\cos x$ $\left[\frac{dy}{dx}=\frac{x\cos x-\sin x}{x^2}-\sin x\right]$

j. $y=\frac{x}{\sin x}-\cos x$ $\left[\frac{dy}{dx}=\frac{\sin x-x\cos x}{\sin^2 x}+\sin x\right]$

k. $y=\frac{\sin x}{\cos x}-\tan x$ $\left[\frac{dy}{dx}=0\right]$

l. $y=\frac{\sin x}{\cos x}-\tan x$ $\left[\frac{dy}{dx}=\begin{cases}0 & x\neq\frac{\pi}{2}+n\pi\\ \text{undefined} & x=\frac{\pi}{2}+n\pi\end{cases}\right]$

m. $y=\frac{\sin x}{\cos x}-\tan x$ $\left[\frac{dy}{dx}=\begin{cases}0 & x\neq\frac{\pi}{2}+n\pi\\ \text{undefined} & x=\frac{\pi}{2}+n\pi\end{cases}\right]$

Calculus Connections: analysis of circular diagrams

14. Determine the circumference of a circle of diameter d in terms of d. $[C=\pi d]$

15. Determine the circumference of a circle of radius r in terms of r. $[C=2\pi r]$

P1. Determine the current equivalent to the combination in the following:

(a) a resistor, R, $20\,\Omega$ in series with a $0.5\,\mathrm{A}$ capacitive

(b) an inductance of $10\,\mathrm{mH}$ in resistance of $50\,\mathrm{k}\Omega$ and a capacitance of $100\,\mathrm{nF}$ in series

(c) a $4\,\Omega$ resistance in parallel with a $10\,\mathrm{mH}$ inductor

(d) a $4\,\Omega$ inductance in parallel with a $10\,\mathrm{mF}$ capacitor

$$\left[(a)\ \left(20+j\frac{10^{6}}{0.5}\right)\Omega;\ (b)\ \left(50\times10^{3}+j\left(2\pi\times10^{3}\times10\times10^{-3}-\frac{10^{9}}{2\pi\times10^{3}\times100}\right)\right)\Omega\right.$$

$$\Rightarrow \frac{4(j2\pi\times10^{3}\times10\times10^{-3})}{4+j2\pi\times10^{3}\times10\times10^{-3}}\ \Omega;\ \frac{4(-j/2\pi\times10^{3}\times10\times10^{-3})}{4-j/2\pi\times10^{3}\times10\times10^{-3}}\ \Omega$$

$$\left.\Rightarrow \frac{j80\pi}{4+j20\pi}\ \Omega;\ \frac{-j40\pi}{4-j40\pi}\ \Omega\right]$$

P2. (a) A $50\,\Omega$ resistor is connected in series with a $100\,\Omega$ resistance, a $0.5\,\mathrm{mH}$ inductance and a $50\,\mathrm{pF}$ capacitance. Determine (i) the resonant frequency, (ii) the bandwidth and (iii) the quality factor. (b) Determine the resonant frequency, and (i) the bandwidth and (ii) the quality factor when the components are connected in parallel.

$$\left[(a)\ (i)\ \left(50+100+j\left(2\pi f\times0.5\times10^{-3}-\frac{1}{2\pi f\times50\times10^{-12}}\right)\right)\Omega\right.$$

$$\text{at resonance, } 2\pi f\times0.5\times10^{-3}=\frac{1}{2\pi f\times50\times10^{-12}}$$

$$\left.\Rightarrow f=\frac{1}{2\pi}\sqrt{\frac{1}{0.5\times10^{-3}\times50\times10^{-12}}}=\frac{10^{6}}{\pi}\ \mathrm{Hz}\right]$$

P3. A circuit consists of a $2.5\,\Omega$ resistor in series with a $0.5\,\mathrm{mH}$ inductance and a $20\,\mu\mathrm{F}$ capacitance. Determine (a) the resonant frequency, (b) the bandwidth, (c) the quality factor, (d) the current flowing in the circuit at resonance if the applied voltage is $5\,\mathrm{V}$, (e) the voltage across the capacitor at resonance, (f) the voltage across the inductor at resonance, (g) the power dissipated at resonance, (h) the half-power frequencies, and (i) the voltage across the resistor at the half-power frequencies.

$$\left[(a)\ f_r=\frac{1}{2\pi\sqrt{0.5\times10^{-3}\times20\times10^{-6}}}=\frac{10^{4}}{2\pi}\ \mathrm{Hz};\ (b)\ \Delta f=\frac{2.5}{2\pi\times0.5\times10^{-3}}=\frac{2.5\times10^{3}}{\pi}\ \mathrm{Hz};\right.$$

$$(c)\ Q=\frac{f_r}{\Delta f}=4;\ (d)\ I=\frac{5}{2.5}=2\ \mathrm{A};\ (e)\ V_C=2\times\frac{1}{2\pi\times\frac{10^{4}}{2\pi}\times20\times10^{-6}}=20\ \mathrm{V};$$

$$(f)\ V_L=20\ \mathrm{V};\ (g)\ P=I^{2}R=2^{2}\times2.5=10\ \mathrm{W};$$

$$\left.(h)\ f_{1,2}=\frac{10^{4}}{2\pi}\pm\frac{2.5\times10^{3}}{2\pi};\ (i)\ V_R=5\times\frac{2.5}{\sqrt{2.5^{2}+\left(2\pi\times\frac{10^{4}}{2\pi}\times0.5\times10^{-3}-\frac{1}{2\pi\times\frac{10^{4}}{2\pi}\times20\times10^{-6}}\right)^{2}}}=3.54\ \mathrm{V}\right]$$

P4. A $10\,\mu\mathrm{H}$ inductor is connected in series with a $47\,\mathrm{pF}$ capacitor and a $100\,\Omega$ resistor. Calculate (a) the resonant frequency, (b) the bandwidth, (c) the quality factor, (d) the half-power frequencies, (e) the voltage across the capacitor at resonance if the applied voltage is $5\,\mathrm{V}$, (f) the voltage across the inductor at resonance, (g) the power dissipated at resonance, (h) the current flowing in the circuit at the half-power frequencies, and (i) the voltage across the capacitor at the half-power frequencies.

$$\left[(a)\ f_r=\frac{1}{2\pi\sqrt{10\times10^{-6}\times47\times10^{-12}}}=\frac{10^{8}}{2\pi\sqrt{47}}\ \mathrm{Hz};\ (b)\ \Delta f=\frac{100}{2\pi\times10\times10^{-6}}=\frac{10^{7}}{2\pi}\ \mathrm{Hz};\right.$$

$$(c)\ Q=\frac{f_r}{\Delta f}=\sqrt{47};\ (d)\ f_{1,2}=\frac{10^{8}}{2\pi\sqrt{47}}\pm\frac{10^{7}}{2\pi};$$

$$(e)\ V_C=5\times\frac{1}{2\pi\times\frac{10^{8}}{2\pi\sqrt{47}}\times47\times10^{-12}}=\sqrt{47}\times5=34.27\ \mathrm{V};$$

$$(f)\ V_L=34.27\ \mathrm{V};\ (g)\ P=I^{2}R=\left(\frac{5}{100}\right)^{2}\times100=2.5\ \mathrm{W};$$

$$\left.(h)\ I=\frac{5}{\sqrt{100^{2}+\left(2\pi\times\frac{10^{8}}{2\pi\sqrt{47}}\times10\times10^{-6}-\frac{1}{2\pi\times\frac{10^{8}}{2\pi\sqrt{47}}\times47\times10^{-12}}\right)^{2}}}=0.0354\ \mathrm{A};\ (i)\ V_C=0.0354\times\frac{1}{2\pi\times\frac{10^{8}}{2\pi\sqrt{47}}\times47\times10^{-12}}=24.25\ \mathrm{V}\right]$$

P5. A $20\,\mu\mathrm{H}$ inductor is connected in series with a $100\,\mathrm{pF}$ capacitor and a $50\,\Omega$ resistor. Calculate (a) the resonant frequency, (b) the bandwidth, (c) the quality factor, (d) the half-power frequencies, (e) the voltage across the capacitor at resonance if the applied voltage is $10\,\mathrm{V}$, (f) the voltage across the inductor at resonance, (g) the power dissipated at resonance, (h) the current flowing in the circuit at the half-power frequencies, and (i) the voltage across the resistor at the half-power frequencies.

$$\left[(a)\ f_r=\frac{1}{2\pi\sqrt{20\times10^{-6}\times100\times10^{-12}}}=\frac{10^{8}}{2\pi\sqrt{20}}\ \mathrm{Hz};\ (b)\ \Delta f=\frac{50}{2\pi\times20\times10^{-6}}=\frac{1.25\times10^{6}}{\pi}\ \mathrm{Hz};\right.$$

$$(c)\ Q=\frac{f_r}{\Delta f}=\sqrt{20};\ (d)\ f_{1,2}=\frac{10^{8}}{2\pi\sqrt{20}}\pm\frac{1.25\times10^{6}}{2\pi};$$

$$(e)\ V_C=10\times\frac{1}{2\pi\times\frac{10^{8}}{2\pi\sqrt{20}}\times100\times10^{-12}}=\sqrt{20}\times10=44.72\ \mathrm{V};$$

$$(f)\ V_L=44.72\ \mathrm{V};\ (g)\ P=I^{2}R=\left(\frac{10}{50}\right)^{2}\times50=2\ \mathrm{W};$$

$$\left.(h)\ I=\frac{10}{\sqrt{50^{2}+\left(2\pi\times\frac{10^{8}}{2\pi\sqrt{20}}\times20\times10^{-6}-\frac{1}{2\pi\times\frac{10^{8}}{2\pi\sqrt{20}}\times100\times10^{-12}}\right)^{2}}}=0.1414\ \mathrm{A};\ (i)\ V_R=0.1414\times50=7.07\ \mathrm{V}\right]$$

[illegible] $\left[\int_0^{\infty} \frac{1}{R}\left(1+e^{-\frac{t}{RC}}\right)dt\right]$

19. A rechargeable battery has a terminal voltage of 12.6 V when fully charged and 11.8 V when discharged. It is used to operate a 12 V electronic flash unit that requires a minimum of 11.9 V and a maximum of 12.4 V. Over what fraction of the battery's charge can the flash unit be operated? (*Hint:* Use the equation for the voltage of a discharging capacitor.)

$$\left[\frac{1}{\ln\frac{12.6}{11.8}}\ln\frac{12.6}{11.9} \approx 0.18 \text{ or } 18\%\right]$$

20. Derive an expression for the growth of current that occurs when a resistor R is connected across a capacitor C that has a voltage V_0 across it. (*Hint:* Use the fact that the voltage across a resistor is IR and the voltage across a capacitor is Q/C.)

$$\left[I = \frac{V_0}{R}e^{-\frac{t}{RC}}\right]$$

21. Starting with the expression for the growth of current that you derived in Problem 20, derive an expression for the growth of charge on the capacitor. (*Hint:* Use the fact that the current is the rate at which charge flows.)

$$\left[Q = CV_0\left(1-e^{-\frac{t}{RC}}\right)\right]$$

22. For the circuit shown in Figure 8-42, derive an equation for the current I as a function of time t after the switch is closed. (*Hint:* Use the fact that the sum of the voltages around a circuit loop must equal zero.)

$$\left[I = \frac{\mathcal{E}}{R}e^{-\frac{t}{RC}} \text{ and } \frac{\mathcal{E}}{R} = I_0\right]$$



Figure 8-42

23. For the circuit in Problem 22, derive an equation for the charge Q on the capacitor as a function of time t after the switch is closed.

$$\left[Q = CV_0\left(1-e^{-\frac{t}{RC}}\right)\right]$$

24. For the circuit in Problem 22, $R = 1.0 \times 10^6\ \Omega$ and $C = 1.0\ \mu F$. Derive an equation for the voltage across the capacitor as a function of time after the switch is closed.

$$\left[V = V_0\left(1-e^{-\frac{t}{RC}}\right)\right]$$

25. In a R–C series circuit like that in Problem 22, capacitor C is initially charged to a value of Q_0. (*Hint:* Use the equation for the voltage across a capacitor.) Derive an equation for the current I as a function of time t after the switch is closed. (*Hint:* Use the fact that the sum of the voltages around a circuit loop must equal zero.)

$$\left[I = \frac{Q_0}{RC}e^{-\frac{t}{RC}}\right]$$

26. In the circuit shown in Figure 8-43, the switch has been connected to position 1 for a long time. It is now suddenly switched to position 2. Derive an expression for the current I flowing through the resistor as a function of time t after the switch is thrown.

$$\left[I = I_0 e^{-\frac{t}{RC}}\right]$$



Figure 8-43

Assignment 14

Use assignment to revise the material mentioned in chapters 16 to 18.

Write down your work, questions you have or your findings in the form of short answers.

1. In Wien experiment it is found that photoelectrons emitted from a Potassium (K) ($\phi = 2.25$ eV) and a Lead (Pb) ($\phi = 4.25$ eV) surface have maximum kinetic energies of 0.4 eV and 2.4 eV respectively. Show that Planck's constant has a universal value. (3)

2. In Wien experiment the graph of frequency and stopping potential is found to be a straight line with slope 4.1×10^{-15} Vs. Calculate the value of Planck's constant. (2)

3. Planck's law explains "ultraviolet" catastrophy and the possibility for quantum mechanics to explain Rayleigh-Jeans law at long wavelengths. If $\lambda \to \infty$ then $e^{\frac{hc}{\lambda k_B T}} \approx 1 + \frac{hc}{\lambda k_B T}$. Show that Planck's law reduces to Rayleigh-Jeans law. (2)

4. Two poles of a battery are connected by a platinum wire and a gold wire having radii of 0.0400 mm and 0.0500 mm respectively. The resistivity of platinum is 10.6×10^{-8} Ωm and that of gold is 2.44×10^{-8} Ωm. If the current in the platinum wire is 2.00 A, find the current in the gold wire. Assuming that the electric field has the same magnitude in each wire, find the length of the platinum wire if the length of the gold wire is 5.00 m. (4)

5. Two resistors, resistor 1 with R and 2 with $3R$, are connected in series to a 12 V battery. (a) Find the equivalent resistance of the circuit and the current in each resistor. (b) Find the potential difference across each resistor. (c) Which resistor has the greater potential difference? (d) Find the power dissipated in each resistor. (e) Which resistor has the greater power dissipated? (f) Find the power delivered by the battery. (g) Find the power delivered to the equivalent resistance of the circuit and verify that it is equal to the power delivered by the battery. (7)



Figure 1 to 4

6. For the simple circuit shown in Figure 4 to 1, calculate the current in each resistor. (7)

7. In Wien experiment how does the photoelectric intensity, I, vary with λ? If the photoelectric current is the saturation current of the tube, i, is directly proportional to the intensity of the incident radiation and the photoelectric effect is instantaneous. Explain why the magnitude and direction of the charge on the metal plate will be unchanged when light of wavelength λ falls on the negatively charged plate of the vacuum tube. (5)

8. The primary coil of a transformer has 400 turns and is connected to 120 V ac line voltage. If 1.5 A flows in the primary coil, how many turns must the secondary coil have if it is to deliver 15 A? (3)

[illegible] [illegible] [illegible] [

Main formulae for part 3 advanced circuit theory and technology

Complex amplitudes

$$\text{[illegible]}$$

Inductor

$$\text{[illegible]}$$

R–L–C series circuit

$$\text{[illegible]}$$

2×1 matrices. $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \quad \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix} \quad \begin{bmatrix} \frac{\sqrt{3}}{2} \\ -\frac{1}{2} \end{bmatrix}$

2×2 matrices. $\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$

Determinants. $\det \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix} = -1 \quad \det \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} = 1$

Orbit map. $\phi_x : G \to X, \quad g \mapsto g \cdot x$

Class equation. $|X| = |X^G| + \sum_{[x] \in [X] \setminus X^G} |[x]|$

Equivalence relation. $x \sim y \iff \exists g \in G : g \cdot x = y$

Complete quadrangle.

$$\begin{aligned} &\text{Let } A, B, C, D \text{ be points in general position in } \mathbb{P}^2(\mathbb{C}). \\ &\text{Let } P = AB \cap CD, \quad Q = AC \cap BD, \quad R = AD \cap BC. \\ &\text{Then } P, Q, R \text{ are not collinear.} \end{aligned}$$

Riemann analysis. $\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} \quad \text{for } \operatorname{Re}(s) > 1$

$$\zeta(s) = \prod_{p \text{ prime}} \frac{1}{1 - p^{-s}} \quad \text{for } \operatorname{Re}(s) > 1$$

Riemann and Eddy current. Let $\mathbf{E}$ be a vector field on $\mathbb{R}^3$ such that $\nabla \cdot \mathbf{E} = 0$. Let $\mathbf{B}$ be a vector field on $\mathbb{R}^3$ such that $\nabla \cdot \mathbf{B} = 0$.

Ohm's Law for Simple Circuits Ohm's law for a resistor: $V = IR$ or $I = \frac{V}{R}$

Resistors in series: $R_{eq} = R_1 + R_2 + R_3 + \cdots$

Resistors in parallel: $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$

Power dissipated in a resistor: $P = IV = I^2R = \frac{V^2}{R}$

Energy used: $E = Pt$

Electrical cost: $\text{cost} = \text{energy} \times \text{rate}$

Kirchhoff's Laws Junction rule: $\sum I_{\text{in}} = \sum I_{\text{out}}$

Loop rule: $\sum \Delta V = 0$

Capacitors in series: $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$

Capacitors in parallel: $C_{eq} = C_1 + C_2 + C_3 + \cdots$

Time constant for RC circuits: $\tau = RC$

Charging a capacitor: $q(t) = C\mathcal{E}\left(1 - e^{-t/RC}\right)$ and $I(t) = \frac{\mathcal{E}}{R}e^{-t/RC}$

Discharging a capacitor: $q(t) = Q_0 e^{-t/RC}$ and $I(t) = -\frac{Q_0}{RC}e^{-t/RC}$

Energy stored in a capacitor: $U = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$

Alternating Current Instantaneous current: $i = I_0 \sin(\omega t + \phi)$

Instantaneous voltage: $v = V_0 \sin(\omega t + \phi)$

Capacitive reactance: $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

Inductive reactance: $X_L = \omega L = 2\pi f L$

Impedance: $Z = \sqrt{R^2 + (X_L - X_C)^2}$

Resonant frequency: $f_0 = \frac{1}{2\pi\sqrt{LC}}$

Phase angle: $\tan\phi = \frac{X_L - X_C}{R}$ $\qquad \cos\phi = \frac{R}{Z}$

Average power: $P_{\text{avg}} = I_{\text{rms}} V_{\text{rms}} \cos\phi$

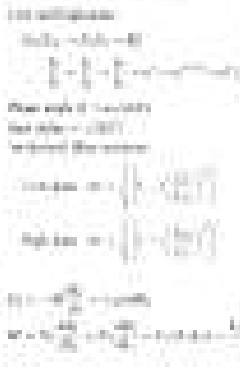
RMS current: $I_{\text{rms}} = \frac{I_0}{\sqrt{2}}$

RMS voltage: $V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$

Offset centre bob



Right-angled V-centre



Unit cell lightweight

Plane angle

Bar plate

Isometric (flat centre)

Hyperbolic cosplot circular

Thermodynamics Sheet

Gibbs free energy $G = H - TS$

Enthalpy change $\Delta H = \Delta U + P\Delta V$

Entropy of a microstate $S = k_B \ln \Omega$

$$\Delta G = \Delta G^\circ + RT \ln Q$$

$$K_c = \frac{[C]^c [D]^d}{[A]^a [B]^b}$$

$$\Delta G = -nFE_{cell}, \quad \Delta G^\circ = -RT \ln K$$

$$E = E^\circ - \frac{RT}{nF} \ln Q$$

$$E = E^\circ - \frac{0.0592}{n} \log Q \quad (25^\circ C)$$

$$pH = pK_a + \log \frac{[A^-]}{[HA]}$$

Buffer capacity $\beta = \frac{dB}{dpH} = 2.303 \frac{K_a [H^+] C_T}{(K_a + [H^+])^2}$

Henderson–Hasselbalch $pH = pK_a + \log \frac{[base]}{[acid]}$

$$pH = pK_a + \log \frac{n_{base}}{n_{acid}}$$

$$pH = \frac{pK_{a1} + pK_{a2}}{2}$$

Kinematics

$v = u + at$

$s = ut + \frac{1}{2}at^2$ $v^2 = u^2 + 2as$ $s = \frac{(u+v)}{2}t$

$s = vt - \frac{1}{2}at^2$

$F = ma$ (Newton's 2nd law)

Centripetal acceleration $a_c = \frac{v^2}{r} = r\omega^2$

$a_t = r\alpha$ $v = r\omega$

$a = \sqrt{a_t^2 + a_c^2}$

Centripetal force $F_c = \frac{mv^2}{r} = mr\omega^2$

Part 4 General Reference

Standard electrical quantities — their symbols and units

Quantity	Usual letter symbol	Unit	Unit abbreviation
Admittance	Y	siemens	S
Angular frequency	ω	radians per second	rad/s
Area	A	square metres	m^2
Attenuation coefficient (or constant)	α	nepers per metre	Np/m
Capacitance	C	farad	F
Charge	Q	coulomb	C
Charge density	ρ	coulombs per cubic metre	C/m^3
Conductance	G	siemens	S
Current	I	ampere	A
Current density	J	amperes per square metre	A/m^2
Efficiency	η	per unit, or per cent	p.u. or %
Electric field strength	E	volts per metre	V/m
Electric flux	Ψ	coulomb	C
Electric flux density	D	coulombs per square metre	C/m^2
Electromotive force	E	volt	V
Energy	W	joule	J
Field strength, electric	E	volts per metre	V/m
Field strength, magnetic	H	amperes per metre	A/m
Flux, electric	Ψ	coulomb	C
Flux, magnetic	Φ	weber	Wb
Flux density, electric	D	coulombs per square metre	C/m^2
Flux density, magnetic	B	tesla	T
Force	F	newton	N
Frequency	f	hertz	Hz
Frequency, angular	ω	radians per second	rad/s
Frequency, fundamental	ω_0	fundamental per second	rad/s
Impedance	Z	ohm	Ω
Inductance, self	L	henry	H

[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]

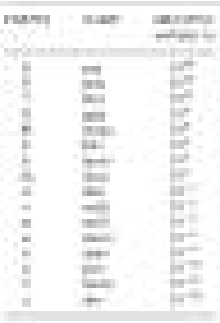
Schwarz-Weiß-Version für Text- und Bildmarke	1	Nur Regular, Farbe: Schwarz	313
Negativ-Version	1	Nur Regular	Weiß
Farbe	1	Nur Text- und Bildmarke	Blau 313
Einfarbig Anthrazit	1		
Reproduktion	2	Text und	3
Farbvariationen, modifiziert	4	Nur Regular, Farbe: Blau	313
Farbvariationen	5	Text	5
Text	1	Text	1
Farbe	2	Negativ-Version	Blau
Farbe Blau	3	Text und Bildmarke	Blau
Farbe Blau, negativ	4	Text und Bildmarke	Weiß
Schwarz	5	Text, Version	5
Weiß Negativ	6	Text	6

Abbildung 313 Text- und Bildmarke Logo 313 313 313 313 313 313 313 313 313

Greek alphabet



Common prefixes



Resistor colour coding and ohmic values

Colour code for fixed resistors

colour	significant digit	decimal multiplier	tolerance
black	0	10^0 = 1	± 20%
brown	1	10^1 = 10	± 1%
red	2	10^2 = 100	± 2%
orange	3	10^3 = 1 000	
yellow	4	10^4 = 10 000	
green	5	10^5 = 100 000	± 0.5%
blue	6	10^6 = 1 000 000	± 0.25%
violet	7	10^7 = 10 000 000	± 0.1%
grey	8	10^8 = 100 000 000	
white	9	10^9 = 1 000 000 000	
gold		10^{-1} = 0.1	± 5%
silver		10^{-2} = 0.01	± 10%

Each resistor has a colour code to indicate its value and tolerance. The first band is the first digit of the value, the second band is the second digit of the value, the third band is the number of zeros that follow the first two digits, and the fourth band is the tolerance. For example, a resistor with the colour code yellow, violet, red, gold has a value of 4 700 Ω and a tolerance of ± 5%.

Some four-band resistors have a fifth band to indicate the reliability of the component. For a 1 000 hour test, a resistor with a reliability factor of 1% will fail in less than 1 000 hours.

Some five-band resistors have a sixth band to indicate the temperature coefficient. This is the change in resistance when the temperature changes by 1 °C.

Letter vast flight milk (Ice breakfast)

[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]

Valve bear is motivated as [illegible]:

> [illegible] — [illegible] — [illegible] — [illegible] and [illegible] — [illegible].

Fast, ice breakfast: [illegible] — [illegible] — [illegible]
[illegible] — [illegible] — [illegible]
[illegible] — [illegible] — [illegible]
[illegible] — [illegible] — [illegible]
[illegible] — [illegible] — [illegible]
[illegible] — [illegible] — [illegible]

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